An estimation of the Euro Area potential output with a semi-structural multivariate Hodrick-Prescott filter

Odile Chagny, Matthieu Lemoine

▶ **To cite this version:**

Odile Chagny, Matthieu Lemoine. An estimation of the Euro Area potential output with a semi-structural multivariate Hodrick-Prescott filter. 2004. hal-00972840

HAL Id: hal-00972840

https://sciencespo.hal.science/hal-00972840

Preprint submitted on 22 May 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Document de travail

AN ESTIMATION OF THE EURO AREA POTENTIAL OUTPUT WITH A SEMI-STRUCTURAL MULTIVARIATE HODRICK-PRESCOTT FILTER

N° 2004-14
Décembre 2004

ODILE CHAGNY
Commissariat Général du Plan and OFCE
e-mail : odile.chagny@ofce.sciences-po.fr

MATTHIEU LEMOINE
OFCE
e-mail : matthieu.lemoine@ofce.sciences-po.fr

This paper has benefited from research conducted for the Eurostat A6 division project on "Short-term indicators of the euro-zone".

Observatoire Français des Conjonctures Économiques
69, Quai d'Orsay 75340 Paris Cedex 07
Tel : 01 44 18 54 00 Fax : 01 45 56 06 15
E-mail: ofce@ofce.sciences-po.fr Web: http://www.ofce.sciences-po.fr

Abstract

In this paper, we develop an analytical framework for the estimation of potential output and output gaps for the euro area combining multivariate filtering techniques with the production function approach. The advantage of this methodology lies in the fact that it combines a model based approach to explicit statistical assumptions concerning the estimation of the potential values of the components of the production function. We discuss the production function approach and the main issues raised by this approach. We then present the main empirical studies which have estimated production function based output gaps with multivariate filtering techniques. The production function approach will be implemented with Multivariate Hodrick-Prescott filters (HPMV). The advantage of the multivariate production function approach will also be assessed through using a variety of statistical criteria.

Keywords: potential output, output gap, production function, multivariate filters, unobserved components models.

JEL *Classification*: C32, E23, E32.

Résumé

Dans cet article, nous développons un cadre analytique pour l'estimation du potentiel de production et de l'écart de production de la zone euro. La méthode d'estimation combine des techniques de filtrage multivarié et l'approche par fonction de production. Cette méthodologie présente l'avantage d'expliciter à la fois le modèle économique sous-jacent et les hypothèses statistiques permettant d'estimer la valeur potentielle de chaque facteur de production. Nous présentons dans un premier temps l'approche par fonction de production et les problèmes soulevés par cette approche. Puis est proposé un survol des études empiriques ayant estimé un potentiel de production avec des filtres multivariés. L'approche par fonction de production est ensuite mise en oeuvre avec un filtre de Hodrick-Prescott multivarié (HPMV). L'intérêt de cette approche est enfin évalué à l'aide de divers critères statistiques.

Keywords: potentiel de production, écart de production, fonction de production, filtres multivariés, modèles à composantes inobservables.

JEL *Classification*: C32, E23, E32.

Contents

1 Introduction **4**

2 The production-function approach **5**
2.1 The theoretical framework 5
2.2 Main empirical applications using multivariate filters 7
2.2.1 Production-function applications with the HPMV 7
2.2.2 Production functions using UCM specifications 8

3 Analytical framework for empirical developments **10**
3.1 Production function specification 10
3.2 State-space model specification 12

4 Empirical results for EU5 aggregates **14**
4.1 Interpretation of the empirical results 14
4.2 Assessment of the output gap with standard quantitative criteria 14

5 National production function models **15**

6 Conclusion **16**

References **17**

Appendices **20**
A Description of the dataset 20
B Measures of forecasting performance 20
C Measures of uncertainty and consistency 22
D Tables and graphs 23

1 Introduction

Potential output is usually defined as the level of output consistent with a stable inflation rate. The appraisal of potential output plays a key role for policy-makers. Indeed, the output gap, i.e. the difference between the actual and potential output, is used by central banks as an indicator of inflationary pressures. This appraisal is also necessary for measuring structural fiscal deficits. However, no consensus exists about an estimation methodology.

Various univariate statistical methods have first been proposed, e.g. the phase-average detrending method, Hodrick and Prescott (1980) or Baxter and King (1995) filters. However, such methods do not explain the economic nature of shocks affecting the trend and various accuracy criteria show that such univariate methods provide poor information about the output gap level in real time (Rünstler, 2002). As a partial answer to these difficulties, multivariate filtering techniques have been proposed, e.g. multivariate Hodrick-Prescott filters (HPMV), multivariate unobserved components models (UCM) and structural vector auto-regressive models (SVAR)[1].

Another strategy for identifying the potential output involves non-statistical structural methods. In this case, the detrending method relies on a specific economic theory. For instance, the production-function-based approach is often used in the literature. This type of method is currently employed by many policy institutions (among others OECD (Giorno *et al.* 1995), IMF (De Masi 1997), European Commission (Mac Morrow and Röger 2001, Denis *et al.* 2002), Banque de France (Banque de France 2002)). It is also the approach recommended by the EU Economic Policy Committee. This approach has the advantage of explicitly identifying the factors which are driving growth (labour, capital, intermediate inputs, technological progress, etc.), and of being used in a forecasting framework. This approach raises a number of issues, such as the choice of an appropriate production function or the measurement of unobservable variables (total factor productivity, equilibrium level of employment and of capital input). Although it is a structural approach, it often relies on simple detrending methods such as HP filters.

Recent empirical papers (Butler 1996, Haltmaier 1996, Rünstler 2002, Proietti *et al.* 2002, for instance) have proposed combining multivariate filtering techniques with the production-function approach. The advantage of this methodology is to combine a model-based approach to estimate potential output with explicit statistical assumptions concerning the estimation of the potential values of the components of the production function. Estimating these production functions with Kalman filter techniques also makes it possible to show confidence bands for potential output and output gaps. In this respect, this type of approach permits a good deal of economic structure to be applied to the disentangling of supply and demand shocks, which is one of the main objectives of the development of multivariate techniques. We might hence also expect the production-function approach to improve the end-of-sample properties of the output gap estimates when compared to univariate filters, or multivariate filters using non disagregated specifications.

The paper is organised as follows. The second section describes the production-function approach and presents the main empirical studies that have estimated production-function-based

[1] For an assessment of these methods, see for instance Chagny, Lemoine and Pelgrin (2003).

output gaps with multivariate filtering techniques. The third section presents the analytical framework adopted for the empirical work. The production-function approach is implemented within a HPMV framework, assuming a Cobb-Douglas functional form for the production function. The production-function based models are estimated with the methodology adopted for the estimation of state-space models. This strategy makes it possible to obtain confidence intervals for the output gap, and to compare them with the results obtained with aggregated models.The fourth section presents the results for the the euro area (understood here as the agregation of the five major economies of the area: Germany, France, Italy, Spain and the Netherlands, accounting for slightly more than 85 percent of the euro area GDP). The advantage of the multivariate production function over the univariate HP filter approach will be assessed through using a variety of statistical criteria (standard errors of the output gaps, comparison of quasi real time estimates with two-sided estimates, predictive power of the output gap estimates as regards inflation). The fifth section deals with the issue of comparing the aggregation of country specific-models to the area wide model.

2 The production-function approach

2.1 The theoretical framework

The key assumption is that the production process can be represented by an aggregate production function. Potential output is then calculated as the output of this function when all factors are at their normal, or equilibrium value. The production function can have several functional forms (constant elasticity of substitution, translogarithmic for instance), but the Cobb-Douglas form is the most widely used.

Assuming for simplicity a Cobb-Douglas technology, the production function takes the following form:

$$Y_t = TFP_t \left(N_t H_t\right)^{\alpha} \left(C_t K_t\right)^{1-\alpha} \tag{1}$$

where Y_t is output, TFP_t is total factor productivity, N_t is employment, H_t is average hours per head, K_t is the capital stock, C_t is the degree of capital utilization, α is the elasticity of output with respect to labour input. Under perfect competitive market assumptions, α is equal to the labour share in the output.

Labour input is here defined as the total number of hours worked, and capital input is measured by the capital stock, corrected for the degree of excess capacity. Total factor productivity is not directly observable, and is usually derived as the so-called Solow's residual from a growth accounting framework. Assuming that all components are at their equilibrium value, i.e. their non-inflationary level, potential output Y_t^* can be written as:

$$Y_t^* = TFP_t^* \left(N_t^* H_t^*\right)^{\alpha} \left(C_t^* K_t^*\right)^{1-\alpha} \tag{2}$$

where * denotes an equilibrium value. Taking the logarithms of both sides of equation 2 gives:

$$y_t^* = tfp_t^* + \alpha(n_t^* + h_t^*) + (1-\alpha)(c_t^* + k_t^*) \tag{3}$$

As it is in many cases difficult to obtain data on hours worked and to estimate the degree of excess capacity utilization, the Solow's residual often include h_t^* and c_t^*, and potential output becomes:

$$y_t^* = f_t^* + \alpha \cdot n_t^* + (1-\alpha)k_t^* \tag{4}$$

where:

$$f_t^* = tfp_t^* + \alpha \cdot h_t^* + (1-\alpha)(c_t^*).$$

The equilibrium value for n_t^* requires estimating both an equilibrium unemployment rate U_t^* and an equilibrium participation rate P_t^*:

$$\exp(n_t^*) = POP_t.P_t^*.(1-U_t^*), \tag{5}$$

where POP_t is the working age population, assumed to be invariant to the business cycle, and U_t^* is estimated as a NAIRU (or NAWRU) using structural equations or as a time-varying NAIRU within the Gordon's triangle model framework [2] (Gordon 1997, Richardson *et al.* 2000), which links the inflation rate to excess supply or excess demand in the labour market and to supply shocks:

$$\Pi_t = \Pi_t^e + a\,(L)\,(U_t - U_t^*) + \sum_{i=1}^{n} \Phi_i^{\pi}\,(L)\,X_{i,t}^{\pi} + \varepsilon_{\pi,t}, \tag{6}$$

where Π_t is the inflation, Π_t^e the anticipated inflation, generally assumed to be a function of the lagged inflation, and $X_{i,t}^{\pi}$ is a vector of temporary supply shocks (like the terms of trade).

Using as equilibrium value of the capital stock the observed capital stock provides a short-to-mid-term potential output, whereas assuming a constant capital/output ratio provides a long-term evaluation of the potential output.

The Cobb-Douglas technology takes an extremely simple form. It assumes: constant returns to scale; per capita output grows at a constant rate; the capital-output ratio is constant; the share of labour and capital in national income are constant (the so-called Kaldor facts in the growth litterature (Kongsamut *et al.,* 1997)). These assumptions have, however, not been confirmed by recent empirical evidence. More particularly, Cobb-Douglas technology assumes that the elasticity of substitution is fixed and equal to unity, which is not consistent with the empirical evidence concerning the evolution of the labour share, especially in Europe (see for instance Bentolila and Saint-Paul, 2003). To account for this, technology can be assumed to take a Constant-Elasticity-of-Substitution form:

$$Y_t^* = \left[\delta\,(B_t N_t H_t)^{-\rho} + (1-\delta)(C_t K_t)^{-\rho}\right]^{\frac{-1}{\rho}} \tag{7}$$

where B_t is the technical progress (here labor-augmenting), δ is the distribution factor indicating the labour intensity of output, and ρ is the substitution parameter. If the substitution parameter

[2] See Chagny et al.(2001) for a theoretical discussion of the equilibrium unemployment rate, Heyer and Timbeau (2001) for a recent empirical extension of the triangle model.

ρ approaches zero, the CES production function reduces to the Cobb-Douglas in which the distribution parameter δ is given, assuming profit maximization, by the labour income share. The advantage of this type of technology is that it does not restrict the substitution elasticity to be equal to unity, but its non-linear nature makes the estimation more difficult than in the Cobb-Douglas case. To overcome this problem, it is however possible to use translog production functions, initiated by the Kmenta's 1967 log-linearization of the CES production function, and further developed by Christensen *et al.* (1973) and Brown *et al.* (1979).

2.2 Main empirical applications using multivariate filters

Production-function-based output gaps and potential output estimates using multivariate filters have been mainly developed in two directions. The first extends the HPMV framework, the second has been implemented within the class of UCM methods.

2.2.1 Production-function applications with the HPMV

Applications of the production function using the HPMV filter have been proposed by Butler (1996) and Haltmaier (1996) (for a description of the HPMV filter, developed by Laxton and Tetlow 1992, see Chagny, Lemoine and Pelgrin 2003) .

In Butler 1996, the estimates are obtained by manipulation of the Cobb-Douglas function and applied to the Canadian economy. The main idea is that, even if the considerable uncertainties about the precise nature of the structural relationships within the economy can justify the use of semi-structural methods for estimating the potential output, decomposing potential output into different components increases the consistency with structural models and "allows an easier interpretation of the sources of changes in the gap or the potential" (St-Amant and van Norden, (1997)).

Estimates are built up from the decomposition of output into the labor-input, the marginal product of labor and the labor-output elasticity:

$$y_t = n_t + lp_t - a_t \tag{8}$$

where y, n, lp and a are the logs of output, labour input, marginal productivity of labour and labour share.

The potential level of y is obtained via estimating equilibrium values for each of the components.

The equilibrium level of employment is obtained using equation 5. P_t^* and U_t^* are estimated separatly, using HPMV filters. In the case of P_t^* , the HPMV filter incorporates additional information about experts' estimates of potential participation rates, and the smoothing parameter is set at a very high level (16,000), in order to provide very smooth estimates. In the case of U_t^*, the HPMV filter incorporates the residual of a Phillips curve, it also adds additional constraints (a recursive updating restriction and a steady-state growth restriction).

The equilibrium value of a is obtained using an univariate HP filter, with a large smoothing parameter, in order to remove high-frequency variations.

The equilibrium value for tfp is estimated adding into the minimization program of the HPMV the residuals of a modified Okuns'law and of an (inflation)/(marginal product of labour) relationship. The first equation is intended to capture the quantity adjustment process that firms undertake when their marginal product deviates from its short-run equilibrium value. The presence of the second equation is motivated by the idea that deviation of the marginal product from its equilibrium level provides an alternative information of excess demand pressure. The HPMV minimization program incorporates also a steady-state restriction, a recursive updating restriction, and deviation of the marginal product from the real wage.

When the output gap estimates are compared to univariate HP filter estimates, substantial differences emerge at the beginning of the seventies (more excess demand with the production-function approach) and at the beginning of the eighties (more excess supply). Rolling estimates show that the production-function-based filter results are revised much less than the comparable results from the univariate HP filters. Moreover, the reasons for the better end-of-sample properties lies in the decomposition of output. Specifically, current (rolling) estimates of the equilibrium participation rate and of the NAIRU generally do a good job in predicting the full-sample estimates.

In Haltmaier (1996), the production-function approach is applied to produce estimates of output gap and potential output for the G7 countries. Output is decomposed into labour productivity and labor input:

$$y_t = n_t + lp_t \tag{9}$$

where y, n and lp are the logs of output, labour input and labour productivity. The equilibrium value for n is obtained through estimating an equilibrium value for the (employment)/(working age population). Variations in the unemployment rate and in the participation rate are hence consolidated into the employment-population ratio r. Estimates are obtained using the following HPMV minimization program:

$$X = \underset{lp^*, r^*}{\arg\min} \begin{array}{l} \Big[\sum_{t=1}^{T}(lp_t - lp_t^*)^2 + \lambda \sum_{t=2}^{T-1}\left[(lp_{t+1}^* - lp_t^*) - (lp_t^* - lp_{t-1}^*)\right]^2 + \\ \sum_{t=1}^{T}(r_t - r_t^*)^2 + \lambda \sum_{t=2}^{T-1}\left[(r_{t+1}^* - r_t^*) - (r_t^* - r_{t-1}^*)\right]^2 + \alpha\epsilon_t\Big] \end{array} \tag{10}$$

where ϵ_t is the residual of an inflation equation linking inflation to its lagged values, supply shocks and the deviations of the employment-population ratio and of the labor productivity from their equilibrium values.

Empirical results suggest that taking account of the major components within a production-function approach provides additional information, as substantial differences emerge in some cases with the estimates obtained using a non-disaggregated version of the HPMV.

2.2.2 Production functions using UCM specifications

Applications of the production function using the UCM framework (for a description of the UCM filter, see Chagny, Lemoine and Pelgrin 2003) have been proposed by Rünstler (2002) and Proietti *et al.* (2002).

The main objective of Rünstler 2002 is to investigate the properties of real-time estimates for the euro area obtained from various unobserved component models. To this end, two basic versions of multivariate UCM models using information about inflation and two extensions of the multivariate UCM models are compared[2]. One of the multivariate UCM is based on a Cobb-Douglas production function, where the output-capital ratio is used in place of the output:

$$y_t - k_t = tfp_t + (1-\alpha)\left[n_t - k_t\right] \tag{11}$$

Capital stock k_t and the labor force l_t are taken as exogenous, and the output gap is related to the cyclical components of the unemployment rate and the total factor productivity:

$$y_t^C = tfp_t^C - (1-\alpha)u_t^C \tag{12}$$

where y_t^C is the output-capital ratio cycle, tfp_t^C and u_t^C are the cyclical components of the total factor productivity and of the unemployment rate[3].

The output-capital ratio is hence decomposed into a cyclical and a trend component as follows:

$$y_t - k_t = (y_t - k_t)^* + y_t^C \tag{13}$$

where the trend component $(y_t - k_t)^*$ is modeled as a local linear trend (Harvey ,1989), and restricted to follow a random walk without drift, accounting for the theoretical assumptions of the Cobb-Douglas function. y_t^C is modeled as an AR(2) process.

The model also incorporates an inflation equation linking inflation to lagged values of y_t^C , together with equations which make it possible to obtain the decomposition of tfp and an equation linking the capacity utilization CAP to the tfp cycle:

$$\begin{aligned} tfp_t &= tfp_t^* + tfp_t^C \\ (1-\Phi L)tfp_t^C &= (a_0 + a_1 L)y_t^C + \epsilon_t^{tfp} \\ (1-wL)CAP_t^C &= (b_0 + b_1 L)tfp_t^C + \epsilon_t^C \end{aligned} \tag{14}$$

The tfp_t^* trend component is again modelled as a local linear trend, while the tfp_t^C cycle is related to the output gap. Empirical results show that the extended multivariate filters are subject to considerably smaller standard errors of the filtered estimates when compared to the simple multivariate methods, and that they show better inflation forecasting properties.

Empirical estimates of Proeitti *et al.* 2002 are based on the decomposition of the inputs of the Cobb-Douglas into three components:

$$\begin{aligned} tfp_t &= tfp_t^* + tfp_t^C \\ n_t &= n_t^* + n_t^C \\ k_t &= k_t^* \end{aligned} \tag{15}$$

The equilibrium value for the capital stock is hence assumed to be equal to it's current value.

[2]The paper also uses univariate HP and Baxter and King filters estimates.

[3]With: $N_t = (1-U_t)\,L_t$ and the approximation $\ln(1-U_t) \approx -u_t$.

Employment has three determinants:

$$n_t = pop_t + p_t + e_t \tag{16}$$

where pop_t, $p_t$and e_t are the logarithms of the working age population, the participation rate and the employment rate. Each component of employment can itself be decomposed into a permanent and a cyclical component, except for POP_t, which is assumed to be invariant to the business cycle. The permanent component of employment is hence obtained as follows:

$$n_t^* = pop_t + p_t^* + e_t^* \tag{17}$$

The decomposition of output becomes:

$$\begin{aligned} y_t &= y_t^* + y_t^C \\ y_t^* &= tfp_t^* + \alpha \cdot n_t^* + (1-\alpha)k_t \\ y_t^C &= tfp_t^C + \alpha \cdot n_t^C \end{aligned} \tag{18}$$

Finally, in accordance with the notion that potential output is consistent with stable inflation, the model incorporates a Phillips-type equation, which relates the nominal price or wage inflation rate to the output gap and to a set of exogenous supply shocks. The model also incorporates an estimation of the equilibrium value of the capacity utilization rate CAP_t. Trend and cycles of the variables tfp_t, $p_t, -e_t, CAP_t$ are estimated using multivariate unobserved components models proposed in Harvey (1989). The paper investigates an explorative approach. It first specifies a system of seemingly unrelated equations without assuming common cycles or trends for the total factor productivity, the participation rate and the contribution of the unemployment rate. The authors also propose a common cycle specification, with the capacity utilization cycle being defined as the reference cycle. The paper also discusses the hysteresis hypothesis, according to which the cyclical variation permanently affects the trend in the participation and unemployment rates. It therefore finally introduces the pseudo-integrated cycle model, which provides an effective way of capturing the cyclical variability in the labor market variables. Empirical results show that although the production-function approach cannot outperform the forecasting power of a simple bivariate UCM model of output and inflation, it reduces substantially the uncertainty in the estimates of the output gap.

Graphs presented in the paper suggest that the choice of a common cycle specification might be justified by the fact that with a SUTSE specification, all cycles are trapped by the capacity utilization dynamics. Indeed, the estimated output gap can not be distinguished from the capacity utilization cycle in this case. It raises the question of using a sophisticated multivariate model instead of a simple indicator of capacity utilization.

3 Analytical framework for empirical developments

3.1 Production function specification

In view of the preceding development, it appears to be of relevant interest to develop output gap estimates based on the combined use of a production-function approach and of multivariate filtering

techniques. The analytical framework adopted for estimating the output gap of the EU5 euro area is as follows (for a description of the data set, see Appendix A).

The production function is assumed to have a Cobb-Douglas functional form:

$$Y_t = TFP_t \left(N_t H_t\right)^{\alpha} \left(K_t\right)^{1-\alpha} \tag{19}$$

Transforming equation 19 gives:

$$Y_t = TFP_t \left(\frac{K_t}{N_t H_t}\right)^{1-\alpha} N_t H_t \tag{20}$$

where output Y_t is decomposed into three components: the hourly labour productivity LP_t (which itself depends on the total factor productivity TFP_t and the capital-labour ratio $\left(\frac{K_t}{N_t H_t}\right)^{1-\alpha}$); employment N_t and the average hours actually worked H_t.

As the employment depends on the population (pop_t), on the potential participation rate (p_t) and on the potential employment rate (e_t), taking the logarithms of both sides of the equation 20, output can be decomposed as follows:

$$y_t = lp_t + pop_t + p_t + e_t + h_t \text{ with } lp_t = tfp_t + \alpha \cdot ki_t$$

Assuming that the working age population is invariant to the cycle, the potential output (y_t^*) estimation might rely on the multivariate estimation of potential values ($lp_t^*, p_t^*, e_t^*, h_t^*$). However, we consider here a simplified decomposition, where $nh = n + h$ is the log of the hours worked by all employed persons:

$$y_t = lp_t + nh_t \text{ with } lp_t = tfp_t + \alpha \cdot ki_t$$

and assuming that all variables are at their potential value, when the output gap is centered, the potential output ($\bar{y}_t^*$) and output gap ($\bar{y}_t^c$) can be written as follows:

$$\bar{y}_t^* = lp_t^* + nh_t^* \text{ with } lp_t^* = tfp_t^* + \alpha \cdot ki_t^* \tag{21}$$

$$\bar{y}_t^c = lp_t^c + nh_t^c \tag{22}$$

The potential value of the hourly labour productivity for the total economy is written as a function of an exogenous technical progress trend tfp_t^* and of the capital-output ratio in the business sector ki_t^*. The gaps are stationary variables.

The model hence requires the estimation of the trends and gaps of two variables (lp_t^*,nh_t^*), and takes into account the impact of the capital-labour ratio on labour productivity. Finally, in accordance with the definition of the potential output as the level of output consistent with stable inflation, the model incorporates an additional equation, linking the inflation (measured on the basis of the private consumption deflator) to the output gap level:

$$\Delta\pi_t = c + \alpha(L)\bar{y}_{t-i}^c + \beta(L)\Delta\pi_{t-i} + \gamma(L)\Delta s_{t-i} + \varepsilon_t^{\pi} \tag{23}$$

This equation is formulated as a "triangle model" (Gordon, 1997), which explains the inflation rate as a linear combination of the anticipated inflation rate π_t^e, assumed here equal to $\beta(L)\pi_{t-i}$

(backward anticipation), the output gap and a vector of temporary supply shocks s_t (here, the relative import price growth rate, measured as the difference between the quarterly growth rate of the goods and services deflator index and the quarterly growth rate of the private consumption deflator). In order to be consistent with the non-accelerating inflation definition of the potential output, $\beta(1)$ has been constrained to be equal to unity. In contrast to other empirical studies, the capacity utilization rate does not enter the model. This is justified by the fact that in many cases (see the previous section, and also Chagny, Lemoine and Pelgrin 2003), the estimated output gaps can be "over-influenced" by this variable. The constant c included in the inflation equation takes into account a possible acceleration or deceleration of the inflation during the period. This constant might be considered as a component of an uncentered output gap (y_t^c), by computing:

$$\begin{aligned} y_t^c &= \bar{y}_t^c + c/\alpha(1) \\ y_t^* &= \bar{y}_t^* - c/\alpha(1) \end{aligned}$$

In other words, this re-computation allows the output to be in average below its potential (y_t^*).

The advantage of this model is twofolds. First, it relies on an integrated framework: all components of the production function are assumed to follow specific stochastic processes and assumptions about these processes are explicit. Second, it leaves room for explaining the potential output path by economic determinants. This is specifically the case of the hourly labour productivity trend, for which the contribution of the labour-capital substitution is considered separately, and where other potential factors could be integrated, in order to allow a better identification of the technical progress (e.g. instruction level, research and development expenses, ...). For instance, in countries where specific policies of employer's social contribution reductions on the lowest wages have been implemented (for the panel of countries considered in this paper, this is the case of the Netherlands and of France[4]), we may expect to underestimate the technical progress trend, as these policy measures may induce a substitution between qualified labour and low skilled labour, which goes in hand with a lower labour productivity. To illustrate this, we have estimated two models for France. The first relies on the specification given hereabove, the second decomposes the trend hourly labour productivity into three componants (the exogenous technical progress trend tfp_t^*,the capital-output ratio in the business sector $ki_t and$ the impact of the reduction of employer's social contribution for low skilled labour lsk_t^*, as measured by the financial resources allocated to these measures in percentage of the GDP):

$$lp_t^* = tfp_t^* + \delta \cdot ki_t^* + \omega \cdot lsk_t^* \tag{24}$$

3.2 State-space model specification

Considering the convergence problems associated with UCM models, when the number of parameters is too large, the HPMV framework has been chosen for estimating the model[5].

We propose also to estimate the production-function-based models with the methodology adopted for the estimation of state-space models. This strategy makes it possible to obtain confi-

[4]For a description of these employment policies, see for instance Bourdin and Marini (2003).

[5]This kind of convergence problem is illustrated by the results of Proeitti *et al.* 2002, who have identified the cycle with the capacity utilization cycle.

dence intervals for the output gap, and to compare them with the results obtained with aggregated models .

Rewriting the model in a state-space form gives the following equations. Three observable variables are used (in logarithm) as endogenous variables: the average worked hours ($nh = n + h$), the hourly labour productivity (lp) and the inflation (π), and are modelled with the three following measurement equations:

$$\begin{aligned} nh_t &= nh_t^* + nh_t^c \\ lp_t &= (tfp_t^* + +\delta ki_t^* + \omega lsk_t^*) + lp_t^c \\ \Delta\pi_t &= \alpha \cdot y_t^c + \beta_1 \Delta\pi_{t-1} + \beta_2 \Delta\pi_{t-2} + (1 - \beta_1 - \beta_2)\Delta\pi_{t-3} + \gamma \Delta s_t + \varepsilon_t^\pi \end{aligned}$$

The output (y_t), the potential output (y_t^*) and the output gap (gap_t) can be deduced from the cycles extracted from the endogenous variables:

$$\begin{aligned} y_t &= nh_t + lp_t \\ y_t^* &= nh_t^* + lp_t^* - c/\alpha \\ y_t^c &= nh_t^c + lp_t^c + c/\alpha \end{aligned}$$

For each variable, the trend is modeled as a processes integrated of order 2 and the cycle as a white noise, leading to the 8 following state equations:

$$\begin{aligned} \Delta^2 nh_t^* &= (\sigma_{nh^*})\, \varepsilon_t^{nh^*} \\ nh_t^c &= (\sigma_{nh^c})\, \varepsilon_t^{nh^c} \\ \Delta lp_t^* &= \Delta tfp_t^* + \delta \Delta ki_t^* \\ lp_t^c &= (\sigma_{lp^c})\, \varepsilon_t^{lp^c} \\ \Delta^2 tfp_t^* &= (\sigma_{tfp^*})\, \varepsilon_t^{tfp^*} \end{aligned}$$

with $\varepsilon_t^{nh^*}, \varepsilon_t^{nh^c}, \varepsilon_t^{lp^c}, \varepsilon_t^{tfp^*}$ Gaussian normalised white noises. The worked hours trend is modelled as smooth, following the Hodrick-Prescott specification. The labour productivity trend is a function of the total factor productivity trend (tfp_t^*) and of the capital intensity trend (ki_t^*). Both gaps are modelled as Gaussian white noises and both variance ratios are constrained. Trend innovations are considered as independent of each other and with cycle innovations. But the employment/productivity interaction is taken into account with a correlation coefficient of cycle innovations ($c_{nh,lp}$). Variance ratio restrictions can be written as follows:

$$\begin{aligned} &\sigma_{nh^c}^2 = 1600 \cdot \sigma_{nh^*}^2 \\ &\sigma_{lp^c}^2 = 1600 \cdot \left[\sigma_{tfp^*}^2 + \delta^2 \sigma_{ki^*}^2\right] \\ &\sigma_{y^c}^2 = 160 \cdot \sigma_\pi^2 \end{aligned}$$

with

$$\sigma_{ki^*}^2 = Var(\Delta ki^*)$$

and

$$\sigma_{y^c}^2 = \sigma_{nh^c}^2 + \sigma_{lp^c}^2 + 2\sigma_{nh^c}\sigma_{lp^c} c_{nh,lp}$$

For trend-cycle decompositions, ratios have been set at the usual value of 1600, in order to select cycles with a period around 8 years. The estimation of the inflation equation with a univariate HP

output gap provides a residual variance, which has been used to fix a weight equal to 160 for the inflation equation. Such a weight with quarterly growth rates is intermediate relative to weights used by Laxton and Tetlow (1992) and Haltmaier (1996) (see Chagny , Lemoine and Pelgrin 2003)). Besides, to help the convergence of the estimation process, the correlation coefficient has been imposed to the value computed by using univariate HP filters. In order to apply the Kalman filter and the maximum likelihood estimation in the HPMV model, we need to make some assumptions on the initial values of the state variables and their matrix of variance-covariance. In this paper, a diffuse prior is used and the initial coefficients are computed by estimating separately each additional equation. Once the values of the parameters have been set, and given initial values of the state variables, optimal estimates of the state variables based on the information available at time t (refered to as filtered estimates) and on the information available from the full sample of observations to time T (referred to as smoothed estimates) are obtained from the Kalman filter.

4 Empirical results for EU5 aggregates

4.1 Interpretation of the empirical results

As shown by the descriptive statistics and the graphs, the multivariate HP production-function approach provides output gap estimates that can differ quite substantially from the univariate HP estimates. For instance, excess supply and excess demand periods are more pronounced, which goes in hand with a higher standard deviation of the output gaps.

(insert graph 1 and table 2)

The difference is more striking than in the case of multivariate HPMV models based on an aggregate approach of the GDP. This is mainly due to the specification of the model. Allowing the hourly labour productivity trend lp_t to be decomposed into an exogenous trend tfp_t and the contribution of the capital-labour ratio (ki_t) gives hourly labour productivity trends and gap estimates quite different from those of an univariate HP filter (graph 13). The δ parameter estimated value is consistent with the value one obtaines when calculating the capital share with the national accounts (table 1), and the choosen specification of the hourly labour productivity trend presents the advantage of identifying separately the impact of the labour-capital subtitution on the labour productivity. According to our model, most of the inflexions in the trend labour productivity growth observed in the eighties where explained by the capital-labour ratio, whereas the TFP trend was upwards-oriented until the end of the eighties (graphs 14 and 15). In contrast, both a decreasing substitution of capital to labour and a slowdown of the TFP trend have contributed to the slowdown in the estimated trend productivity growth rate in the nineties.

(insert table 1, graph 7)

4.2 Assessment of the output gap with standard quantitative criteria

The semi-structural production-function (PF) model provides a multivariate output gap that can be assessed using various quantitative criteria, in comparison with the univariate HP output gap.

Two sets of criteria are considered, which concern the accuracy of the output gap estimates (table 4) and their forecasting power regarding inflation (table 5) (see Appendices C and B for more details on these criteria).

(insert tables 4 and 5)

Concerning the accuracy of the output gap estimates, the confidence bands are generally larger with the production-function approach. The one-sided RMSE is equal to 0.82, as against 0.46 in the case of the univariate HP filter. The two-sided RMSE is equal to 0.46, as against 0.25 in the case of the univariate HP filter. However, one-sided and two-sided estimates of the PF output gap also have larger standard deviation (1.60 and 1.18), than those of HP (0.99 and 0.90). We therefore have to look at the Student statistics, which test whether the output gap estimate is equal to zero (hypothesis H0). Student statistics of HP and PF estimates have very close values: they stand above 1.8 for one-sided estimates and around 3 for two-sided estimates. Thus, H0 might be rejected with a risk equal to 90% with one-sided estimates and to 95% with two-sided estimates. Concerning revisions between quasi-real-time and smooth estimates, multivariate PF estimates have higer standard revisions (0.89) than univariate HP ones (0.76).

Although the accuracy of inflation prediction displays significant differences over the forecasting horizons, the forecasting performance of the production-function approach can be considered as broadly satisfactory. At a 10% level, the production-function model performs indeed better than a naive random walk model over most of the horizons, and better than an AR(3) over long horizons[6].

5 National production function models

Parameter estimates, descriptive statistics and graphs of the country-specific production-function models output gaps are displayed in tables 1, 2 , 3 and graphs 2-6). Parameter estimates are relatively heterogenous among countries, specifically for what concerns the inflation dynamics and the labour market variables, a result which is consistent with the view that the euro area is still characterized by a strong cross-country heterogeneity of the labour market institutions (see for instance Fitoussi and Passet 2000). Despite this, the aggregation of the country-specific models estimates provides potential output and output gap estimates which are very similar to those obtained at the EU5 widel level (graph 1).

(Insert tables 1, 2, 3, graphs 1 to 6)

Even if they are functionnally more appropriated to capture economic structure differences across countries, the production-function based approach exhibits agregation properties which are in conclusion very similar to those observed with multivariate models based on a direct measure of the GDP. The specification choosen for the model allows nevertheless to identify strong national specificities with regard to the estimated hourly labour productivity trends (graphs 16 to 23). In most of the countries considered here, the path of capital-labour substitution has been decreasing since the mid-nineties, reflecting among others the effects of the wage moderation. But the

[6]However, one must keep in mind that the weight of the inflation equation has been arbitrarily fixed at a value corresponding to a weight of 10 for year on year inflation.

slowdown is much less pronounced in Italy than in Spain for instance, which results in a smaller contribution of the capital-labor ratio to the decrease of the labour productivity trend growth rate. With regard to the capital-labour ratio, the Netherlands appear to be an outlier. The capital-labour substitution has been practically stopped from the mid-eighties to the mid-nineties, and this is one of the main factor explaining the slow growth rate of the total hourly labour productivity trend in comparison to the other countries (1.4 percent per year in average over the period 1982-1993, to compare with 2.4-2.6 percent in the other countries). In contrast, the capital-labour ratio has been oriented upwards since 1998 in the Netherlands, which has contributed to moderate the slowdown in the total labour productivity growth.

(Insert graphs 7 to 13)

Taking in account the capital-labour substitution reduces indeed substantially the cross-country divergences. The average standard deviation of the total hourly labour productivity growth rate trend (Δlp_t^*) is 0.86 over the period 1978:1 2002:4, but reduced to 0.66 for the the exogenous technical progress trend (Δtfp_t^*). It also allows in principle to obtain a more consistent estimates of the technical progress trend. But as displayed by the graphs 24 to 29, the 95 percent confidence bands based on the filter uncertainty are large. For instance, estimated value for the exogenous technical progress growth rate is -0.3% in 2002:4 in Italy, but 95% confidence upper and lower limit are respectively 1.5% and -2.1%.

(Insert graphs 14 to 19)

A comparison of the two models estimated for France is proposed in graph 10 (see table 1 for the parameter estimates). When the trend hourly labour productivity incorporates the impact of the reduction of employer's social contribution for low skilled labour lsk_t^*,our estimates exhibit an acceleration in the exogenous technical progress trend tfp_t^* which is more pronounced since 1993. Moreover, our model suggest that 300 000 job creations have been induced by this measure, an estimation which is consistent with other sources (see DARES 2003 for a recent synthesis of the employment policy in France).

6 Conclusion

Our results show that the multivariate HP production-function approach provides output gap estimates that can differ quite substantially from the univariate HP estimates. As they are not centered, they allow to take into account the desinflation which occured during the eighties in some European countries. Concerning the accuracy of the output gap estimates, the confidence bands are generally larger with the production-function approach, but the forecasting performance of the production-function approach can be considered as broadly satisfactory. Concerning the comparison of euro area wide and country-specific model, our results show that the aggregation of the country-specific models estimates provides potential output and output gap estimates which are very similar to those obtained at the EU5 wide level. The specification chosen for the model allows nevertheless to identify national specificities. The model leaves room for economic determinants to

explain the potential output path, specifically for the hourly labour productivity trend. We illustrate this point in the case of France by identifying separately the effect of specific policy measures in favour of low skilled labour on the labour productivity trend. Our results are consistent with other sources. The approach presented in this paper can be subject to many further developments. Use of translog production function would for instance permit to use a more complex functional form of the production function than the Cobb-Douglas framework. We may also seek to improve the measure of the capital-labour ratio, in order to remove the movements due solely to economic fluctuations. Additional exogenous factors could also be integrated in the model, allowing, for instance, to assess the impact of the underlying determinants of the capital-labour ratio.

References

BANQUE DE FRANCE, 2002, "Croissance potentielle et tensions inflationnistes", *Bulletin de la Banque de France* No 103.

BAXTER M. and KING R., 1995, "Measuring business cycles: approximate band-pass filters for economic time series", *NBER Working Paper* No 5022.

BENTOLILA S. and Gilles SAINT-PAUL, 2003, "Explaining movements in the Labor Share", *Contributions to macroeconomic*s, Vol. 3 No 1.

BOURDIN J. and P. MARINI, 2003, "Une décennie de réformes fiscales en Europe : la France à la traîne", *Rapport du Sénat* No 343.

BROWN, R.S., D.W. CAVES, and L.R. CHRISTENSEN, 1979, "Modelling the structure of cost and production for multiproduct firms", *Southern Economics Journal,* 46, 2.

BUTLER L. ,1996, "A semi-structural method to estimate potential output: combining economic theory with a time-series filter", *Bank of Canada Technical Report* No. 76.

CAMBA-MENDEZ G. and D. RODRIGUEZ-PALENZUELA, 2003, "Assessment criteria for output gap estimates", *Economic modelling*, pp. 529-562.

CAYEN J.P and S. VAN NORDEN, 2002, "La fiabilité des estimations de l'ecart de production au Canada", *Working paper Bank of Canada.*

CHAGNY O., M. LEMOINE and F. PELGRIN, 2003, "An assessment of multivariate output gap estimates in the Euro area", *Eurostat report*, Project "Short term indicators for the Euro zone".

CHRISTENSEN, L.R., D.W. JORGENSEN, and L.J. LAU, 1973, "Transcendental logarithmic production frontiers", *The Review of Economics and Statistics*, 55, 1.

CLARK, T. E. and M. W. McCRACKEN, 2001, "Tests of forecast accuracy and encompassing for nested models", *Journal of Econometrics*, Vol 105, pp85-110.

DARES, 2003, "Les politiques de l'emploi et du marché du travail", *La Découverte, collection Repères.*

De MASI P., 1997, "IMF estimates of potential output: theory and practice." *Staff Studies for the World Economic Outlook.*

DENIS C., K. Mc MORROW and W. ROEGER, 2002, "Production function approach to

calculating potential growth and output gaps. Estimates for the EU Member States and the US", *European Commission Economic Papers* No 176.

DIEBOLD, F. X., and R. S. MARIANO, 1995, "Comparing predictive accuracy", *Journal ofBusiness and Economic Statistics*, Vol..13, No.3, pp. 253-63.

FITOUSSI J.P. and O. PASSET, 2000, "Réduction du chômage, les réussites en Europe", *Rapport du Conseil d'Analyse Economique* No 23.

GIORNO , P. RICHARDSON, D. ROESEVAERE and P. van den NOORD, 1995, "Potential output, output gaps and structural balance budgets", *OECD Economic Studies* No 24.

GORDON R.J., 1997, "The Time Varying NAIRU and its implications for economic policy", *Journal of Economic Perspectives*, Vol. 11; N°1, winter.

HAMILTON J.D., 1986, "A standard error for the estimated state vector of a state-space model", *Journal of Econometrics*, 52:129-57.

HALTMAIER J.,1996, "Inflation-adjusted potential output", *Board of Governors of the Federal Reserve System International Finance Discussion Papers*, No 561.

HARVEY A. C., 1989, *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press.

HARVEY, D. I., S. J. LEYBOURNE and P. NEWBOLD, 1997, "Testing the equality of prediction mean squared errors", *International Journal of Forecasting* Vol. 13 No 2, pp281-291.

HODRICK, R. and PRESCOTT, E.C., 1980, "Post-war U.S. business cycles: an empirical investigation", *Discussion Paper* No *451*, Carnegie-Mellon University, Pittsburgh, P.A.

KMENTA J., 1967, "On estimation of the CES production function", *International Economic Review*, Vol. 8 No 2.

KONGSAMUT P., S. REBELO and D. XIE, 1997, "Beyond Balanced Growth", *NBER Working Paper* No 6159.

LAXTON, D., and R. TETLOW, 1992, "A Simple Multivariate Filter for the Measurement of Potential Output", *Technical Report No. 59.* Bank of Canada.

Mc MORROW K. and W. RÖGER, 2001, "Potential output: measurement methods, new economy influences and scenarios for 2001-2010", *European Commission Economic Papers* No 150.

OKUN, A., 1962, "Potential GDP: its measurement and significance", American Statistical Association, *Proceedings of the Business and Economic Statistics* Section: 98-103. Washington.

ORPHANIDES A. and S. VAN NORDEN, 1999, "The reliability of Output gap estimates in real time", *Economic Discussion Series*, Board of Governors of the Federal Reserve System.

ORPHANIDES A. and S. VAN NORDEN, 2003, "The reliability of inflation forecasts based on output gap estimates in real time", *CIRANO working paper.*

PESARAN M.H. and A. TIMMERMANN, 1992, "A simple nonparametric test of predictive performance", *Journal of Business and Economic Statistics*, Vol. 10, pp561-56.

PROIETTI T., A. MUSSO and T. WESTERMANN, 2002, "Estimating potential output and the output gap for the euro area: a Model-Based Production Function Approach", *ECB Working paper.*

RICHARDSON P., L. BOONE, C. GIORNO, M. MEACCI, D. RAE and D. TURNER, 2000, "The concept, policy use and measurement of structural unemployment: estimating a time varying nairu across 21 OECD countries", *OECD Economic Department Working Paper* No 250.

RÜNSTLER G., 2002, "The information content of real-time output gap estimates. An application to the euro area", *ECB Working Paper*.

St AMANT P. and S. VAN NORDEN, 1997, "Measurement of the output-gap: a discussion of recent research at the Bank of Canada", *Bank of Canada Technical Report* No 79.

Appendices

A Description of the dataset

The database is quarterly. The sample covers the period extending from the first quarter of 1978 to the fourth quarter of 2002. Countries covered are Germany, France, Italy, the Netherlands and Spain. As far as possible, data have been taken from the Eurostat national accounts. When necessary, data have been back-calculated or are taken from the national sets of accounts. Specific attention has been paid to using the same concepts and definitions for each country. EU5 data are calculated as the aggregation of the five national data. German data have been retropolated for the years preceding the reunification (first quarter of 1978 to fourth quarter of 1990) using the data set published by the Statistisches Bundesamt, where national accounts aggregates for west Germany have been harmonized with the ESA95 concepts and definitions.

Gross domestic product at constant prices: Eurostat and national sets of accounts, billions of 1995 ECUs. Data are seasonally and working-day adjusted.

Private consumption and import of goods and services deflators: National sets of accounts (billions of 1995 ECUs at constant prices, billions of current euros converted in ECUs with the 1995 exchange rate for current prices). Data are seasonally and working-day adjusted. Deflators for the EU5 data are calculated as the ratio of the aggregates in current prices to the aggregates in constant prices.

Labour market data (employment, unemployment, working age population): National accounts, European labour force survey (Eurostat), national labour force surveys and own calculations. Employment: thousands of persons. ILO definition for unemployment. Labour market data are fully consistent: labour force = working age population in employment + unemployment ; labour force ratio = ratio of the labour force to the working age population; unemployment rate = ratio of the unemployment to the labour force. Annual data interpolated to quarterly data for the working age population (15-64 years).

Hours worked: hours actually worked per person except for the Netherlands (hours paid per job converted into hours per person) (Groningen University, OECD, national accounts, own calculations). Annual data interpolated to quarterly data.

Capital stock in the business sector: capital stock in constant prices (converted into billions of ECUs of 1995) of the total economy excluding government and housing taken from the national accounts for Germany, and France. OECD time series converted into billions of 1995 ECUs for Italy, the Netherlands, and Spain. Annual data interpolated to quarterly data.

B Measures of forecasting performance

We test the forecasting performance of the different measures of the output gap by the root mean squared error (RMSE) for multi-step ahead inflation forecasts.[7] All models have hence been

[7] Other criteria can be used, as for instance the mean absolute error. Results are robust to alternative measures of forecasting performances.

recursively estimated from 1978Q1 to i=1996Q4...2002Q3, and out-of-samples forecasts from 1 to 10 quarters ahead have been computed over the period 1997Q1-2002Q4.

The RMSE for any forecast is the square root of the arithmetic average of the squared differences between the actual inflation rate and the predicted inflation rate over one-period for which simulated forecasts are constructed. As it is common in the literature, we use two reference models: a "naive" random walk and an AR(3) process.[8] We compare the accuracy of the inflation forecasts to our reference model by comparing the RMSE of the two sets of forecasts. Specifically, we form the ratio (Theil statistics) of the RMSE for each model of output gap to the RMSE for the naive model (or AR(3) model). A ratio greater than one thus indicates that output gap model's forecast is less accurate than the "naive" or AR(3) models.[9]

We use the test of predictive performance proposed by Diebold and Mariano (1995) to introduce a formal statistical procedure. The procedure is designed to test the null hypothesis of equality of expected forecast performance by considering the mean of the difference of the RMSE of a pair of two competing models. Let us suppose that a pair of h-step ahead forecast have produced forecasts errors (e_{1t}, e_{2t}), $t = 1, 2, ..., T$ and the RMSE is the specified function of the forecast error. Then the null hypothesis of the Diebold-Mariano test is:

$$E[RMSE(e_{1t}) - RMSE(e_{2t})] = 0.$$

Defining $d_t = RMSE(e_{1t}) - RMSE(e_{2t}), t = 1, 2, ..., T$, the Diebold-Mariano test statistic is then:

$$S_{DM} = \left[\widehat{V}(\overline{d})\right]^{-1/2} \overline{d} \xrightarrow{d} N(0,1)$$

where $\widehat{V}(\overline{d}) \approx T^{-1}\left[\widehat{\gamma}_0 + 2\sum_{k=1}^{h-1}\left(1 - \frac{k}{h}\right)\widehat{\gamma}_k\right]$ and $\widehat{\gamma}_k = T^{-1}\sum_{t=k+1}^{T}(d_t - \overline{d})(d_{t-k} - \overline{d})$.[10]

However, as is noted by Diebold and Mariano (1995), simulation evidence suggests that their test could be seriously over-sized in the case of two-steps ahead prediction and the problem become more acute as the forecasting horizon increases. In this respect, Harvey, Leybourne and Newbold (1997) proposed employing an approximately unbiased estimator of the variance of $\overline{d}$, which leads to a modified Diebold-Mariano test statistic:

$$S^* = \left[\frac{T + 1 - 2h + T^{-1}h(h-1)}{T}\right] S_{DM}.$$

Harvey et al. (1997), Clark and McCracken (2001) show that this modified test statistic performs better than the DM test statistic even if it still performs poorly in finite samples (Clark and McCracken, 2001). They also show that the power of the test is improved, when p-values are computed with a Student distribution.

[8] In the case of SVAR models, we use an AR(3) in first differences in order to be consistent.

[9] It is to to be noted that the significance of the relative mean squared error (RRMSE) is not directly tested. Indeed, the one-sided test H_0 :RRMSE=1 againt H_0 :RRMSE < 1 can be used and the corresponding p-values measure type-I error associated with the test.

[10] Diebold and Mariano (1995) initially proposed an estimate of $\hat{V}(\overline{d})$ that was not always positive semi-definite. In this paper, a consistent estimate of the standard deviation is constructed from a weighted sum of the available sample autocovariances of the loss differential vector - the difference between the squared forecast error of the models and that of the reference model. Chosen weights insure that the matrix $\hat{V}(\overline{d})$ is positive semi-definite (Newey and West, 1987).

C Measures of uncertainty and consistency

The statistical reliability of the output gap is also assessed by distinguishing the filter uncertainty and the parameter uncertainty . As it is common in the literature, we use the decomposition of Hamilton (1986):

$$E\left(C_{t,t}^{y}(\widehat{\theta})-C_{t}\right)^{2}=E\left(C_{t,t}^{y}(\widehat{\theta})-C_{t}(\theta)\right)^{2}+\Sigma_{t,t}(\theta)$$

where $C_{t,t}^{y}(\widehat{\theta})$ is the estimate of the output gap given $(y_t)_{1..T}$ and the parameter estimate $\widehat{\theta}$. The left-hand side term is the parameter uncertainty, directly obtained from the Kalman filter at each date t. The second term of the right-hand side expression is the filter uncertainty ($\Sigma_{t,t}^{P}$). It needs to be computed by a Monte-Carlo simulation procedure as follows:

- Draw K parameters $(\theta_k)_{k=1..K}$ from a normal distribution $N(\widehat{\theta},\widehat{\sigma}_{\theta}^{2})$;
- Compute the corresponding K cycles $C_t(\theta_k)$ with the Kalman filter;
- For each date t, the parameter uncertainty is defined by a mean square error criterion:

$$\Sigma_{t,t}^{P}=\frac{1}{K}\sum_{k=1}^{K}\left[C_{t}(\theta_{k})-C_{t}(\widehat{\theta})\right]^{2}.$$

In addition, we also compare the one-sided/filtered estimates and the two-sided/smoothed estimates by computing the parameter and filter uncertainty of smoothed estimates $\left(\Sigma_{T,t},\Sigma_{T,t}^{P}\right)$. Both sources of uncertainty are also characterised. Orphanides and Van Norden (1999), Cayen and Van Norden (2002) proposed a battery of descriptive statistics but no formal testing in order to evaluate the estimates of the output gap using real time data, final data or quasi-real time data. Specifically, Orphanides and Van Norden (2003) find large discrepancies between the sequentially estimated measures of the output gap when compared with final estimates. The disparities are explained by the unreliability of the models in estimating end of sample values and to a lower extent by data revisions. However, Orphanides and Van Norden (2003) mainly compare univariate methods. Camba-Mendez and Rodriguez-Palenzuela (2003) assess the consistency of output gap estimates by implementing the Pesaran and Timmermann (1992) test of directional change and a Fisher test to compare the variances of recursively estimated output gap sequence and the finally estimated sequence. Camba-Mendez and Rodriguez-Palenzuela (2003) show that the concern of reliability of estimates may be to some extent overdone in the Euro-area. However, Van Norden (2003) casts some doubts on their conclusion.

D Tables and graphs

Table 1 : Production function HPMV estimates

	UE5	Germany	France 1	France 2	Italy	Spain	Netherlands
Inflation dynamics							
β_1	0.54 (0.01)	0.29 (0.01)	0.41 (0.04)	0.35 (0.008)	0.67 (0.05)	0.27 (0.00)	0.24 (0.03)
β_2	0.35 (0.01)	0.46 (0.01)	0.27 (0.04)	0.35 (0.009)	0.18 (0.07)	0.37 (0.00)	-
α	0.04 (0.00)	0.10 (0.00)	0.04 (0.00)	0.05 (0.008)	0.04 (0.01)	0.03 (0.08)	0.12 (0.08)
γ_1	0.08 (0.00)	0.14 (0.00)	0.05 (0.009)	0.05 (0.002)	0.01 (0.01)	-	0.08 (0.02)
γ_2	-	0.08 (0.00)	-	-	-	-	-
c	-1.74e-4 (2.66e-5)	-1.34e-4 (5.15e-5)	-3.28e-4 (3.71e-4)	-3.28e-4 (3.71e-4)	-2.88e-4 (2.73e-4)	-6.34e-4 (5.01e-4)	1.49e-5 (6.68e-4)
Trend dynamics							
δ	0.40 (0.06)	0.24 (0.05)	0.31 (0.00)	0.37 (0.01)	0.4 -	0.42 (0.06)	0.40 (0.14)
ω				-0.26 (0.00)			
$100\times\sigma_{tfp}$	1.37e-2	2.62e-2	6.02e-2	1.77e-2	7.74e-2	2.63e-2	2.32e-2
$100\times\sigma_{nh}$	1.31e-4	2.55e-4	2.46e-4	2.46e-4	3.51e-4	2.35e-4	1.38e-4
Correlation between cycles							
$c_{nh,lp}$	-0.11	0.17	0.04	0.04	-0.42	-0.86	-0.78

Note : Standard deviation in parentheses. Estimation period : 1978:1 2002.4

Table 2: Descriptive Statistics of UE5 output gaps and trends[a]

		UE5 aggregate (production function)	UE5 aggregate (univariate HP filter)	UE5 national approach (production function)
(1)	Mean	-0.43	0.02	-0.67
(2)	Standard deviation	1.01	0.89	1.03
(3)	Minimum	-2.14	-1.71	-2.46
(4)	Maximum	2.14	2.34	1.96
(5)	Actual potential output growth rate[b]	2.05	1.99	2.07
(6)	Standard deviation of potential output growth rate	0.53	0.59	0.54

Note : statistics are reported for the period 1978:2 2002:4.

a : data are in percentage points. Growth rates have been annualised in the raw (5) and (6).

b : calculated as the growth rate of the trend output at the fourth quarter of the year 2002.

Table 3: Descriptive Statistics of national output gaps and trends[a]

		Germ. (prod. func.)	Germ. (univ. HP)	France 1 (prod. func.)	France 2 (prod. func.)	France (univ. HP)	Italy (prod. func.)	Italy (univ. HP)	Spain (prod. func.)	Spain (univ. HP)	Neth. (prod. func.)	Neth. (univ. HP)
(1)	Mean	-0.13	0.02	-1.03	-0.80	0.01	-0.75	0.03	-2.39	-0.01	0.00	0.01
(2)	Standard deviation	1.20	1.19	1.25	1.24	0.90	1.22	1.00	1.40	1.04	0.94	1.19
(3)	Minimum	-2.75	-2.43	-3.30	-3.06	-1.80	-3.27	-2.09	-5.25	-2.32	-3.32	-3.65
(4)	Maximum	3.08	3.57	1.55	1.90	2.15	2.23	2.78	1.46	3.27	2.12	2.52
(5)	Actual pot. output growth r.[b]	1.51	1.35	2.81	2.74	2.62	1.71	1.72	3.31	3.23	1.16	2.08
(6)	St. dev. of pot. output growth r.	0.92	0.81	0.46	0.49	0.61	0.49	0.49	0.97	1.13	1.01	0.87

Note : statistics are reported for the period 1978:2 2002:4.

a : data are in percentage points. Growth rates have been annualised in the raw (5).

b : calculated as the growth rate of the trend output at the fourth quarter of the year 2002.

Table 4 : UE5 output gaps accuracy (1978:1 2002:4) and revisions statistics (1997:1 2002:4)

	UE5 aggregate (function production)	UE5 aggregate (univariate HP filter)
Smoothed estimation		
Total uncertainty	0.460	0.253
Filtered uncertainty	0.449	0.253
Parametric uncertainty	0.091	-
Filtered estimation		
Total uncertainty	0.820	0.460
Filtered uncertainty	0.809	0.460
Parametric uncertainty	0.116	-
Output gap standard deviation		
Smoothed[1]	1.181	0.897
	(2.56)	(3.54)
Filtered[1]	1.596	0.994
	(1.95)	(2.16)
Standard deviation of revisions		
Quasi-real time	0.890	0.764

Note: t-student in parentheses.

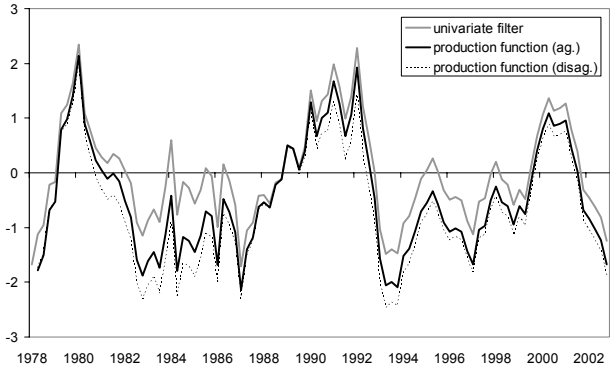
Table 5 : Out-of-sample inflation forecasting performances (UE5 aggregate estimates)

	Horizon	AR(3)				Random walk			
Naïve model		RMSE	RRMSE	DM	P-value	RMSE	RRMSE	DM	P-value
	1	0.219	1.000	0.000	1.000	0.260	1.000	.000	1.000
	2	0.221	1.000	0.000	1.000	0.280	1.000	.000	1.000
	3	0.225	1.000	0.000	1.000	0.244	1.000	.000	1.000
	4	0.217	1.000	0.000	1.000	0.327	1.000	.000	1.000
	5	0.220	1.000	0.000	1.000	0.337	1.000	.000	1.000
	10	0.238	1.000	0.000	1.000	0.295	1.000	.000	1.000
UE5 aggregate model									
	1	0.178	0.813	-1.208	0.247	0.178	0.686	-1.622	0.127
	2	0.171	0.771	-1.615	0.129	0.171	0.609	-2.647	0.019
	3	0.173	0.770	-2.019	0.063	0.173	0.712	-1.850	0.085
	4	0.164	0.756	-1.565	0.140	0.164	0.503	-3.098	0.008
	5	0.159	0.726	-2.328	0.035	0.159	0.472	-3.561	0.003
	10	0.159	0.670	-2.100	0.054	0.159	0.541	-2.758	0.015

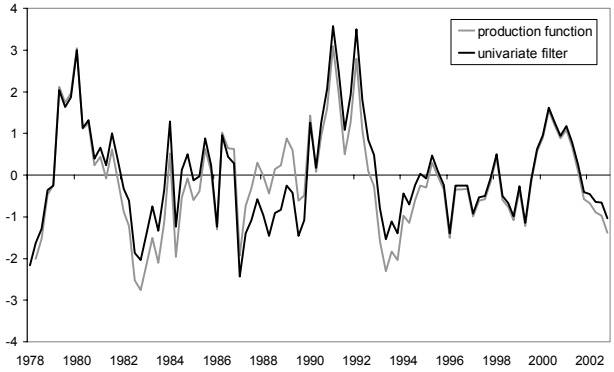
N.B. The table compares the one to ten steps ahead inflation forecasts(out of sample) of the production function estimates to the one to ten steps ahead inflation forecasts of two naive models (an AR(3) model and a random walk).
The RMSE (root mean squared error) is the square root of the mean prediction errors. The RRMSE is the relative root mean squared error of the models against that of the naive models. The D-M statistics is the modified Diebold-Mariano (Diebold and Mariano 1995, Harvey Leybourne and Newbold 1997) test statistics. P value is the marginal probability of the Diebold-Mariano test of the null hypothesis of equal predictive accuracy of the alternative models and of the naive model.

Output gaps

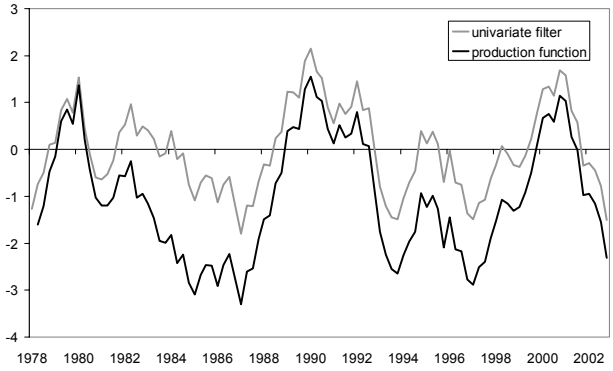
Graph 1 : UE5 output gap estimates



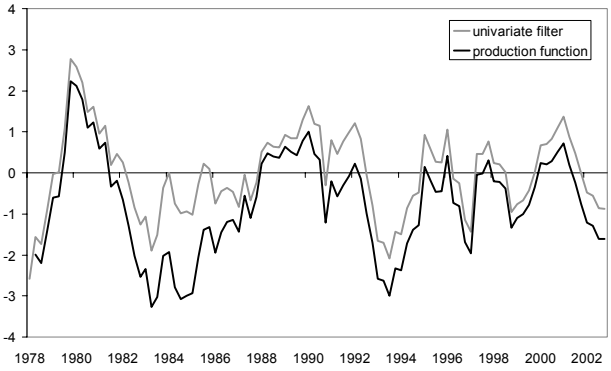
Graph 2 : Germany output gap estimates



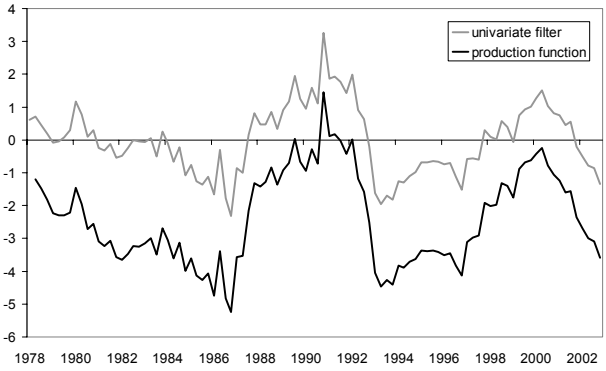
Graph 3 : France output gap estimates



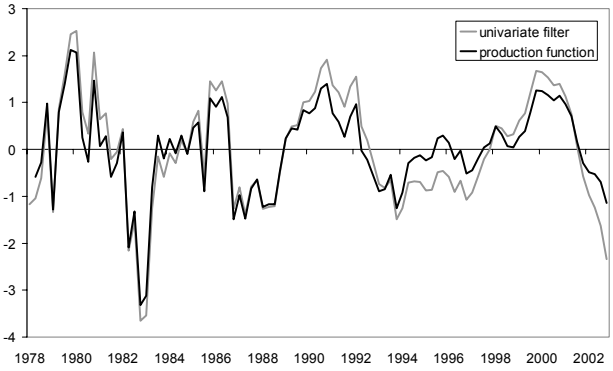
Graph 4 : Italy output gap estimates



Graph 5 : Spain output gap estimates

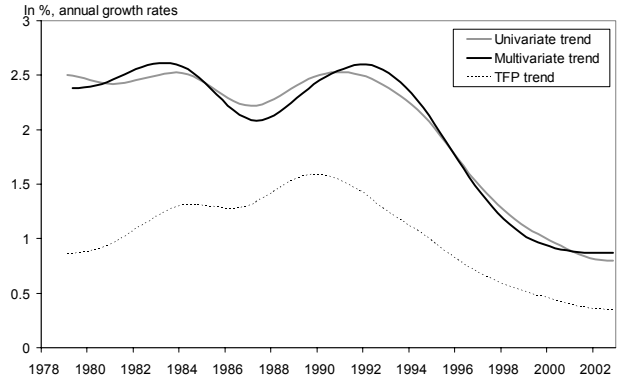


Graph 6 : Netherlands output gap estimates

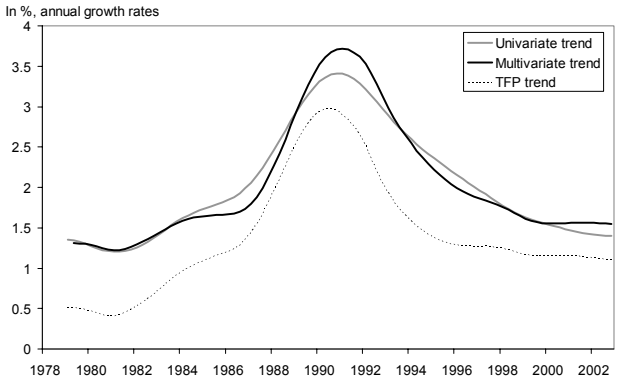


Labour productivity gaps and trends

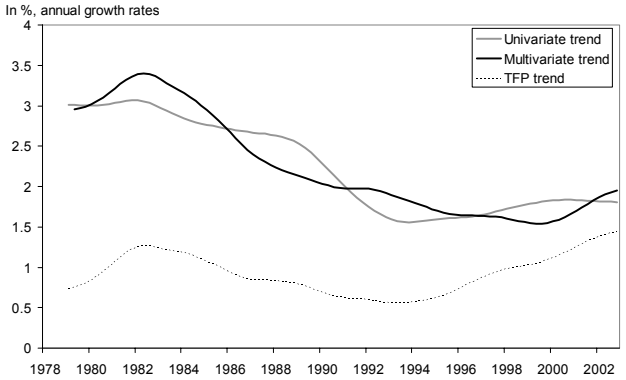
Graph 7 : UE5 trend hourly labour productivity



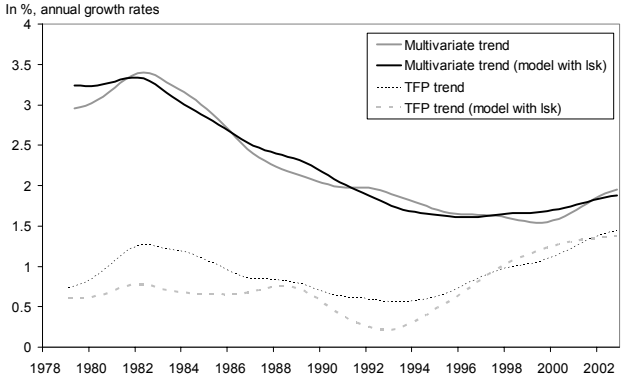
Graph 8 : Germany trend hourly labour productivity



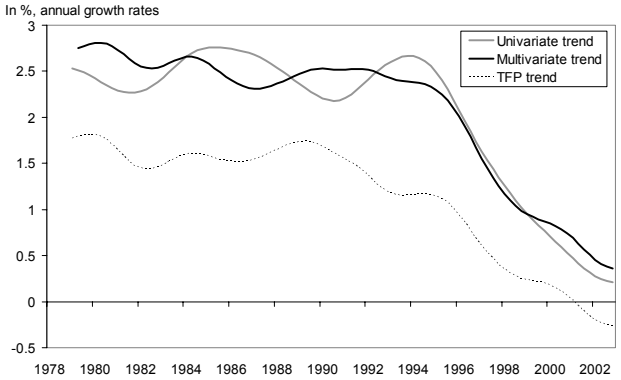
Graph 9 : France (1) trend hourly labour productivity



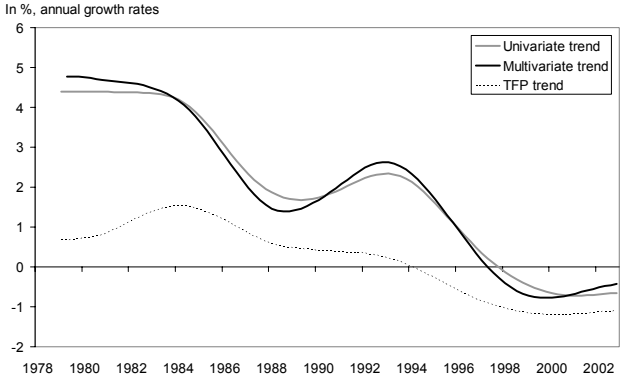
Graph 10 : France (1) and (2), comparison of trend hourly labour productivity



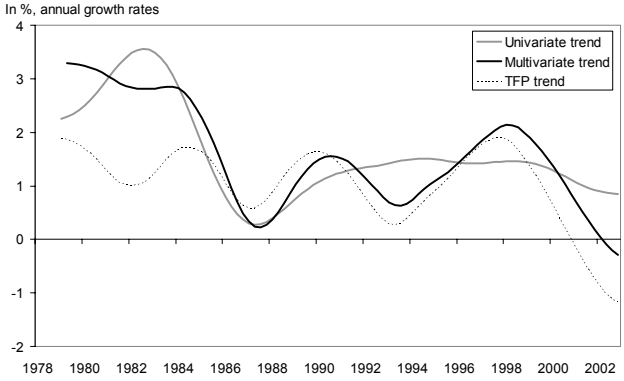
Graph 11 : Italy trend hourly labour productivity



Graph 12 : Spain trend hourly labour productivity

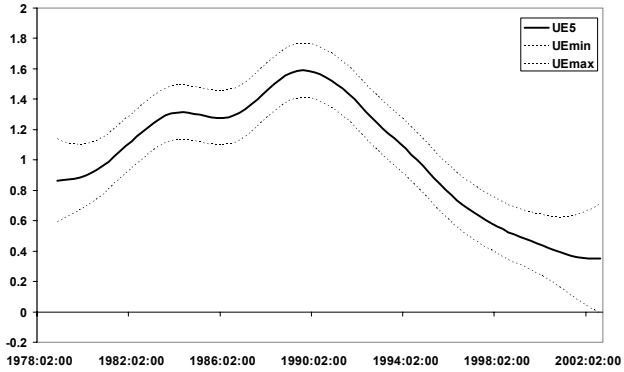


Graph 13 : Netherlands trend hourly labour productivity

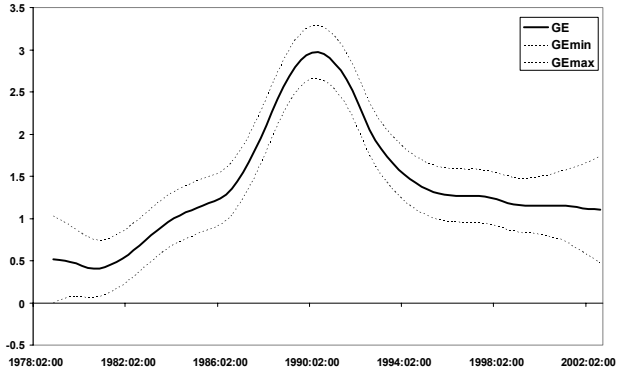


Confidence bands of the endogenous productivity trends growth rates

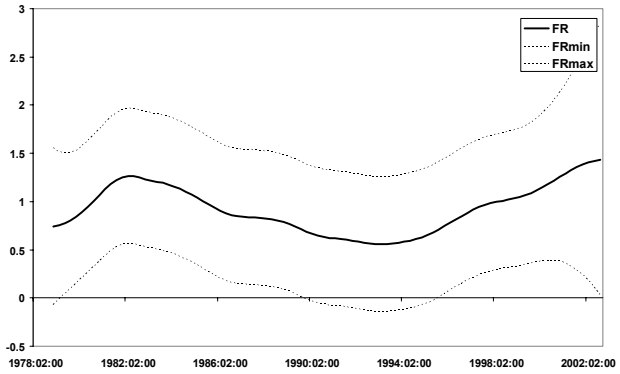
Graph 14 : Confidence band UE5



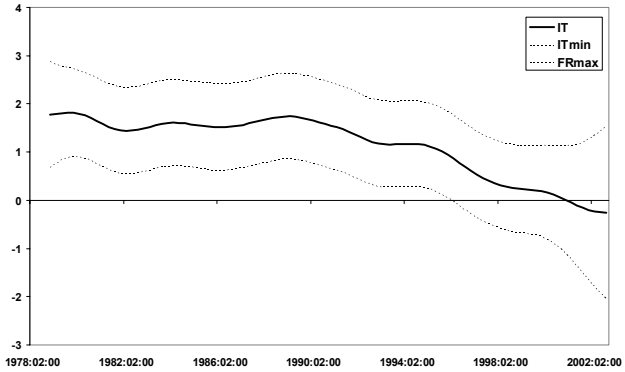
Graph 15 : Confidence band Germany



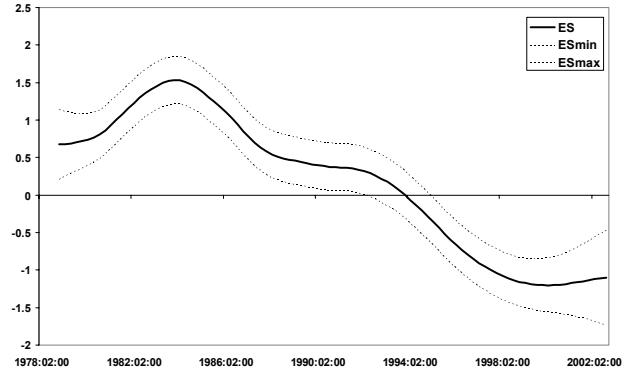
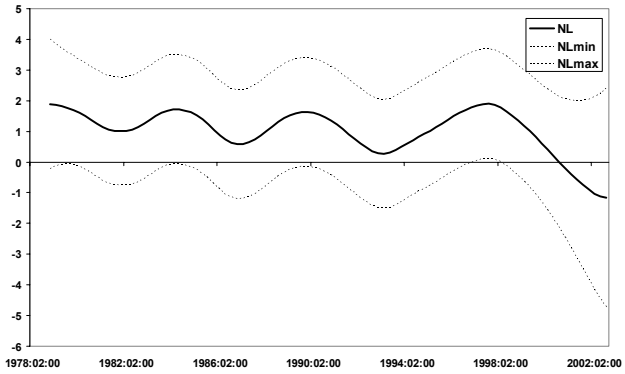
Graph 16 : Confidence band France (1)



Graph 17 : Confidence band Italy



Graph 18 : Confidence band Spain



Graph 19 : Confidence band Netherlands