The Cyclical Volatility of Labor Markets under Frictional Financial Markets

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Abstract

In an economy with search on credit and labor markets, a financial multiplier raises the elasticity of labor market tightness to productivity shocks. This multiplier increases with total financial costs, which are minimized under a credit market Hosios-Pissarides rule. Relaxing that condition leads to either a small "bank " or a small "firm" surplus in the credit market, and larger multipliers which can match or even overshoot the elasticity of market tightness in the data. Furthermore, when wages are endogenous, it is possible to relax Hagedorn and Manovskii's (2008) small *labor* surplus assumption in order to match the data

1 Introduction

Cole and Rogerson (1999) and Shimer (2005) have investigated the cyclical properties of the search matching models following Pissarides (1985) and Mortensen and Pissarides (1994). The celebrated Shimer's puzzle is the demonstration of the inability of the conventional matching model to replicate the US statistics regarding the volatility of job vacancies, unemployment and their ratio (called labor market tightness), in response to productivity shocks. His main finding is that the elasticity of labor market tightness to productivity shocks is around 20 in the data, and around 1 in a calibration of the Mortensen-Pissarides model. Several calibration improvements have been proposed, including raising the model value of non-employment utility (Hagedorn and Manovskii 2008), wage rigidity (Hall 2005) and on-the-job search (Mortensen and Nagypàl 2007).

One line of research that has so far been ignored but seems promising is the existence of credit market imperfections. In this paper we pursue this logic, following two previous papers. On the one hand, Petrosky-Nadeau (2009) shows that introducing credit market imperfections, with in particular costly state verification, in a search model can lead to a large amplification of the volatility of labor market tightness. The standard deviation in his model of the vacancy-unemployment ratio approaches 12.5 relative to that of output, while it is 15.4 in US data and merely 3.7 in the standard search model. Credit market imperfections induce an amplification factor of 3.5.

On the other hand, Wasmer and Weil (2004), who develop financial imperfections in a Mortensen-Pissarides economy with two matching functions (one in the labor market, one in the credit market), show that the steady-state volatility of labor market tightness to profit shocks is augmented by a factor 1.7 by the existence of moderate credit market imperfections. They call this a financial accelerator, in line with an earlier literature.

Despite recent papers attempting to bring together credit market imperfections and the search-matching approach, the macro-labor literature has been slow to incorporate the well-known message of an earlier literature. Indeed, it has been known for a while that credit market imperfections generate additional volatility of the business cycle. Early papers such

as Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and subsequent papers (such as Bernanke and Gertler 1995, Bernanke Gertler and Gilchrist 1996, and several others), have emphasized the amplification role of credit markets and the existence of a *financial accelerator*. Although part of this literature is centered on the role of credit shocks and the credit channel of monetary policy, the ingredients generating the amplification of credit shocks can very well be adapted to the amplification of business cycle shocks to labor markets.

Firms in our model arise the result of the meeting of an entrepreneur and a banker on a frictional credit market. The average cost of creating a firm is the sum of all prospecting costs on the credit market which, compared to the world with perfect credit markets in Mortensen and Pissarides (1994), imposes a lower limit on the value of a job vacancy to a firm. Consequently, frictional credit markets limit firm entry on the labor market, resulting in a greater equilibrium rate of unemployment. Our results regarding the amplification of productivity shocks in this double matching economy can be summarized as follows.

First, consistent with Wasmer and Weil (2004), financial imperfections raise the calibrated elasticity of labor market tightness to productivity shocks by a factor M_f called the financial multiplier, which is an increasing function of total financial costs in the economy. Second, a Hosios-Pissarides rule exists in the credit market : the bargaining power of firms vis-à-vis banks is equal, at the social optimum, to the elasticity of the finding rate of banks with respect to credit market tightness. Third, under the Hosios rule, the search costs in the credit market are minimized, and so is M_f. Relaxing that condition leads to a larger financial accelerator, which can match or even overshoot the elasticity of market tightness in the data.

Fourth, away from the Hosios rule, four situations generate a large or very large volatility: i) When the matching function in the credit market is "balanced", e.g. a Cobb-Douglas with elasticity around half for each segment of the market (banks and entrepreneurs) and when entrepreneurs have a very low bargaining power in the bargaining relation with the bank; ii) The symmetrical case with balanced matching in the credit market but when entrepreneurs have a very high bargaining power in the bargaining relation with the bank, which corresponds to, and corroborates the results of, Petrosky-Nadeau (2009); iii) When instead the matching function is unbalanced, corresponding to a situation in which credit creation is limited by the

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"ideas of entrepreneurs", that is an excess of liquidity; iv) When the matching function is unbalanced on the other side, corresponding to a situation in which credit creation is limited by "supply of liquidity," that is a scarcity of liquidity. Finally, these results hold with endogenous or exogenous wages and in a deterministic or stochastic setup, e.g., one can obtain any high elasticity of labor market volatility to productivity shocks in each of the four cases identified above.

The last result with regards to endogenous wages speaks strongly to what we will call the "small labor surplus" assumption often followed in the literature in order to raise the elasticity of labor market tightness to productivity. This approach, suggested by Hagedorn and Manovskii (2008), rests on choosing high values of non-employment activities and very low values for the bargaining power of workers. However, as Mortensen and Nagypal (2007) point out, this assumption implies that there is very little utility gain to accepting a job, nor does it fit well with estimates of the value of non-employment. Financial imperfections in our model enable usto relax the "small surplus" assumption in order to match the elasticity of market tightness to productivity found in the data. Therefore, our results are readily interpreted as a generalization of the small surplus assumption: when the credit market is either very tight or very slack for firms, one side of the market has a very small surplus to entering the relationship. Consequently, the entry of that side of the credit market is restricted and even small productivity shocks can generate large relative increases in the number of agents on the restricted side of the market.

This paper is organized as follows. In Section 2, we summarize the main equations in Wasmer and Weil (2004) and calculate the volatility of labor market tightness to productivity shocks. In Section 3, we show how the Hosios rule in the credit market affects the volatility of the labor market. In Section 4, we proceed to the calibrations of the stochastic variants of the model with and without credit market imperfections, and we show how changing the parameters of the model related to the credit matching technology substantially raises this elasticity. In Section 5, we extend the model to endogenous wages. In Section 6 we conclude.

2 An economy with credit and labor market frictions

2.1 Model

Time is continuous and there are three types of agents: entrepreneurs with no capital; banks with no ability to produce; and workers with no capital and no ability to start a business. The timing of events for entrepreneurs is asthat they initially need to find a "banker" in order to start a business. This search process costs e units of effort per unit of time. Search is successful with probability p. The newly formed firm, from the successful meeting of entrepreneur and banker, then goes to the labor market. The bank finances the vacancy posting cost γ to attract workers (the so-called recruitment costs) for the firm. This search process succeeds with probability q. The firm is then able to produce and sell in the good market, which generates a flow profit $y - w - \rho$ where y is the marginal product, w is the wage (assumed exogenous in this section, bargained in Section 5), r is the flow rate of discount, and ρ is the flow repayment to the bank (determined through bargaining). Jobs are subject to destruction shocks with Poisson parameter s. The steady-state asset values of the entrepreneurs are denoted by E_j with $j = c, l$ or g the market in which the entrepreneur is operating, standing respectively for the credit, labor and good markets. We also assume free entry at the first stage, that is $E_c \equiv 0$. We therefore have the following Bellman equations:

$$rE_c = 0 = -e + pE_l \tag{1}$$

$$rE_l = 0 + q(E_g - E_l) \tag{2}$$

$$rE_g = y - w - \rho + s(0 - E). \tag{3}$$

In the last line, it was assumed that job destruction also leads to the destruction of the firm and the lending relation with the bank.

Symmetrically, the bank's asset values are denoted by B_j, $j = c, l$ or g for each of the stages. We also assume free entry of the banking relationship: $B_c = 0$. We denote by k the screening cost per unit of time of banks in the first stage, and by $\widehat{p}$ the Poisson rate at which a bank finds

a firm to be financed. We have:

$$rB_c = 0 = -k + \widehat{p}B_l \tag{4}$$

$$rB_l = -\gamma + q(B_g - B_l) \tag{5}$$

$$rB_g = \rho + s(0 - B_g). \tag{6}$$

The matching rates p and $\widehat{p}$ are made mutually consistent by the existence of a matching function $M_c(\mathscr{B}, \mathscr{E})$, where $\mathscr{B}$ and $\mathscr{E}$ are respectively the number of bankers and of entrepreneurs in stage c. This function is assumed to have constant returns to scale. Hence, denoting by ϕ the ratio $\mathscr{E}/\mathscr{B}$, which is a reflection of the tension in the credit market and that we shall call credit market tightness from the point of view of entrepreneurs, we have

$$p = \frac{M_c(\mathscr{B}, \mathscr{E})}{\mathscr{E}} = p(\phi) \text{ with } p'(\phi) < 0.$$
$$\widehat{p} = \phi p(\phi) \text{ with } \widehat{p}'(\phi) > 0.$$

After the contact, the bank and the entrepreneur engage in bargaining about ρ which is such that

$$(1 - \beta)B_l = \beta E_l \tag{7}$$

where β is the bargaining power of the bank relative to the entrepreneur. With $\beta = 0$ the bank leaves all the surplus to the entrepreneur.

Combining (1), (4) and (7), we obtain the equilibrium value of ϕ denoted by ϕ^* with

$$\phi^* = \frac{k}{e} \frac{1 - \beta}{\beta}.$$

Matching in the labor market is denoted by $M_l(\mathscr{V}, u)$ where u is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1. $\mathscr{V}$ is the number of "vacancies", that is the number of firms in stage l. The function is also assumed to be constant return to scale, hence the rate at which firms fill vacancies is a function of the ratio $\mathscr{V}/u$, that is tightness of the labor market. We have

$$q(\theta) = \frac{M_l(\mathscr{V}, u)}{\mathscr{V}} \text{ with } q'(\theta) < 0.$$

Further using (2), (3) and (5), (6), we finally simultaneously solve for ρ :

$$\frac{\rho}{r+s} = \beta \frac{y-w}{r+s} + (1-\beta)\frac{\gamma}{q(\theta)}$$

and obtain the two main equations of the model :

$$\text{(EE)} \quad : \quad \frac{e}{p(\phi)} = \frac{q(\theta)}{r+q(\theta)}\left(\frac{y-w}{r+s} - \frac{\gamma}{q(\theta)}\right)(1-\beta) \tag{8}$$

$$\text{(BB)} \quad : \quad \frac{\kappa}{\phi p(\phi)} = \frac{q(\theta)}{r+q(\theta)}\left(\frac{y-w}{r+s} - \frac{\gamma}{q(\theta)}\right)\beta \tag{9}$$

Each equation provides a link between θ and ϕ that is of opposite sign. There is therefore at most one equilibrium set of (θ^*, ϕ^*).[1] Finally, summing up (EE) and (BB), one obtains a single market equation denoted by (CC) for θ^* describing a job creation condition for this double matching economy:

$$\text{(CC):}\frac{e}{p(\phi^*)} + \frac{k}{\phi^* p(\phi^*)} = \frac{q(\theta)}{r+q(\theta)}\left(\frac{y-w}{r+s} - \frac{\gamma}{q(\theta)}\right) \tag{10}$$

where the left-hand side is a measure of the total amount of search costs in financial markets. These are the total financial costs associated with the creation of a firm and that we shall denote by $K \equiv \frac{e}{p(\phi^*)} + \frac{k}{\phi^* p(\phi^*)}$.

2.2   Steady-state volatility of θ to shocks

We now want to calculate the elasticity of θ to profit shocks, denoted by $\zeta_{\theta/\pi}$. Let θ^P be the value of tightness solving for

$$\frac{y-w}{r+s} = \frac{\gamma}{q(\theta^P)} \tag{11}$$

The value of θ^P defined here is the credit frictionless world in Pissarides (1985), which one would obtain from (10) when $K = 0$. In using (CC), one has:

$$\frac{\gamma}{q(\theta^P)} - \frac{\gamma}{q(\theta^*)} = K\left(\frac{r+q(\theta^*)}{q(\theta^*)}\right) > 0$$

Hence, given that q' is downward sloping, we have that $\theta^* < \theta^P$, as was shown in Wasmer and Weil (2004) and arises in Petrosky-Nadeau (2009), and the difference is precisely due to the

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[1]Wasmer and Weil (2004) provide a condition for existence.

existence of search costs in the credit market. Let $\pi = (y-w)/(r+s)$ be the present discounted value of profits. Posing $r = 0$ to marginally simplify the analysis, we have an equilibrium job creation condition under frictional credit markets which states that the profit flows from a job net of the total financial costs to creating a firm must equal the average cost of filling a job vacancy:

$$\pi - K = \frac{\gamma}{q(\theta^*)}. \tag{12}$$

Taking logs and differentiating, we have

$$-\frac{q'(\theta^*)\theta^*}{q(\theta)^*}\frac{d\theta}{\theta^*} = \frac{d\pi}{\pi}\frac{\pi}{\pi - K}$$

or, reusing (11) and (12) and where $\eta = -q'(\theta)\theta/q(\theta)$ is the (non-necessarily constant) elasticity of q to θ, we have

$$\zeta_{\theta/\pi} = \frac{d\ln\theta}{d\ln\pi} = \frac{1}{\eta}\frac{\frac{\gamma}{q(\theta^P)}}{\frac{\gamma}{q(\theta^*)}} = \frac{1}{\eta}\frac{q(\theta^*)}{q(\theta^P)}$$

Two remarks are in order. First, in the (credit) frictionless world in Pissarides, the elasticity is simply the inverse of the elasticity of q to θ, that is $1/\eta$. Second, the existence of credit market imperfections reduces θ^* relative to θ^P, and therefore raise the volatility $\zeta_{\theta/\pi}$ by a factor due to the financial accelerator identified in Wasmer and Weil (2004): higher profits raise the entry of firms, hence banks make faster profits, which in turn benefits firms, and so on. Denote by

$$M_f = \frac{q(\theta^*)}{q(\theta^P)}$$

the value of the financial accelerator, which can more generically be defined as the ratio of the elasticity in a world with credit frictions and the elasticity in a world where credit frictions disappear.

Under the assumption of an exogenous wage, the response of this economy to productivity shocks on y is therefore :

$$\zeta_{\theta/y} = \frac{d\ln\theta}{d\ln y} = \frac{d\ln\theta}{d\ln\pi}\frac{d\ln\pi}{d\ln y} = \frac{1}{\eta}\frac{y}{y-w}M_f$$

The first component of this elasticity is the amplification due to the existence of search frictions on the labor market. The second component is the gap between wages and marginal product -

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the smaller the gap, the more responsive job creation is to productivity shocks; and finally, the third is the financial accelerator.

With the parameters values in Wasmer and Weil (2004), $\eta = 0.5$, $y = 1$, $w = 2/3$, and $M_f = 1.74$, such that

$$\zeta_{\theta/y} = 2 \times 3 \times 1.74 = 10.44.$$

This is a large factor compared to the conventional Pissarides model elasticity as in Shimer (2005), for example, who found a much smaller number value of 1.13. This difference is due to three factors :

1. the choice of the matching elasticity in Shimer (1/0.72) ; assuming $\eta = 0.5$ instead raises the elasticity with respect to Shimer by a factor 2*0.72=1.44.

2. the assumption of wage rigidity in our model (see Hall 2005): in the absence of rigidity in wages, the factor $\frac{y}{y-w} = 3$ would have to be replaced by a more complex term, derived and discussed later on in the part devoted to endogenous wages. In short, wage rigidity raise volatility by a factor of 4 to 5.

3. The last part of the difference is due to the existence of a financial accelerator $M_f = 1.74$, consistent with the literature initiated by Bernanke and Gertler (1989).

The labor literature has attempted to raise the elasticity of market tightness to productivity with either wage rigidities (Hall 2005) or by making what we will call hereafter the "small labor surplus" assumption by choosing higher values of non-employment utility and lower values for the bargaining power of workers (Hagedorn and Manovskii 2008) and reducing the gap between wages and marginal product. While acknowledging the interest of these approaches, we pursue another avenue here and attempt to understand the determinants of M_f.

3 Entry costs and efficiency in the credit market

3.1 Hosios-Pissarides in the credit market

We start by noting that frictions in the credit market may lead to a second best efficiency condition similar to that in Hosios (1990) and Pissarides (1990).

To see this, we can calculate the social welfare function as output net of all search costs. We have :

$$\Omega = y(1-u) + zu - \gamma\theta u - k\mathcal{B} - e\mathcal{E}$$

where z is the value of non-employment utility and $\theta u = \mathcal{V}$ is the number of firms prospecting in the labor market. To obtain a simpler expression for Ω, we can note that in a steady-state, we have $\mathcal{E}p(\phi) = q(\theta)\mathcal{V}$ which states that inflows into the financing stage are compensated by outflows out of that stage. It follows that

$$\mathcal{E} = \frac{q(\theta)\theta u}{p(\phi)} \text{ and } \mathcal{B} = \frac{\mathcal{E}}{\phi} = \frac{q(\theta)\theta u}{\phi p(\phi)}$$

Therefore, the social planner's program can be rewritten as

$$\max_{u,\theta,\phi}\Omega = y(1-u) + zu - \gamma\theta u - \left(\frac{k}{\phi p(\phi)} + \frac{e}{p(\phi)}\right)q(\theta)\theta u$$
$$\text{s.t. } u = s/(s + \theta q(\theta))$$

Relative to the choice of the optimal ϕ denoted by ϕ^{opt}, the problem is simple and block-recursive in ϕ and then in u and θ. For the first block that we only consider here, the optimal choice of ϕ amounts to minimizing total search costs $K(\phi) = \frac{k}{\phi p(\phi)} + \frac{e}{p(\phi)}$:[2]

$$\frac{\partial\Omega}{\partial\phi} = q(\theta)\theta u\frac{\partial}{\partial\phi}K(\phi) = 0$$
$$\Leftrightarrow \phi^{opt} = \frac{1-\varepsilon}{\varepsilon}\frac{k}{e} \text{ where } \varepsilon = -\frac{\phi p'(\phi)}{p(\phi)}$$

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[2]Intermediate steps are :

$$\frac{\partial\Omega}{\partial\phi} = 0 \Leftrightarrow \frac{k}{\phi p(\phi)}\frac{\phi p'(\phi) + p(\phi)}{\phi p(\phi)} + \frac{e}{p(\phi)}\frac{p'(\phi)}{p(\phi)} = 0$$
$$\Leftrightarrow \frac{k}{\phi p(\phi)}(1-\varepsilon) = \frac{e}{p(\phi)}\varepsilon$$

Hence, since $\frac{\partial^2}{\partial \phi^2} K(\phi) > 0$, the socially optimal value of credit market tightness is the one that minimizes search costs on credit markets. The Hosios-Pissarides rule, which states that there is a value of the bargaining parameter over ρ that internalizes the matching externalities due to the search frictions, applies here :

$$
\begin{aligned}
\phi^* &= \phi^{opt} \\
&\Leftrightarrow \beta = \varepsilon : \text{Hosios condition in the credit market}
\end{aligned}
$$

3.2 Minimizing the financial costs and the gap between θ^* and θ^P

One may think that the Hosios condition is the one that minimizes entry costs in the credit market. One can check this formally. The left-hand side of job creation condition (CC) is a function of β and ε denoted by $K(\beta, \varepsilon)$; the right-hand side is increasing in θ. It is therefore enough to show that $K(\beta, \varepsilon)$ is minimized in $\beta = \varepsilon$. Before doing so, we can use two intermediate steps. First, note that $K(\beta, \varepsilon) = \frac{\frac{e}{p(\phi^*)}}{1 - \beta}$ from equation (EE) divided by $(1 - \beta)$. Second, we have $\frac{\partial \phi^*}{\partial \beta} = \frac{-1}{\beta^2} \frac{k}{e}$ hence

$$
\frac{\partial K}{\partial \beta} = \frac{\frac{-ep'}{p^2(\phi^*)} \frac{\partial \phi^*}{\partial \beta}}{1 - \beta} + \frac{\frac{e}{p(\phi^*)}}{(1 - \beta)^2} = 0 \Leftrightarrow \varepsilon = \beta
$$

Given that M_f, and hence $\zeta_{\theta/y}$, is increasing in the gap between θ^* and θ^P, at any ϕ^*, the Hosios condition in the credit market is the one minimizing the volatility induced by financial imperfections. Away from this equation, one has a larger financial accelerator.

4 A stochastic extension and calibrations

In this Section, we study the model with departures from the above Hosios condition in the credit market, i.e., with $\beta \neq \varepsilon$. For that, and in order to provide the most general results, we relax the assumption that $r = 0$ and further calibrate the stochastic evolution of the double-matching economy.

We make the following assumptions for convenience. First, time is discrete and labor productivity is assumed to follow a stationary AR(1) process $y_t = \rho_y y_{t-1} + v_t$, where $0 < \rho_y < 1$

and v_t is white noise. Second, an entrepreneur meeting a banker begins the recruiting process within the period. A successful meeting between a firm and worker begins production the following period. Maintaining our assumption of free entry on both sides of the credit market and bargaining over ρ, we find that the equilibrium credit market tightness ϕ^* is time invariant and of the same form as earlier.[3] Moreover, ρ is assumed to be determined when a banker and an entrepreneur meet and is solved as

$$E_t[\rho_{t+1}] = \beta \mathbb{E}_t[y_{t+1} - \overline{w}] + (1-\beta)\mathbb{E}_t\left[\frac{(1+r)\gamma}{q(\theta_t)} - \frac{(1-s)\gamma}{q(\theta_{t+1})}\right] \tag{13}$$

where $\mathbb{E}_t$ is an expectations operator over productivity and $\overline{w}$ is a fixed wage. The next section will allow for an endogenous wage.

From the constant values of being in the recruiting stage, $B_{l,t} = \frac{\kappa}{\phi^* p(\phi^*)}$ and $E_{l,t} = \frac{e}{p(\phi^*)}$, we can combine the (EE) and (BB) curves in this stochastic environment,

$$\frac{e}{p(\phi)} = \frac{q(\theta_t)}{1+r}\mathbb{E}_t\left[E_{g,t+1}\right] + \frac{(1-q(\theta_t))}{1+r}\frac{e}{p(\phi)}$$

$$\frac{\kappa}{\phi p(\phi)} = -\gamma + \frac{q(\theta_t)}{1+r}\mathbb{E}_t\left[B_{g,t+1}\right] + \frac{(1-q(\theta_t))}{1+r}\frac{\kappa}{\phi p(\phi)}$$

to obtain a job creation condition in the presence of frictional credit markets

$$\frac{\Gamma_t}{q(\theta_t^*)} = \frac{1}{1+r}\mathbb{E}_t\left[y_{t+1} - \overline{w} + (1-s)\frac{\Gamma_{t+1}}{q(\theta_{t+1}^*)}\right] \tag{14}$$

where $\Gamma_t \equiv \gamma + K\left(1 - \frac{1}{1+r}(1-q(\theta_t^*))\right)$ are vacancy costs augmented for frictional credit markets and $K = \frac{e}{p(\phi^*)} + \frac{\kappa}{\phi^* p(\phi^*)}$ is once again total search costs on the credit market.

It is worth noting two special cases. First, when $r = 0$, Γ_t is simply the sum of all prospection costs in credit and labor markets, unadjusted for discounting. Second, when credit markets are perfect, Γ_t boils down to γ, and the job creation condition reduces to

$$\frac{\gamma}{q(\theta_t^P)} = \frac{1}{1+r}\mathbb{E}_t\left[y_{t+1} - \overline{w} + (1-s)\frac{\gamma}{q(\theta_{t+1}^P)}\right] \tag{15}$$

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[3]Time invariance follows from the sharing rule $(1-\beta)B_{l,t} = \beta E_{l,t}$ which implies a constant ratio $\frac{E_{l,t}}{B_{l,t}} = \frac{1-\beta}{\beta}$.

4.1 Elasticity of θ_t to productivity shocks

Define period profits from labor as $\Pi_t = y_t - \overline{w}$. Taking log-linear deviations around a steady state of equation (15), deviations in market tightness in the credit frictionless world can be expressed as a discounted sum of deviations in future expected profits

$$\widehat{\theta}_t^P = \frac{q(\theta^P)\Pi}{\eta\gamma(1+r)}\mathbb{E}_t\sum_{i=0}^{\infty}\left(\frac{1-s}{1+r}\right)^i\widehat{\Pi}_{t+1+i}$$

Given a fixed wage and the assumption on productivity, this is simply $\widehat{\theta}_t^P = \frac{q(\theta^P)}{\eta\gamma(1+r)}\sum_{i=0}^{\infty}\rho_y^{i+1}v_t$ such that the elasticity of market tightness to a productivity shock in the Pissarides world with a fixed wage is

$$\frac{\partial\widehat{\theta}_t^P}{\partial v_t} = \frac{q(\theta^P)\rho_y}{\eta\gamma[(1+r)-(1-s)\rho_y]} \tag{16}$$

By the same steps, the elasticity in the presence of credit frictions is given by

$$\frac{\partial\widehat{\theta}_t^*}{\partial v_t} = \frac{q(\theta^*)\rho_y}{\eta\gamma^T[(1+r)-(1-s)\rho_y]} \tag{17}$$

where $\gamma^T \equiv \left[\gamma + K\left(\frac{r}{1+r}\right)\right] > \gamma$ is a measure of total frictional costs in both credit and labor markets.

The financial multiplier in this dynamic setting is thus:

$$M_f \equiv \frac{\partial\widehat{\theta}_t^*/\partial v_t}{\partial\widehat{\theta}_t^P/\partial v_t} = \frac{q(\theta^*)}{q(\theta^P)}\frac{\gamma}{\gamma^T}$$

which is identical to the accelerator derived in Section 2 when $r = 0$. Finally, the elasticity of the job finding rate is simply: $\frac{\partial f(\theta_t)}{\partial v_t} = \frac{\partial f(\theta_t)}{\partial\theta_t}\frac{\partial\theta_t}{\partial v_t} = (1-\eta)\frac{\partial\theta_t}{\partial v_t}$

4.2 Calibration and results

We follow an incremental strategy, building on a calibration of the Pissarides model to a set of steady state labor market outcomes at a quarterly frequency. In this section the wage is assumed exogenous and equal to three quarters of labor productivity. The steady state rate of job separation is set to $s = 0.1$. We assume an the elasticity of the labor matching function with

respect to unemployment of $\eta = 0.5$.[4] The labor matching function is assumed to be a Cobb-Douglas $M_l(\mathcal{V}, u) = \chi \mathcal{V}^{1-\eta} u^{\eta}$ and, given a value for unit recruitment costs of $\gamma = 0.25$, we adjust the level parameter χ to achieve a desired level of unemployment, approximately 10% in this calibration. Finally, the risk free rate is set to 4%, corresponding to a 3-month treasury bill, and the persistence coefficient in the process for productivity is set to 0.975, a commonly used value in the real business cycle literature.

The calibration of the credit market requires choosing parameters of the credit matching function, assumed to be of the form $M_c(\mathcal{B}, \mathcal{E}) = \varsigma \mathcal{E}^{1-\varepsilon} \mathcal{B}^{\varepsilon}$, the costs of prospecting on credit markets and the bargaining weight β. The baseline calibration adopts a "balanced" credit matching function and the credit market Hosios condition; i.e., $\beta = \varepsilon = 0.5$, and symmetry in prospecting costs $\kappa = e = 0.05$. The remaining parameter, ς, is set such that the excess gross rate of return on a business loan $R - r$ equal an annualized 15%.[5]

Table 1: Baseline results

	$q(\theta)$	Wage	Elasticity of θ	Elasticity of $f(\theta)$	Financial accelerator M_f
Pissarides					
- fixed wage	0.11	0.75	6.48	3.24	1
Credit friction - fixed wage					
$\beta = 0.5, \varepsilon = 0.5$	0.20	0.75	11.48	5.74	1.77
$\beta = 0.2, \varepsilon = 0.8$	0.72	0.75	39.11	19.56	6.04
$\beta = 0.8, \varepsilon = 0.2$	0.72	0.75	39.11	19.56	6.04

[4]See Petrongolo and Pissarides (2001) for a survey of estimates of the labor matching function.

[5]The internal rate of return of loans to firms is the interest rates R that equalizes the expected present discounted value of the loan $\gamma/[R + q(\theta^*)]$ and the expected present discounted repayment on the loan $\{q(\theta^*)/[R + q(\theta^*)]\}\{\rho/(R+s)\}$. See Wasmer and Weil (2004).

Table 1 presents the results for several scenarios. The first row shows that the Pissarides model with a fixed wage yields an elasticity of labor market tightness of 6.48, which is 4.7 times greater than when wages are flexible (see Table 2). The next three rows present the results for a multi-frictional economy. In the baseline calibration, the matching function is "balanced" and the economy is at a Hosios condition on the credit market. The resulting minimized financial accelerator has a value of 1.79, and the elasticity of labor market tightness to productivity shocks is 11.57, values that are close to those obtained from our steady state calculations.

When we move away from the credit market Hosios condition, the value of the financial accelerator can become very large and the elasticity of market tightness to productivity shocks overshoots the value in the data. Figure 1 presents the values of the elasticity of labor market tightness to productivity shocks keeping a "balanced" credit matching function, i.e., $\varepsilon = 0.5$, and varying the bargaining weight β between 0.1 and 0.9. As was known from the analysis in Section 3, Figure 1 illustrates that the financial accelerator is minimized at $\beta = \varepsilon = 0.5$ and increases away from this point.

At one extreme, e.g., when β is small, firms in the entry stage get a very low share of the surplus. Hence, any small positive productivity shock leads to a large impact on the entry of firms on the credit market. This is essentially a generalization of the small surplus idea in the literature, and in particular of Hagedorn and Manovskii (2008), although applied to firms in a different stage, that is before the production of the final good. Conversely, when β is close to 1 it is banks who get a small surplus. In that case, any small profit shock would lead to a large relative increase in the number of prospecting banks and hence to a large amplification of the same shock. The small surplus here corresponds to that of banks, and their optimal response to shocks is therefore large.

We explore this further with two cases in which there are large departures in the degree of matching externalities and creditor's bargaining weight. In both cases, there is near linearity in the matching function; first in the supply of creditors, second in the supply of entrepreneurs. The results are shown in the last two rows of Table 1, and in the first ($\varepsilon = 0.8$) and second ($\varepsilon = 0.2$) panels of Figure 4. Case 1 shows that if matching is near linear in supply of creditors
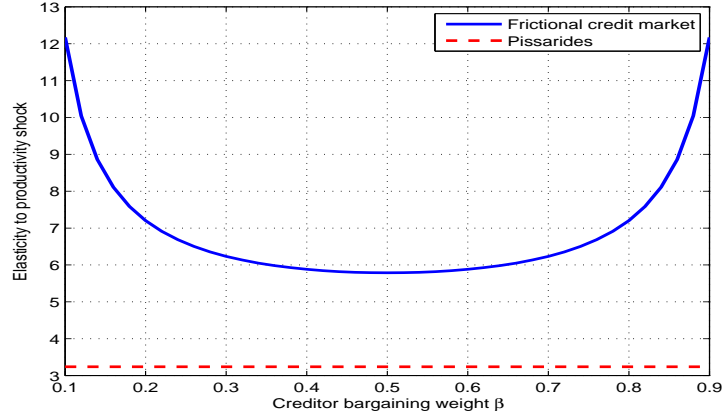
Figure 1: Elasticity of job finding rate to productivity shocks.

and we depart from the credit market Hosios condition by reducing the bargaining weight of bankers, the elasticity of labor market tightness to productivity shocks becomes potentially very large. Intuitively, when entrepreneurs extract a larger share of the surplus generated by a firm they respond more strongly to the change in productivity by wanting to enter the credit market. On the other side of the market, the average search cost for creditors is relatively unresponsive to the bankers entering the market from the near linearity in the matching function. As a result, when $\varepsilon = 0.8$ and $\beta = 0.2$ the elasticity of labor market tightness to productivity shocks nears 40, and the financial accelerator contributes a factor $M_f = 6.04$. A step further, at $\beta = 0.15$, the elasticity exceeds 80. The exact inverse is observed when the matching function is near linear in the supply of entrepreneurs (see the second panel of Figure 4).[6]

5   Extension: endogenous wages

Endogenous wages seriously reduce the elasticity of labor market tightness to productivity shocks. This paper must therefore address the question of, and replicate the analysis of the previous section with, endogenous wages. We assume that the worker bargains the wage with

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[6]The two regimes imply very different annualized excess return on the loan. In the first case, when the firm has most of the weight in sharing the surplus, the excess return is 3.5%. In the second case, the excess return is 26%.
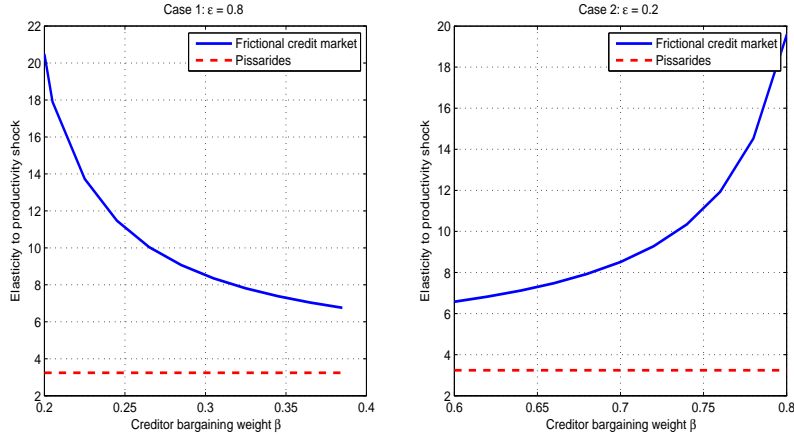
Figure 2: Two cases for the credit market and the financial multiplier.

a firm, defined as the entrepreneur-banker block, at the time of meeting. There are two related reasons for this choice.

The first one is that the natural alternative, bargaining between the entrepreneur and the worker, leads to complex strategic interactions illustrated in Wasmer and Weil (2004, Section IV-A): the entrepreneur and the bank wish to raise the debt of the firm above what is needed in order to reduce the size of total surplus to be shared between the firm and the worker at a later time. Hence, wages are driven down to the reservation wage of workers and do not vary with the firm's productivity, which is counterfactual. This leads to the second reason, which is that we want our endogenous wage extension to be comparable to the classical wage solution in the labor search literature in order to compare the volatility in the model to other elasticities found in the literature.

Define the values of employment and unemployment in a discrete time stochastic setting as

$$
\begin{aligned}
U_t &= z + f(\theta_t)\beta\mathbb{E}_t W_{t+1} + (1 - f(\theta_t))\beta\mathbb{E}_t U_{t+1} \\
W_t &= w_t + \beta\mathbb{E}_t\left[(1 - s)W_{t+1} + sU_{t+1}\right]
\end{aligned}
$$

where z is the value of non-employment activities and $f(\theta) = \theta q(\theta)$ the job finding rate. The Pissarides wage is $w_t^P = \alpha\left(y_t + \gamma\theta_t^P\right) + (1 - \alpha)z$ where α is the bargaining power of workers vis-à-vis the firm. Taking log-deviations, movements in labor market tightness to

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future productivity are given by

$$\widehat{\theta}_t^P = \frac{q(\theta^P)(1-\alpha)}{\eta\gamma(1+r)}\mathbb{E}_t\sum_{i=0}^{\infty}\Psi^i\widehat{y}_{t+1+i}$$

where the second term in $\Psi = \left(\frac{1-s}{1+r}\right) - \frac{\alpha\theta^P q(\theta^P)}{\eta(1+r)}$ reflects the share of the change in productivity accruing to the worker through the wage. The latter strongly reduces the elasticity of labor market tightness to productivity shocks which, with our specification, is[7]

$$\frac{\partial\widehat{\theta}_t^P}{\partial v_t} = \frac{q(\theta^P)(1-\alpha)\rho_y}{\eta\gamma(1+r)-\gamma[\eta(1-s)-\alpha f(\theta^P)]\rho_y} \tag{18}$$

Compared to the elasticity when wages are fixed, only a share $(1-\alpha)$ of the rise in productivity accrues to the firm. In addition, the equilibrium rise in labor market tightness following a positive productivity shock improves the outside option of the worker and his bargaining position in the wage determination. This appears in the denominator as the term $\alpha f(\theta^P)$, further reducing the elasticity of labor market tightness to productivity shocks.

Turning now to the responsiveness of labor market tightness under frictional credit markets, we begin by detailing the determination of the wage. As discussed earlier, we assume that the wage negotiated in a worker-firm pair, and in the presence of credit market frictions, it must satisfy a sharing rule $\alpha F_{g,t} = (1-\alpha)(W_t - U_t)$, where $F_{g,t} = E_{g,t} + B_{g,t}$ is the joint value of the firm to the entrepreneur-banker pair. Under this assumption the wage is

$$w_t = \alpha\left[y_t + \Gamma_t\theta_t^*\right] + (1-\alpha)z$$

and differs from the Pissarides wage by the coefficient Γ_t on market tightness. To the extent that this term is negatively correlated with productivity, credit market frictions induce a certain degree of wage rigidity by limiting the effect of a rise in market tightness on wages, a feature also present in Petrosky-Nadeau (2009). To see why this is the case, recall that $\Gamma_t \equiv \gamma + K\left(1 - \frac{1}{1+r}(1-q(\theta_t^*))\right)$ are vacancy costs augmented for frictional credit markets. Since q is decreasing in market tightness, so is Γ.

---

[7]To check the result, note that if $\rho_y = 1$ this is the elasticity obtained when comparing steady states, or to a permanent productivity shock, as in Shimer (2005), i.e. $\varepsilon_{\theta,y} = \frac{(1-\alpha)}{\gamma\left[\frac{\eta(r+s)}{q(\theta^P)}+\alpha\theta^P\right]}$. The details for deriving the elasticities can be found in the appendix.

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Finally, the elasticity of labor market tightness under frictional credit markets and an endogenous wage is

$$\frac{\partial \widehat{\theta}_t^*}{\partial v_t} = \frac{q(\theta^*)(1-\alpha)\rho_y}{\eta \gamma^T (1+r) - [\eta \gamma^T (1-\delta) - \alpha f(\theta^*)(\gamma^T + (1-\eta)\widetilde{\kappa})]\rho_y} \tag{19}$$

where $\widetilde{\kappa} \equiv K \frac{q(\theta^*)}{1+r}$.

Table 2 presents the results when wages are endogenous following a similar calibration exercise as in the previous section, along with a Hosios condition on the labor market to determine the bargaining weight α. We find an elasticity of labor market tightness when credit markets are perfect and wages endogenous of 1.37. For the economy with a frictional credit market, we focus on two sets of results: 1) the results for our baseline calibration and, 2) calibrating to a small labor surplus. Table 2 reveals, first, that size of the financial accelerator is greater in the presence of endogenous wages, rising from 1.79 to 2.14, the additional amplification arising from the slight degree of wage rigidity outlined above. Second, amplification is less sensitive to departures from the credit market Hosios condition. For $\beta = 0.2$ and $\varepsilon = 0.8$ the financial accelerator reaches a factor of 2.54. Note that the magnitude of the multiplier in this scenario also corroborates the results in Petrosky-Nadeau (2009). More extreme departures are necessary for a large financial accelerator: at $\beta = 0.1$ and $\varepsilon = 0.9$ we obtain $M_f = 8.6$.

The next section of Table 2 investigates the proposition that has appeared in the literature of calibrating to a small labor surplus. Such a strategy, with $\alpha = 0.05$ and $z = 0.95$, yields an elasticity of 22.84 for the Pissarides model, approximately the elasticity in the data. When we perform the same exercises in our model with both labor and credit market frictions under the symmetric case $\beta = \varepsilon = 0.5$, we obtain an elasticity of labor market tightness to productivity shocks of 49.27, overshooting the value in the data by a factor of 2.16. We then ask whether frictional credit markets can explain the elasticity in the data without the stringent small labor surplus assumption and propose two scenarios to achieve this. First, keeping the bargaining weight at $\alpha = 0.05$, the value of non-market activities necessary to match the elasticity of market tightness in the data drops from 0.95 to 0.75, and the equilibrium wage from 0.96 to 0.83. Second, with a mixed strategy, it is possible to match the data with a greater weight to

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the worker in wage bargaining, 0.1, and a lower value of non-market activities z of 0.845.[8]

Table 2: Robustness to endogenous wages

	$q(\theta)$	Wage	Unemp.	Elast. of θ	Elast. of $f(\theta)$	Financial accel. M_f	Elast. $f(\theta)$ to z
Pissarides							
- flexible wage	0.04	0.75	0.10	1.37	0.68	1	-0.104
Credit friction - flexible wage							
$\beta = 0.5, \varepsilon = 0.5$	0.29	0.74	0.10	2.94	1.47	2.14	-0.225
$\beta = 0.2, \varepsilon = 0.8$	0.62	0.72	0.11	3.49	1.75	2.54	-0.265
$\beta = 0.8, \varepsilon = 0.2$	0.62	0.72	0.11	3.49	1.75	2.54	-0.265
Small labor surplus (Hagedorn and Manovskii 2008)							
- Pissarides, $\alpha = 0.05, z = 0.95$	0.27	0.96	0.10	22.84	11.42	1	-15.289
- Credit friction $\beta = 0.5, \varepsilon = 0.5$							
$\alpha = 0.05, z = 0.95$	0.74	0.96	0.10	49.27	24.64	2.16	-26.929
$\alpha = 0.05, z = 0.75$	0.98	0.83	0.11	22.38	11.18	-	
$\alpha = 0.10, z = 0.845$	0.91	0.92	0.11	21.50	10.75	-	

6   Conclusion

Financial imperfections raise the calibrated elasticity of labor market tightness to productivity shocks by a factor M_f called the financial multiplier. With exogenous wages, and without assuming a small labor surplus, it is easy to generate a plausible large elasticity of labor market tightness to productivity shocks, if one relaxes the Hosios-Pissarides rule in the credit market.

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[8]It is important to note that in each of these exercises we have kept the discipline of maintaining the equilibrium rate of unemployment at approximately 10%.

Under the assumption of a large enough difference between the bargaining power of banks vis-à-vis entrepreneurs (β) with the elasticity of the rate at which entrepreneurs meet bankers with respect to credit market tightness (ε), one can obtain an elasticity around 20 or even larger.

Under endogenous wages with bargaining power α of workers relative to firms, defined as the joint bank-entrepreneur entity, all elasticities are divided by a factor 4 to 5, as was established by Shimer (2005) and Hall (2005). Hence, the model requires more extreme values of β or ε, (e.g., 0.95 or 0.05) to match the data. Alternatively, even with the "balanced" values of β and ε, the model can generate large volatility in labor market tightness with less stringent assumptions on α or the value of leisure of the unemployed z as compared to the 'small labor surplus assumptions' in Hagedorn and Manovskii (2008). Another way of stating this is to say that the "small surplus assumption" is no longer necessary on firms in the production stage: it may be either on firms in the credit-prospection stage or on banks in the project-screening stage.

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Technical Appendix to: The Cyclical Volatility of Labor Markets under Frictional Financial Markets
Not intended for publication

A Introduction

This appendix details the derivation of the various equations and elasticities presented in the main text. We begin by fully describing the stochastic model in discrete time.

A.1 Asset values of an entrepreneur

$$E_{c,t} = -e + p_t E_{l,t} + (1 - p_t)\frac{1}{1+r}\mathbb{E}_t E_{c,t+1}$$

$$E_{l,t} = -\gamma + \gamma + \frac{1}{1+r}\mathbb{E}_t\left[q_t E_{g,t+1} + (1 - q_t)E_{l,t+1}\right]$$

$$E_{g,t} = y_t - w_t - \rho_t + \frac{1}{1+r}\mathbb{E}_t\left[s E_{c,t+1} + (1 - s)E_{g,t+1}\right]$$

The cost of convincing a bank to fund future negative cash flows is e, and with probability $0 < p_t < 1$ this results in a successfull match within the period. During the second stage, the bank covers the cost of recruiting a worker, γ, who is met with probability $0 < q_t < 1$. During the production stage, y goods are produced which must cover both the wage rate w and interest payments ρ. During the last stage, firms are subject to death shocks with probability s.

An assumption of free entry for entrepreneurs leads $\frac{e}{p_t} = E_{l,t}$ such that the final stage may be simplified to

$$E_{g,t} = y_t - w_t - \rho_t + (1 - s)\frac{1}{1+r}\mathbb{E}_t E_{g,t+1}$$

A.2 Matching on credit markets

We follow the matching literature and assume that the total number of matches is governed by a matching technology associating the total number of banks in stage 0, denoted by $\mathscr{B}$, and the total number of entrepreneurs in stage 0, denoted by $\mathscr{E}$. Let $M_C(\mathscr{E}, \mathscr{B})$ be the matching process

in the credit market. We have that $p = M_C(\mathscr{E}, \mathscr{B})/\mathscr{E}$. Symmetrically, the rate at which banks find a project they are willing to finance is $M_C(\mathscr{E}, \mathscr{B})/\mathscr{B} = \phi p$ where $\phi = \mathscr{E}/\mathscr{B}$. Under the assumption of constant returns to scale of $M_C(\mathscr{E}, \mathscr{B})$, we have that $p = p(\phi)$ with $p'(\phi) < 0$, elasticity $\varepsilon(\phi) = -\phi p'(\phi)/p(\phi)$, and it follows that ϕ is a natural measure of the tightness of the credit market . We also make the assumptions

$$\lim_{\phi \longrightarrow 0} p(\phi) = 1$$
$$\lim_{\phi \longrightarrow +\infty} p(\phi) = 0$$

The first line states that in the relative scarcity of competing firms relative to banks, matching with a banker is instantaneous, and the second line states that in the relative abundance of competing firms relative to banks, matching with a banker is ifinitely slow.

A.3 Asset values for a banker

$$B_{c,t} = -\kappa + \phi_t p(\phi_t) B_{l,t} + (1 - \phi_t p(\phi_t)) \frac{1}{1+r} \mathbb{E}_t B_{c,t+1}$$
$$B_{l,t} = -\gamma + \frac{1}{1+r} \mathbb{E}_t \left[q_t B_{g,t+1} + (1 - q_t) B_{l,t+1} \right]$$
$$B_{g,t} = \rho_t + \frac{1}{1+r} \mathbb{E}_t \left[s B_{c,t+1} + (1-s) B_{g,t+1} \right]$$

Bankers search for a suitable investment at a cost of κ and enter the recruiting stage with probability $\phi_t p(\phi_t)$ during which the vacancy cost γ must be disbursed. Meeting a worker occurs at the rate q_t, at which point a banker enters the production stage and the remuneration ρ is received. An assumption of free entry for bankers leads $\frac{\kappa}{\phi_t p(\phi_t)} = B_{l,t}$,

A.4 Time invariant credit market tightness

Free entry on both sides of the credit market, along with Nash bargaining over the surplus of a credit relationship, results in a time invariant tightness. To show this, note first that we had under free entry

$$B_{l,t} = \frac{\kappa}{\phi_t p(\phi_t)}; \text{ and } E_{l,t} = \frac{e}{p(\phi_t)}$$

Denoting the banker's bargaining weight by β, and defining the credit relationship surplus as $S_{C,t} = (E_{l,t} - E_{c,t}) + (B_{l,t} - B_{c,t})$, results in $\frac{E_{l,t}}{B_{l,t}} = \frac{1-\beta}{\beta}$ and

$$\phi^* = \frac{1-\beta}{\beta} \frac{\kappa}{e}$$

A.5 Deriving a job creation condition:

It will be convenient at this stage to express the joint value of recruiting a worker to banker and entrepreneur as $F_{l,t} = E_{l,t} + B_{l,t}$, which corresponds to the surplus from the credit relationhsip, as

$$\frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} = -\gamma + q_t \frac{1}{1+r} \mathbb{E}_t \left[E_{g,t+1} + B_{g,t+1} \right] + (1-q_t) \frac{1}{1+r} \left[\frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} \right]$$

Define total costs on the credit market as $K(\phi) = \frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)}$ and $\Gamma_t \equiv \gamma + K(\phi) \left(1 - \frac{1}{1+r}(1-q_t) \right)$, then

$$\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t \left[E_{g,t+1} + B_{g,t+1} \right]$$

Using the Bellman equations for entrepreneur and banker during production to define $[E_{g,t} + B_{g,t}] = F_{g,t} = y_t - w_t + (1-s) \frac{1}{1+r} \mathbb{E}_t \left[F_{g,t+1} \right]$, we obtain a job creation condition in the presence of frictional credit and labor markets

$$\frac{\Gamma_t}{q_t} = \frac{1}{1+r} \mathbb{E}_t \left[y_{t+1} - w_{t+1} + (1-s) \frac{\Gamma_{t+1}}{q_{t+1}} \right]$$

Note that when the credit market is perfect $K(\phi) = 0$ and $\Gamma_t = \gamma$ such that the job creation condition collapses to the familiar

$$\frac{\gamma}{q_t} = \frac{1}{1+r} \mathbb{E}_t \left[y_{t+1} - w_{t+1} + (1-s) \frac{\gamma}{q_{t+1}} \right]$$

A.6 Rental rate

This section provides the details in deriving the rental rate

$$\mathbb{E}_t\left[\rho_{t+1}\right] = \beta\mathbb{E}_t\left[y_{t+1} - w_{t+1}\right] + (1-\beta)\mathbb{E}_t\left[\frac{(1+r)\gamma}{q(\theta_t)} - \frac{(1-s)\gamma}{q(\theta_{t+1})}\right]$$

Define the surplus to the credit relationship as $S_{C,t} = E_{l,t} + B_{l,t}$. The sharing rule under Nash bargaining implies $B_{l,t} = \beta S_{C,t}$ and $E_{l,t} = (1-\beta)S_{C,t}$. Expanding on the former,

$$
\begin{aligned}
-\gamma + \frac{1}{1+r}\mathbb{E}_t\left[q_t B_{g,t+1} + (1-q_t)B_{l,t+1}\right] &= -\beta\gamma + \beta q_t\frac{1}{1+r}\mathbb{E}_t\left[E_{g,t+1} + B_{g,t+1}\right] \\
&\quad + \beta(1-q_t)\frac{1}{1+r}\mathbb{E}_t\left[E_{l,t+1} + B_{l,t+1}\right]
\end{aligned}
$$

Rearranging terms,

$$
\begin{aligned}
\mathbb{E}_t B_{g,t+1} + \frac{(1-q_t)}{q_t}\mathbb{E}_t B_{l,t+1} &= (1-\beta)\frac{\gamma(1+r)}{q_t} \\
&\quad + \beta\mathbb{E}_t\left(\left[E_{g,t+1} + B_{g,t+1}\right] + \frac{(1-q_t)}{q_t}\left[E_{l,t+1} + B_{l,t+1}\right]\right) \\
\mathbb{E}_t\left[\rho_{t+1} + \frac{1}{1+r}(1-s)B_{g,t+2}\right] &= (1-\beta)\frac{\gamma(1+r)}{q_t} \\
&\quad + \beta\mathbb{E}_t\left[y_{t+1} - w_{t+1} + (1-s)\frac{1}{1+r}\left[B_{g,t+2} + E_{g,t+2}\right]\right] \\
&\quad + \beta\frac{(1-q_t)}{q_t}\mathbb{E}_t\left[E_{l,t+1} + B_{l,t+1}\right] - \frac{(1-q_t)}{q_t}\mathbb{E}_t B_{l,t+1}
\end{aligned}
$$

Since $B_{l,t} = \beta\left[E_{l,t} + B_{l,t}\right]$, $\mathbb{E}_t B_{g,t+1} = (1-\beta)\frac{\gamma(1+r)}{q_t} + \beta\mathbb{E}_t\left[E_{g,t+1} + B_{g,t+1}\right]$, or

$$\mathbb{E}_t\left[(1-\beta)B_{g,t+1} - \beta E_{g,t+1}\right] = (1-\beta)\frac{\gamma(1+r)}{q_t}$$

then

$$\mathbb{E}_t\left[\rho_{t+1} + \frac{1}{1+r}(1-s)B_{g,t+2}\right] = (1-\beta)\frac{\gamma(1+r)}{q_t} + \beta\mathbb{E}_t\left[y_{t+1} - w_{t+1} + (1-s)\frac{1}{1+r}\left[B_{g,t+2} + E_{g,t+2}\right]\right]$$

and

$$\mathbb{E}_t\left[\rho_{t+1}\right] = \beta\mathbb{E}_t\left[y_{t+1} - w_{t+1}\right] + (1-\beta)\mathbb{E}_t\left[\frac{(1+r)\gamma}{q(\theta_t)} - \frac{(1-s)\gamma}{q(\theta_{t+1})}\right]$$

A.7 Workers and wages

An individual may be unemployed and earning income $z < y$. The unemployed meet job offers at rate $f(\theta) = \theta q$. Once employed, workers earn wage w until separation, which occurs with probability s per unit of time. The Bellman equations describing each of these stages are

$$U_t = z + f(\theta_t)\frac{1}{1+r}\mathbb{E}_t W_{t+1} + (1 - f(\theta_t))\frac{1}{1+r}\mathbb{E}_t U_{t+1}$$

$$W_t = w_t + \frac{1}{1+r}\mathbb{E}_t\left[(1-s)W_{t+1} + sU_{t+1}\right]$$

We assume that the wage negotiated in a worker-firm pair in the presence of credit market frictions, with surplus $S_{L,t} = F_{g,t} + W_t - U_t$, satisfies $\alpha F_{g,t} = (1 - \alpha)(W_t - U_t)$, where $F_{g,t} = E_{g,t} + B_{g,t}$ is the joint value of the firm to the entrenpreneur-banker pair. Applying this sharing rule to the worker-firm surplus, first we have

$$S_{L,t} = y_t - w_t + (1-s)\frac{1}{1+r}\mathbb{E}_t F_{g,t+1}$$

$$w_t + \frac{1}{1+r}\mathbb{E}_t\left[(1-s)W_{t+1} + sU_{t+1}\right] - z - \frac{1}{1+r}\mathbb{E}_t\left[\theta_t q_t W_{t+1} - (1 - \theta_t q_t)U_{t+1}\right]$$

$$S_{L,t} = y_t - z + (1-s)\frac{1}{1+r}\mathbb{E}_t\left[F_{g,t+1} + W_{t+1} - U_{t+1}\right] - \theta_t q_t \frac{1}{1+r}\mathbb{E}_t\left[W_{t+1} - U_{t+1}\right]$$

$$S_{L,t} = w_t - z + (1-s)\frac{1}{1+r}\mathbb{E}_t S_{L,t+1} - \alpha\theta_t q_t \frac{1}{1+r}\mathbb{E}_t S_{L,t+1}$$

and second, using $F_{g,t} = (1 - \alpha)S_{L,t}$ and $\frac{\Gamma_t}{q_t} = \frac{1}{1+r}\mathbb{E}_t(1-\alpha)F_{g,t+1}$,

$$y_t - w_t + (1-s)\frac{1}{1+r}\mathbb{E}_t F_{g,t+1} = (1-\alpha)\left(y_t - z + (1-s)\frac{1}{1+r}\mathbb{E}_t S_{L,t+1}\right) - \alpha\theta_t\Gamma_t$$

Rearraging terms yield the wage rule under frictional labor and credit markets:

$$w_t = \alpha(y_t + \theta_t\Gamma_t) + (1-\alpha)z$$

B Deriving the elasticity of market tightness to a productivity shock

B.1 Cannonical framework

Assume the matching function is Cobb-Douglas such that $q(\theta_t) = \chi \theta_t^{-\eta}$. Define period profit flows as $\Pi = y - w$. Taking log linear deviations around a stationary steady state, $\eta \frac{\gamma(1+r)}{q(\theta^P)} \widehat{\theta}_t^P = \Pi \mathbb{E}_t \widehat{\Pi}_{t+1} + \eta \frac{\gamma(1-s)}{q(\theta^P)} \mathbb{E}_t \widehat{\theta}_{t+1}^P$, and using the forward operator we have that $\left[1 - \frac{1-s}{1+r} \mathbb{E}_t L^{-1}\right] \widehat{\theta}_t^P = \frac{q(\theta^P)\Pi}{\eta\gamma(1+r)} \mathbb{E}_t \widehat{\Pi}_{t+1}$ such that

$$\widehat{\theta}_t^P = \frac{q(\theta^P)\Pi}{\eta\gamma(1+r)} \mathbb{E}_t \sum_{i=0}^{\infty} \left(\frac{1-s}{1+r}\right)^i \widehat{\Pi}_{t+1+i}$$

Deviations of market tightness are forward looking, discounting future deviations of profits. Using the definition of the wage $w_t = \alpha(y_t + \gamma \theta_t) + (1-\alpha)z$:

$$\widehat{\theta}_t^P = \frac{q(\theta^P)(1-\alpha)}{\eta\gamma(1+r)} \mathbb{E}_t \sum_{i=0}^{\infty} \Psi^i \widehat{y}_{t+1+i}$$

where $\Psi = \left(\frac{1-s}{1+r}\right) - \frac{\alpha \theta^P q(\theta^P)}{\eta(1+r)}$. Assuming that productivity follows an AR(1) with persistence parameter $0 < \rho_y < 1$ and innovation v_t as white noise, then

$$\widehat{\theta}_t^P = \frac{q(\theta^P)(1-\alpha)}{\eta\gamma(1+r)} \sum_{i=0}^{\infty} \Psi^i \rho_y^{i+1} v_t$$

so that $\widehat{\theta}_t^P = \frac{q(\theta^P)(1-\alpha)}{\eta\gamma(1+r)} \left(\frac{1}{1-\Psi\rho_y}\right) \rho_y v_t$, and

$$\frac{\partial \widehat{\theta}_t^P}{\partial v_t} = \frac{(1-\alpha)q(\theta^P)\rho_y}{\eta\gamma(1+r) - \gamma[\eta(1-s) - \alpha f(\theta)]\rho_y} \tag{20}$$

To check the result, note that if $\rho_y = 1$ this is the elasticity obtained when comparing steady states, or to a permanent productivity shock, as in Shimer (2005), i.e. $\frac{(1-\alpha)}{\gamma\left[\frac{\eta(r+\delta)}{q(\theta)} + \alpha\theta\right]}$.

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B.2 Frictional credit markets - fixed wage

Recall the job creation condition $\frac{\Gamma_t}{q_t} = \frac{1}{1+r}\mathbb{E}_t\left[y_{t+1} - \overline{w} + (1-s)\frac{\Gamma_{t+1}}{q_{t+1}}\right]$, with $\Gamma_t \equiv \gamma + K\left(1 - \frac{1}{1+r}(1-q_t)\right)$ and $q(\theta_t) = \chi\theta_t^{-\eta}$. Taking log linear deviations around a stationary steady state: $\frac{\gamma(1+r)}{q(\theta)}\left[\gamma + K(\frac{r}{1+r})\right]\widehat{\theta}_t = \mathbb{E}_t\widehat{y}_{t+1} + \frac{\eta(1-s)}{q(\theta)}\left[\gamma + K(\frac{r}{1+r})\right]\mathbb{E}_t\widehat{\theta}_{t+1}$. Call $\gamma^T \equiv \left[\gamma + K(\frac{r}{1+r})\right]$, then $\left[1 - \frac{1-s}{1+r}\mathbb{E}_tL^{-1}\right]\widehat{\theta}_t = \frac{q(\theta)}{\eta\gamma^T(1+r)}\mathbb{E}_t\widehat{y}_{t+1}$, and

$$\widehat{\theta}_t = \frac{q(\theta)}{\eta\gamma^T(1+r)}\mathbb{E}_t\sum_{i=0}^{\infty}\left(\frac{1-s}{1+r}\right)^i\widehat{y}_{t+1+i}$$

If productivity follows the same AR(1) process, then $\widehat{\theta}_t = \frac{q(\theta)}{\eta\gamma^T(1+r)}\mathbb{E}_t\sum_{i=0}^{\infty}\left(\frac{1-s}{1+r}\right)^i\rho_y^{i+1}v_t$, $\widehat{\theta}_t = \frac{q(\theta)}{\eta\gamma^T(1+r)}\left(\frac{\rho_y}{1-\frac{1-s}{1+r}\rho_y}\right)v_t$ and

$$\frac{\partial\widehat{\theta}_t}{\partial v_t} = \frac{q(\theta)\rho_y}{\eta\gamma^T[(1+r)-(1-s)]\rho_y} \tag{21}$$

B.3 Frictional credit markets -flexible wage

If the wage outcome is $w_t = \alpha[y_t + \Gamma_t\theta_t] + (1-\alpha)z$, we can write the job creation condition as

$$\frac{\Gamma_t}{q_t} = \frac{1}{1+r}\mathbb{E}_t\left[(1-\alpha)(y_{t+1}-z) - \alpha\Gamma_{t+1}\theta_{t+1} + (1-s)\frac{\Gamma_{t+1}}{q_{t+1}}\right].$$

The following preparatory steps are usefull. First, write $\Gamma_t = \gamma^T + \frac{K}{1+r}q(\theta_t)$. Then take log-linear deviation around a stationary steady state of the job creation condition:

$$\frac{\eta(1+r)}{q(\theta)}\gamma^T\widehat{\theta}_t = (1-\alpha)\mathbb{E}_t\widehat{y}_{t+1} - \alpha\left(\gamma^T\theta + \frac{K}{1+r}(1-\eta)f(\theta)\right)\mathbb{E}_t\widehat{\theta}_{t+1}$$

$$+\frac{\eta(1+s)}{q(\theta)}\gamma^T\mathbb{E}_t\widehat{\theta}_{t+1}$$

$$\widehat{\theta}_t = \frac{(1-\alpha)q(\theta)}{\eta\gamma^T(1+r)}\mathbb{E}_t\widehat{y}_{t+1}$$

$$+\left[\frac{(1+s)}{(1+r)} - \frac{\alpha q(\theta)}{\eta\gamma^T(1+r)}\left(\gamma^T\theta + \frac{K}{1+r}(1-\eta)f(\theta)\right)\right]\mathbb{E}_t\widehat{\theta}_{t+1}$$

Calling $\Phi \equiv \left[\frac{(1+s)}{(1+r)} - \frac{\alpha q(\theta)}{\eta\gamma^T(1+r)}\left(\gamma^T\theta + \frac{K}{1+r}(1-\eta)f(\theta)\right)\right]$, we then follow similar steps by obtaining:

$$\widehat{\theta}_t = \frac{(1-\alpha)q(\theta)}{\eta\gamma^T(1+r)}\mathbb{E}_t\sum_{i=0}^{\infty}\Phi^i\widehat{y}_{t+1+i}$$

and making use of the specification for labor productivity, $\widehat{\theta}_t = \frac{(1-\alpha)q(\theta)}{\eta\gamma^T(1+r)}\mathbb{E}_t \sum\limits_{i=0}^{\infty} \Phi^i \rho_y^{i+1} v_t$. Finally $\widehat{\theta}_t = \frac{(1-\alpha)q(\theta)}{\eta\gamma^T(1+r)}\left(\frac{\rho_y}{1-\Phi\rho_y}\right) v_t$ and

$$\frac{\partial \widehat{\theta}_t}{\partial v_t} = \frac{(1-\alpha)q(\theta)\rho_y}{\eta\gamma^T(1+r) - [\eta\gamma^T(1+s) - \alpha f(\theta)(\gamma^T + (1-\eta)\widetilde{\kappa})]\rho_y}$$

where $\widetilde{\kappa} \equiv K\frac{q(\theta)}{1+r}$.

C Additional numerical results

The baseline results kept the unemployment rate constant by adjusting the labor matching function's level parameter χ. We show in the following table the this level parameter in the labor matching function has no incidence on the propagation of productivity shocks when wages are fixed.

Table A1: Keeping χ fixed

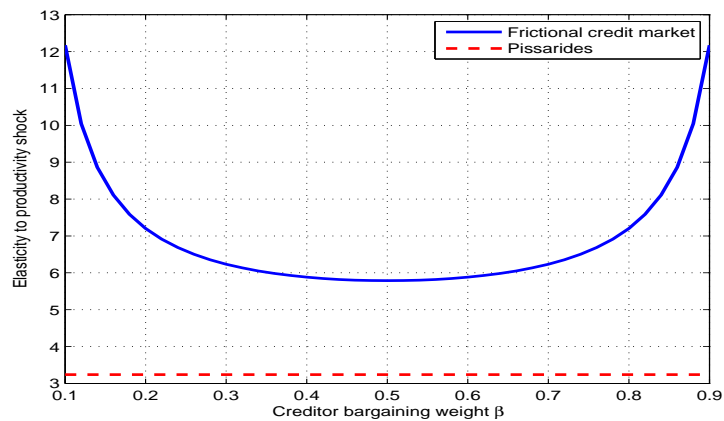|	$q(\theta)$	Wage	U rate	Elasticity of θ	Financial accelerator M_f
Pissarides					
- fixed wage	0.11	0.75	0.08	6.48	1
Credit friction - fixed wage					
$\beta = 0.5, \varepsilon = 0.5$	0.20	0.75	0.08	11.57	1.79
$\beta = 0.2, \varepsilon = 0.8$	0.72	0.75	0.24	39.11	6.04
$\beta = 0.8, \varepsilon = 0.2$	0.72	0.75	0.24	39.11	6.04

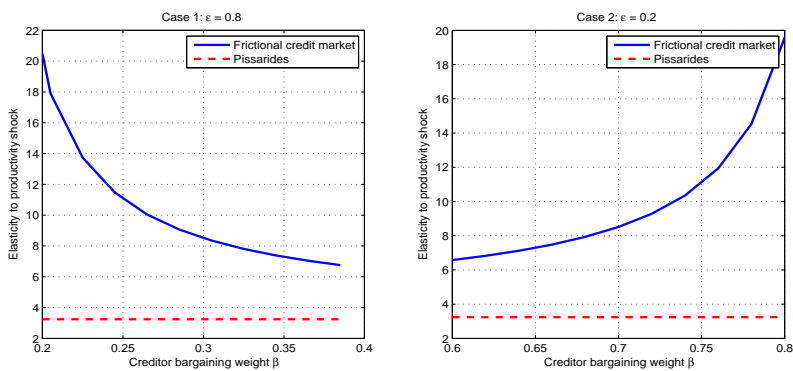Figure 3: Elasticity of labor market labor tightness to productivity shocks.

Figure 4: Two cases for the credit market and the financial multiplier.