Review Essay: The Netz-Works of Greek Deductions

Bruno Latour

The Netz-Works of Greek Deductions*

A review of Reviel Netz (2003) The Shaping of Deduction in Greek Mathematics: A Study in Cognitive History. (Cambridge: Cambridge University Press).

Bruno Latour, Sciences Po, Paris
(to be published in ***Social Studies of Science)***

For Ian Hacking

> *"We may now say that the mathematical* apodeixis *is, partly, a development of the rhetorical* epideixis." p. 293

This is, without contest, the most important book of science studies to appear since Shapin and Schaffer's *Leviathan and the Air-Pump*. I say this even though its author, Reviel Netz, a serious classicist from Stanford, prefers to take his distance from our field (which in all likelihood he considers as somewhat disreputable). From the beginning he asserts, with an added pun on his own title, that "This book should not be read as if it were 'the Shapin of deduction'" (p. 3). Instead, he prefers to call his endeavor "cognitive history" — which means exactly what we mean by "science studies", namely an obsessive attention to the material, historical, and practical conditions necessary for the discovery of new cognitive skills.

[1]* Once again, I have to thank Martha Poon for being so patient with my English and for her comments.

Even though Netz is not a card-carrying socio-historian of science, a look at another of his stunning books (Netz, 2004) – this time on the history of barbed wire from the cattle ranches of the Great Plains, through to Pearl Harbor, the trenches of the World War I, and then all the way to Auschwitz – would be enough to reassure any reader of *SSofS* that he is one of us, and one of our best. That the field of science studies is much larger (and also, alas, much *smaller*!) than the list of its official members should not come as any great surprise. I have a vivid memory of Thomas Kuhn accepting the Bernal Prize from our society with more than a slight embarrassment… Netz's achievements are likewise of such crucial importance for our field that, even though I am neither a classicist nor a philosopher of mathematics, I must take the risk of similarly embarrassing him by placing his book at the centre of the STS corpus.

A non-formalist description of formalism

Netz's book does exactly what he says he doesn't want it to do: it offers for the origin of *formalism* what Shapin and Schaffer have done for the origin of *experimental* science.

If, in order to propose and alternative history of science, the cradle of experimental science —Boyle's estate *qua* laboratory— had to be revisited, it's clear that this revision would have remained grossly incomplete as long as the cradle of deduction were not be revisited in the same manner. In *The Leviathan*, the divide between a scientific and a literary style, the very distinction between science and politics, the autonomy of scientific reasoning, and the invention of a new form of persuasion were taken as the *topics* of the historical inquiry instead of as the *resources* with which to write this history. In that work we were witnessing the emergence of those new cognitive skills and this new form of life: the laboratory, the experimental style. This is exactly what we can witness again in Netz's book, only this time with a much older, tougher, less documented and yet even more influential discipline: the very heart of what it is to deduce, to demonstrate and to reason, as they say, "rigorously".

> Little wonder, then, that Greek mathematics stresses form. Throughout the book, I have stressed form rather than content, partly as a method of getting at the cognitive reality behind texts, but partly –and this is the fundamental justification of my approach- because this is the place where stress should be placed, if we are to be sensitive to the historical context of Greek mathematics. Greek mathematics, to put it briefly, was a cultural practice in which the dominant was the form. p. 311

Instead of taking the "Greek Miracle" as the resource for retelling, once again, the glorious story of apodictic reasoning, it is the very invention of this style of reasoning that is chosen as the topic of the inquiry. This is a great surprise indeed, because the sources seem to be totally lacking, at first sight. If it was a great achievement to have retold the Scientific Revolution of the 17th century as though we had the same type of data as we have for contemporary laboratory lives, how much greater would the difficulty be of retrieving the practice of deduction of a few hundred badly known mathematicians from Antiquity for whom we possess only fragmentary and corrupted texts? And yet, you get the same feeling of practicality from this book that you get, for instance, from Ed Hutchins' *Cognition in the Wild* (1995) on a not so dissimilar topic −collective calculation with instruments. The similarity holds even though Hutchins had the benefit of videos, tape recorders, and archives of all sorts, standing there, on site, as a living ethnographer. Thanks to Netz, the reader is genuinely transported into the "flat laboratory" of Greek mathematics and is allowed to witness its step by step inventions in a way that very few ethnographies of mathematicians at work have been able to emulate (Rosental, 2003). As I say, there may have been no "Greek Miracle" at work back then, but we find in the here and now a Netz's miracle of some proportions…

This miracle resides in the level of practice that is taken as a focal point of investigation: scripto-visual inventions.[2]

> I will argue that the two main tools for the shaping of deduction were the diagram, on the one hand, and the mathematical language on the other hand. Diagrams -in the specific way they are used in Greek mathematics- are the Greek mathematical way of tapping human visual cognitive resources. Greek mathematical language is a way of tapping human linguistic resources (...) But note that there is nothing universal about the precise shape of such cognitive methods. They are not neural; they are a historical construct. (...) One need studies in *cognitive history*, and I offer here one such study. p. 6-7

As can be seen from this quotation, Netz's materialism is not to be found as in some "social construction of mathematics", in the economical background of classical Greece, but in the *intellectual technologies*[3] in which so much of science studies today consists. How do you *de-monstrate* something to someone? That is, how do you *show* it? How do you *draw* it? How do you point *your finge*r at it while speaking? How do you *letter* it? How do you *gain* assent in the absence of your correspondents? How do you *share* conviction? It's largely the semiotic level, once properly focused in Netz's clever hands, that a stunning amount of information about the now absent practice can be provided. Which proves, once again, that "practice" is not something that you observe *de visu*, but is more an explanatory paradigm. It is a genre that may retrieve as much from dead documents and immensely distant times as from visitable sites ("My plan is to proceed, as usual, from the practice" p. 241). The meanings that Netz is able to extract from papyri and parchments are as stunning as those that paleo-archeologists are able to retrieve from scattered silex in some distant stone tool quarry − the minute by minute, gesture for gesture chain of actions carried out by people they know nothing about.

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[2] This is a common venue for the small number of analysts daring enough to deal with a non formalist description of formalism. See, for instance, Bastide, 1990; Rotman 1987, 1993.

[3] The paradigm is more or less that of Jack Goody's master book (1977). For a general presentation, see Latour, 1990.

This book might not be the first non-formalist description of formalism, but it is certainly the first instance in which a non-formalist description bears upon the very origin of deduction, geometry and apodictic reasoning in the 5th century BCA. These of course are the very forms of reasoning at the core of the scientific imagination. No matter how excellent the other studies of mathematical practices at later periods have been, they have always relied (like their subject matter) on an already rich repertoire of techniques and literary genres some of which could be traced all the way to the Greek. This means that they have taken for granted what it meant to *deduce* something from something else, to *demonstrate* a result, to *convince* through a figure.[4] What Netz's does is to transport us back in time to where there was no geometry, no apodictic reasoning, no deduction, and to when each of those practices had to be devised from scratch without relying on any precedent. If you have had to suffer through geometrical demonstrations at school (which is certainly the case for me), there is something exhilarating about witnessing, page after page, in this luminous and lucid book, the difficulty that Greek geometers had themselves in inventing, one after the other, the micro-techniques necessary to navigate diagrams and the "transport of necessity" (a crucial term I will explain later) to carry a proof through from the beginning to its end.[5] Once again, to shift the very notion of form and formalism from a resource to a topic possesses a liberating effect —what could be called the illumination proper to "science studies"!

Because of my lack of credentials in Greek mathematics, the present paper can be no more than a sign post for the science study community to be directed to a book they might have otherwise missed. I will therefore restrict myself to extracting three main arguments from Netz's masterpiece (without following the order of the chapters) which seem to me especially decisive: a) the misuse of Greek mathematics by philosophers; b) the autonomy of formalism and finally, c) the core of the book, the technology of lettered diagrams and what these do for the transfer of necessities.

When Plato goes to Hollywood

It is easy to study laboratory practices because they are so heavily equipped, so evidently collective, so obviously material, so clearly situated in specific times and spaces, so hesitant and costly. But the same is not true of mathematical practices: notions like "demonstration", "modeling", "proving", "calculating", "formalism", "abstraction" resist being shifted from the role of indisputable resources to that of

[4] For some recent examples see Dear 1995, Galison, 1997, MacKenzie 2001, Warwick, 2003, but their mathematicians already knew so much that they could not possibly put the elementary components of what it was when no one yet knew what it is to think mathematically on center stage.

[5] "One should make an effort to realize how mundane Greek mathematical words are. We translate *tome* by 'section', *tmema* by 'segment', *tomeus* by 'sector'. Try to imagine them, as, say, 'cutting', 'cut' and 'cutter'. The Greeks had no Greek or Romans to borrow their terms from." p. 124.

inspectable and accountable topics. It is as if we had no tool for holding such notions under our eyes for more than a fleeting moment, or simply no metalanguage with which to register them. We seem to be inevitably contaminated by them, as if abstraction has rendered us abstract as well! A moment of inattention, and sure enough, they will have evaporated, they will have bounced back again, so to speak, behind us instead of remaining under our gaze. Instead of being *what* is to be described by a new, still to be invented descriptive language, abstractions all too easily slip back into providing the metalanguage of our descriptions. Thus the materialist account of the act of abstracting, has become an abstract rendering of abstraction.[6] Many a reliable science student has veered back into being an epistemologist and has ended up simply piling up formalism upon formalism… This is why, on the whole, the field of science studies is so heavily skewed in favor of experimental sciences; for two dozens studies of experiments and machineries, we find only one about equations, modelisation, formalism or logics.

The great liberating decision of Netz's book is that this state of affairs might not be due to the inherently "abstract" nature of deduction, but rather to a strange operation of channeling (not to speak of kidnapping), by Platonist philosophers of a narrowly specialized set of skills, nurtured inside tiny networks of cosmopolites practitioners of Greek geometry… A rather long but hilarious quote, will be enough to see where his argument is going:

> To be more precise: we all know the fate of a book which suddenly becomes a best seller after being turned into a film –in the version 'according to the film'. This process originated in South Italy in the late fifth century BC, but it was Plato who turned 'Mathematics: the Movie' into a compelling vision. This vision remained to haunt western culture sending it back again and again to 'The Book according to the Film' –the numerology associated with Pythagoreanism and Neoplatonism. A few people, especially in the Aristotelian tradition, went back to the original, until, emerging from the last Platonic revival of the Renaissance, mathematics exploded in the sixteenth century and left Platonism behind it with the rest of philosophy and the humanities. We now take the centrality of mathematics for granted; we should not project it into the past. (p. 290)

To the great surprise of those who believe in the Greek Miracle, the striking feature of Greek mathematics, according to Netz, is that it was completely peripheral to the culture, even to the highly literate one. Medicine, law, rhetoric, political sciences, ethics, history, yes; mathematics, no. "There is something very radical about the isolationism of Greek mathematics, compared with the general background" p. 309. With one exception: the Plato-Aristotelian tradition. But what did this tradition (itself very small at the time) take from mathematicians? Not the lettered diagrams (*horresco referens*), not much of the vocabulary, almost none of the intellectual technologies, but only one crucial feature: that there might

[6] This is probably what has entangled Eric Livingston (1985) in the loop out of which Netz has marvelously extracted himself: there is more than one way to reach "unique adequacy".

exist one way to convince which is *apodictic* and *not* rhetoric or sophistic. The philosophy extracted from mathematicians was not a full-fledged practice. It was only a way to radically differentiate itself through the right manner of achieving persuasion.

The words *apodeixis* and *epideixis* have almost the same etymological root and for many centuries were utterly indistinguishable (Cassin, 1995). It's only the Platonist philosophers who, patterning their speech from the resulting *effect* on persuasion of geometrical demonstrations, introduced into philosophy a radical differentiation between one way of being convinced (by rigorous demonstrations), *apodeixis*, and another way, which relied on flourishes of rhetoric, sophistry, poetry, imagination and political maneuvering, *epideixis*. Was this the real effect of philosophical practices? Good heavens, no! The major scandal for philosophers in Antiquity, a scandal that is still with us today, is that no two philosophers agree with one another. Was the Platonic philosophy a real emulation of geometer's practices which produced conviction around the collective inspection of lettered diagrams, sticking to the conclusions that forms, and only forms, could lead to?[7] Of course not, since there was no diagram to begin with. Philosophy did not carefully limit itself to forms, as geometers did (more of this below), but instead claimed to be talking about *contents*: the Good Life, the proper way of searching for Truth, the Laws of the City, etc. It is as though Plato extracted no more than a style of conviction from geometry and added to it a totally unrelated content; it is as though the type of persuasion mathematicians obtained at great pains (because they limited themselves to forms) could nonetheless be reached, at almost no demonstrative cost, by philosophers with regards to what they saw as the only relevant content! A mimicry of mathematics, just sufficient enough to boot the Sophists out of philosophy. A prowess indeed, that remains the secret spring of so many of the Science Wars, past and present.

And yet, the "Book according to the film", to invoke Netz's simile again, the one that has been read and taught for twenty six centuries, states that "there is a radical difference between conviction and persuasion". It further states that this difference is what defines philosophy, epistemology, science and even Reason with a capital S… When we admire Socrates' admonition to the sophist Callicles: "*geômetrias gar ameleis*!"[8] we continue quoting from the book that was made *after* the blockbuster film. We might screen the film from time to time, but no one, no one

[7] Here is Netz's assesment of Plato's connection with the technique: "Plato's works suggest that mathematicians had a certain terminology –Plato does this by allowing his mathematical passages to be filled with what looks like jargon. This jargon is often different from the Euclidean one, but there is no reason to suppose Plato is trying to use the *correct* jargon. Otherwise, Plato is strangely reticent about such aspects of mathematical practice as, for instance, the use of letters in diagrams. In general, his use of mathematics is done at a considerable distance from it, and does not allow us to see clearly what was the shape of the mathematics he knew." p. 276

[8] "You've failed to notice how much power geometrical equality has among gods and men, and this neglect of geometry has led you to believe that one should try to gain a disproportionate share of things" (Gorgias 508 a) and my commentary of this most famous dialog in Latour, 1999.

that is except Netz, still reads the tiny esoteric script out of which the Hollywood film and later bestseller were produced… After being freed from the best seller of the Scientific Revolution, we might now be freed from the best seller of the Greek Miracle. Back to the originals. Once again, the history of Reason turns out to be even more enlightening than what the Enlightenment hinted at.

Inventing a new autonomy

We should not be too harsh with philosophers, though: after all, even bad peplum films may trigger the vocations of great classicists! Centuries before physics could take over philosophy foresaw that what Greek mathematicians succeeded in doing might work for them as well. Sticking to forms could provide a fabulous new source of certainty if only a certain type of content could be treated in the same way. When "Galilean objects" would be added to Archimedean proofs, contents, would at last fit forms and demonstration would take over. In a way, philosophy (of the Platonist sort at least) was a placeholder for that future development. But still, this is not, definitely not, what Greek mathematicians wished to do at that time. To the contrary, they took considerable pains to make just the sort of kidnapping philosophers indulged in, impossible. As Netz documents time and time again, geometers maintained a strict separation between first-order mathematics (work done on forms) and second-order mathematics (the search for contents left entirely to outsiders).

> The important thing is not *how* the second-order lexicon is different, but *that* it is different. The two are separated. Indeed, they are sealed off from each other, literally. Second order interludes between proofs, not to mention within proofs, are remarkably rare. The two are set as opposites. And it is of course the first-order discourse which is marked by this, since the second-order discourse is simply the continuation of normal Greek prose. P. 120

Here again, the parallel with Shapin and Schaffer's method is striking: autonomy is not the starting point from which to explain how deduction miraculously appears like Athena from her father's thigh. What has to be explained comes about through the intrusion of a set of new and highly specialized techniques. Just as Boyle invented a boring style of virtual witnessing that permitted a relative autonomization of scientific prose from the rest of literature, Netz shows that in the same way, Greek mathematicians deliberately invented a style which allowed them to differentiate themselves from any other intellectual practice —*especially* from philosophy. *Not* doing any second-order reflexion about the proofs is a central aspect of that style, "sealing it off" from the rest of literature:

> The lettered diagram supplies a universe of discourse. Speaking of their diagrams, Greek mathematicians need not speak about their ontological principles. This is a characteristic feature of Greek mathematics. Proofs were done at an object-level, other questions being pushed aside. One went directly to the diagrams, did the dirty work, and, when asked what the ontology behind it was, mumbled something about the weather and went

> back to work. (...) There is a certain single-mindedness about Greek mathematics, a deliberate choice to do mathematics and nothing else. That this was at all possible is partly explicable through the role of the diagram, which acted, effectively, as a *substitute* for ontology. p. 57

This is why Netz is so important for allowing us to study abstraction with ethnographic methods at last. Too often we are paralyzed by the ontology of deduction (the best seller from the film) and are thus rarely able to focus on the "universe of discourse" supplied, in this case, by the scripto-visuals techniques. To understand the "shaping of deduction" we have to reverse the substitution back again, we have to replace the ontology by the practice. Then, and then only, striking features appear: the constant reference in the text to the diagrams and especially the letters on them,[9] the limited vocabulary ("The entire Archimedean corpus is made up of 851 words". p. 107), the use of well encapsulated and constantly repeated formulae. Here is how Netz sums up how his corpus manages to achieve proofs:

> -Around 100- 200 words used repetitively, responsible for 95% or more of the corpus (most often, the article, prepositions and the pseudo-words 'letters').
>
> –A similar number of formulae –structure of words- within which an even greater proportion of the text is written (most often, lettered object-formulae). These formulae are extremely repetitive.
>
> –Both words and formulaes are an economical system (tending, especially with words but also with formulae, to the principle of one lexical item per concept).
>
> –The formulae are flexible, without losing their clear identity. The flexibility usually takes the form of gradual ellipsis, which in turn makes the semantics of the text 'abnormal'.
>
> – Further about half the text is made up of strongly semantically marked formulae, which serves further to mark the text as a whole.
>
> – The flexibility sometimes takes the form of transformations of one formula into another and, more generally, formulae are structurally related (either vertically – one formula is constituent in another – or horizontally – the two formulae are cognate).
>
> – Thus a web of formulae is cast over the corpus. (p. 161).

This is clearly not what epistemologists would say the material heart of deduction consists of. Yet this is what becomes visible from the practice of deduction, *once* the ideology of rigor has been pushed aside. More exactly, instead of "deducing rigorously", we are discovering, one by one, the various ingredients with which "rigor" is made. De-duction, becomes an activity as difficult as walking on a treacherous shore, trying not to stray from the tiny path where our few predecessors have passed, so as not to be swallowed in by quick stands. The

[9] This is a key semiotic distinction "The letters in the diagrams are useful signposts. They do not stand *for* objects, they stand *on* them." p. 47

"trouble with deductions" make for as fascinating a read as the "trouble with experiments" in Shapin and Schaffer's hands. If there is one thing that you cannot do while stepping along a deduction, is to look around and admire the landscape and to boast that you have found a new way to truth! One step away, and here you go…

> I suggest therefore that one part of the answer to 'why are Greek mathematical proofs the way they are?' is that proofs are compartmentalised from broader discussions, so that their structure is wholly autonomous. When doing mathematics, one does nothing else. Instead of the multidimensional structure of interests and implications of natural discourse, Greek mathematics abstracts mathematical relationships. This is perhaps obvious for a science, but the Greek mathematics had no earlier science to imitate in this respect. p. 214

Yes, science is autonomous, but this autonomy has to be achieved. And achieved at a great cost. Exactly the sort of cost that Plato's philosophy tried to avoid by bringing deduction (or the imitation thereof) to bear on all the Great Problems of the Time. What fascinates Netz, is how few mathematicians there were: he even goes as far as doing their prosopography (on average three are born per year in the whole Mediterranean basin –p.285!). The scarcity of them is so great that even Archimedes does not manage to have any colleagues…[10]

> It was an enterprise pursued by *ad hoc* networks of amateurish autodidacts –networks for which the written form was essential; constantly emerging and disappearing, hardly ever obtaining any institutional foothold. The engine does not glide forward evenly and smoothly: it jolts and jerks, ever starting and restarting. Our expectations of a 'scientific discipline' should be forgotten. An 'intellectual game' will be a closer approximation. p. 291-292

Again, "the centrality of mathematics, should not be projected back in time". But then there is a problem, in the end, in insisting so much on the building of autonomy: if this new deductive style is so difficult to master, so esoteric, if it is a game pursued by so few people, who abstain from any general comments, any second-order claims, any application to practice (by fear of appearing too banausic), who withdraw from public affairs, what is it that fascinated Platonist philosophers so much that they saw in such a "game" the crucial invention that could help kick the Sophists out of the City? There might not be any Greek Miracle, but there is a mystery indeed: how come that the least likely candidate for public–the shaping of deduction- was yanked out of its proper usage and brought to bear on public affairs as The Way to Reason? And one could add: the miracle is even greater, since this ploy has succeeded to this day: there is no public official that would dare to forget what Callicles forgot: that mathematics holds the key to the Public Good and to the Good Life. That bestseller is still on sale at any newsstand.

[10] "Archimedes, in the *Method* and elsewhere, gives a sense of boundless intellectual energy, crying out for some collaboration; the world did not collaborate" p. 286.

Like all good inquiries in science studies, Netz does not take the context as an explanation for the science, but, on the contrary, shows how any specific science, elaborates its own highly specific way of being related to a context. Even though, I have to admit, his answer is less original on this than the rest of the book. What he does is to confirm Geoffrey Lloyd's well known argument (1990; 2005): it is precisely *because* the public life in Greece was so invasive, so polemical, so inconclusive, that the invention, by "highly specialized networks of autodidacts", of *another way* to bring an endless discussion to a close took such a tantalizing aspect. "Those guys, who live in utter obscurity, seem to have come upon a totally new way to conclude an argument! What a relief it is! Can we use it as well?" Nowhere but in Greece, would this have been seen as a welcome new resource.

> (…) the development of rigorous arguments in both philosophy and mathematics must be seen against the background of rhetoric, with its own notions of proof. It was the obvious shortcomings of rhetoric which led to the bid for incontrovertibility, for a proof which goes beyond mere persuasion**.** p. 309 **(…)**
>
> Within Greek polemical culture, this feature of mathematics acquired a meaning which it did not possess in China or in Mesopotamia.[11] For the Greeks, mathematics was radically different in this respect from other disciplines and therefore mathematicians pursued their studies with a degree of isolationism. p. 310

All great books in science studies pertain to "political epistemology", that is, they don't extend politics to science, nor science to politics. Instead they try to understand where the difference comes from and how the distribution of skills among the different domains has been adjudicated. Netz's book manage to do just that for the most resilient of all the distinctions, the one between being convinced and being persuaded, demonstration and rhetoric, *apo*- and *epi-deixis*. Hence, this most audacious sentence: "We may now say that the mathematical *apodeixis* is, partly, a development of the rhetorical *epideixis*." p. 293. He does not say they are different from the start, he points out where and why they begin to diverge. With this, science studies should be able to move forward: even this most enshrined of all distinctions can be explained and brought back to its precise material origin.

But on one condition. The most radical and original condition of this most radical and original book is that the precise "deixis", that which is designated by the forefinger, be itself designated… This is what we now have to understand.

> Return now to the Greek mathematician: we see him phrasing to himself -silently, aloud or even in writing- Greek sentences. Most probably, he does not write much -after all there is nothing specifically written about his use of language. For four chapters, we have looked for the Greek mathematician. Now we have finally found him: thinking aloud, in a few formulae made up of a small set of words, staring at a diagram, lettering it. This is the material reality of Greek mathematics. We now move to see how deduction is shaped out of such material. p. 167

[11] See the counter case on China in Chemla, et al 1999.

How to transfer necessities through transformations

What do we really see when confronted with a scientific phenomenon? We are led to a scene roughly like this one: —"This, here", says a practitioner designating the window of some instrument with his or her forefinger. —"I don't see anything" a colleague retorts. —"But yes surely here, see this spike?" — "Ah, yes this is what you mean, that's great, I now see it". So much of science studies has been able to retrieve this type of emerging visibility, those deictics and the crucial importance they have in simplifying perceptions provided they are accompanied by an attentive and concentrated intersubjective monitoring (Garfinkel, et al. 1981; Goodwin, 1995; Knorr and Amann, 1990; Lynch, 1985). The experimental form of life is constantly creating those sort of scenes at the interface between what the instruments inscribe, what the local group of colleagues manage to extract from the inscriptions, what is finally defined as the stable phenomenon that has been collectively witnessed and progressively hardened into a genuine fact —or quickly dissipated as an artifact. A large part of the strength of conviction provided by experimental sites comes from this possibility of step by step deictics surveying the constantly inspectable and collectively accountable scripto-visual local worlds.[12] This is roughly what the adjective "scientific" now means.

What was true of observation and experimentation, was precisely what was not supposed to be true of deduction. Here, it was argued that there was no deictic, and that there *should* be none.[13] Thought itself takes over and makes your mind go through the logical steps unaided by any rope, any diagram, any inscription — except, at school, for purely pedagogical purposes (Lakatos 1976). This is why mathematics, the argument continued, is so different from the rest of science. It does not rely on step by step inspection of a previously transformed material reality to extract new intuitions about the empirical world from its instruments. This is why only a formalist account of formalism is called for: no amount of emphasis on the intellectual techniques will explain how one mind suddenly manages to avoid reference and takes its leave from daily reality to access a superior reality – one that no amount of empirical manifestation may ever express. As every French kid has learned from Poincaré "*la géométrie est l'art de raisonner juste sur une figure fausse*" ("geometry is the art of reasoning correctly about figures which are poorly constructed" —Netz's translation p.33). No matter how unreliable the figure the reasoning flows correctly and effortlessly from it, for it is in another dimension altogether.

This might be true, today, among well trained mathematicians. After years of practice they may no longer see what is needed for them to think, no more than an acrobat remembers what it took her to catch a trapeze in full swing fifteen

[12] This collaboration between the scripted and the visual is a feature of diagrams even in their more recent instantiations (Bastide, 1990; Kaiser 2005; Lynch, 1991), except when they are not about anything (Lynch, 1990)!

[13] Peter Galison (2002) has told the history of the specific form of iconoclasm proper to the formalist tradition.

meters above ground. But it was certainly not true in Greece, Netz reminds us, when every single aspect of this new "intellectual game" had to be invented bit by bits. At the time, diagrams were essential to achieve step by step collectively inspectable certainty. When he wrote the book under review, the indispensable presence of diagrams was deduced by him from the semiotic of the texts (they alluded to features which made no sense unless a now absent figure had been designated by the finger of the mathematicians at this very moment in the argumentation). And he had pursued this against much of the common wisdom of specialists of Greek mathematics. But then something extraordinary happened to Netz: he had the luck of being vindicated later by the discovery of actual diagrams from a very early Archimedes copy (Netz, 2006).

> Why is the diagram reliable? First, because references to it are references to a construction, which, by definition, is under our control. Had one encountered an anonymous diagram, it would have been impossible to reason about it. The diagram which one constructed oneself, however, is also known to oneself because it is verbalized. Note the combination: the visual presence allows a synoptic view, an easy access to the contents; the verbalization limits the contents. The text alone is too difficult to follow[14]; the diagram alone is wild and unpredictable. The unit composed of the two is the subject of Greek mathematics. p. 181

Here goes another radical division, the one, this time, between experimentation and deduction, the empirical world of physics and the "purely logical" world of mathematics. For the Greek mathematicians at least, it was not an ideal world, it was indeed an experiment, a highly specific and totally surprising one at that. What happens when this unique combination of a limited vocabulary and formulaic syntax is obsessively applied to figures, and figures only, not because of the quality of their drawings, but because of their relations? Or rather the opposite (see how easy it is to slip?): what we call relations, and logical relations at that, are precisely the *discovery* made by Greek mathematicians when extracting *that kind* of newly visible phenomenon from the empirical world, at the exclusion of all others. What we now take for granted as a "logical relationship" is what you elicit when you retrace the steps of Greek geometers. The logical relationship too, has a history.

> Deduction, in fact, is more than just deducing. To do deduction, one must be adept at noticing relevant facts, no less than combining known facts. The eye for the obviously true is no less important than the eye for the

[14] Remember that the Greeks did not separate letters and had the most cumbersome notation system for calculations. Because of that, diagrams were like oases of clarity. This is also related to a technical reason due to scrolls: scrolls happened to be much easier to read for demonstrations than volumens since diagrams being at the end you could easily fold or unfold the scrolls to always have synoptically the connection between texts and visuals —something which is very hard to do with books. Remember also, that before print no one had any sort of access to any mathematical text with diagrams (Eisenstein, 1979).

> obvious result and, as is shown by the intertwining of starting-points and argued assertions, the two eyes act together. p. 171

Mathematics is empirical through and through. If it was odd of Boyle to describe what happened to birds and candles inside the artificial trapping of an air-pump, think of how utterly bizarre mathematics experiments might have been. Think of experiments in which badly drawn (but carefully lettered) diagrams were subjected to inspection in order to extract only one type of connection, that of the transitive relations between different parts of the diagrams at different moments in the written proof. If you think it odd that Pasteur's microbes, once cultivated in a dish, were able to become visible and accountable, then you should find the fact that "logical relations" emerging from the Petri dish of Greek geometers' "flat laboratories" no more and no less odd.[15]

Actually, Netz is slightly more prudent here than I am about the ontological impact of such a discovery (remember he is a serious classicist not a feverish science student (Latour, 2007)):

> We are historians –we do not have to answer such questions. All we have to note is that there is a decision here, to focus on relations in so far as they are transitive. Whether they really exist independently of the decision is a question left for the philosopher to answer; the historian registers the decision.[16] At some stage, some Greeks –impelled by the bid for incontrovertibility, described in Lloyd (1990)- decided to focus on relations in so far as they are transitive, to demand that in discussions of relations of areas and the like, the make-believe of ideal transitivity should be entertained. Here is finally the make-believe, the abstraction truly required by Greek mathematics. Whether the sphere is made of bronze or not is just immaterial. The important requirement –the point at which mathematics takes off from the real world- is that if the sphere is equal in volume to some other objects, say 2/3 the circumscribed cylinder, and this cylinder in turn is equal to some other object *X*, then the sphere will be equal to *X*. This is true of 'equal' only in an ideal sense, a sense divorced from real-life applications and measurements. And this is the *qua* operation, the make-believe at the heart of Greek mathematics. p. 197-198

We should be careful at this juncture. Just one second of inattention and we risk losing the point: the "ideal" should not be treated "ideally" but "materially". Actually, "make-believe" is a somewhat unfortunate term, a possible reversion to a

[15] It should be recalled that the extraction of a constant through transformations is possible only because the tiny artificial world of the proof has been itself incredible shrunk : "To sum up, then, our first description: the Greek mathematical lexicon is tiny, strongly skewed towards particular objects (whose properties and relations are only schematically given) and is invariant, within work and between authors." p. 108

[16] Yet, the whole tenor of Netz's book clearly puts to rest, like so much of science studies, the traditional opposition between constructivists and idealists: mathematical objects are real *because* they are constructed. The world does respond to that practice with fecundity because of the artificiality of the flat laboratory. This is Netz' solution to the Kantian puzzle of "synthetic a priori judgment".

looser (de)constructivist idiom, but the key feature of Netz description is undisputable. The intellectual technology of sticking to forms in this highly specific fashion allows the "ideal world" to emerge —or rather allows the empirical world to be educed as "ideal". What it this "ideal" made of? As Poincaré says, of the, oh so slight distance between "reasoning correctly" and "a badly drawn figure"…

We could say that the ideal has finally landed safely! Quite a feat. Everyone else —well almost everyone— takes the ideal world of logical relations as *another* world, one that has been simply "discovered" by the Greek, and that has always existed in itself effortlessly. And yet the whole idea of an ideal world of formalism is just as unrealistic as if you had continued imagining that all the planes that cross your blue summer skies never land nor take off and are never serviced by any air line ground staff. Here we witness the very laying of the very macadam of the very air-strip out of which the "ideal plane" takes off and we hear the roar of the engines that allows it to fly! Such is the decisiveness of that amazing book.

But what exactly is the phenomenon that made the Greek mathematicians so enthusiastic and so quickly productive, at least at first? Can we name more precisely the materiality of this ideal plane?

> My argument is simple. Some statements and arguments are seen as directly necessary – they are the building-blocks, the 'atoms' of necessity. These then combine in necessity-preserving ways to yield the necessity of Greek mathematics. (p. 168)

Greek mathematicians invented a completely new path: the preservation of necessities through successive transformations. They suddenly realized that by extracting only the relations that the text could describe, you could transfer necessity from the beginning of a proof to its end. Providing that is, that you indicate every single transformation through the diagrams with the point of the finger, and that you never leave this step by step procedure for the "content" or for "what it really means". Surely, every reader, no matter how badly they were taught, must have felt the same stupefaction when doing geometry at school: —You see that this is correct —Yes, okay, I don't move, —Okay, it's the same all along, yes, this is the same as this one, and this again is the same as this one, yes —Okay, and, oh, surprise, surprise, *that* is also true of *this*. Everything has changed, nothing has changed. If bad pupils of the 20th century were so surprised by this feat of *salva veritate*, imagine what it should have been like to discover this amazing path-blazing way in the middle of the agora in the 5th century BCA. Enough indeed to have stupefied the little slave of the *Meno*.

But why is this so extraordinary? Why does focusing on only one type of relations allow one to take off from reality? Because of the chain. It invents a "chain that will never break down".

> The majority of *assertions* [in Euclid] is that of equivalences. I have already noted in the previous chapter the enormous repetitiveness of the relation 'equality' in Greek mathematics. We now see the logical significance of this centrality. (…)

> We can say that Greek mathematics is ultimately deductive, because it deals with transitive relations. This answer is partly valid. The empirical world is recalcitrant, it does not yield to logic, and this is because it behaves by degrees, by fine shades, by multiple dimensions. Shading into each other, the chains of relations operating in the real world break down after a number of steps; the quantity of liquids transferred again and again from vessel to vessel, will finally reduce; the preferability of *A* to *B* and of *B* to *C* does not always entail that of *A* to *C*. Mathematical objects are different. (p. 197)[17]

That's the "ideal" – a glass of water that you can keep emptying into another glass endlessly without any drops falling away and that no burning sun will ever evaporate. In the "real world" there is no transformation that does not adulterate what is transported. In the "ideal world", the one that mathematicians are inventing, or discovering, you may have as many transformations as you wish (Netz is very careful in actually counting the steps in the always locally achieved proofs: there are never more than forty) and yet you manage to conserve the constant. In other words, you are able to have at the same time mobility and yet immutability (Latour, 1990).[18] Enough to be totally absorbed for the rest of your life —so absorbed that, like Archimedes himself, you might not even notice the shadow of the sword of the Roman soldier that is going to kill you!

Yes, but we all know this already. This is the Western trivia about the difference between real and ideal. No, because what Netz does is to show you that "ideal" is an *effect* generated by a non-ideal and wholly practical experimental work on the very surface of the lettered diagram. It is not that mathematics is "abstract" —abstraction is no more made of abstractions than cheese is made from cheese. It is that doing abstraction is what mathematics have learned to extract from the empirical world. The chains will never break down and the immutability will be obtained through all the transformations only on condition that the path be strictly limited to forms.

Even generalization requires practical tools, an essential tenet of science studies, to be sure, but one that no one had dared to apply to the "generality" of theorems and proofs that we usually take for granted. And yet, generality, too, has to be achieved step by step. We have to come back to an earlier point Netz's

[17] And again: "The crucial thing is that Greek mathematics relies so much upon relations of equivalence, such as identity, equality, proportionality. These formulaes operate in a double role: once as a substratum for manipulation, once again as a licence for manipulation *a:b::c:d* is both a set of objects, in which '*a:b*', for instance is ready for substitution by other, equivalent ratios, and it is also a statement about objects, asserting the substituability of '*a:b*' and '*c:d*'. Equivalence relations are both the raw material and the machines in the factory of Greek proofs." (p. 196)

[18] "Greek mathematics is the trading of properties between objects. Arguments often start from the existence of a set of properties, to conclude that another property obtains as well. Theorems in general claim that when a certain property obtains, so does another. Definitions are at their most practical where they supply the building blocks for such structures." p. 92-93

makes here, and which is totally counterintuitive for those who take the ideal world of mathematics as already fully made:

> Greek geometric propositions are not about universal, infinite space. As is well known, lines and planes in Greek mathematics are always finite sections of the infinite line and plane which *we* project. They are, it is true, indefinitely extendable, yet they are finite. Each geometrical proposition sets up its own universe -which is its diagram. p. 32

In other words, Euclid's proofs do not unfold *in a Euclidian space* —at least, not yet, not before centuries of work have ended up building it. The great paradox of Netz careful demonstration of what it is to do demonstration, is that the extensibility of the proof is directly correlated to the tiny enclosed world on which it relies. It's precisely because it is not all encompassing that it can spread "everywhere" —although only "locally"! A point that should not surprise those of us who study networks, but which is still marvelously refreshing when applied to theorems…

The difference is especially revealing when contrasted with the type of generality that philosophical language claims to achieve (remember that Plato's dialogs try to imitate the step by step practice of mathematicians).

> Well may Socrates argue that the art of medicine does not study its own interest but the needs of the body (…). Yes we tend to respond, this has some truth in it. But how much? How general? How well could the statement be repeated, with other arts substituted for medicine? Checking a few cases (as Socrates does) is helpful, but does not solve our problem. We just cannot foresee how the terms may stretch, because the borders and the very constituents of the conceptual universe they inhabit are vaguely marked. The simplicity of the mathematical lexicon, on the other hand, makes it inspectable. We know not only what the text asserts, but what are the available options were we to try to manipulate it, to stretch it.
>
> In short, then, the simplification of the universe, both in terms of the qualitative diagram and in terms of the small and well-regulated language, makes inspection of the entire universe possible. Hence generality is made possible. p. 266.

Socrates and the Platonists may try to hype the advantages of the newly discovered "intellectual game" —generality, necessities, universalities— but they will never be able to hide that to get at all those goodies you must first incredibly restrict the universe and stick to the forms of the lettered diagrams without ever jumping to contents —exactly what they crave to do. The "flat laboratory" will yield results only on the conditions that it remains flat.

Conclusion: Poincaré's dictum

I have only skimmed the surface of Netz's book. Nearly every paragraph offers a treasure trove of methods as well as of results for science studies. If we ever manage to write an alternative history of Reason, the metalangage for redescribing this type of scientific practice will certainly resemble – in precision, in tone, in humor too – a great deal of what this book has achieved on the most difficult topic of all: what it is to deduce rigorously a consequence from a premise.

I would like to end on the following point. One of the great lessons of this work is that even though it spends so much time on the diagrams, it is not about the visual per se, and is certainly not about the imaginary dimension of science. Rather it's about the obsessive literality of diagramatic works, the main focus of Greek mathematicians, and precisely the sort of work and focus that "abstraction" and "formalism" were *not* supposed to need in order to achieve results.

> By delegating some, but not all, action to 'imagination', the mathematicians imply that, in the ordinary run of things, they literally mean what they say: the circle of the proof is drawn, not imagined to be drawn. It will not do to say that the circle was drawn in some ideal geometrical space; for in that geometrical space one might as easily draw a sphere. Thus, the action of the proof is literal, and the object of the proof must be the diagram itself, for it is only in the diagram that the acts of construction literally can be said to have taken place. p. 53

In science studies there is now a great attention (to avoid saying a fashion) for the image dimension of science. To be sure, it is a nice counter weight to having paid so much attention to "ideas" only. And yet, the new attention to images might not lead anywhere, because strictly speaking, there is no image in science, but only cascades of transformations (Pinch, 1985) from one inscription to the next. Isolated, or taken out of its series of transformations, a scientific image has no referent. The idealism of a science made of ideas, risks simply being replaced by an "imaginism" of a science made of images. In other words, the real phenomenon to concentrate upon is neither the ideas nor the images and what they might refer to, but is the trade off between what is conserved and what is discarded when going from one scripto-visual trace to the next in line. This is not a visual dimension, but, on the other hand, it is exactly the sort of necessity-conservation transfer that deduction consists of.[19] Netz provides the tools to simultaneously focus on the material and the visual properties of diagrams without being fascinated by their imagery.

Poincaré's dictum, does not say that we don't need *any* diagram at all. What it actually says that there is a distance between "thinking correctly" and "drawing badly". Poincaré's position, therefore, is not one of iconoclasm in that he doesn't preach abstinence from all figurations. The question is to define this distance between thinking and drawing (this "make-believe" aspect in which Netz saw the proper way to educe the "ideal plane") with some precision.

> It is just because there is an inherent make-believe in the diagram that the make-believe of transitivity is naturally entertained. 'This is equal to that, and this to that, so this to that' – 'Oh really? Have you measured

[19] It is also at the centre of Hutchins' attention : "Within this larger unit of analysis, what used to look like internalization now appears as a gradual propagation of organized functional properties across a set of malleable media" (1995, p.312) and more generally of what I have called, for this reason, "immutable mobiles".

> them?' – 'Come on, don't be a fool. There's nothing to measure here – it's only a diagram.'
>
> 'Nothing to measure here': I have invented this retort. But it is there in the original –in the behaviour of the diagram. It is precisely this metric aspect, these relations of measurement, that the diagram does not set out to represent. Diagrams and formulaes are thus functionnaly related in a single structure. p. 198

If, as I have argued, there is a "war on images" in science as well as in religion (Latour, 2002), Netz's attempt might be one of a very few works to navigate its way, to use Galison's analysis (2002), between iconoclasm and idolatry: "If only we had no image; we cannot do without images". For the first time, then, we have a truly iconophiliac study of formalism.

Bibliography

Bastide, Françoise. (1990) 'The Iconography of Scientific Texts: Principle of Analysis', in M. Lynch and S. Woolgar (eds), *Representation in Scientific Practice*, (Cambridge, Mass: MIT Press): 187-230.

Cassin, Barbara (1995) *L'effet sophistique*. (Paris: Gallimard).

Chemla, Karine, Donald Harper and Marc Kalinowski (1999) *Divination et rationalité en Chine ancienne*. (Paris: PUF).

Dear, Peter (1995) *Discipline and Experience: The Mathematical Way in the Scientific Revolution*. (Chicago: University of Chicago Press).

Eisenstein, Elizabeth (1979) *The Printing Press as an Agent of Change*. (Cambridge: Cambridge University Press).

Galison, Peter (1997) *Image and Logic. A Material Culture of Microphysics*. (Chicago: The University of Chicago Press).

Galison, Peter. (2002) 'Images Scatter into Data. Data Gather into Images', in B. Latour and P. Weibel (eds), *Iconoclash*, (Cambridge: MIT Press): 300-323.

Garfinkel, Harold, Michael Lynch and Eric Livingston (1981) 'The Work of a Discovering Science Construed with Materials from the Optically Discovered Pulsar', *Philosophy of Social Sciences*:131-158.

Goodwin, Charles (1995) 'Seeing in Depth', *Social Studies of Science* 25:237-284.

Goody, Jack (1977) *The Domestication of the Savage Mind*. (Cambridge University Press: Cambridge).

Hutchins, Edwin (1995) *Cognition in the Wild*. (Cambridge, Mass: MIT Press).

Kaiser, David (2005) *Drawing Theories Apart : The Dispersion of Feynman Diagrams in Postwar Physics*. (Chicago: The University of Chicago Pres).

Knorr-Cetina, Karin and Klaus Amann (1990) 'Image dissection in natural scientific inquiry', *Science, Technology, & Human Values* 15: 259-283.

Lakatos, Imre (1976) *Proofs and Refutations*. (Cambridge: Cmabridge U.P.).

Latour, Bruno. (1990) 'Drawing Things Together', in M. Lynch and S. Woolgar (eds), *Representation in Scientific Practice*, (Cambridge, Mass: MIT Press): 19-68.

· (1999) *Pandora's Hope. Essays on the reality of science studies.* (Cambridge, Mass: Harvard University Press).

· (2007). 'A Textbook Case Revisited. Knowledge as Mode of Existence' in E. Hackett, O. Amsterdamska, M. Lynch and J. Wacjman (eds) *The Handbook of Science and Technology Studies -Third Edition*. (Cambridge, Mass, MIT Press: 83-112).

— and Peter Weibel (eds) (2002) *Iconoclash. Beyond the Image Wars in Science, Religion and Art* (Cambridge, Mass: MIT Press).

Livingston, Eric (1985) *The Ethnomethodological Foundations of Mathematical Practice*. (London: Routledge).

Lloyd, Geoffrey (1990) *Demystifying Mentalities*. (Cambridge: Cambridge University Press).

Lloyd, GER (2005) *The Delusions of Invulnerability. Wisdom and Morality in Ancient Greece, China and Today*. (London: Duckworth).

Lynch, Michael (1985) *Art and Artifact in Laboratory Science A Study of Shop Work and Shop Talk in a Research Laboratory.* (London: Routledge).

— l (1990) 'Pictures Of Nothing? Visual Construals In Social Theory', *Sociological Theory* 9:1-22.

— (1991) 'Science in the Age of Mechanical Reproduction: Moral and Epistemic Relations between Diagrams and Photographs', *Biology and Philosophy* 6:205-226.

MacKenzie, Donald (2001) *Mechanizing Proof: Computing, Risk, and Trust (Inside Technology)*. (Cambridge, Mass: MIT Press).

Netz, Reviel (2003) *The Shaping of Deduction in Greek Mathematics : A Study in Cognitive History*. (Cambridge: Cambridge University Press).

— (2004) *Barbed Wire: An Ecology of Modernity*. (??: Wesleyan University Press).

— (2006)

Pinch, Trevor (1985) 'Observer la nature ou observer les instruments', *Culture technique*:88-107.

Rotman, Brian (1987) *Signifying Nothing. The Semiotics of Zero*. (London: Macmillan).

— (1993) *Ad Infinitum. The Ghost in Turing Machine. Taking God out of Mathematics and Putting the Body Back In*. (Stanford: Stanford Unversity Press).

Rosental, Claude (2003) *La Trame de l'évidence*. (Paris: P.U.F.).

Warwick, Andrew (2003) *Masters of Theory: Cambridge and the Rise of Mathematical Physics*. (Chicago: The University of Chicago Press).