

# WEALTH EFFECTS AND PUBLIC DEBT IN AN ENDOGENOUS GROWTH MODEL

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*The debate on public finances' sustainability has long focused on the conditions for the accumulation of debt. This implied that, empirically, the analyses revolved around estimations of dynamic versions of the debt accumulation equation, through unit root tests and cointegration tests between e.g. revenues and primary expenditures, or debt and deficit. Bohn (2007, Journal of Monetary Economics), has forcefully argued in favour of a stronger focus on theory. The model of this paper shows to which extent and under which conditions earlier results considering fiscal policy in an endogenous growth setting are modified if government spending is not entirely tax-financed. Therefore the model uses Barro's (1990, Journal of Political Economy) production function and Blanchard (1985, Journal of Political Economy)-type consumers to assess fiscal sustainability and the determinants of long-run (or potential) growth, in presence of productive capital services. The main conclusion is that, provided public spending is not too high, it will be growth-enhancing. This feature does not hurt fiscal sustainability if taxes are adjusted appropriately. We also calibrate the model to show that the current level of public capital is low in France, the UK and the USA.*

## 1 Introduction

The issue of fiscal sustainability is the source of a vast literature since the seminal empirical investigation of Hamilton and Flavin (1986) on US public debt. Broadly speaking, the empirical literature has followed five approaches. First, time-series properties can be investigated and their consistency with the intertemporal budget constraint (IBC) checked. According to Hamilton and Flavin (1986), stationarity of the debt time series was required for sustainability. Trehan and Walsh (1988) and Quintos (1995) respectively advocated that total deficit should be stationary and difference-stationary. Second, Blanchard (1993) and Blanchard *et al.* (1999) proposed to compute a medium run tax rate or a primary surplus which would be consistent with sustainability, *i.e.* that would fulfil the IBC. Fiscal policies can be assessed in terms of their sustainability depending on expected future tax rates and surpluses being above or below the computed thresholds. Third, reliance on fiscal rules has been used to check the interactions between flow variables like public spending or taxes and public debt: a sufficiently low (high) response of spending (taxes) to a positive shock on public debt is required for sustainability. Barro (1986) and Bohn (1998) assessed the sustainability of US public finances following this methodology. Fourth, Canzoneri *et al.* (2001) studied the interactions between primary (net of interest) deficit and public debt within a VAR approach, and concluded that US public finances were on a sustainable path. Creel and Le Bihan (2006) extended their methodology to European countries and reached the same conclusion. Fifth, the generational accounting approach, originally proposed by Auerbach, Gokhale and Kotlikoff (1992, 1994), computes for each generation the costs and benefits of projected trends in public finances, thus allowing potential conflicts to emerge.

In a recent contribution, Bohn has cast serious doubts on the appropriateness of empirical investigations relying on the first broad methodology. His paper proves that if the relevant debt variable is stationary after any finite number of differencing operations, then the IBC is satisfied. The IBC is also satisfied if revenues and with-interest spending are difference-stationary of

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arbitrary order, and this without cointegration requirement. (Bohn, 2007, p. 1838). Bohn shows that fiscal sustainability is no longer a time-series properties' issue, even if he acknowledges that in some cases, sustainability may be labelled absurdly weak.

As a consequence, a return to economic thinking (Bohn, 2007, p. 1845) is largely encouraged. In this respect, the other empirical methodologies also need to devote attention to the long-run properties of the underlying economic model. Blanchard (1993)'s computation, for example takes for granted that, at least the rate of growth and the rate of interest are constant and not correlated, whereas fiscal rules and VAR approaches have generally been embedded in short-run specifications. Last, the generational accounting framework is a-theoretical.

In fact, long after Arrow and Kurtz (1970) emphasized the importance of fiscal policy as a determinant of long-run growth, a substantial literature has used the features of endogenous growth models to reinvestigate this issue. Most papers belonging to this literature extended the seminal model by Barro (1990) in which tax-financed government services affect balanced-growth along a hump-shaped curve: increases in government services are associated with higher long-run growth rates up to a certain threshold.

Barro (1990) uses a constant returns to scale production function incorporating the public sector. Assuming a role for public services as an input to private production, the production function takes the following form:

$$Y = K\phi\left(\frac{G}{K}\right) \quad (1)$$

where  $Y$  is output per worker,  $K$  is capital per worker,  $G$  is the per capita quantity of government purchases of goods and services, and  $\phi$  satisfies the usual conditions for positive and diminishing marginal products ( $\phi' > 0$  and  $\phi'' < 0$ ).

With a Cobb-Douglas production function, equation (1) becomes:

$$\frac{Y}{K} = A\left(\frac{G}{K}\right)^\alpha \quad (2)$$

where  $A > 0$  is the constant net marginal product of capital and  $0 < \alpha < 1$ .

Under the assumption of tax-financed public services (*i.e.*, of a balanced government budget), different sizes of governments have two effects on the steady-state growth rate. The increase in taxes reduces investment and growth, whereas the increase in public services raises it through capital productivity increases. The second force dominates when government size is small, whereas for increasing levels of public spending the negative effect of taxes on investment and growth eventually becomes more important.

In the vein of Arrow and Kurtz (1970), Futagami *et al.* (1993) assumed that public capital, rather than the flow of public services, enters in the production function. Contrary to Barro (1990), they showed that the economy has a transitional dynamics and, consequently, the tax rate which maximises welfare is lower than the tax rate which maximises the growth rate of the economy. As a corollary, the level of public capital in the economy may be suboptimal if households pay insufficient attention to its productive features.

Brauninger (2005) and Yakita (2008) extended the model to the case of unbalanced budget, encompassing therefore debt dynamics. Brauninger (2005) assumed an AK production function and concluded on the existence of a threshold public deficit ratio. If the ratio stays below the threshold,

an increase of public deficit reduces the economic rate of growth; if the deficit ratio exceeds the threshold, there is no steady state. Yakita assumed a Futagami *et al.* (1993) production function and concluded that the threshold of public finance sustainability is increasing in public capital stock: countries with already high levels of public capital are more prone to sustain their public finances than countries with smaller ones. Moreover, reducing the public investment-to-GDP ratio, while keeping the debt finance ratio constant, raises the threshold of the public debt-to-public capital ratio: the range of sustainable initial public debt is therefore enlarged. In both Brauning (2005) and Yakita (2008), increased public investment ratios not only require higher taxes but also increases in bond issuance, which drive interest rates upward. Crowding-out effects are thus prominent in their analyses and it is all the more so that public debt has no direct incidence on the long run steady state. Thus, it is not surprising that their conclusions are relatively weaker for public capital as a growth-engine than Barro (1990) or Futagami *et al.* (1993).

Ghosh and Mourmouras (2004) also extended Futagami *et al.* (1993) to the case of welfare-maximising fiscal rules, while taking into account government debt. They concluded that under a golden rule of public finance (GRPF), a less strict budgetary stance may lead to lower steady state welfare if public consumption increases and produces crowding-out effects. However, under the GRPF regime, the ratio of public capital to private capital is lower than under other fiscal regimes so that crowding out effects are minimised in this context. Greiner (2008) studied the implication of publicly-financed education in an endogenous growth model with human capital where governments pay attention to fiscal sustainability. All the models described so far, are characterized by infinitely lived consumers.

Blanchard (1985) showed that, after relaxing the assumption of infinite horizons for households, aggregate consumption is a linear function of aggregate financial and human wealth. In his model public bonds are net wealth for households, in contradiction with Barro (1974). Blanchard also introduced a government which spends on goods but does not affect production directly. Not surprisingly, he concluded on crowding-out effects of fiscal policy.

Blanchard-type consumers and Barro-type production as in equation 1 have already been introduced in endogenous growth models. Mourmouras and Lee (1999) introduced the production function 2 in the Blanchard model, but they assumed a balanced budget rule in the vein of Barro (1990). They were finally unable to find an analytical solution. Saint-Paul (1992) introduced an AK production function in the Blanchard model and showed that higher public debt produced a lower economic rate of growth, even under the assumption that the balanced-growth rate remains above the interest rate: future generations will be harmed by fiscal policy. Reinhart (1999) used a simplified version of Saint-Paul's model to assess the incidence of higher life expectancy on economic growth and fiscal sustainability. With an AK production, higher debt involves lower public spending at the steady-state, and economic growth rate may rise.

In this paper, we want to show to what extent and under which conditions these results are modified if government services are introduced but are not entirely tax-financed. Therefore the model uses Barro's production function 2 and Blanchard-type consumers to assess fiscal sustainability and the determinants of long-run (or potential) growth, in presence of productive public services. Finally, the model is calibrated using data from national accounts. We assess the degree of optimality of public finances in three large countries that have different fiscal rules (France, the UK and the USA) taking the theoretical model as a benchmark. Finally, we conclude the paper with a discussion of the optimality of public finances' limitations, as well as the quality of public expenditures.

## 2 The model

### Households

Following Blanchard, households maximize a standard lifetime utility function, discounting the probability of death  $p$ :

$$U = E\left(\int_j^\infty \ln(c_s) e^{(\theta+p)(j-s)} ds\right)$$

where  $\theta$  is the subjective rate of time discount. Maximizing, using the budget constraints, and aggregating across consumers, Blanchard obtains:

$$\begin{aligned}\dot{C} &= (r - \theta)C - p(p + \theta)W \\ \dot{W} &= rW + L - C\end{aligned}$$

The first equation states that consumption grows with the interest rate, and decreases with total wealth. It may be noted that a positive probability of death implies that Ricardian equivalence does not hold, as current wealth affects aggregate consumption. This wealth effect would disappear if households were infinitely lived (*i.e.* if  $p = 0$ ). A larger discount rate or probability of death of course increase current consumption for a given wealth, and hence reduces consumption growth. With finite-horizon households ( $p \neq 0$ ), both government bonds and capital are part of net wealth: they thus dampen substitution effects. The second equation states that nonhuman wealth increases thanks to interest rate accruals and labour income  $L$ , while it is reduced by consumption.

Wealth can take the form of bonds ( $B$ ) or physical capital ( $K$ ); furthermore, capital accumulation is a residual from consumption choices and depreciations:

$$\begin{aligned}W &= K + B \\ \dot{K} &= Y - C - G - \delta K\end{aligned}$$

where  $G$  is public spending and  $\delta$  is the depreciation rate. Public debt dynamics is standard, and depends on interest rate payments and primary surpluses or deficits:

$$\dot{B} = rB + G - T$$

As in Barro (1990), we assume the production function to have constant returns to scale, and to be subject to an externality linked to public spending  $G$ , the quantity of public services provided to each household-producer which enters as an input to private production:

$$Y = AK\left(\frac{G}{K}\right)^\alpha$$

The marginal product of capital is thus:

$$\frac{\partial Y}{\partial K} = (1 - \alpha)A\left(\frac{G}{K}\right)^\alpha$$

In equilibrium, the interest rate has to be equal to the net marginal product of capital:

$$r = \frac{\partial Y}{\partial K} - \delta = (1 - \alpha)A \left( \frac{G}{K} \right)^\alpha - \delta$$

The complete dynamic system can therefore be written as follows:

$$\begin{aligned} \frac{\dot{C}}{C} &= (1 - \alpha)A \left( \frac{G}{K} \right)^\alpha - \delta - \theta - p(p + \theta) \frac{B + K}{C} \\ \frac{\dot{K}}{K} &= A \left( \frac{G}{K} \right)^\alpha - \frac{C}{K} - \frac{G}{K} - \delta \\ \frac{\dot{B}}{B} &= (1 - \alpha)A \left( \frac{G}{K} \right)^\alpha - \delta + \frac{G}{B} - \frac{T}{B} \end{aligned} \quad (3)$$

Define lower case letters as per unit of capital variables:  $x = X/K$ . Then, we can rewrite (3) as:

$$\frac{\dot{C}}{C} = (1 - \alpha)A g^\alpha - \delta - \theta - p(p + \theta) \frac{b + 1}{c} \quad (4a)$$

$$\frac{\dot{K}}{K} = A g^\alpha - c - g - \delta \quad (4b)$$

$$\frac{\dot{B}}{B} = (1 - \alpha)A g^\alpha - \delta + \frac{g}{b} - \frac{t}{b} \quad (4c)$$

To derive the steady state values we take as given the stock variables,  $B$ ,  $K$ , and the exogenous decision on the amount of public spending,  $g = G/K$ . This implies that we have to determine endogenously two jump variables: the ratio of taxes to capital  $t = T/K$  (such that debt increases at the same rate as the other variables), and consumption to capital  $c = C/K$ . The solution can be found recursively by imposing equal growth rates in the system above ( $\sigma = \dot{C}/C = \dot{K}/K = \dot{B}/B$ ) and eliminating one equation:

$$\begin{aligned} c + g &= \alpha A g^\alpha + \theta + p(p + \theta) \frac{b + 1}{c} \\ t &= g(b + 1) + b(c - \alpha A g^\alpha) \end{aligned}$$

From the first equation we can derive the steady state consumption to capital ratio:

$$c^* = \frac{\alpha A g^\alpha - g + \theta + \sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}}{2}$$

(the other root being always negative). Notice that if consumers were infinitely lived, we would have the standard steady state consumption per unit of capital:  $p = 0 \Rightarrow c_{NC}^* = (\alpha A g^\alpha - g + \theta)$ . A positive probability of death introduces a wealth effect, which is increasing in the ratio of public debt to capital,  $b$ . We can now easily prove the following:

**Lemma 1** – The effect of  $g$  on  $c^*$  is positive below a threshold  $\bar{g} = (\alpha^2 A)^{\frac{1}{1-\alpha}}$ , and negative above it.

**Proof** – See the Appendix.

In fact, an increase of public spending has two opposite effects on consumption: The first negative, through the increased taxation needed to sustain public spending. The second, positive,

through the increased productivity of capital.  $\bar{g}$  denotes the threshold beyond which the former effect dominates.

The effect on the growth rate can be inferred from equation 4b. We can then prove the following:

**Proposition 2** – A sufficient condition for the effect of  $g$  on  $\sigma$  to be positive is that:

$$g < \bar{g} = (\alpha A(2 - \alpha))^{\frac{1}{1-\alpha}} \quad (5)$$

**Proof** – See the Appendix.

Equation 5 is not a necessary condition, because the effect on the growth rate could be positive for values of  $g > \bar{g}$ . The threshold for the necessary condition, nevertheless, is impossible to derive analytically. The growth maximizing level of  $g$ , that we denote with  $g^*$ , will have to be found through calibration and numerical simulations.

The intuition for this result is the following. For values of  $g < \bar{g}$ , an increase of public spending induces an increase of consumption, with negative effects on the growth rate. The direct effect of increasing productivity nevertheless dominates. For intermediate values ( $\bar{g} < g < \hat{g}$  as defined in the Appendix) the two effects push in the same direction, as increasing  $g$  both increases productivity and savings. Finally, above  $\hat{g}$  the effect of further increases of  $g$  on productivity is negative. This effect is initially dominated by the effect on savings, but as  $g$  goes beyond the threshold  $\bar{g}$  the loss of productivity grows larger, and the effect of increases of  $g$  on the growth rate eventually becomes negative.

### 2.1 Calibration on National Accounting Data

We took data from the French, British and US national accounts<sup>1</sup> to calibrate the model and find the optimal rate of public spending to capital,  $g^*$ . The comparison between these three countries is interesting because they have quite different fiscal frameworks: France and the UK are constrained by the Stability and Growth Pact; nevertheless, the latter has adopted a specific framework, the Golden rule of public finances which gives more fiscal leeway to finance public investment. Finally, US public finances face no rule at all.

The technology parameters for the three countries can be estimated from equation 2. Thanks to the Cobb Douglas formulation, the value of  $\alpha$  can be approximated by the share of public capital services on GDP. The level of public capital services is computed as general government gross capital formation plus acquisitions of produced non financial assets, less disposals of non produced non financial assets. Once estimated  $\alpha$ , we computed  $A = Y(G^\alpha K^{1-\alpha})^{-1}$  for each country.

We assumed the discounting and preference parameters to be equal for the three countries, and we took the values commonly found in the current literature. Table 1 summarizes the calibration parameters.

<sup>1</sup> French data were taken from the web site of French National Institute (INSEE), UK data were taken from the Blue Book 2007 and US data were taken from the web site of the Bureau of Economic Analysis (BEA).

Table 1

## Calibration Parameters

	US	FR	UK
$b = B/K$	0.182	0.187	0.084
$K/Y$	2.41	3.20	4.72
$g = G/K$	0.014	0.018	0.004
$\alpha$	0.033	0.070	0.018
$A$	0.478	0.383	0.234
$\delta$	0.1	0.1	0.1
$\theta$	0.05	0.05	0.05
$p$	0.05	0.05	0.05

Using these parameters, we computed for each country the different thresholds and the growth maximizing level of public spending,  $g^*$ . Table 2 reports the computed values, and the difference between actual and optimal levels of public spending (both as a ratio to capital and as a ratio to GDP). Notice that while the first two thresholds ( $\bar{g}$  and  $\bar{\bar{g}}$ ) only depend on technological factors, the last one depends on the level of indebtedness of the country. Thus, the growth maximizing level of public expenditure cannot be computed independently of the debt conditions.

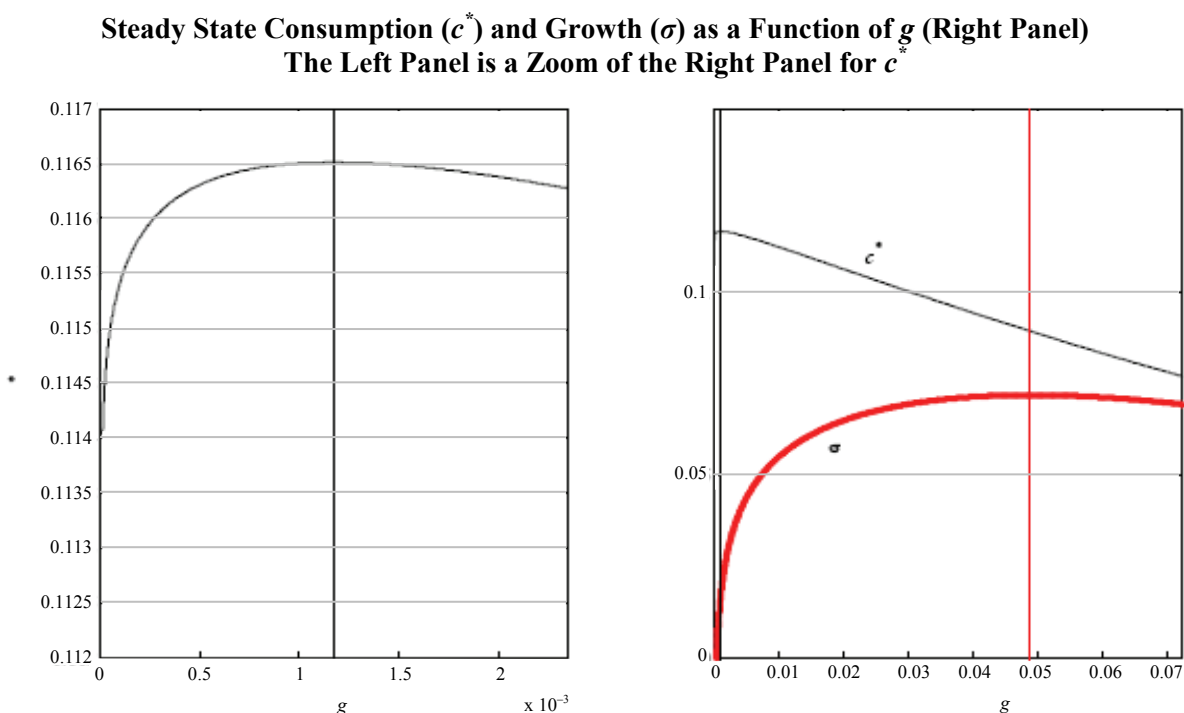
Figure 1 shows the relationship between consumption, the growth rate, and the level of public services  $g$  in France. As in Barro (1990), we have a hump shaped curve both for consumption and for the growth rate (the other countries have similar patterns).

Table 2

## Thresholds of Optimal Public Spending Ratios for the Calibrated Model

	US	FR	UK
$\bar{g}$	0.0004	0.0012	0.0001
$\bar{\bar{g}}$	0.0276	0.0414	0.0076
$g^*$	0.0342	0.0489	0.0106
$g$	0.014	0.018	0.004
$g^* - g$	0.0206	0.0309	0.0068
$(G/Y)^* - G/Y$	0.049	0.099	0.032

Figure 1



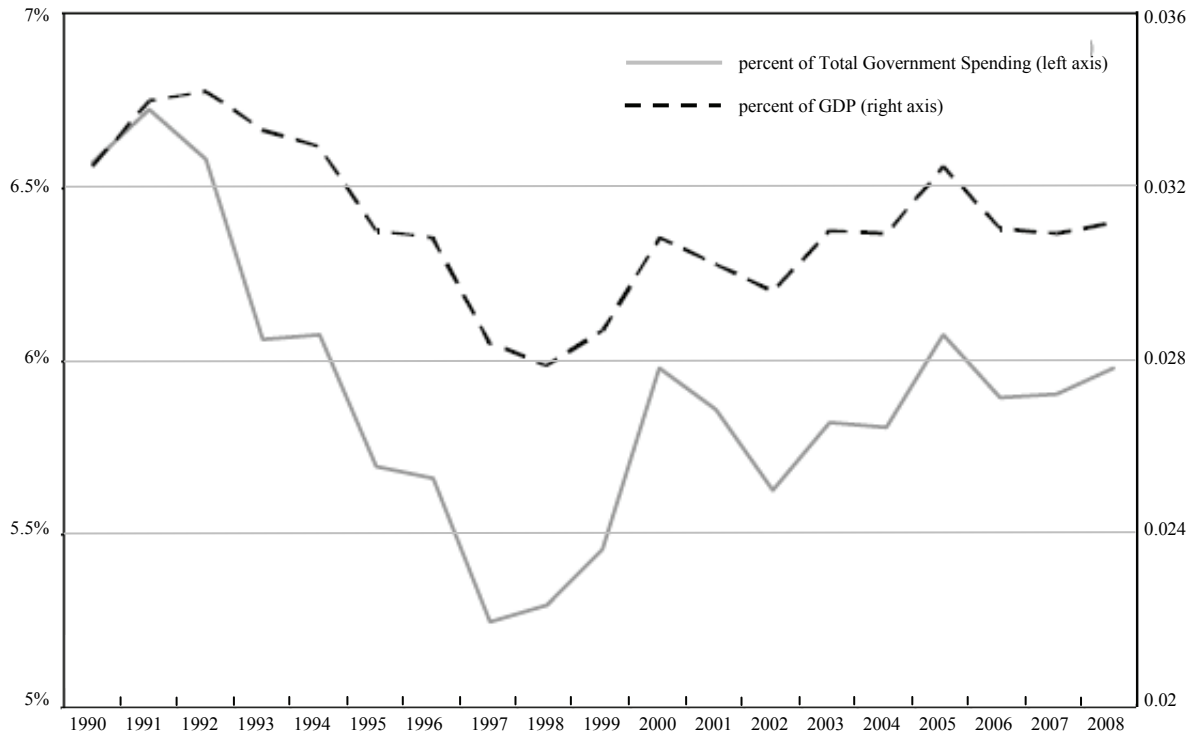
The first immediate consideration related to the results of Table 2 is the fact that the three countries are below their optimal level of public expenditure. Comparing  $g$  with the computed optimal level leads to conclude that budgetary efforts are necessary to bridge the gap. For example, the actual level of public capital services is 0.018 in France (with a historical peak of 0.022 in 1991 and an astonishingly low variance of  $1.6 \cdot 10^{-6}$  since 1978), well below the optimal value  $g^* = 0.0489$ . Even discounting for the roughness of the calibration, the gap between optimal and actual  $g$  – 3 per cent of the total value of gross private capital in France, *i.e.* almost 10 per cent of GDP – is sufficiently high to question the relevance of public finances limitations in the case of France. As a matter of fact, the UK with its Golden rule of public finances is the country that comes closest to the optimal level of public capital services: the gap between optimal and actual  $g$  is less than 1 per cent, *i.e.* around 3 per cent of the British GDP. The USA perform relatively well also with a gap of around 5 per cent of GDP. These differences are of course influenced by technology, capital and so on, but the institutional setting plays a crucial role (public investment in France declined dramatically in the 1990s, as reported in Figure 2).

Our rough calibration exercise also allows to pursue the analysis one step further. The assumption that  $g^*$  is a benchmark for the economy allows to infer the quality of public spending in each of the three countries. The quality of public spending has generally been assessed with respect to a given level of output. Most studies related to that issue take as given the production frontier (see, e.g., Afonso *et al.*, 2005), using one country (or a group of countries) as a benchmark, and computing distance from the benchmark for the other countries. Within the present framework, it is possible to evaluate the efficiency or quality of public spending when the production frontier is moving, thanks to the endogenous growth setting. Then, the model can serve as a normative instrument to gauge the efficiency of all public spending.



Figure 2

**Government Public Investment (OECD Definition)  
as a percent of Total Government Disbursement and of GDP**



Source: OECD Economic Outlook.

As a matter of fact, one can assess this efficiency from three different, although related, perspectives. First, one can compare the actual level of public capital services with its optimum level. Above, we concluded that the gap was worth 5 per cent of GDP for the USA, or equivalently, that the ratio  $g/g^*$  is equal to 40 per cent. This, can be interpreted as saying that in the US public capital services have reached 40 per cent of the full capacity (for which their efficiency would be 100 per cent). The corresponding figures for France and the UK are 37 per cent and 36 per cent respectively (see Table 3, column 3).

Second, one can compare all public spending ( $g_{Tot}$ ) with the optimal level for the economy,  $g^*$ . Take for example France, where  $g_{Tot}$  (equal to 0.137) largely exceeds the optimal value of 0.0489. If following our model the latter is interpreted as the proportion of total spending which is efficient, and if we assume that France could reach this optimal level, taking the levels of public debt and private capital at their actual levels, only  $g^*/g_{Tot} \approx 36\%$  of total public spending can be labelled as “productive”. The corresponding values for the US and the UK are respectively 23 per cent and 12 per cent (see Table 3, column 4). The differences can be explained by a standard decreasing marginal productivity argument, public spending in these two countries having been shown to be already closer to their optimum.

Table 3

## Efficiency Scores

(column 2 taken from Table 3 in Afonso et al. (2005), column 3 to 5: our computations)

Country	Input efficiency (percent)	Public Capital Services' efficiency (percent)		
		$g/g^*$	$g^*/g_{Tot}$	$x = \frac{g^* - g}{g_{Tot} - g}$
US	100	40	23	16
France	64	37	36	26
UK	84	36	12	8

Third, we can go a step further and gauge the efficiency of current expenditures. Taking as given the former efficiency assessment of total spending ( $g^*/g_{Tot}$ ), we can assume that a composition effect between current and capital expenditures does exist. The latter, approximated by public capital services, are assumed to be a 100 per cent productive. Then the corresponding efficiency of current expenditures  $x$  can be computed from a simple formula:

$$\frac{g^*}{g_{Tot}} = 1 \cdot \frac{g}{g_{Tot}} + x \cdot \frac{(g_{Tot} - g)}{g_{Tot}}$$

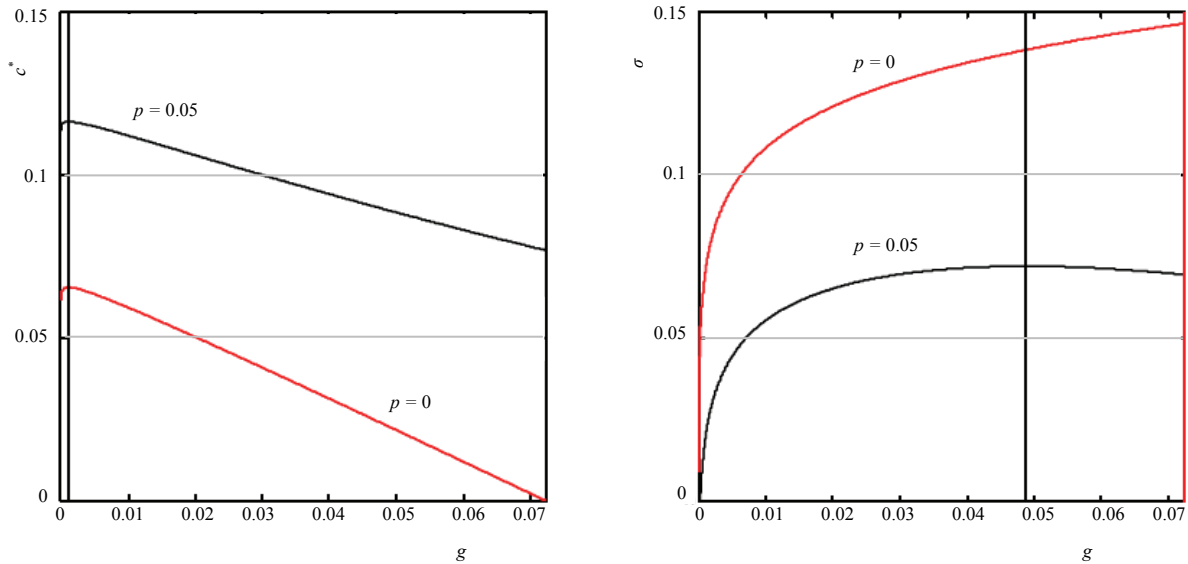
If we apply that formula, we find that 26 per cent of French current expenditures are productive in the sense of Barro (1990). The corresponding values for the US and the UK are respectively 16 per cent and 8 per cent (Table 3, column 5).

As we said above, our approach allows to compare the quality of public spending without assuming a constant production frontier nor giving some countries more weight than others in the estimations. Afonso *et al.* (2005), for instance, use a non-parametric production frontier technique, and their results are used to rank countries according to the distance of their efficiency score *vis-à-vis* a benchmark. It is obvious that it is impossible to evaluate the score of those countries which form the benchmark, *i.e.* which are somewhat arbitrarily assumed to be on the production possibility frontier. Thanks to the use of an underlying model, we don't have such a limitation in our framework.

Table 3 reports the estimations by Afonso *et al.* (2005) for our three countries and compares them with ours. It is clear that for non benchmark countries like France and the UK, efficiency "scores" are high when one assumes a constant production frontier. In the case of France, for instance, Afonso *et al.* find that total output in 2000 could have been reached with only two thirds of actual public spending, *i.e.* 66 per cent of French public spending were "productive". In sharp contrast, we report a score of 36 per cent for all expenditures after we take into account the fact that public services endogenously cause an increase in steady-state production. We can also report a score of 26 per cent for French current expenditures. We can further compute the difference in efficiency between capital and current spending, as the percentage decrease between column 3 and 4 of Table 3. This is largest in the UK, 33 per cent (compared with 30 and 28 per cent in the US and France respectively). We can interpret this as an indicator of the more pronounced preference of UK authorities for public capital services. It can also be interpreted as reflecting the cost of the UK

Figure 3

**Consumption (left panel) and Growth Rate (right panel)  
in the Case with Finite ( $p = 0.05$ ) and Infinite ( $p = 0$ ) Lives**



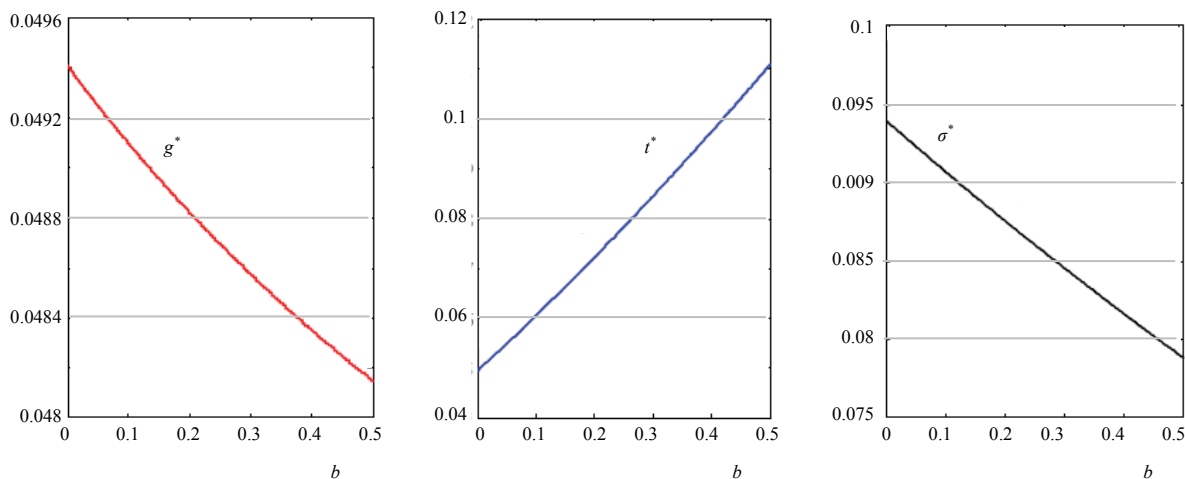
fiscal framework. Whereas the UK is close to the optimal level of public capital services, their current expenditures are dramatically “unproductive”; they are four times and two times more productive in France and in the USA respectively.

It is also interesting to see how our results compare with the original Barro (1990) model. In Figure 3 we plot consumption and the growth rate in our case ( $p = 0.05$ ) and in the case of infinitely lived households ( $p = 0$ ), that corresponds to the Ricardian equivalence case. We only report figures with French parameters and values, as plots are similar whatever the country. As the figure shows, the introduction of a wealth effect increases consumption for each level of public services, but it also decreases the growth rate (as is intuitive from equation 4b). The lower savings, the lower economic growth in this neoclassical context. In comparison with the infinitely-lived agents’ case, the value-added of our results can be twofold: First, they stem from a realistic assumption, a finite horizon; second, growth-maximising consumption at the steady-state is positive and not nil as in the infinite horizon case.

Finally, we investigated how the growth rate and optimal taxation react to different level of initial indebtedness. In Figure 4 we plotted for different levels of  $b = B/K$  the growth maximizing level of public services, and the corresponding growth rate and level of taxation. It can be observed that, in spite of the positive effect on the wealth of the households, an increasing level of debt causes an increase in the tax burden, and a decrease in the optimal level of public services. The maximum attainable growth rate is also decreasing in  $b$ . Only countries with low initial public debt level can afford high levels of public capital services and low taxes. In this respect, no “one-size-fits-all” fiscal strategy can exist: the optimal strategy must be conditional on the initial level of debt.

Figure 4

**Growth Maximizing Level of Public Services ( $g^*$ , left panel),  
and Taxation ( $t^*$ , center panel), as a Function of Initial Debt Stock.  
The Corresponding Growth Rate  $\sigma^*$  is also Reported (right panel)**



### 3 Conclusion

Although our results abstract from important issues like a sustainable growth respectful of the environmental issue, they show that in some cases, increasing public spending will be growth-improving, without hurting the sustainability of public finances. Moreover, the theoretical model has been used as a benchmark for assessing the efficiency of public spending. In the cases of France, UK and US, it has first been shown that the actual level of general government gross fixed capital formation was below its optimal level. Second, total public spending lacks efficiency and a reallocation in favour of productive public expenditures would be beneficial for all economies, although the UK has performed better in terms of productive expenditures. It has also been shown that such a result has been reached at the expense of current expenditures: whereas the UK has been performing the best for productive expenditures, its current expenditures have proven to be far less productive than in the other two countries.

Possible extensions of the model are manifold. First, we need to control empirically that the threshold public spending is compatible with a sustainable tax rate, in a Blanchard (1993) manner. Then, the sensitiveness of the results to the constant probability of death could be analysed. Finally, we could extend the model to incorporate nominal rigidities in the mid-run and of fiscal and monetary rules in the vein of new Keynesian models.

## APPENDIX – PROOFS

**Proof of Lemma 1**

**Lemma 1** – *The effect of  $g$  on  $c^*$  is positive below a threshold  $\bar{g} = (\alpha^2 A)^{\frac{1}{1-\alpha}}$ , and negative above it.*

**Proof** – The derivative of  $c^*$  with respect to  $g$  is:

$$\begin{aligned} \frac{\partial c^*}{\partial g} &= \frac{(\alpha A g^\alpha - g + \theta) + \sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}}{2\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}} (\alpha^2 A g^{\alpha-1} - 1) \\ &= \frac{c^* (\alpha^2 A g^{\alpha-1} - 1)}{\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}} \end{aligned}$$

This implies that:

$$\frac{\partial c^*}{\partial g} > 0 \Leftrightarrow (\alpha^2 A g^{\alpha-1} - 1) > 0$$

If we define  $\bar{g} = (\alpha^2 A)^{\frac{1}{1-\alpha}}$ , we have that:

$$\frac{\partial c^*}{\partial g} > 0 \Leftrightarrow g < (\alpha^2 A)^{\frac{1}{1-\alpha}} = \bar{g}$$

**Proof of Proposition 2**

**Proposition 2** – *A sufficient condition for the effect of  $g$  on  $\sigma$  to be positive is that:*

$$g < \bar{\bar{g}} = (\alpha A (2 - \alpha))^{\frac{1}{1-\alpha}}$$

**Proof** – Starting from equation 4b, we can write  $\frac{\partial \sigma}{\partial g}$  as follows:

$$\begin{aligned}
\frac{\partial \sigma}{\partial g} &= (\alpha A g^{\alpha-1} - 1) - \frac{\partial c^*}{\partial g} \\
&= (\alpha A g^{\alpha-1} - 1) - \frac{c^*(\alpha^2 A g^{\alpha-1} - 1)}{\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}} \\
&\Rightarrow \\
\frac{\partial \sigma}{\partial g} > 0 &\Leftrightarrow \underbrace{(\alpha A g^{\alpha-1} - 1)}_{LHS} > \underbrace{\frac{c^*(\alpha^2 A g^{\alpha-1} - 1)}{\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}}}_{RHS}
\end{aligned}$$

Thus, we have to study under what conditions  $LHS > RHS$ .

We can notice that  $\frac{\partial \sigma}{\partial g} > 0$  in one of the following three cases:

- 1)  $LHS > 0 > RHS$
- 2)  $LHS > RHS > 0$
- 3)  $0 > RHS > LHS$

Denote  $\hat{g} = (\alpha A)^{\frac{1}{1-\alpha}}$  as the threshold below which  $LHS > 0$  (cases 1 and 2). For  $\bar{g} < g < \hat{g}$ ,  $RHS < 0$  and the proposition is trivially proven (case 1).

For  $g < \bar{g}$ ,  $RHS > 0$ , but we can prove that  $LHS > RHS$ . With some manipulation, we have that:

$$\frac{\partial \sigma}{\partial g} > 0 \Leftrightarrow \underbrace{\frac{\alpha A g^{\alpha-1} - 1}{\alpha^2 A g^{\alpha-1} - 1}}_X - \frac{c^*}{\underbrace{\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}}_Z} > 0$$

The proposition is proved if  $X > Z$ .  $\alpha < 1$  implies that  $X > 1$ . Then, to prove the proposition we just need to prove that  $Z < 1$ . Substituting  $c^*$ , we have:

$$\begin{aligned}
\frac{(\alpha A g^\alpha - g + \theta) + \sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}}{2\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}} &< 1 \\
&\Leftrightarrow \\
\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)} &> (\alpha A g^\alpha - g + \theta)
\end{aligned}$$

which is always true provided  $g < \bar{g}$ .

We are left with case 3, in which  $g > \hat{g}$ , and both  $LHS$  and  $RHS$  are negative.

Multiplying both sides of equation 6 for  $-1$  we obtain:

$$\frac{\partial \sigma}{\partial g} > 0 \Leftrightarrow (1 - \alpha A g^{\alpha-1}) < \frac{c^*(1 - \alpha^2 A g^{\alpha-1})}{\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}}$$

The threshold  $\bar{g}$  can be found analytically. Rewriting steady-state consumption like:

$$c^* = \frac{c_{NC}^* + \sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}}{2}$$

one needs to demonstrate that:

$$X = \frac{\alpha A g^{\alpha-1} - 1}{\alpha^2 A g^{\alpha-1} - 1} < \frac{c_{NC}^*}{2\sqrt{(\alpha A g^\alpha - g + \theta)^2 + 4p(p + \theta)(b + 1)}} + \frac{1}{2}$$

Since the first element on the *RHS* is always positive, a sufficient condition for  $\frac{\partial \sigma}{\partial g} > 0$  is:  $X < 1/2$ . With trivial algebra, we can show that this is always true as long as  $g < \bar{g}$  where:

$$\bar{g} = (\alpha A (2 - \alpha))^{\frac{1}{1-\alpha}}$$

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