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# A Link Between Housing Markets and Labor Markets: Time to Move and Aggregate Unemployment

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## **Abstract**

A model is developed that allows for interaction between the labor market and the housing market. A job location has an associated commuting time that may affect the job acceptance decision. Obstacles to mobility, such as regulations in the housing market will affect the reservation strategy of workers. Thus, aggregate unemployment will depend, at least partly, on the functioning of the housing market. Data from the U.S. and E.U. reveals that individuals in the U.S. are about three times more likely to experience a change in residence within a given year. At the same time, unemployment in the E.U. is roughly twice that in the U.S. This paper seeks to understand, both qualitatively and quantitatively, how housing market frictions might affect the functioning of the labor market.

# 1 Introduction

A model is developed that allows for interaction between the labor market and the housing market. A job location has an associated commuting time that may affect the job acceptance decision. Obstacles to mobility, such as regulations in the housing market will affect the reservation strategy of workers. Thus, aggregate unemployment will depend, at least partly, on the functioning of the housing market.

Data for the U.S. shows that on average between 1990 and 2000 about 16% of residents move yearly.<sup>1</sup> In comparison, data for fifteen European Union countries show that less than 5% of residents move yearly. In addition, data on reasons for moving allows us to distinguish between job-related and family (or housing-related) moves, and this distinction is built into the model.<sup>2</sup>

Although residents in the U.S. move about three times the rate of those in the E.U., the reasons why they move are roughly similar. This observation suggests that the shocks that affect mobility are similar across the countries yet there is substantially less overall mobility in the E.U.

In terms of labor markets, in July of 2007 the unemployment rate in the United States was 4.6 %. In contrast, in the four large continental countries – France, Germany, Spain, and Italy – the unemployment rate was substantially higher. The Spanish unemployment rate was over 8%, Italy around 7%, Germany over 8% and France above 9%. Moreover, as noted by Blanchard (<http://www.nber.org/reporter/summer04/blanchard.html>) “...at a given unemployment rate, individual unemployment duration is substantially longer, and flows in and out of unemployment substantially lower, in Europe than in the United States.”

This paper develops a model that links the labor market and the housing market and shows that the functioning of the housing market can affect unemployment and job vacancies. The theory predicts that greater frictions in the housing market makes individuals more choosy about the jobs they take.

We also extend the model to address short (intra-county) vs. long (inter-county) moves. This

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<sup>1</sup>Geographical Mobility, Current Population Reports, P20-538, May, 2001, U.S. Department of the Census.

<sup>2</sup>Why People Move: Exploring the March 2000 Current Population Survey, Current Population Reports, P23-204, May, 2001, U.S. Department of the Census.

is of interest because the reasons for moving short distances as compared to long distances are different.

## 2 Mobility Data

Mobility data for the U.S. and E.U. comes from two sources: the U.S. Census 2000 data and the European Community Household Panel (1999-2001) which contain similar questions.

Table 1: Mobility in the U.S. and E.U.

	US	EU15
Mobility rate	15.5%	4.95%
Share within county / area	0.67	0.83
Share between county / area	0.33	0.17

[Table 1](#) reveals that 15.5% of American residents move yearly for one reason or another. About 2/3 of these moves are within county. In contrast, mobility in the EU15 is about 1/3 of that in the U.S. The share of moves within an area/locality is higher, although at this level of disaggregation it is difficult to strictly compare to US counties. [Table 2](#) provides an overview of reasons for moving for those that moved. In the U.S., most of the intra-county mobility is house related (65.4%), and only a small fraction (5.6%) move within a county for job related reasons. In contrast, work related mobility is approximately 33% of inter-county moves. Family related mobility is a fairly constant fraction of all moves. In the E.U., perhaps surprisingly, there is a very similar pattern. The Census questions are actually more precise about the exact reasons for mobility, while no detailed questions are available in the ECHP. [Table 3](#) gives the details for the U.S.

Finally, [Table 4](#) shows that the reasons for work-related mobility are quite different within and across counties. As expected, intra-county work-related moves bring workers closer to their job, while 70% of inter-county work-related moves are related to a new job.

Table 2: Reasons for Moving, US and EU15

	Proportions, US				Proportions, EU15		
	intra-county	inter-county	all		intra-area	inter-area	all
All pop. (1+)							
Work related	5.6%	31.1%	16.2%	Job related	7.61%	40.0%	14.3%
Family related	25.9%	26.9%	26.3%	Personal Reason	31.6%	29.8%	31.3%
House related	65.4%	31.9%	51.6%	House Related	59.1%	28.1%	52.7%
Others	3.0%	10.1%	6.0%	Not Available	1.7%	2.11%	1.8%
All reasons	100%	100%		All reasons	100%	100%	100%

Table 3: Reasons for Moving, U.S.

<b>Work</b>		<b>House</b>	
new job or job transfer	60.5%	wanted own home, not rent	22.2%
look for work or lost job	9.7%	wanted new or better home/apartment	35.8%
closer to work / easier commute	19.6%	wanted better neighborhood/less crime	8.6%
retire	2.4%	wanted cheaper housing	10.8%
other job related reason	7.7%	other housing reason	22.6%
All work related reasons	100%	All house related reasons	100%
<b>Family</b>		<b>Other</b>	
change in marital status	23%	attend or leave college	38.2%
establish own household	27.2%	change of climate	11.0%
other family reason	49.8%	health reason	17.2%
All family related reasons	100%	other reasons	33.6%
		All other related reasons	100%

Table 4: Work-related Moves, U.S.

Work related	All	Intra	Inter
new job or job transfer	60.5%	24.5%	69.4%
look for work or lost job	9.7%	9.4%	7.7%
closer to work / easier commute	19.6%	53.5%	13.6%
retire	2.4%	1.4%	3.0%
other job related reason	7.7%	11.1%	6.3%
total	100%	100%	100%

## 3 Model

### 3.1 Preferences and Search in the Housing Sector

A dwelling is a bundle of services generating utility to individuals or a household. The defining characteristic of a dwelling, however, is that the services it provides are attached to a fixed location. The services can, of course, depend on the quality of the dwelling and its particular location. Amenities such as space, comfort, proximity to theaters, recreation, shops and the proximity to one's job increase the utility of a given dwelling. The dwelling may also be a factor of production of home-produced goods. In addition, the dwelling could be a capital asset. For these services, individuals pay a rent or a mortgage. We assume that the rent or mortgage is such that utility across dwellings will be equalized to reflect any differences in amenities.

In this paper we focus on one particular amenity, distance to work. Because a dwelling is fixed to a location, the commuting distance to one's job,  $\rho$ , becomes an important determinant of both job and housing choice. We assume that space is symmetric, in the sense that the unemployed have the same chance of finding a job wherever their current residence, implying that  $\rho$  is a sufficient statistic determining both housing and job choice.

Time is continuous and individuals discount the future at rate  $r$ . Agents face two types of housing shocks. First, they may receive a family shock that arrives according to a Poisson process with parameter  $\delta$ . The shock changes the valuation of the current location, necessitating a move. This shock can be thought of as a marriage, divorce, the arrival of children, deterioration of the neighborhood, and so on. Upon the arrival of the shock they make one draw from the existing stock of housing vacancies, distributed as  $G_S(\rho)$ .<sup>3</sup> Note that agents may sample from the existing stock of houses at any time.

Second, agents randomly receive new housing opportunities that (possibly) allows them to relocate closer to their job. These arrivals are assumed to be Poisson with parameter  $\lambda_H$ . The distribution of *new* vacancies is given as  $G_N(\rho)$ .

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<sup>3</sup>The one draw assumption is not very strong. It is equivalent to making up to  $N$  independent draws, in which case this would be like one single draw from a distribution  $(G_S)^N$ . See Lemma 1 in Quentin et al. (2006).

In the model,  $\lambda_H$  is the parameter determining frictions in the housing market. An increase in  $\lambda_H$  means there are more arrivals of opportunities to find housing. As  $\lambda_H$  approaches infinity, housing frictions go to zero. Obviously, if  $\lambda_H = 0$  there is no mobility. The main idea behind  $\lambda_H$  is that agents may not move instantaneously to their preferred location. Such restrictions might arise from length of lease requirements or eviction policies.

### 3.2 Labor market

Individuals can be in one of two states: employed or unemployed. While employed, income consists of an exogenous wage,  $I = w$ . We assume no on-the-job search and all separations from the job occur exogenously with Poisson arrival rate,  $s$ .

The unemployed receive income  $I = b$ , where  $b$  can be thought of as the utility from not working. While unemployed, job offers arrive at Poisson rate  $p$ , indexed by a distance to work,  $\rho$ , drawn from the cumulative distribution function  $F_J$ .

Let  $E(\rho)$  be the value of employment for an individual residing at distance  $\rho$  from the job. Let  $U$  be the value of unemployment, which does not depend on distance given the symmetry assumption made above. We can now express the problem in terms of the following Bellman equations;

$$(r+s)E(\rho) = w - \tau\rho + sU + \lambda_H \int \max[0, (E(\rho') - E(\rho))] dG_N(\rho') + \delta \int \max[U - E(\rho), E(\rho'') - E(\rho)] dG_S(\rho'') \quad (1)$$

$$(r+p)U = b + p \int \int \max[U, E(\rho'), E(\rho'')] dF_J(\rho') dG_S(\rho''), \quad (2)$$

where  $\tau$  is the per unit cost of commuting and  $\rho$  is the distance of the commute. Eq. 1 states that workers receive a utility flow  $w - \tau\rho$ ; may lose their job and become unemployed – in which case they stay where they are; they receive a housing offer among the new vacancies  $G_N$ , which happens with intensity  $\lambda_H$ , in which case they have the option of moving closer to their job; and finally may receive a family shock  $\delta$ , and need to relocate.

Eq. 2 states that the unemployed enjoy  $b$ ; receive a job offer with Poisson intensity  $p$  at a distance  $\rho'$  from the distribution  $F_J(\rho)$ . They have the option of rejecting the offer if the distance is too far, but also have the option to move instantaneously if they find a residence in the stock of existing vacant units. To the extent that  $\rho'$  and  $\rho''$  are independent draws, this means that there is a distribution,  $F$ , combining  $F_J$  and  $G_S$  such that the integral terms can be rewritten as  $\int \max[U, E(\rho)] dF(\rho)$  where  $\rho$  will simply be the min of the two draws:  $\rho = \text{Min}(\rho', \rho'')$ .<sup>4</sup>

### 3.3 Reservation strategies

We now derive the job acceptance and moving strategies of individuals. Observe that  $E$  is downward sloping in  $\rho$ , with slope

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r + s + \lambda_H P_W + \delta P_\delta}, \quad (3)$$

where  $P_W$  is the probability of moving conditional on receiving a housing offer and  $P_\delta$  is the probability of moving conditional on receiving a family shock. Note that  $0 < P_W < 1$  and  $0 < P_\delta < 1$  and possibly depends on  $\rho$ . The function  $E(\rho)$  is monotonic so that there exists a well-defined reservation strategy for the employed, with a reservation distance denoted by  $\rho^E(\rho)$ , below which a *housing offer* is accepted. In addition, there is a reservation strategy  $\rho^\delta(\rho)$ , below which an agent relocates when hit by a family shock. Note that there is state-dependence in the reservation strategy of the employed,  $\rho^E(\rho)$ , with presumably  $d\rho^E/d\rho > 0$ . The further away the tenants live from their job, the less likely they will be to reject a housing offer, therefore,

$$P_W = G_N(\rho^E(\rho)).$$

Similarly, for a family shock, we have

$$P_\delta = G_S(\rho^E(\rho)).$$

We can now rewrite Eq. 1 and Eq. 2 as:

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<sup>4</sup>We prove in the appendix that  $1 - F(\rho) = (1 - F_J(\rho))(1 - G_S(\rho))$ .



$$(r+s)E(\rho) = w - \tau\rho + sU + \lambda_H \int_0^\rho [E(\rho') - E(\rho)] dG_N(\rho') \quad (4)$$

$$+ \delta \int_0^{\rho^u} [E(\rho') - E(\rho)] dG_S(\rho'') + \delta \int_{\rho^u}^{+\infty} [U - E(\rho)] dG_S(\rho'')$$

$$(r+p)U = b + p \int_0^{\rho^u} [E(\rho')] dF(\rho') + pU(1 - F(\rho^u)) \quad (5)$$

Note that in addition to the exogenous separation from the match,  $s$ , there is an additional source of entry into unemployment. It arises due to workers receiving a family shock, redrawing in the vacant housing stock distribution, but are unable to find a sufficiently close dwelling to the current job and (optimally) quit. That is,  $s' = s + \delta(1 - G_S(\rho^u))$ . The slope of  $E(\rho)$  is now written as

$$\frac{\partial E}{\partial \rho} = \frac{-\tau}{r+s+\lambda_H G_N(\rho) + \delta G_S(\rho^u)}. \quad (6)$$

Next, in the absence of relocation costs (this case is studied in the Appendix), tenants move as soon as they get a dwelling offer closer to their current one, implying

$$\rho^E(\rho) = \rho.$$

Denote by  $\rho^U$  the reservation strategy for the unemployed, below which any *job offer* is accepted, it is defined by

$$E(\rho^U) = U.$$

Using the fact that  $E(\rho^U) = U$ ,

$$b + p \int_0^{\rho^u} [E(\rho') - U] dF(\rho')$$

$$= w - \tau\rho^U + \lambda_H \int_0^{\rho^U} [E(\rho') - U] dG_N(\rho') + \delta \int_0^{\rho^u} [E(\rho'') - U] dG_S(\rho''). \quad (7)$$

Integrating [Eq. 7](#) by parts gives:

$$\rho^U = \frac{w-b}{\tau} + \int_0^{\rho^U} \frac{\lambda_H G_N(\rho) + \delta G_S(\rho) - pF(\rho)}{r+s+\lambda_H G_N(\rho) + \delta G_S(\rho^U)} d\rho. \quad (8)$$

The determination of  $\rho^U$  is shown in Figure 1.

With this specification the model is quite parsimonious, since a single variable,  $\rho$ , determines several dimensions of choice:

1. *job acceptance*:  $F(\rho^U)$  ;
2. *residential mobility rate*:  $\int \lambda_H G_N(\rho)$  over the distribution of commute distance of employed workers  $d\Phi$ ;
3. *quit rate due to family shock*:  $\delta(1 - G_S(\rho^U))$ .

Figure 1: Determination of  $\rho_u$

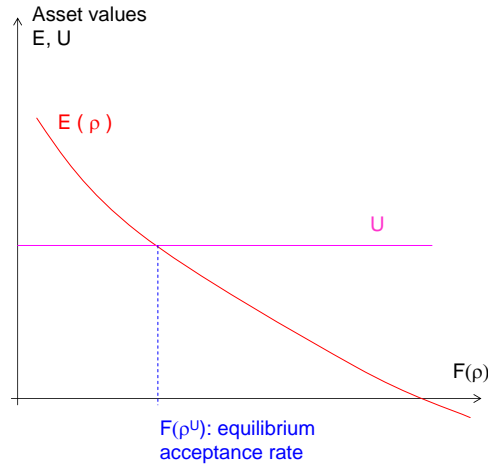


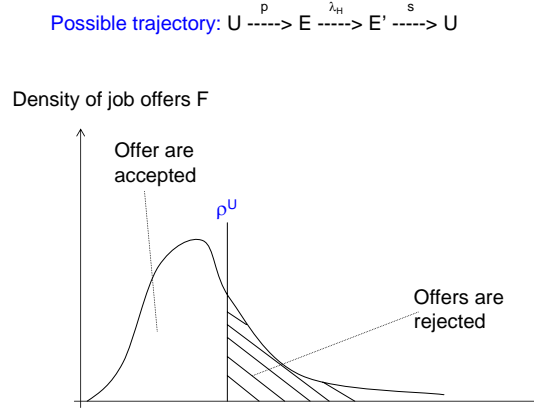
Figure 2 provides an overview of the transitions and the acceptance decisions of the unemployed: jobs closer than distance  $\rho^U$  are accepted and jobs farther are rejected by workers.

### 3.4 Free entry

Assuming free entry of firms, and defining  $\theta$  as the tightness of the labor market,  $\theta = \frac{V}{U}$ , we have

$$\frac{y - w}{r + s'} = \frac{c}{q(\theta)P_F}$$

Figure 2: Transitions and Acceptance Decision

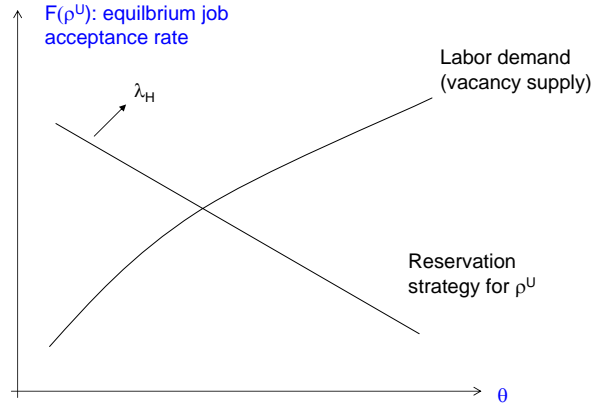


where  $P_F$  is the rate of acceptance of job offers by the unemployed, as expected from the viewpoint of the firm. We assume, still by symmetry, that the distribution of contacts between the firm and unemployed workers has distribution  $F(\rho)$ , so that  $P_F = F(\rho^U)$ . This generates a positive link between  $\theta$  and  $\rho^U$  since  $q'(\theta) < 0$ , characterized by:

$$q(\theta)F(\rho^U) = \frac{c(r + s')}{y - w}. \quad (9)$$

The intuition is quite simple. The firm's iso-profit curve at the entry stage depends negatively on both  $\theta$  (as a higher  $\theta$  implies more competition between the firm and the worker) and on  $\rho^U$  (as more of their offers will be rejected because of distance). The zero-profit condition thus implies a positive link between  $\theta$  and  $\rho^U$ . Note that this relation is independent of  $\lambda_H$ . On the other hand,  $\rho^U$  is determined through (Eq. 8). It is decreasing in  $p(\theta)$  and thus in  $\theta$ , as can be seen in (Eq. 11). When there are more job offers (higher  $\theta$ ) workers can wait for offers closer to their current residential location; they are pickier. The two curves are represented in  $(\rho^U, \theta)$  space in Figure 3.

Figure 3: Vacancies and Reservation Strategy



### 3.5 Unemployment and the Beveridge curve

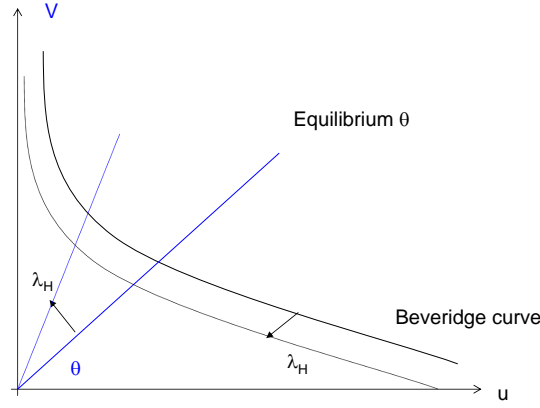
To sum up, the unemployment effect of an increase in  $\lambda_H$  is twofold: it raises the acceptance rate of job offers and raises  $\theta$ , thus increasing job offers by firm. In terms of a Beveridge curve representation, this is like an inward shift of the curve (less structural mismatch) and a counter-clockwise rotation of  $\theta$ .

Letting  $p = p(\theta) = \theta q(\theta)$ , the unemployment rate is given as

$$u = \frac{s'}{s' + p(\theta)F(\rho^U)}. \quad (10)$$

A graphical representation of this result in the Beveridge space is shown in [Figure 4](#).

Figure 4: Beveridge Curve



## 4 Housing frictions and mobility

To determine how housing frictions affect the decisions of workers and firms, it is necessary to determine how the reservation distance is affected. Fully differentiating  $\rho^u$  gives

$$d\rho^u \left( \frac{r+s+pF(\rho)}{r+s+\lambda_H G_N(\rho) + \delta G_S(\rho^u)} \right) = \left( \int_0^{\rho^u} \frac{G_N(\rho)(r+s) + pF(\rho)G_N(\rho)}{[r+s+\lambda_H G_N(\rho) + \delta G_S(\rho^u)]^2} d\rho \right) d\lambda_H \quad (11)$$

$$- \left( \int_0^{\rho^u} \frac{F(\rho)d\rho}{r+s+\lambda_H G_N(\rho) + \delta G_S(\rho^u)} \right) dp + d \left( \frac{w-b}{\tau} \right)$$

Equation (11) indicates that  $\rho^u$  depends positively on  $\lambda_H$ , negatively on  $p$  and positively on  $w-b$ . An increase in  $\lambda_H$  (more housing offers) will shift the curve in Figure 1 upward, raising  $\rho^u$  and thus the acceptance rate of the unemployed. The intuition is simply that they take the job offer hoping to find a better location in the near future because housing offers arrive very frequently. Hence,

**Proposition 1** *An increase in  $\lambda_H$  makes the unemployed less choosy about jobs:  $\partial \rho^u / \partial \lambda_H > 0$ .*

Next, differentiating (9) and using Proposition 1, we obtain:

**Proposition 2** *More housing frictions reduce job creation:  $\partial \theta / \partial \lambda_H > 0$ .*

This is an indirect effect caused by more job creation through the higher job acceptance rate of workers. Another interpretation of this effect is that firms are reluctant to create jobs where workers have no place to live.

**Proposition 3** : *An increase in  $\lambda_H$  has three effects on unemployment:*

- *it reduces the quit rate from a  $\delta$ -shock,*
- *it raises the job acceptance rate of workers (through a higher threshold  $\rho^U$ ),*
- *it raises  $\theta$  (Proposition 2) and thus job creation.*

The proof is found by first differentiating  $u$ :

$$\begin{aligned} \frac{du}{d\lambda_H} &= \frac{-\delta g_S(\rho^U) \omega_\rho (s' + p(\theta) F(\rho^U)) - s' [-\delta g_S(\rho^U) \omega_\rho + p(\theta) f(\rho^U) \omega_\rho + p'(\theta) F(\rho^U) \omega_\theta]}{[s' + p(\theta) F(\rho^U)]^2} \\ &= \frac{-\delta g_S(\rho^U) p(\theta) F(\rho^U) \omega_\rho - s' [p(\theta) f(\rho^U) \omega_\rho + p'(\theta) F(\rho^U) \omega_\theta]}{[s' + p(\theta) F(\rho^U)]^2}. \end{aligned}$$

Next,

$$\frac{ds'}{d\lambda_H} = -\delta g_S(\rho^U) \frac{\partial \rho^U}{\partial \lambda_H} = -\delta g_S(\rho^U) \omega_\rho < 0,$$

where  $\omega_\rho$  is simply a convenient notation for the partial derivative of  $\rho^U$  with respect to  $\lambda_H$ . Finally, note that

$$\begin{aligned} \frac{d[p(\theta) F(\rho^U)]}{d\lambda_H} &= p'(\theta) F(\rho^U) \frac{\partial \theta}{\partial \lambda_H} + p(\theta) f(\rho^U) \frac{\partial \rho^U}{\partial \lambda_H} \\ &= p(\theta) f(\rho^U) \omega_\rho + p'(\theta) F(\rho^U) \omega_\theta > 0 \end{aligned}$$

where  $\omega_\theta$  is another convenient notation for the partial derivative of  $\theta$  with respect to  $\lambda_H$ .

## 4.1 Mobility rate

We can now investigate the mobility rate. Let  $\Phi(\rho)$  be the steady-state distribution of employed workers living at a distance closer than  $\rho$ .  $\Phi$  is governed by the following law of motion:

$$(1-u)\frac{\partial\Phi(\rho)}{\partial t} = u p F(\rho) + (1-u)\lambda_H G_N(\rho)[1-\Phi(\rho)] + (1-u)\delta G_S(\rho)[1-\Phi(\rho)] - (1-u)\Phi(\rho)s. \quad (12)$$

Eq. 12 states that the number of people residing in a location closer to a given  $\rho$  from their job changes (either positively or negatively) due to:

- (+) to the unemployed  $u$  receiving a job offer at rate  $p$  with a distance closer to  $\rho$  with probability  $F(\rho)$
- (+) due to the employed  $1-u$  who are further away from the current distance  $\rho$  (a fraction  $1-\Phi(\rho)$ ) and receive an offer on the housing market with intensity  $\lambda_W$  closer to  $\rho$  with probability  $G_N(\rho)$ ;
- (+) due to the employed  $1-u$  who are further away from the current distance  $\rho$  (a fraction  $1-\Phi(\rho)$ ) and need to relocate in the instant due to a demographic shock  $\delta$  and happen to sample a new distance smaller than  $\rho$  with probability  $G_S(\rho)$ ;
- (-) there are  $s$  exits per unit of time due to people losing their job.

In steady state and for all  $\rho < \rho^u$

$$\Phi(\rho) = \frac{pF(\rho)\frac{u}{1-u} + \lambda_H G_N(\rho) + \delta G_S(\rho)}{s + \lambda_H G_N(\rho) + \delta G_S(\rho)} \quad (13)$$

$$= \frac{\frac{F(\rho)}{F(\rho^u)}s + \lambda_H G_N(\rho) + \delta G_S(\rho)}{s + \lambda_H G_N(\rho) + \delta G_S(\rho)} \leq 1 \quad (14)$$

with equality in  $\rho = \rho^U$ :  $\Phi(\rho^U) = 1$  as no unemployed ever accept a job offer further away from a job than  $\rho^U$  and then, no employed worker subsequently moves away from his/her job, only closer from the current job. The second line above is obtained in replacing  $u$  by its steady-state

expression in (10). We see that, as expected, all workers have a house at a distance below  $\rho^U$  since they won't move for a house at a longer distance than that of the job they eventually accepted. Now, two special cases:

- if firms receive no offer ( $\lambda_H G_N = \delta G_S(\rho) = 0$ ), the distribution of distance of the employed  $\Phi$  converges to  $\frac{F(\rho)}{F(\rho^U)}$ , that is, the distribution of  $\rho$  conditional on a job being accepted
- if jobs are not destroyed,  $s = 0$ , the distribution of distance of the employed  $\Phi$  converges to 0, that is,  $\Phi(\rho) = 1$  for all  $\rho > 0$ : all workers eventually find a house infinitely close to their job.

We can now write the various mobility rates of the different sub-samples of the population. Denote by  $M_K^S$  the number of movers of status  $S=(U,E)$  (unemployed, employed) and for a reason  $K=(J,D)$  (job-related or divorce-related), we have:

1. job-related mobility of the employed: they have a job and relocate once they sample a better housing location:

$$M_J^E = (1-u)\lambda_H \int_0^{\rho^U} G_N(\rho) d\Phi(\rho) \quad (15)$$

$$= (1-u)\lambda_H \left[ G_N(\rho^U) - \int_0^{\rho^U} g_N(\rho) \Phi(\rho) d\rho \right] \quad (16)$$

where the second line is found from integrating by parts and noticing that  $\Phi(\rho^U) = 1$ .

2. job-related mobility of the unemployed: they have a job, accept it with probability  $G(\rho^U)$  and may relocate if they drew a location from  $G_S$  closer from their current  $\rho$ :

$$M_J^U = up \int_0^{\rho^U} G_S(\rho) dF_J(\rho)$$



3. demography-related mobility:

$$\begin{aligned} M_D^U &= u\delta \\ M_D^E &= (1-u)\delta \\ M_D^{E+U} &= \delta \end{aligned}$$

Note that in  $M_D^E$ , a part of workers have to quit their job (a fraction  $1 - G_S(\rho^U)$  to be precise).

## 4.2 Special Case: $\lambda_H \rightarrow \infty$ .

In this case, the model collapses to  $\Phi(\rho) = 1$ , meaning that all workers will be located infinitely close to their job. The job acceptance decision is indeterminate since we now have

$$\rho^U = \frac{w-b}{\tau} + \int_0^{\rho^U} d\rho$$

The economics is simple though: if  $w > b$ , all job offers are accepted, meaning that  $\rho^U$  goes to infinity. Therefore, we obtain the standard Pissarides value for tightness:  $q(\theta^*) = \frac{c(r+s')}{y-w}$  with  $\theta^* > 1$ . In addition,

$$\frac{q(\theta^*)}{q(1)} = F(\rho^U) < 1$$

and therefore, with  $q(\theta + d\theta) = q(\theta) + q'(\theta)d\theta = q(\theta)(1 + \eta_q d\theta/\theta)$ , we have

$$\frac{q(\theta^*)}{q(1)} = 1 + \eta_q d\theta/\theta = F(\rho^U)$$

hence

$$\frac{d\theta}{\theta} = \theta^* - 1 = \frac{1 - F(\rho^U)}{-\eta_q} > 0$$

We can decompose unemployment into two parts:

$$u = \frac{s}{s+p}$$

implying

$$\frac{du}{u} = -\frac{dp}{s+p} = -(1-u)\frac{dp}{p} = -(1-u)\frac{d\theta}{\theta}\eta_p$$

Thus

$$\frac{du}{u} = -(1-u) \frac{1-F(\rho^U)}{-\eta_q} \eta_p$$

With  $\eta_q = -0.5$  and  $\eta_p = 0.5$ , we have that the variation in unemployment purely due to imperfect housing markets is of the order of magnitude of the fraction of rejected offers  $1 - F(\rho^U)$ .

## 5 Calibration

The time period is one month. We set the interest rate,  $r$ , to 0.0033, corresponding to an annual rate of 0.04. We calibrate to the mobility rate of the employed, which was 17% between March 1999 and March 2000, so the target is (17/12)%. The number for the employed that move comes from the Bureau of the Census.<sup>5</sup> Of the roughly 31 million persons who moved during that year, 22.3 million of them were employed, 1.5 million unemployed and 7.8 million out of the labor force.

We set  $\delta = 0$  (no demographic shock) and the stock of vacant housing  $G_S = 0$ . Therefore, we are left with two distributions:  $G_N$ , new housing offers and  $F$ , job offers. We assume that they are represented by the same exponential function with parameter  $\alpha$ :  $F = G_N = 1 - e^{-\alpha p}$ .

The program finds the parameters of the model such that:

- We match the unemployment rate in the U.S., which averaged about 4.2% between March 1999 and March 2000, and the job hiring rate,  $p = 1/2.4$  monthly, meaning an average duration of unemployment of 2.4 months; this imposes a value for  $s$  given that  $u = s/(s + p)$  when  $\delta = 0$ :  $s = p(u/(1 - u)) = 0.0183$ .
- We set  $p(\theta) = A\theta^{0.5}$ . If we fix

$$\theta = 1, \tag{17}$$

we obtain  $A = 0.5$ . Together with the free-entry condition, (Eq. 9), this fixes a value for recruiting costs  $c$ : we will set  $y = 1$ ,  $w = 0.8$ ,  $q(\theta) = A\theta^{-0.5}$ .

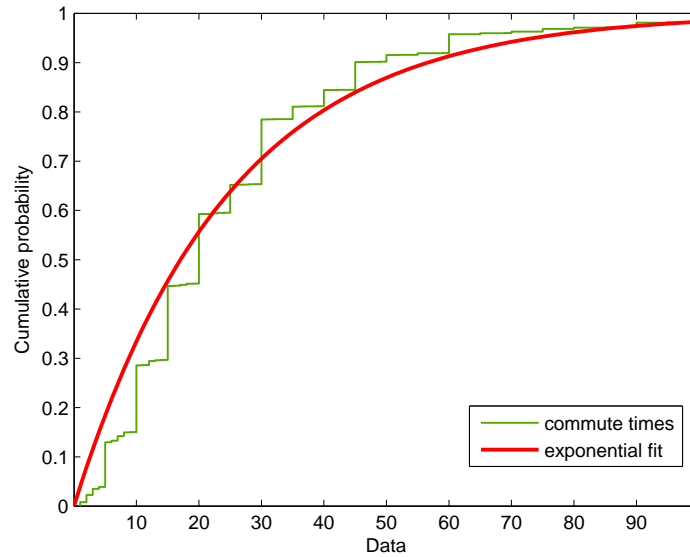
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<sup>5</sup>Why People Move: Exploring the March 2000 Current Population Survey, P23-204, Bureau of the Census, May, 2001.

- We match the mobility rate to a target value of 17% annually (17/12% monthly) with (Eq. 16). The program finds the values of  $\alpha$  and  $\lambda_H$  which lead to the right value of mobility given  $\rho^U$  obtained from (Eq. 8).

To find a value for  $\alpha$  we use data on the distribution of commute times from the Census 2000. Figure 5 shows that the distribution is close to an exponential. The mean commute time is about 24 minutes. Let  $\rho$  be the distance expressed in monthly commuting time. We then have  $F(\phi\rho)$ , where  $\phi$  is a scale parameter reflecting the change in units since the data is for one-way commuting time. Assuming that people commute twice a day and 20 days a month,  $\phi = 1/40$ . When estimating the slope of  $\log(1-F(\phi\rho))$  which, under the assumption of an exponential distribution with coefficient  $\alpha$ , is  $\alpha\phi$ , we obtain 0.0548 (or 0.065 if we drop commute times of zero) as the estimate. It follows that we should fix a value of  $\alpha = 0.0548 * 40 = 2.19$  (or  $2.6=0.065*40$  under the second estimate).

Figure 5: Distribution of Commute Times



Next, we repeat the above exercise using European values. In particular, we set the mobility target to one third of its US value, set the unemployment duration to one third of the US value, set unemployment benefits to 0.55 instead of 0.25, finally set the unemployment target to 10%.

## 5.1 Findings

The findings for the benchmark economy are given in [Table 5](#) where each column represents a different arrival rate of housing offers. The value of the scale parameter of the matching function,  $A$ , is 0.4657, and the recruiting cost,  $c$ , is 3.858. These values do not change as  $\lambda_h$  changes.

Table 5: U.S. Calibration

	$\lambda_h = 0.0533$	$2 * \lambda_h$	$3 * \lambda_h$	$10 * \lambda_h$
$\theta$	1.0000	1.1522	1.2148	1.2493
$\rho^U$	1.0278	1.4738	1.9521	5.6238
$F^U$	0.8947	0.9603	0.9861	1.0000
rej. rate	0.1053	0.0397	0.0139	4.5E-6
unemployment	0.042	0.0367	0.0348	0.0339
mobility	0.0136	0.0215	0.0269	0.0445

Reducing frictions in the housing market by increasing the value of  $\lambda_h$  results in only a modest gain to improving mobility (that is, multiplying the speed of arrival of housing offers by 2, 3, or 10). The model converges rapidly to the "frictionless" (no housing frictions) rate of unemployment which is 0.034 instead of 0.042. In short, housing frictions increase unemployment by a little more than half a percentage point.

We now calibrate the model to a "typical" European situation where mobility is a third of that of the US, and unemployment duration is also three times higher than in the US. The findings are shown in [Table 6](#). We obtain a lower value of  $\lambda_h$ , 0.0153, so that there are nearly twice the housing offers in the U.S. as compared to Europe,  $\lambda_h^{US}/\lambda_h^{EU} = 3.48$ . We find  $A = 0.1836$  that is, the scale parameter of the matching function is about one third lower in Europe. The program also finds lower vacancy costs in Europe ( $c = 1.4803$ ) which implies that total setup costs of firms are similar to that of the US: Hiring costs are equal to  $c/q = 3.8579/0.4657 = 8.28$  in the US, and are equal to  $1.4803/0.1836 = 8.06$  in Europe.

In the European case, since the housing market was calibrated with a lower mobility rate,  $\lambda_h$  is smaller. Hence, workers reject more job offers since they find it more difficult to move subsequently. The rejection rate is 0.21. Further, reducing frictions in the housing market, that is,

Table 6: European Calibration

	$\lambda_h = 0.0153$	$2 * \lambda_h$	$3 * \lambda_h$	$10 * \lambda_h$
$\theta$	1.000	1.1436	1.2647	1.6684
$\rho^U$	0.6446	0.7554	0.8678	1.7196
$F^U$	0.7563	0.8088	0.8505	0.9769
rej. rate	0.2437	0.1912	0.1495	0.0231
unemployment	0.1000	0.0886	0.0808	0.0624
mobility	0.0042	0.0078	0.0108	0.0234

Europe	Benchmark	$\lambda_h^{US}$	$A^{US}$	$A^{US}, c^{US}$	$b^{US}$
$\theta$	1.000	1.3153	2.4385	0.6122	1.4177
$\rho^U$	0.6446	0.9224	0.2862	0.4278	1.0535
$F^U$	0.7563	0.8673	0.4657	0.6082	0.9005
rej. rate	0.2437	0.1327	0.5343	0.3918	0.0995
unemployment	0.1000	0.0779	0.0436	0.0651	0.0727
mobility	0.0042	0.0121	0.0030	0.0037	0.0051

raising  $\lambda_h$ , now delivers a quite significant improvement in the labor market as unemployment is reduced by approximately 2 percentage points.

### 5.1.1 More counterfactuals for Europe

A last exercise we can run is to start from European values and change the various parameters to US levels: in order,  $\lambda_h$ ,  $A$ ,  $A$  and  $c$  together and  $b$ .

We can see that the Europe calibrated economy could gain almost as much in raising the efficiency of the housing market to US levels as in cutting unemployment benefits to the (very low) US levels. The most efficient margin of action is however to raise the effectiveness of matching in the labor market in Europe.

We now allow for a demographic shock.

For the U.S, we find  $c = 4.6303$  and  $A = 0.4182$ .

The calibration for Europe finds  $c = 1.6440$  and  $A = 0.1389$ .

Table 7: U.S. Calibration,  $\delta > 0$ 

	$\lambda_h = 0.0048$	$2 * \lambda_h$	$3 * \lambda_h$	$10 * \lambda_h$
$\theta$	1.0000	1.0039	1.0057	1.0073
$\rho^U$	0.4109	0.4669	0.5244	0.9403
$F^U$	0.9964	0.9983	0.9992	1.0000
rej. rate	0.0036	0.0017	7.6E-4	2.6E-6
unemployment	0.0420	0.0407	0.0397	0.0356
mobility	0.0018	0.0035	0.0049	0.0123

Table 8: Europe,  $\delta > 0$ 

	$\lambda_h = 0.0143$	$2 * \lambda_h$	$3 * \lambda_h$	$10 * \lambda_h$
$\theta$	1.0000	1.0003	1.0003	1.0003
$\rho^U$	0.6506	0.8806	1.1113	2.7180
$F^U$	0.9999	1.0000	1.0000	1.0000
rej. rate	1.3E-4	5.8E-6	2.5E-7	1.1E-16
unemployment	0.1000	0.0949	0.0921	0.0890
mobility	0.0047	0.0081	0.0107	0.0212

### 5.1.2 Additional Experiments

The model also displays complementarities. The relationship between  $\lambda_H$  and  $u$  varies depending on the value of the utility from not working,  $b$ . Alternatively it is possible to vary  $b$  and look at the link between the rejection rate,  $(1 - F_U)$ , and unemployment,  $u$ . [Figure 6](#) and [Figure 7](#) show the findings of these experiments.

## 6 What is $\lambda_H$ ?

### 6.1 A model

Landlords post vacancies and screen applicants. They offer a lease to the “best applicants”, in a sense defined below. In case of a default on the rent by the tenant, however, landlords incur a loss due to the length of litigation and eviction procedures. The asset value of owning a dwelling with

Figure 6: Unemployment and Housing Market Frictions

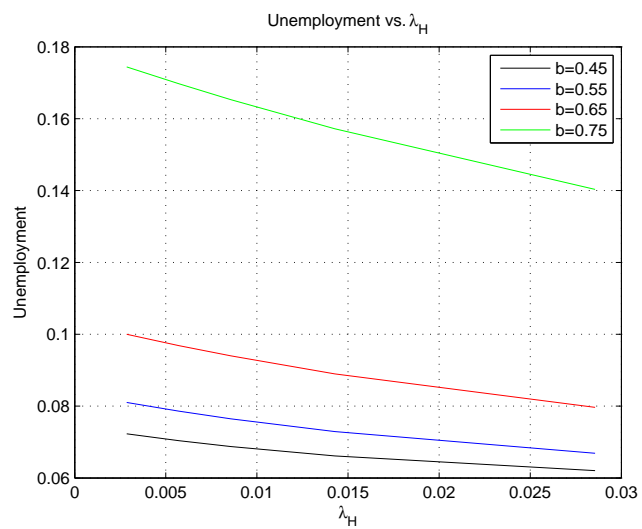
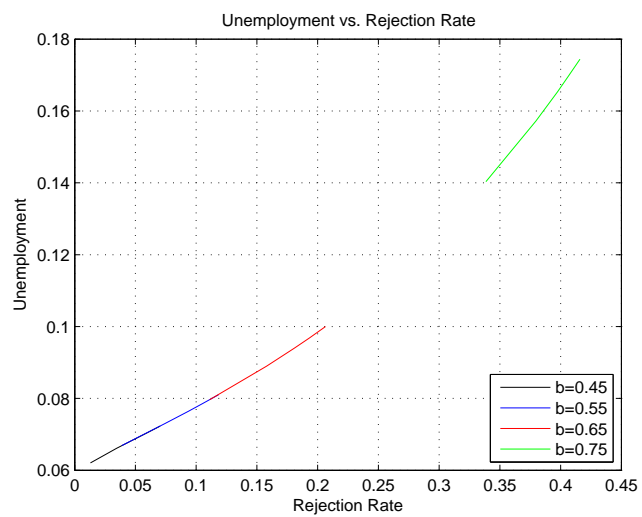


Figure 7: Unemployment and the Rejection Rate



a tenant defaulting on the rent is denoted by  $\Lambda$ . Therefore,  $\Lambda$  will decline with more regulation in the rental housing market, due to the length of the procedures to recover the unpaid rents and the dwelling.

Potential tenants (applicants) are all ex-ante homogenous but ex-post may represent a default risk for the landlord. More precisely, at the time of the contact between the applicant, the landlord gets a random signal on the tenant and postulates that the particular applicant has a specific default rate  $\delta_i$  (a Poisson rate in continuous time). It follows that for such a tenant, the value of a filled vacancy for the landlord is:

$$rH_F = R + \delta_i(\Lambda - H_F)$$

where  $R$  is the rent and  $H_F$  is the value of a filled vacancy.<sup>6</sup> We therefore have:

$$H_F = \frac{R + \delta_i \Lambda}{r + \delta_i} \quad (18)$$

The derivative of  $H_F$  with respect to  $\delta_i$  is given as  $\Lambda r - R$ , which is negative since the landlord's value of a default,  $\Lambda$ , cannot be higher than the capitalized value of the rent  $R/\delta_i$ . Therefore,  $H_F$  is decreasing in  $\delta_i$ , from  $R/r$  to  $\Lambda$ . This implies that there will be a well defined reservation strategy by landlords given the signal they receive.

To derive this reservation strategy, we denote by  $H_V$  the value of a vacant housing unit prior to screening. This value is exogenous and is given, in the long-run, by the cost of construction of new housing units. Therefore, it is independent of rental regulations and in particular of  $\Lambda$ . At the time of a contact with a tenant, landlords decide to offer a lease if the perceived value of default  $\delta_i$  is below a cut-off value  $\bar{\delta}$  with

$$\begin{aligned} \frac{R + \bar{\delta} \Lambda}{r + \bar{\delta}} &= H_V \\ \text{or } \bar{\delta} &= \frac{R - r\Lambda}{H_V - \Lambda} \end{aligned}$$

---

<sup>6</sup>Recall that we assumed in the model that the flow of service of the dwelling to the tenant was exactly compensating the rent so we did not need to include the rent in the Bellman equations of workers. We also assume that workers do not benefit from defaulting on the rent: in case of default, they may have a disutility exerted by landlords or their lawyers so that, after default, the value of being in a dwelling is not higher than it was when paying. Therefore, the expression for the Belmann equations of tenants derived previously is unchanged .



Note that  $\bar{\delta}$  is increasing in  $\Lambda$ : the higher is  $\Lambda$ , the easier it is to accept a tenant since the risk of default is lower.

Hence, the screening rate,  $\alpha$ , of tenants is  $\text{prob}(\delta_i > \bar{\delta})$ . Denoting by  $L(\delta_i)$  the c.d.f. of default-rates, we have

$$\alpha = 1 - L\left(\frac{R - r\Lambda}{H_V - \Lambda}\right)$$

The screening rate is therefore increasing when  $\Lambda$  is lower, that is, when rental housing market regulations are higher.

Finally, denote by  $\phi$  the Poisson rate at which tenants receive dwelling offers. We assume that this contact rate is exogenous. It then follows that

$$\lambda_H = \phi \alpha$$

Therefore:

**Proposition 4:**  *$\lambda_H$  is decreasing in the amount of housing market regulations.*

We will now investigate in the data whether there is indeed a link between mobility, unemployment and housing market regulations.

## 6.2 Within-EU mobility differences in rental housing market regulations.

Europe has so far been considered as a homogenous block. However, there are fairly large geographical mobility differences between countries. Scandinavian countries report mobility rates which are close to the US rates, while Southern European countries exhibit the lowest of all rates. Part of the differences may be due to socio-cultural factors, such as family attachment and to differences in regions leading to higher mobility costs. However, our model provides a natural starting point for an analysis of mobility.

We first report indices of rental housing market regulations collected by Djankov et al. (2002). This is a composite index based on the difficulty to evict a tenant, reflecting the complexity and the length of the procedure at various stages (pre-trial, process of trial, execution of the court decision). We report the numbers for 13 European countries and contrast the indices with mobility rates and

with average unemployment rates.

	Mobility rate outside the area within 3 yrs.	Housing market regulation index	Average unemployment rate 1995-2001
Denmark	0.054	3.6	5.3
Netherlands	0.029	3	4.2
Belgium	0.013	3.17	8.5
France	0.042	3.6	10.4
Ireland	0.010	3.2	7.9
Italy	0.011	4.24	10.7
Greece	0.011	4.31	10.5
Spain	0.009	4.81	14.5
Portugal	0.007	4.54	5.6
Austria	0.015	3.62	4.02
Finland	0.058	2.53	11.9
Germany	0.021	3.76	8.3
UK	0.072	2.22	6.5

Source of data: mobility figures based on ECHP. Regulation indices based on Djankov et al. (2002).

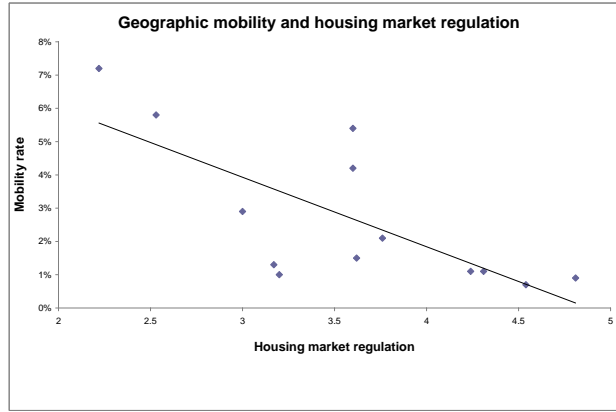
Unemployment based on Eurostat statistics.

Figure 8 displays a strong, negative correlation between housing market regulations and mobility rate across European countries. This is a good illustration of Proposition 4: higher regulation lower  $\lambda_H$  which in turn reduces mobility.

The strong and negative correlation suggests that indeed, our model has the potential to explain mobility patterns using the parameter  $\lambda_H$ .

The next question is whether regulations can explain, per se, a large share of unemployment differences? The answer lies in Figure 9 the next chart and appears to be yes to some extent, at least based on this scatter plot. The correlation coefficient between the series is 0.31 and the R-sq

Figure 8: Housing Regulations and Mobility



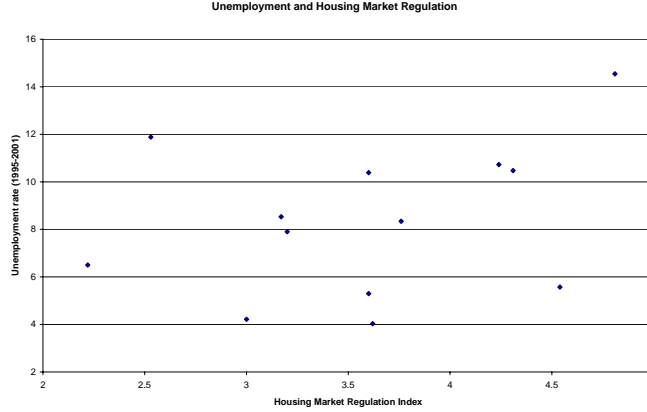
is 0.09.

## 7 Concluding comments

In this paper we have taken seriously the idea that labor market frictions, and in particular the reservation strategies of unemployed workers when they decide whether or not to accept a job offer, depends strongly on the functioning of the housing market. This interconnection between two frictional markets (housing and labor) is, per se, an interesting problem and deserves analysis.

This paper has offered such a model, based on decisions to accept or reject a job offer, given the commuting distance to jobs. The model is relatively parsimonious, thanks to simplifying assumptions such as the isotropy of space, an unrealistic assumption but which conveniently provides closed form solutions and makes it possible to explain quit, job acceptance and geographic mobility decisions with a decision rule on a single dimension. Our model is yet rich enough and can be extended to deal with new issues such as discrimination in the housing market, mobility allowances or “moving toward opportunity” schemes, spatial mismatch issues and so on, as in the urban economics literature.

Figure 9: Housing Regulations and Unemployment



We have attempted to explain differences in mobility rates between Europe and the US. A calibration exercise of “Europe” and the US is able to account for differences in unemployment and mobility thanks to a parameter which captures the speed of arrival of housing offers to households.

In our calibration, we find that housing frictions account for 0.6 percentage points of US unemployment and at most 2 percentage points (out of 10) of European unemployment. However, policy inducing mobility might never be able to reduce unemployment by 2 percentage points. Taking the  $\lambda_H$  parameter to infinity is not an available option, given the nature of frictions. Tripling the European value of  $\lambda_H$  to reach the US value is already an ambitious objective. Doing so reduces unemployment in Europe by 0.6 percentage points, which is large already, but suggests that after all, housing market imperfections may not be a major cause of aggregate unemployment. Of course, the lack of mobility is a more specific problem for outsiders in the labor market (young workers in particular) and relatively low effects in aggregate could hide massive effects for some categories of the population.

Finally, this conclusion of large effects of  $\lambda_H$  on mobility and positive but not massive effects on unemployment is coherent with intra-European differences in housing market regulations. Based on indices of rental housing market regulation, we find a strong negative correlation of the indices

with geographical mobility and a small and positive correlation with unemployment rates.

Future work should attempt to enrich the model to introduce more specific urban features such as anisotropy of space and the existence of centers in cities and suburbs, as well as different groups of the labor force. Our work is a first step in integrating housing and labor markets in a coherent macroeconomic model.

## 8 Appendix

### 8.1 Proof of the determination of $\rho^U$

Rewrite the Bellman equations as:

$$\begin{aligned} rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int_0^{\rho^E(\rho)} (E' - E) dG_N(\rho') \\ rU &= b + p \int_0^{\rho^U} (E' - U) dF(\rho') \end{aligned}$$

Take the value of  $E$  in  $\rho^U$ , we have:

$$\begin{aligned} rE(\rho^U) &= w - \tau\rho^U + s(U - E(\rho^U)) + \lambda_H \int_0^{\rho^U} (E' - E(\rho^U)) dG_N(\rho') \\ &= rU = b + p \int_0^{\rho^U} (E' - U) dF(\rho') \end{aligned}$$

so

$$\rho^U = \frac{\lambda_H}{\tau} \int_0^{\rho^U} (E' - U) dG_N(\rho') - \frac{p}{\tau} \int_0^{\rho^U} (E' - U) dF(\rho') + \frac{w - b}{\tau} \quad (19)$$

This equation simplifies a bit after integrating by parts. Noting that

$$\begin{aligned} \frac{\partial E}{\partial \rho} &= \frac{-\tau}{r + s + \lambda_H G_N(\rho)} \\ \int_0^{\rho^U} (E' - U) dH(\rho') &= \int_0^{\rho^U} \frac{\tau H(\rho)}{r + s + \lambda_H G_N(\rho)} d\rho \end{aligned}$$

where  $H$  is any distribution such that  $H(0) = 0$ , we can thus rewrite  $\rho^U$  as in the text.

## 8.2 Link between $F$ , $F_J$ and $G_S$

We start from the integrand  $A = \int \int \max[U, E(\rho'), E(\rho'')] dF_J(\rho') dG_S(\rho'')$ . Noting that

$$\frac{dE}{d\rho} = \frac{-\tau}{r+s+\lambda_H G_N(\rho)}$$

we can rewrite  $A$  as

$$\begin{aligned} A &= \int \int \{I[\rho' > \rho_U] I[\rho'' > \rho_U] U \\ &+ I[\rho' < \rho''] I[\rho' < \rho^U] E(\rho') \\ &+ I[\rho'' < \rho'] I[\rho'' < \rho^U] E(\rho'')\} \\ & dF_J(\rho') dG_S(\rho'') \end{aligned}$$

or

$$\begin{aligned} A &= U \int_{\rho^U} dF_J(\rho') \int_{\rho^U} dG_S(\rho'') \\ &+ \int_0^{\rho^U} E(\rho') \left( \int_{\rho'} dG_S(\rho'') \right) dF_J(\rho') \\ &+ \int_0^{\rho^U} E(\rho'') \left( \int_{\rho''} dF_J(\rho') \right) dG_S(\rho'') \end{aligned}$$

or

$$\begin{aligned} A &= U(1 - F_J(\rho^U))(1 - G_S(\rho^U)) \\ &+ \int_0^{\rho^U} E(\rho') (1 - G_S(\rho')) dF_J(\rho') \\ &+ \int_0^{\rho^U} E(\rho'') (1 - F_J(\rho'')) dG_S(\rho'') \end{aligned}$$

Denote by  $F = 1 - (1 - F_J(\rho))(1 - G_S(\rho))$ . We have that

$$dF = (1 - F_J) dG_S + (1 - G_S) dF_J$$

and we can thus rewrite

$$\begin{aligned}
A &= U(1 - F_J(\rho^U))(1 - G_S(\rho^U)) \\
&\quad + \int_0^{\rho^U} E(\rho) dF(\rho) \\
&= \int \max(U, E(\rho)) dF(\rho)
\end{aligned}$$

The last equality comes from the observation that  $1 - F(\rho^U) = (1 - F_J(\rho^U))(1 - G_S(\rho^U))$ .

### 8.3 Adding up moving costs

Let  $C$  be a relocation cost paid by workers. We ignore here  $G_S$  assumed to be degenerate and fix  $\delta = 0$ . We have thus:

$$\begin{aligned}
rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int \max[0, (E' - E - C)] dG_N(\rho') \\
rU &= b + p \int (E' - U) dF(\rho')
\end{aligned}$$

$E$  is downward sloping in  $\rho$  with slope

$$\frac{dE}{d\rho} = \frac{-\tau}{r + s + \lambda_H P_W}$$

where  $P_W \geq 0$  is the probability to move conditional on receiving a housing offer, with  $0 < P_W < 1$  possibly depends on  $\rho$ . The function  $E(\rho)$  is thus monotonic and there is thus a well-defined reservation strategy, with a reservation distance denoted by  $\rho^E$  for the employed above which a **housing offer** is rejected. Note in addition that there is NOW state-dependence in the reservation strategy of the employed: we have that  $\rho^E(\rho)$  with presumably  $d\rho^E/d\rho > 0$ : the further away the tenants live from her job, the less likely they will reject an offer. (NB: to be shown in the general case). e can rewrite

$$P_W = G_N(\rho^E(\rho))$$

and obtain the Bellman equations as:

$$\begin{aligned}
 rE(\rho) &= w - \tau\rho + s(U - E(\rho)) + \lambda_H \int_0^{\rho^E(\rho)} (E' - E - C) dG_N(\rho') \\
 rU &= b + p \int_0^{\rho^U} (E' - U) dF(\rho')
 \end{aligned}$$