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Improving “National Brands”: Reputation for Quality and Export Promotion Strategies

Julia Cagé^a, Dorothée Rouzet^{b,*}

^aHarvard University

^bOECD

Abstract

This paper studies the effect of firm and country reputation on exports when buyers cannot observe quality prior to purchase. Firm-level demand is determined by expected quality, which is driven by the dynamics of consumer learning through experience and the country of origin’s reputation for quality. We show that asymmetric information can result in multiple steady-state equilibria with endogenous reputation. We identify two types of steady states: a high-quality equilibrium (HQE) and a low-quality equilibrium (LQE). In a LQE, only the lowest-quality and the highest-quality firms are active; a range of relatively high-quality firms are permanently kept out of the market by the informational friction. Countries with bad quality reputation can therefore be locked into exporting low-quality, low-cost goods. Our model delivers novel insights about the dynamic impact of trade policies. First, an export subsidy increases the steady-state average quality of exports and welfare in a LQE, but decreases both quality and welfare in a HQE. Second, there is a tax/subsidy scheme based on the duration of export experience that replicates the perfect information outcome. Third, a minimum quality standard can help an economy initially in a LQE moving to a HQE.

Keywords: product quality, product differentiation, export promotion, industrial policy, trade, asymmetric information

JEL: F12, F13, L15, L52, O14, O24

*Corresponding author

Email addresses: cage@fas.harvard.edu (Julia Cagé), drouzet@post.harvard.edu (Dorothée Rouzet)

1. Introduction

Why do country-of-origin labels matter to consumers? This question finds no clear answer in standard models of international trade, which assume that consumers are perfectly informed about the characteristics of every available product and leave no role for country reputations. However, as a large literature on experience goods has shown, quality is not fully known to consumers prior to purchase for a wide range of goods. Inferring the quality of a good requires time and is achieved both through search and through experience. For these categories of goods, country-of-origin affects product evaluations and consumers' decisions.¹

In this paper, we argue that a “national brand image” matters because it provides an anchor for the expected unobservable quality of imports such that a bad country reputation can become self-fulfilling. Consumption decisions, in practice, are based on a limited information set available to consumers at the time of purchase: information gathered as a result of past consumption experience and word-of-mouth diffusion, but also the country where the good was manufactured. For new and unknown foreign goods, the main piece of information available to consumers besides observable characteristics is the “made in” label, which indicates the country of manufacturing and creates a key role for national reputations. We call “national reputation” the common component of consumers' expectations of the quality of goods produced within a given country. Country reputations determine the quality that buyers expect before they learn any information specific to a product. We show how the dynamics of consumer learning and country reputation can create multiple equilibria with self-fulfilling expectations.

More specifically, we consider an infinite-horizon two-country model with a continuum of potential foreign exporters heterogeneous in quality, and a constant flow of new potential exporters per period. Each new firm draws a quality parameter from a fixed distribution of firm technologies and has the option to produce a good of this quality. The decision to produce is endogenous: potential foreign exporters decide whether to enter the market and when to exit, taking into account the impact of their decisions on expected future sales. We assume that the cost of producing one physical unit

¹Schott (2008) documents that the prices that US consumers are willing to pay for Chinese exports are substantially lower than the prices they are willing to pay for OECD exports of the same products. Many survey-based studies in the marketing literature, summarized by Roth and Diamantopoulos (2009) also emphasize the role of country-of-origin labels in setting consumer perceptions of quality.

of the good is monotonically increasing in quality, but the cost per quality-adjusted unit is decreasing in quality. Quality is known to firms but not observed by consumers before purchase. Hence, import demand depends on perceived quality, which has two components. Goods imported from a given country are first evaluated according to a country-wide prior, which is endogenously determined by the average quality of the country's exports in a long-run industry equilibrium. Importers then learn about the true quality of firms that have exported in the past. The fraction of informed consumers increases with the time a firm has been active on the market. The effect of the country prior will thus prevail for new exporters and fade over time as buyers gain familiarity with individual foreign brands. As a consequence, asymmetric information fosters entry by low-quality firms, which earn higher profits than under perfect information by free-riding on high-quality expectations. It depresses profits of the highest-quality firms, forced to incur initial losses in order to reveal information about their type.

We characterize the equilibrium structure. There are two types of steady states with endogenous reputation: a high-quality, high-reputation equilibrium (HQE), and a low-quality, low-reputation equilibrium (LQE). In a LQE, only the lowest-quality and the highest-quality firms are active; a range of relatively high-quality firms are permanently kept out of the market by the informational friction. Fly-by-night firms export only for one period in this equilibrium; on the contrary, in a HQE, low-quality firms that enter initially as fly-by-nights can last longer than one period. We show that there can be multiple equilibria, such that countries with bad quality reputation can be locked into exporting low-quality, low-cost goods. Where multiple equilibria exist, reputation shocks can then have self-fulfilling effects.

This last result challenges the effectiveness of some export-led growth strategies which rely on exporting low-quality, low-cost goods and gradually moving up to higher quality, higher unit-value goods. A number of East Asian economies have pursued such strategies in the past. Without policy intervention, we show that it may not be a feasible path if the economy is trapped in a self-fulfilling equilibrium in which the country's reputation for low quality prevents some high-quality firms from entering export markets. We therefore consider various policies that can be implemented in a low-quality equilibrium to promote exports and improve a country's reputation abroad.

Our model delivers novel insights about the impact of the following trade policies. First, an export subsidy increases the steady-state average quality of exports and welfare of the exporting country in a LQE, but decreases both quality and welfare in a HQE. This policy raises the incentives to

export for all firms, but in a LQE it has a larger effect on high-quality firms which have a longer effective horizon. Conversely, in a HQE, it only leads to additional entry by low-quality firms, which creates a negative reputation externality on all exporters. Second, there is a tax and subsidy scheme based on the duration of export experience that replicates the perfect information outcome. Finally, a minimum quality standard can help an economy initially in a LQE move to a HQE.

The remainder of this paper proceeds as follows. Section 2 reviews the literature relevant to the present study. Section 3 lays out our modelling framework. Section 4 analyzes high-quality and low-quality steady-state equilibria with endogenous reputation. Section 5 explores the effects of different policy instruments on quality, reputation and welfare. Section 6 concludes.

2. Related Literature

This paper relates to the international trade and industrial organization literature on vertical quality differentiation and asymmetric information. This section briefly explains how our dynamic approach with a continuum of quality types and self-fulfilling reputations differs from the existing models of asymmetric information in exports.

Informational barriers to entry in international trade have been studied by Mayer (1984), Grossman and Horn (1988), Bagwell and Staiger (1989), Bagwell (1991), Chen (1991), Dasgupta and Mondria (2012, 2013) and Chisik (2003). Mayer (1984) was the first to investigate export subsidies in the presence of initially uninformed consumers but did so without modeling explicitly the process of consumer learning and expectations formation, and relied on pessimistic consumer beliefs. Dasgupta and Mondria (2013) develop a two-period model with similar features to ours, where the quality of new exporters is unobservable and that of continuing exporters is known by a fraction of consumers. They, however, focus on the role of intermediaries in providing quality assurance and do not analyze the formation of country reputations.

The articles that are the most closely related to the present paper are Bagwell and Staiger (1989) and Chisik (2003) who both explore the interplay between country reputation, exporting firm quality and optimal trade policy. While our paper builds on these pioneering works, it departs from them both in the assumptions and in the policy implications. In our model, exporting firms cannot signal their quality, so that information acquisition

is entirely driven by the dynamics of consumer learning through experience and the evolution of country reputations. We also introduce a richer quality structure with a continuum of quality types rather than only two types; and a richer time structure with an infinite horizon model, while Bagwell and Staiger (1989) use a two-period model and Chisik (2003) considers a static game. We are then able to model the process of reputation formation and investigate the transition dynamics of quality and reputation between steady states, between an uninformed prior and the steady state, or following shocks.

In contrast to the existing literature, we obtain two distinct regimes (HQE and LQE). In the LQE regime, non-exporters are in the middle of the quality distribution, while the lowest-quality firms and the highest-quality firms are not precluded from entering the market. While Grossman (1990) has anticipated the hollowing out of the middle of the quality distribution, our paper is the first to formalize these results.²

Note moreover that our paper is one of the very few that focus on country-of-origin reputations rather than on pure informational barriers to entry. Together with Chisik (2003) and Dasgupta and Mondria (2012) we are the first to model self-fulfilling country reputations – and we improve on these previous works by introducing a dynamic model with a more full-fledged quality dimension. Country-of-origin reputations are certainly related to the topic of exporters of an unknown quality, but also encompass a different set of issues. In particular, their policy implications can depart from those of informational asymmetries as a policy goal is to push the economy on a path to a different, higher welfare equilibrium. In that respect, our modeling of country-of-origin reputations is related in spirit to the approach of the statistical discrimination literature, though policy instruments naturally differ from those considered in a labor market framework (for a survey of the literature on discrimination in the labor market, see Lang and Lehmann, 2012).

Furthermore, our model delivers novel insights about the impact of trade policies. It is well-understood in the literature that export subsidies can be welfare-improving or -decreasing depending on whether information externalities lead to a problem of insufficient or excessive entry, respectively. Bagwell and Staiger (1989) focus on the welfare-improving case: they explore a situation in which asymmetric information yields socially insufficient

²In Bagwell and Staiger (1989) and in Chisik (2003), there are only two types of firms, so by construction there cannot be the hollowing out of the middle that we obtain.

entry. A subsidy can induce high-quality firms to enter the export market, whereas absent the subsidy their entry may be blocked by their inability to sell at prices reflecting their true quality. In contrast, Grossman and Horn (1988) point out that asymmetric information may lead to socially excessive entry when the quality choice is endogenous. In their model, an export subsidy does encourage entry but the marginal entrants are those with the greatest incentive to produce low-quality goods, reducing average quality (see Grossman, 1990). Our model encompasses both effects. We obtain a positive effect of an export subsidy in the LQE case: the subsidy induces firms in the middle of the quality distribution to export, and the overall effect on average quality is positive.³ In a HQE, however, the dominant effect of a subsidy is to worsen the socially excessive entry of low-quality firms. We also depart from Chisik (2003) where export subsidies are of no use since they do not alter the relative payoffs of high-quality and low-quality firms. In our model, high-quality firms face a longer effective time horizon, so that even a subsidy granted indiscriminately to all exporters tilts the incentive to export in favor of high-quality firms.

Next, we analyze the effect of other trade policies, some of which have been overlooked in the literature. We show that if the government can observe how many periods a firm has been in the market, it can design a tax and subsidy program that similarly changes the relative incentives of high-quality and low-quality firms and improves both quality and reputation. This result extends the finding by Bagwell and Staiger (1989) that an introductory phase tax on exporters followed by a mature phase subsidy improves welfare. The first-period tax can be interpreted as an export license. Chisik (2003) explores the effect of a similar policy – an optional tax that is rebated only to firms that produce high quality – but imposes more stringent information requirements by assuming that the government perfectly learns the quality of every firm’s product after the initial period. Alternatively, we show that a minimum quality standard can help move to a more favorable equilibrium.

Our paper also relates to the industrial organization literature dealing

³This would still be the case if we allowed for an endogenous quality choice, although fly-by-night firms would then always choose the minimum quality level.

with experience goods⁴ as well as with labels and quality certification.⁵ Lastly, the formation of collective reputations had been modelled by Tirole (1996) in a moral hazard setting, where group reputation is both the outcome and a determinant of individual behavior.⁶

3. Model Set-Up

We develop a model with two countries. We focus on the industry equilibrium in an export-oriented sector in the home country, for which the foreign country is the importer. In the home country, there is a continuum of potential exporters j of mass 1 being born every period. In the foreign country, there is a pool of importers indexed by i , each of whom demands one unit of the good. All firms in the industry produce for export only.⁷

3.1. Firms

Each new potential exporter j draws a quality parameter θ from a Pareto distribution $G(\theta)$ with support on $[\theta_m, \infty)$ and shape parameter $\alpha > 1$. We denote the unconditional expectation of quality draws by μ_0 , equal to $\frac{\alpha}{\alpha-1}\theta_m$. Each firm j has the option to produce one unit of a good of quality level $\theta(j)$.⁸

⁴Several early papers (Shapiro, 1983; Riordan, 1986; Farrell, 1986; Liebeskind and Rumelt, 1989) have studied entry and pricing strategies for experience goods in a closed economy framework. Bergemann and Välimäki (1996, 2006) incorporate the experimentation and learning processes by consumers. We develop these insights further by considering the demand for imports, where initial priors depend on country-of-origin, and reputations are built not only for specific firms but also for exporting countries as a whole.

⁵A strand of it assuming that quality is unknown to consumers (see e.g. Perrot and Linnemer, 2000). This literature focuses on the signaling role of labels: labels act as quality signals for individual firms which seek to signal quality and build consumer trust (see e.g. Grossman, 1981; Klein and Leffler, 1981; Milgrom and Roberts, 1986; Lizzeri, 1999), but there are no reputation externalities between firms.

⁶His model generates a high- and a low-reputation steady-state equilibrium, but for given initial conditions, there is a unique equilibrium. An interesting result is that a one-time shock can have permanent effects on collective reputation, as in our model.

⁷Or, equivalently, markets are segmented. We could easily extend the model to allow firms to serve their domestic market as long as the decisions to enter the domestic and export markets are separable. The key assumption is that there is no information flowing between buyers located in different geographic markets.

⁸For simplicity we do not model the quality choice. We can think of the exogenous quality draw as determined on the domestic market before considering the decision to export, or as a technology blueprint which comes from an R&D process with uncertain outcome.

At the beginning of every period, firms decide whether to stay active and export, or shut down. If it produces and sells, firm j incurs a cost $w\theta(j) + k$, including both production and transport costs (with $k > 0$ and $0 < w < 1$). Hence, profits at period $t + s$ of an active firm j born at date t are:

$$\pi_{t+s}(j) = p_{t+s}(j) - w\theta(j) - k \quad (1)$$

where $p_{t+s}(j)$ is the price at which firm j sells its output.

A firm can freely exit at any period and realize zero profits from this period onwards. However if it chooses to exit the export market in a given period, it cannot re-enter later.⁹ Moreover, each firm has a probability $(1 - \delta)$ per period of suffering from an exogenous shock that forces it to exit, independent of both quality and the firm's age. This exogenous "exit shock" acts as a discount rate, as in Melitz (2003).

3.2. Buyers

In the foreign country, each potential importer demands one unit of the good. Potential demand for imported goods is assumed to be large, in the sense that the market size is sufficient for all home exporters to find a buyer at a price that does not exceed the expected value of their goods. The true utility from consuming a good of quality level $\theta(j)$ is $\theta(j)$ but is not observable before purchase.¹⁰

At the beginning of every period, each active firm j is randomly matched to a foreign buyer i . The firm cannot sell to another importer in that period, nor can the buyer purchase from another exporter before the next period. The firm then sets the price equal to the expected value of the good for its buyer.¹¹ As $\theta(j)$ is not observed, the maximum price that an importer i is willing to pay for the output of firm j at time $t + s$ is given by its expected quality from the perspective of the buyer.

⁹This assumption is inconsequential for the steady-state analysis. It rules out coordination problems among high-quality firms along the transition path.

¹⁰The indirect utility buyer i receives from the good sold by firm j at time $t + s$ is $u_{t+s}^i(j) = \theta(j) - p_{t+s}(j)$, which can be derived from an additively separable utility function where buyer i consumes a numeraire good and one unit of the imported differentiated good.

¹¹We assume that firms hold all the bargaining power and receive the full expected surplus of the transaction. Long-term contracts between exporters and importers are ruled out in this setting: all contracts are one-period sales contracts and firms are matched to customers for one period only. In particular, there cannot be price schedules resembling an introductory pricing strategy, whereby buyers would pay a low price in the initial period and offer a sequence of prices contingent on their future consumption experience.

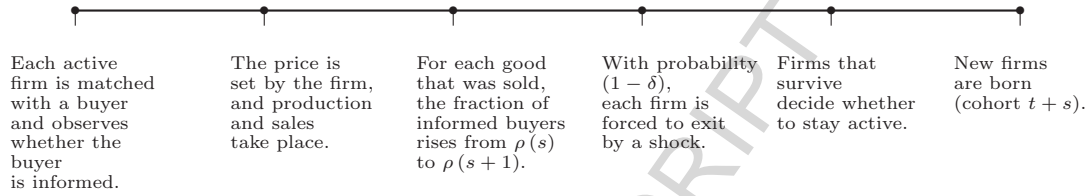


Figure 1: Timing of actions

There are two types of buyers, informed and uninformed. Uninformed buyers have no information specific to firm j . The only information at their disposal is the “national reputation”, i.e. a prior μ_{t+s} about expected quality among all foreign exporters. μ_{t+s} is common across buyers and is endogenized in Section 4. Informed buyers know the true quality of firm j , either because they have past experience from consumption of good j or because they have received information from another importer who has. The price received by firm j matched with buyer i in period $t+s$ is therefore μ_{t+s} if i is uninformed, and $\theta(j)$ if i is informed.

In the first period when a firm j enters the market, all importers are uninformed about j . Then, if firm j has exported s times in the past, a fraction $\rho(s)$ of buyers are informed, where we make the natural assumption that the fraction of informed buyers increases as the firm gains export experience (see Appendix B for a formal microfoundation).

Assumption 1. $\rho' \geq 0$, $\rho(0) = 0$, and $\lim_{s \rightarrow \infty} \rho(s) = 1$.

For expositional simplicity we drop the j notation in the next sections and refer to “firm θ ” instead of “firm j with quality parameter θ ”.

3.3. Timing

For a given cohort of firms born at date t , each new firm draws a parameter θ and decides whether to export or not at t . For each $s \geq 1$, at time $t+s$, the timing is shown in Figure 1.

3.4. Perfect Information

Under perfect information, all θ are observable by all parties. All firms receive a price p_{t+s}^* equal to true quality regardless of how long they have been exporting: $p_{t+s}^*(\theta) = \theta$ for all s .

Therefore, it follows from (1) that firms are active exporters if and only if $\theta \geq \theta^*$, where the perfect information threshold is defined as:

$$\theta^* \equiv \frac{k}{1-w} \quad (2)$$

The perfect information threshold is key for our analysis as we will show that under asymmetric information, per-period profits converge over time towards their perfect information value. Moreover, from a global welfare point of view, it is not optimal for firms with quality below θ^* to enter export markets – in this case, the value of the output to consumers is lower than the opportunity cost of production. However, given that exporters do not internalize the impact of their decisions on consumer surplus, it can be profitable for firms with quality below θ^* to enter temporarily under asymmetric information.

3.5. Imperfect Information: Price and Profits

Under asymmetric information, suppose μ_t is the buyers' prior about the expected quality of foreign goods at time t ("national reputation"), taken as exogenous by individual firms although it is endogenous at the country level. We derive its equilibrium value in Section 4. The price offered to a firm born at date t is either the country-wide prior if the buyer is uninformed, or its true quality if the buyer is informed. In the first period in which a firm is active, no buyer has any information specific to the firm, so that the price only depends on the prior: $p_{t+1} = \mu_{t+1}$. Then in the following periods, conditional on the firm still being active, the price is set according to the following rule:

$$p_{t+s} = \begin{cases} \theta & \text{with probability } \rho(s-1) \\ \mu_{t+s} & \text{with probability } 1 - \rho(s-1) \end{cases} \quad \text{for } s \geq 1 \quad (3)$$

where $\rho(s-1)$ is the fraction of informed buyers for a firm that has previously exported $(s-1)$ times. The expectation of the price converges to the perfect information price θ over time if the firm stays in the market indefinitely.

The expected profits of the firm in future periods, conditional on remaining active, are the difference between its expected price and its production cost:

$$E_t \pi_{t+s} = [\rho(s-1) - w] \theta + [1 - \rho(s-1)] E_t \mu_{t+s} - k \quad (4)$$

Expected profits place a larger weight on true quality and a smaller weight on national reputation as the firm gains tenure into exporting. It immediately follows that if reputation is time-invariant, which will be the case in a steady-state equilibrium, a firm with quality above the country

prior ($\theta > \mu$) expects to realize an increasing sequence of profits over time, while a firm with quality below the country prior ($\theta < \mu$) expects decreasing profits. For all active firms, if μ is constant, the price is monotonically converging towards θ and per-period profits are monotonically converging towards their perfect information value $(1 - w)\theta - k$.¹²

To ensure that expected profits from repeat purchases are increasing in true quality, we assume that the updating parameter is large enough relative to the cost of producing quality:

Assumption 2. $\rho(1) > w$

Firms are free to exit at any date. In each period t , a firm of quality θ having exported s times in the past stays active if the expected present value of doing so, $PV_t(\theta, s)$, is positive:

$$PV_t(\theta, s) = \sum_{u=0}^{T(\theta)} \delta^u \left([\rho(s+u) - w]\theta + [1 - \rho(s+u)] E_t \mu_{t+u} - k \right) \quad (5)$$

where $E_t \mu_{t+u} = \mu_{t+u}$ for all u since there is no aggregate uncertainty, and $T(\theta)$ is the exit date (possibly infinity) that maximizes the firm's intertemporal problem.

4. Industry Equilibrium

4.1. Equilibrium Definition

We define a steady-state industry equilibrium as one in which national reputation is endogenously pinned down by the average quality of a country's exports and the quality distribution is stationary. Country reputations are taken as exogenous by individual firms. In each period t , let $M_t(\theta, s)$ be the number of active firms of quality θ having previously exported s times. Let $N_t(\theta) = \sum_{s=0}^t M_t(\theta, s)$ be the total number of active firms of quality θ . We derive $\bar{\theta}_t$ as the average quality of exports across quality levels and cohorts of firms:

$$\bar{\theta}_t = \frac{\int_{\theta_m}^{\infty} \theta N_t(\theta) d\theta}{\int_{\theta_m}^{\infty} N_t(\theta) d\theta} \quad (6)$$

A new firm of quality θ is active at $t + 1$ if its expected present value of doing so is positive. Hence the number of active new firms per quality level

¹²In Appendix C, we characterize the path of prices for a given cohort of firms.

is:

$$M_{t+1}(\theta, 0) = \begin{cases} g(\theta) & \text{if } PV_{t+1}(\theta, 0) > 0 \\ 0 & \text{if } PV_{t+1}(\theta, 0) \leq 0 \end{cases} \quad (7)$$

where $g(\theta)$ is the PDF of the quality distribution.

Among incumbent firms of quality θ having exported s times, $\delta M_t(\theta, s - 1)$ survive from period t to period $t + 1$. They remain active if $PV_{t+1}(\theta, s) > 0$ in equation (5). Thus the number of active old firms is, for $s \geq 1$:

$$M_{t+1}(\theta, s) = \begin{cases} \delta M_t(\theta, s - 1) & \text{if } PV_{t+1}(\theta, s) > 0 \\ 0 & \text{if } PV_{t+1}(\theta, s) \leq 0 \end{cases} \quad (8)$$

We can then define the industry steady-state as an equilibrium with constant reputation and a constant distribution of quality.

Definition 1 (Steady-State Equilibrium).

$\{\mu, \{M(\theta, s)\}_{s,\theta}\}$ is a steady-state equilibrium if and only if:

- i. For all $\theta \in [\theta_m, \infty)$ and all $s \geq 0$, if $M_t(\theta, s) = M(\theta, s)$ and $E_t \mu_{t+u} = \mu$ for all $u \geq 0$, then $M_{t+1}(\theta, s) = M(\theta, s)$ in (7) and (8);
- ii. If $M_t(\theta, s) = M(\theta, s)$ for all $\theta \in [\theta_m, \infty)$ and all $s \geq 0$, then $\bar{\theta}_t = \mu$ in (6).

Condition (i) ensures that the number of firms in each quality-age segment is constant in the steady state. Condition (ii) states that the average quality resulting from an equilibrium distribution of active firms is equal to the equilibrium country reputation. It guarantees that μ is constant in a steady state. In other words, a steady state with national reputation μ is a rational expectations equilibrium if the average quality of active exporters is equal to buyers' quality expectation. The endogenous entry and exit decisions induced by μ justify the reputation ex post.¹³

¹³In the numerical examples, we assume that country reputations evolve according to the actual quality of exported goods in the previous period as follows: $\mu_{t+1} = \mu_t + \eta(\bar{\theta}_t - \mu_t)$, where $\eta < 1$ and $\bar{\theta}_t$ is the average quality of foreign firms' exports at period t . Setting $\eta < 1$ captures the slow-moving aspect of reputations and only matters for equilibrium stability. This equation is a reduced form for consumers' updating process, where the implied simplifying assumption is that η – which captures the speed at which beliefs are updated – is constant. We adopt this simple rule-of-thumb formulation for beliefs updating rather than modeling explicitly the Bayesian updating process for the sake of simplicity. This hypothesis is not necessary for our main steady-state results and policy implications, and only ensures that equilibrium stability holds under general conditions.

4.2. Equilibrium Steady-State Reputation

Let $\bar{\theta}(\mu)$ be the average quality of exports as a function of constant beliefs μ . An equilibrium steady-state reputation is a time-invariant reputation μ such that $\bar{\theta}(\mu) = \mu$.

To compute the fixed point(s) of $\bar{\theta}(\mu)$, we proceed as follows. First, we characterize firms' entry and exit decisions. Second, we compute $M_t(\theta, s)$, i.e. the number of active firms of quality θ having previously exported s times, and thus the average quality of exports as a function of μ . We then derive the existence conditions for each type of equilibrium.

4.2.1. Sorting of Firms into Non Exporters and Exporters

Steady-state equilibria as defined by Definition 1 fall into one of two regimes: a “high-quality equilibrium” (HQE) or a “low-quality equilibrium” (LQE), depending on the position of the equilibrium country reputation relative to the perfect information threshold.

Definition 2 (HQE & LQE).

A steady-state equilibrium $\{\mu, \{M(\theta, s)\}_{s,\theta}\}$ is a high-quality steady-state equilibrium if $\mu > \theta^*$, and a low-quality steady-state equilibrium if $\mu < \theta^*$.

The two regimes have different entry and exit patterns, due to the impact of asymmetric information and the dynamics of consumer learning. Compared to the perfect information case, the unobservability of quality initially fosters entry by low-quality firms but depresses profits of the highest-quality firms. In a HQE, a firm with quality equal to the country's reputation would be viable in a perfect information setting. All firms receive high prices as they enter the market, which encourages entry. Therefore, imperfect information does not hinder entry of high-quality firms into export markets but generates excess entry by low-quality firms. On the contrary, in a LQE, a firm with quality equal to the country's reputation would never export in a perfect information setting. Some low-quality firms still profit by free-riding on the country reputation, but a range of firms with above-average quality are permanently kept out of the market by the informational friction. In other words, there is a hollowing out of the middle of the quality distribution. Proposition 1 establishes the sorting of firms into non exporters and exporters in a HQE and in a LQE.

Proposition 1 (Sorting of Firms into Exporting: Two Regimes).

In a HQE:

1. All entrants are initially active;

2. Firms with $\theta < \theta^*$ expect to exit after $T(\theta)$ periods where T is weakly increasing in θ ;
3. Firms with $\theta > \theta^*$ stay in the market until hit by the exogenous shock.

In a LQE:

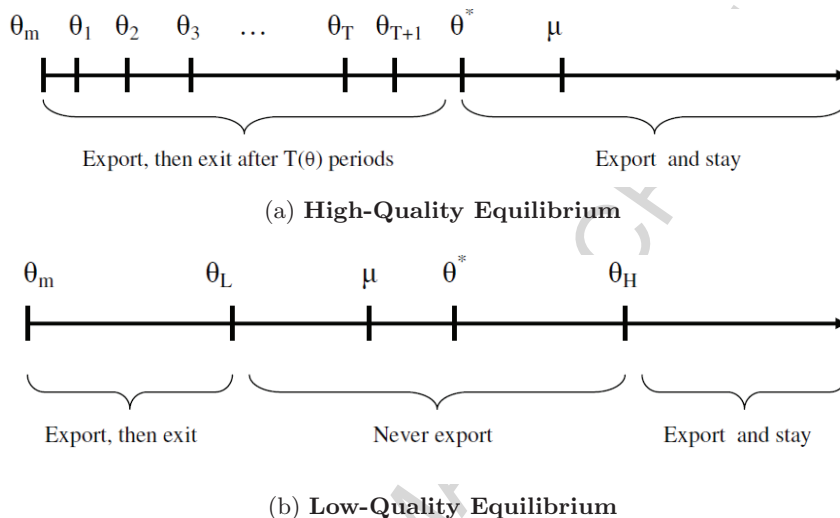
1. Firms with quality $\theta < \theta_L$ enter the export market and exit after selling for one period, where $\theta_L \equiv \frac{\mu-k}{w} < \mu < \theta^*$;
2. Firms with quality $\theta > \theta_H$ enter and stay in the market until hit by the exogenous shock, where $\theta_H \equiv \frac{k-\mu(1-A_\rho)}{A_\rho-w} > \theta^*$ and $A_\rho \equiv (1-\delta) \sum_{s=0}^{\infty} \delta^s \rho(s)$.
3. Firms with quality $\theta_L \leq \theta \leq \theta_H$ never enter the market.

Proof: see Appendix A.1.

Figure 2 shows the sorting of firms according to their quality parameter in each regime. In a HQE (upper figure 2a), all low-quality firms (below θ^*) find it profitable to enter initially as fly-by-nights, as they have low production costs and can therefore reap positive expected profits as long as a small enough number of buyers have information about their type. The higher the country reputation, the higher the price they receive in the first period. However, they become less profitable as consumers gain information about their quality through consumption experience. Low-quality firms thus face a decreasing sequence of expected profits converging to a negative value; they will eventually see their expected present value of profits turn negative and exit. The lowest-quality firms exit first, and θ_T is the highest quality type that exits after selling for T periods.¹⁴ For high-quality firms (above θ^*), it is always profitable to enter and keep exporting. Firms between θ^* and μ have expected profits declining over time, but positive in every period. Firms above μ have expected profits increasing over time. The highest quality firms incur losses in the initial period but recoup these losses in later periods once enough buyers have received information about their type; their expected intertemporal profits are always positive.

In a LQE (bottom figure 2b), there is a range of low-quality firms (below θ_L) that gain from the information asymmetry and realize positive first-period profits. However they would make losses if they were to stay active in

¹⁴Firms below θ^* exit after $T(\theta)$ periods, where $T(\theta)$ is the exit date that maximizes the firm's intertemporal profit (see equation (5)). We can derive $\theta_T = \max \left\{ \frac{k-[1-\rho(T)]\mu}{\rho(T)-w}, \theta_m \right\}$ for $T \geq 1$ and $\lim_{T \rightarrow \infty} \theta_T = \theta^*$ (See Appendix A.1).

Figure 2: Sorting of firms by θ

the second period. These firms therefore exit immediately after selling once, while in a HQE low-quality firms that enter initially as fly-by-nights can last longer than one period. An intermediate range of middle-quality firms $[\theta_L, \theta_H]$ around θ^* never become active exporters. Those with $\theta_L < \theta < \theta^*$ have negative expected profits at all periods, while those with $\theta^* < \theta < \theta_H$ would be profitable in the long run once enough buyers have gathered information about their type. However, for the latter, the present value of their profit stream is negative: losses incurred in the initial periods in order to establish a reputation are not made up for with later expected profits. Hence this range of firms is kept out of export markets by the information asymmetry and the cost of revealing quality. Finally, high-quality firms (above θ_H) are not profitable in the first period but nevertheless choose to enter given that expected profits from sales in later periods, when a larger portion of the price reflects true quality, exceed initial losses. The negative profits in their first export periods can be interpreted as investments in building a firm-specific brand name.

4.2.2. Average Quality

The number $M(\theta, s)$ of active firms of quality θ having already exported s times is derived from Proposition 1 and equations (7) and (8).

High-quality equilibrium. In a HQE, the number $M(\theta, s)$ of active firms of quality θ having already exported s times is:

$$M(\theta, s) = \begin{cases} \delta^s g(\theta) & \text{if } \theta < \theta^* \text{ and } s < T(\theta) \\ 0 & \text{if } \theta < \theta^* \text{ and } s \geq T(\theta) \\ \delta^s g(\theta) & \text{if } \theta \geq \theta^* \end{cases} \quad (9)$$

so that the total number $N(\theta)$ of active firms of quality θ is $\frac{1-\delta^{T(\theta)}}{1-\delta}g(\theta)$ if $\theta < \theta^*$, and $\frac{1}{1-\delta}g(\theta)$ if $\theta \geq \theta^*$. The steady-state average quality of exports in a HQE as a function of μ and exogenous parameters is given by equation (6) as:

$$\bar{\theta}(\mu) = \mu_0 \left(\frac{1 - \sum_{T=0}^{\infty} \delta^{T+1} \left[\left(\frac{\theta_m}{\theta_T} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{T+1}} \right)^{\alpha-1} \right]}{1 - \sum_{T=0}^{\infty} \delta^{T+1} \left[\left(\frac{\theta_m}{\theta_{T+1}} \right)^{\alpha} - \left(\frac{\theta_m}{\theta_{T+1}} \right)^{\alpha} \right]} \right) \quad (10)$$

where $\theta_0 \equiv \theta_m$. The average quality of active firms is higher than the mean of the unconditional distribution of θ , as lower-quality firms exit earlier than higher-quality firms. However, it lies below the perfect information average quality.

Low-quality equilibrium. In a LQE, the number of active firms of quality θ having already exported s times is given by:

$$M(\theta, s) = \begin{cases} g(\theta) & \text{if } \theta < \theta_L \text{ and } s = 0 \\ 0 & \text{if } \theta < \theta_L \text{ and } s \geq 1 \text{ or if } \theta_L \leq \theta \leq \theta_H \\ \delta^s g(\theta) & \text{if } \theta > \theta_H \end{cases} \quad (11)$$

so that the total number $N(\theta)$ of active firms of quality θ is $g(\theta)$ if $\theta < \theta_L$ and $\frac{1}{1-\delta}g(\theta)$ if $\theta > \theta_H$. The steady-state average quality of exports in a LQE as a function of μ and exogenous parameters is given by equation (6):

$$\bar{\theta}(\mu) = \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_L} \right)^{\alpha-1} + \frac{1}{1-\delta} \left(\frac{\theta_m}{\theta_H} \right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_L} \right)^{\alpha} + \frac{1}{1-\delta} \left(\frac{\theta_m}{\theta_H} \right)^{\alpha}} \right) \quad (12)$$

4.2.3. Existence Conditions

A steady-state equilibrium is a fixed point of $\bar{\theta}(\mu)$ defined by equation (12) on $[\theta_m, \theta^*]$ and by equation (10) on $[\theta^*, \infty)$. An equilibrium such that

$\mu = \bar{\theta}(\mu) < \theta^*$ is a LQE. An equilibrium such that $\mu = \bar{\theta}(\mu) > \theta^*$ is a HQE. Proposition 2 establishes existence conditions.

Proposition 2 (Existence Conditions).

1. *There is always at least one steady-state equilibrium.*
2. *There is one HQE (and zero or a positive number of LQEs) if and only if $\bar{\theta}(\theta^*) > \theta^*$, i.e. if and only if:*

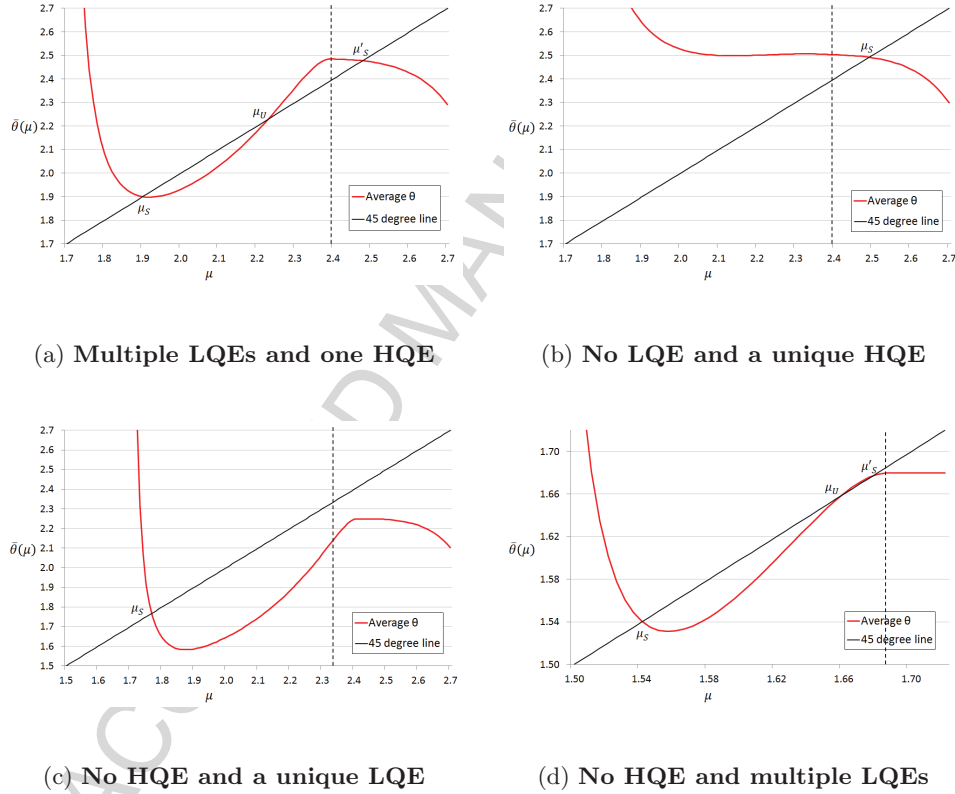
$$\alpha \left(\frac{\theta_m}{\theta^*} \right) + \frac{\delta}{1 - \delta} \left(\frac{\theta_m}{\theta^*} \right)^\alpha > \alpha - 1 \quad (13)$$

3. *There is no HQE and at least one LQE if and only if $\bar{\theta}(\theta^*) < \theta^*$.*

Proof: see Appendix A.2.

Hence, depending on the parameters, the rational expectations steady-state falls into one of two regimes: a LQE or a HQE. The number of equilibria is odd except in the knife-edge case where $\bar{\theta}(\mu)$ is tangent to the 45-degree line. The type of equilibrium depends on whether the (not necessarily unique) fixed point of $\bar{\theta}(\mu)$ falls left or right of θ^* . Figure 3 provides a graphical illustration. The figure represents the steady-state average quality of exports $\bar{\theta}(\mu)$ as a function of μ (red line). The black diagonal is a 45-degree line, and we plot a vertical dotted line at the perfect information threshold θ^* . The upper left figure 3a illustrates the existence of a HQE: $\bar{\theta}(\theta^*) > \theta^*$ and there is one HQE, μ'_S . This equilibrium is not unique: there are also two LQEs, μ_S and μ_U . In the upper right figure 3b, there is also a HQE but it is the unique equilibrium of the economy. On the contrary, the bottom left figure 3c illustrates the fact that when $\bar{\theta}(\theta^*) < \theta^*$, there is no HQE. In this example, there is a unique LQE, μ_S . Finally, the bottom right figure 3d illustrates the case with several LQEs and no HQE.

The existence condition (13) for a HQE holds for δ (the probability that a firm still exists from one period to the next) high enough, α (the shape parameter of the distribution) low enough, and k (the costs that are independent of quality) and w (the costs that increase with quality) low enough. Figure 4 depicts this existence condition. In the dark region, condition (13) holds and there exists one HQE. In subfigure 4a, we fix the values of k and w and show that there exists a HQE if α is low enough and δ is high enough. A high δ implies that exogenous exit is relatively less prevalent than endogenous exit, increasing the relative mass of high-quality firms. A low α means that there is high dispersion in the prior distribution of θ and

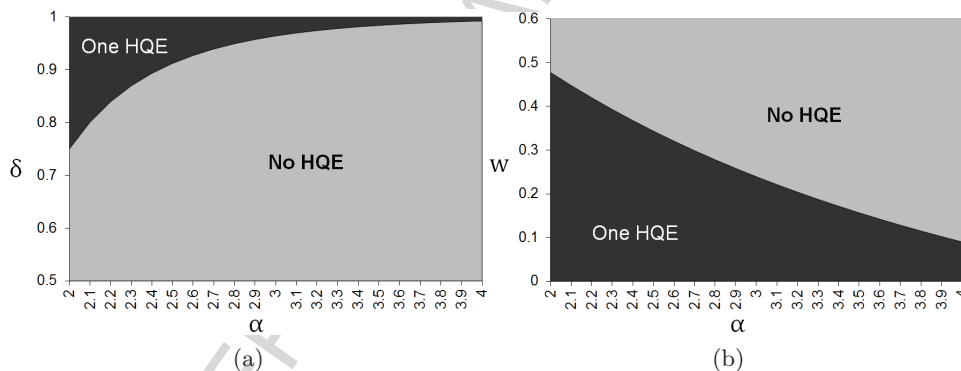


Notes: In subfigure 3a, the fixed parameter values are: $\theta_m = 1$, $\alpha = 2.2$, $\delta = 0.7$, $k = 1.2$, $w = 0.5$. The perfect information threshold is $\theta^* = 2.4$. The steady states of this economy are $\mu_S \approx 1.900$, $\mu_U \approx 2.230$ and $\mu'_S \approx 2.477$. In subfigure 3c, the parameters are the same except $\alpha = 2.4$. The unique steady state of this economy is $\mu_S \approx 1.767$. In subfigure 3b, the fixed parameter values are the same as in subfigure 3c except $\delta = 0.8$. The unique steady state of this economy is $\mu_S \approx 2.492$. In subfigure 3d, the fixed parameter values are: $\theta_m = 1$, $\alpha = 3$, $\delta = 0.5$, $k = 1.18$, $w = 0.3$. The perfect information threshold is $\theta^* \approx 1.686$, and the steady states are $\mu_S \approx 1.540$, $\mu_U \approx 1.660$ and $\mu'_S \approx 1.678$.

Figure 3: Unique and multiple equilibria

therefore more firms at the right tail of the distribution pushing up the mean. Note that this result is related to Chisik (2015), who develops a model of statistical discrimination and productivity signaling with stereotype threat. He finds that the existence of multiple self-fulfilling stereotypes is more likely if there is less variance in the ability distribution.

In subfigure 4b, we fix the values of k and δ . There exists one HQE if w is low enough. Intuitively, a low w reduces the relative cost advantage of low-quality firms, as well as the loss incurred in initial periods by high-quality firms. Similarly, a low k also reduces the initial losses of high-quality firms and lowers the perfect information threshold θ^* , expanding the existence region of a HQE.



Notes: Fixed parameter values in subfigure 4a: $\theta_m = 1$, $k = 1.5$, $w = 0.5$. Fixed parameter values in subfigure 4b: $\theta_m = 1$, $k = 1.5$, $\delta = 0.95$.

Figure 4: Parameter values for the existence of a HQE

4.3. Equilibrium Stability and Reputation Shocks

A steady-state equilibrium is stable if after a small shock to the country reputation μ , the economy reverts to the initial equilibrium. We characterize the stability properties of HQEs and LQEs in Proposition 3.

Proposition 3 (Equilibrium stability).

1. A HQE is always stable.
2. A LQE μ_S is stable if $\frac{\bar{\theta}(\mu)}{\mu}$ is locally decreasing at μ_S .
3. A LQE μ_U is unstable if $\frac{\bar{\theta}(\mu)}{\mu}$ is locally increasing at μ_U .

Proof: see Appendix A.3.

The intuition of the stability results relies on entry and exit decisions. In a HQE, it follows from (10) that $\bar{\theta}(\mu)$ is continuously decreasing in μ on $[\theta^*, \infty)$. Improving national reputation does not affect the incentives of firms above θ^* to stay or exit, but it encourages lower-quality firms to stay longer. Hence, starting from a HQE, an improvement in national reputation μ has a negative effect on actual quality, which ensures that a high-quality steady state is stable.

In a LQE, $\bar{\theta}(\mu)$ as derived in (12) is not necessarily monotonic over $[\theta_m, \theta^*]$. On the one hand, a higher μ reduces the initial loss incurred by high-quality firms, allowing more firms with above-average quality to be active. On the other hand, a better reputation enables more firms to realize positive profits from first-period sales; this fosters entry by firms with below-average quality. The net change in the average quality depends on the balance between these two effects. If $\bar{\theta}(\mu)/\mu$ is locally decreasing in μ (i.e. $\bar{\theta}$ crosses the 45-degree line from above), the former dominates and the LQE is stable. If $\bar{\theta}(\mu)/\mu$ is locally increasing in μ (i.e. $\bar{\theta}$ crosses the 45-degree line from below), the latter dominates and the LQE is unstable.

The equilibrium structure implies that in the presence of multiple equilibria, pure reputation shocks can have long-run effects even without any independent change in fundamentals.¹⁵ Starting from a stable equilibrium, suppose there is a negative reputation shock, i.e. a one-shot decrease in μ_t absent any change in the underlying quality distribution of firms. If the economy has only one long-run equilibrium, it must return to this steady state in the long run. If the economy has multiple steady states, however, reputation shocks can induce a transition to a different equilibrium. A small reputation shock only has short-lived effects as the economy reverts to its initial equilibrium, but a large enough reputation shock can have self-fulfilling effects. Figure 3a above provides an illustration of this case when three equilibria exist. If the initial stable equilibrium is the HQE μ'_S , a “large” shock μ_t below the unstable LQE μ_U deteriorates national reputation both in the short- and in the long-run. The new steady-state equilibrium is the LQE μ_S .

Hence, our model predicts that there can be long-term consequences of a sudden large drop in reputation, which moves a country to a less desir-

¹⁵Negative reputation shocks have been analyzed empirically through event studies, such as recalls of Chinese toys (Freedman et al., 2012) or the negative perception of France in the US at the onset of the Iraq war (Michaels and Zhi, 2010).

able steady-state equilibrium. In particular, large product recalls or heavily mediatized consumer safety scandals concerning exports of one country can permanently affect the structure of its industry, lowering both quality and reputation in the long-run.

5. Policy Implications

How can countries improve their “national brand name” – and is it worth it? First-best policies would involve conducting verifiable quality audits or taxing low-quality firms and subsidizing high-quality ones. These policies are not feasible when policy-makers are not better informed than consumers about firms’ quality levels. We analyze the effects of three policy instruments on reputation, quality and welfare: (i) export subsidies, (ii) a tax scheme based on export tenure, and (iii) minimum quality standards.

5.1. *Export Subsidies*

Export subsidies have been used historically by a number of countries to favor exporting activities. For example in South Korea, public investment subsidies were tied to exporting activity in the 1970s, as Korean governments were determined to favor the emergence of the country on the international trade scene.¹⁶ In our model an export subsidy is a permanent¹⁷ unanticipated subsidy to fixed export costs, resulting in a lower effective k for active exporters, financed by non-distortionary lump-sum taxes.

Interestingly, export subsidies have opposite effects in the two types of equilibria. In a LQE, an export subsidy leads to higher entry by firms in the middle of the quality distribution. We show that the overall effect on average quality, and thus steady-state national reputation, is positive as long as δ is not too low: starting from a LQE, an export subsidy increases long-run equilibrium quality and the welfare of the exporting country.¹⁸ On the contrary, in a HQE, an export subsidy is actually detrimental to welfare as it discourages exit by low-quality firms, worsening the problem of socially excessive entry. These results are summarized in Proposition 4.

¹⁶Pack and Westphal (1986), Westphal (1990), Levy (1991), Rodrik (1995) and Aw et al. (1998) have documented the importance of government investment subsidies in Korea.

¹⁷We are comparing the long-run industry equilibria with and without the policy. With a subsidy of limited duration, if the equilibrium is unique, the economy would return to the initial steady state in the long-run after the subsidy expires.

¹⁸Since there are no domestic consumers in our model and foreign consumers always receive a zero surplus, the welfare of the exporting country is composed of exporters’ profits and fiscal balance.

Proposition 4 (Export Subsidy).

1. *An export subsidy in a LQE increases the steady-state average quality of exports and welfare of the exporting country.*
2. *An export subsidy in a HQE decreases the steady-state average quality of exports and welfare of the exporting country.*

Proof: see Appendix A.4.

More specifically, in a LQE, a decrease in k induces more relatively high-quality firms to start and continue exporting (lower θ_H) and more relatively low-quality firms to export for one period (higher θ_L). The net effect of this entry response is an increase in average quality, which creates a positive externality on firms that would be exporting regardless of the policy. They receive higher prices on their exports due to improved reputation. New exporters also benefit from the better reputation as well as the subsidy, so that the increase in aggregate profits exceeds the tax cost of the subsidy.¹⁹ The welfare gain is a direct outcome of higher long-run reputation.²⁰

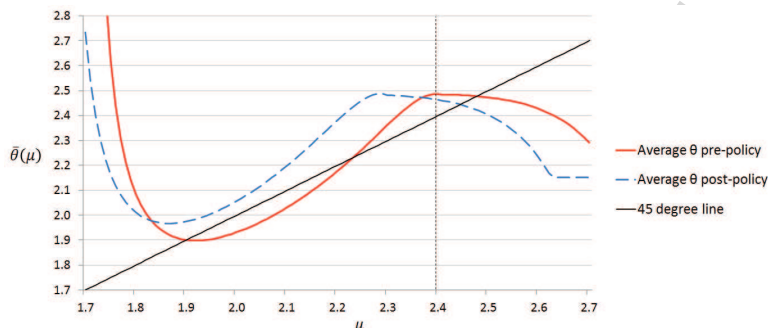
In a HQE, however, a permanent export subsidy lowers average quality by inducing low-quality firms to remain exporters longer, while it does not change the incentives and decisions of high-quality firms. The number of active low-quality firms increases but the number of active high-quality firms remains unchanged. The lower average quality in turn damages the country's reputation, which decreases the profits of all exporters. Because of this negative externality from excessive entry, the overall increase in aggregate profits of all firms receiving the subsidy is not large enough to cover the cost of the policy, despite a higher volume of sales.²¹

Overall, the desirability of an export subsidy depends on the trade-off between encouraging entry by high-quality firms which are deterred by the

¹⁹In a setting where firms would set prices in a competitive way, we would have to balance this gain against the argument that an export subsidy tends to subsidize foreign consumers.

²⁰Figures A.1 and A.2 in the online Appendix provide a numerical examples of the transition paths to the new steady states starting from, respectively, a LQE and a HQE.

²¹We can decompose the welfare effect into two components. For the combination of quality and export experience for which firms are active both with and without the subsidy, the effect is negative: they receive lower prices and the additional profits brought about by the subsidy are taken out of taxes. For the additional periods in which firms below θ^* stay in the market because of the subsidy, their profits fall short of the cost of the subsidy: otherwise, since the price is lower than in the absence of the policy, they would have been exporting without the subsidy. Therefore, the total welfare change is unambiguously negative.



Notes: The initial parameter values are identical to subfigure 3a. The export subsidy is a reduction in k from 1.2 to 1.15. The initial θ^* is 2.4 and the post-policy θ^* is 2.3.

Figure 5: Equilibria before and after an export subsidy

cost of establishing a reputation, and inducing entry by low-quality fly-by-night firms. The former impact dominates for countries initially exporting low-quality goods, while the latter prevails for countries that already export a large share of high-quality goods. Figure 5 illustrates how an export subsidy shifts the average quality function and affects the steady state equilibria. In this example, starting from a situation with two LQEs and one HQE, the export subsidy eliminates all but one equilibrium which is a HQE. The post-policy unique steady state is higher than the pre-policy LQEs but lower than the pre-policy HQE.

The distinction between the two regimes is new compared to the existing literature. It reconciles the argument that export subsidies can help high-quality firms enter a market when they are initially unable to separate from low-quality firms (as in Bagwell and Staiger, 1989) with the criticism by Grossman and Horn (1988) according to which the marginal entrants are those with the greatest incentive to produce low-quality goods. Our model delivers both of these predictions, depending on the initial equilibrium type. It suggests, in particular, that the gain from such policies – or lack thereof – may critically depend on the level of development and export sophistication of the exporting country.

5.2. A Tax and Subsidy Program

Let us now assume that the government is able to observe the “age” of a firm, i.e. the number of periods it has previously exported. Starting from a LQE, is there a tax/subsidy scheme based on the duration of export experience that replicates the perfect information outcome? It is

noteworthy to observe first that a temporary subsidy (subsidizing entrants) can never do better than a permanent subsidy to all exporters. A subsidy that targets new firms only disproportionately benefits low-quality firms, which account for a higher share of entrants than incumbents. As such, infant industry protection in the traditional sense is counterproductive in our model: it would lower average quality, worsen the country's reputation and hurt overall profits. A more promising alternative is to tax entrants and subsidize incumbents, so as to improve the relative payoffs of high-quality firms compared to low-quality firms. In order to replicate the perfect information equilibrium, the tax should deter firms below θ^* from entering the market, but the subsequent subsidy should be sufficient to enable all firms above θ^* to earn positive intertemporal profits. However, the design of the tax/subsidy scheme must take into account the fact that if it is successful, the endogenous reputation change also affects the profitability of entry and exit. In fact, we show that when using taxes and subsidies based on age to replicate the perfect information outcome, the policy involves taxing firms for a number of periods and only subsidizing the most mature exporters.

Proposition 5 (Taxes and Subsidies based on Export Experience).

The perfect information entry and exit decisions by quality and export experience can be replicated by the following tax/subsidy scheme, where τ_s is the tax (possibly negative) levied on a firm that has previously exported $s - 1$ times:

$$\begin{aligned}\tau_1 &= \left(\frac{\alpha}{\alpha-1}\right) \frac{k}{1-w} - k - w\theta_m > 0 \\ \tau_s &= \left[1 - \rho(s-1) + \frac{1-\delta}{\delta}\right] \frac{1}{\alpha-1} \frac{k}{1-w} - \left(\frac{1-\delta}{\delta}\right) \tau_1 \text{ for } s > 1\end{aligned}$$

The resulting steady-state average quality and reputation are $\left(\frac{\alpha}{\alpha-1}\right)\theta^$.*

Proof: see Appendix A.5.

The tax/subsidy scheme needs to satisfy three conditions. First, the first-period tax needs to be high enough that no firm below θ^* earns positive profits in the first period. This is ensured by setting τ_1 at least equal to the first-period profits of the lowest quality firm. Second, the taxes on incumbents need to be large enough not to induce any firm below θ^* to enter and stay beyond the first period despite initial losses. Third, the taxes on incumbents also need to be small enough to allow all high-quality firms above θ^* to realize positive expected profits over time. These three

conditions are achieved by setting the tax and subsidy rates as in Proposition 5. τ_s is decreasing in s and is negative for large enough s . An important point is that there needs not be a large subsidy for continuing exporters, because the endogenous change in reputation acts as an implicit subsidy for firms which are still active exporters after the policy is put in place. By discouraging the entry of low-quality firms, the tax on entrants improves the country reputation, which allows the remaining firms in the market to charge higher prices. The price effect makes up partially or fully for the effect of the tax on profits, in such a way that the government needs to compensate these firms less through later subsidies.

As long as the government can observe the export tenure of each firm and there is no cost of collecting taxes and distributing subsidies, this tax/subsidy scheme is optimal from a global and domestic welfare point of view. However, it has a distributional impact as not all firms benefit from the policy. On the one hand, the low-quality firms that would export without the tax (below the initial θ_L) lose profits. On the other hand, the firms that become exporters because of the policy (initially between θ^* and θ_H) gain as they are now able to realize positive profits; and firms that export in both cases (above θ_H) gain from higher prices brought about by the better country reputation. Overall, the improvement in reputation raises global welfare. Since all the surplus is captured by the home country through the pricing mechanism, the losses incurred by firms pushed out of the market, whose production is socially inefficient, are more than offset by the gains of the new and remaining exporters.

The large first-period tax can be interpreted as an export license. All entrants are required to pay the initial tax, while only high-quality firms benefit in equilibrium from the lower tax rates in later periods and eventually from a subsidy. In this regard, the policy resembles the export license or quality stamp explored by Chisik (2003) whereby an optional tax is rebated only to firms that produce high quality. Chisik (2003) finds that this policy supports a separating equilibrium and improves welfare, but he needs to assume that the government perfectly observes the quality of a firm's exported products after the first period. Our tax and subsidy program relaxes this assumption by only requiring that the government observes the export tenure of each firm. From this point of view, it is closer to the introductory tax and mature phase subsidy described by Bagwell and Staiger (1989), but we generalize their result to a large number of periods and types and take into account the endogenous response of consumer beliefs.

Our solution has the novel feature that it depends on the variance of the quality distribution. The taxes τ_1 and τ_s are decreasing in the shape pa-

parameter α (for all s), which implies that they are increasing in the variance of the distribution of firm quality types. Similarly, the post-policy equilibrium quality and reputation increase with variance. A higher variance does not affect the perfect information threshold but raises the average quality of firms above the threshold. Therefore, the equilibrium country reputation under the policy is higher the higher the variance. Low-quality firms would stand to make larger pre-tax profits if they were to enter, hence a higher path of taxes is needed to deter them and to support the perfect information outcome.

Lastly, a caveat is that this policy may suffer from a time inconsistency problem, a point also raised in Bagwell and Staiger (1989) and Chisik (2003). Once low-quality firms have decided not to enter, the government has no further incentive to carry through with the announced taxes and subsidies on continuing exporters. This concern can be alleviated by the fact that it is a repeated game. In our model with an infinite horizon and new cohorts of firms every period, if new firms can observe the taxes paid and subsidies received by older generations of firms, the announced policies need to be implemented for the government to maintain its credibility with new entrants and sustain the higher steady-state equilibrium. However, such a cooperative equilibrium might not be sustainable, for example if government officials are not re-electable or if there are term limits.

5.3. Minimum Quality Standards

Finally, we examine the effect of imposing minimum quality standards. An advantage of minimum quality standards is that they are not affected by the time inconsistency problem. Quality standards have been used by Japan, for instance, in the aftermath of World War II. At that time, “Made in Japan” goods had a reputation for being cheap low-quality goods. To improve their “national brand”, both Japanese private companies and the government proceeded to impose strict quality norms (Matsushita, 1979).²² Providing product quality assurances to importers stimulated growth in exports, improved terms of trade, and was key to establish a reputation for product quality (Lynn and McKeown, 1988).

We model a minimum quality standard (MQS) as a quality threshold θ_{MQ} . We assume that firms of quality lower than θ_{MQ} are never able to sell

²²Japanese firms formed export cartels which provided product quality guarantees, by setting product design and quality standards, establishing industry brand names, guaranteeing delivery schedules and mediating disputes between individual exporters and foreign buyers (Dyck, 1992).

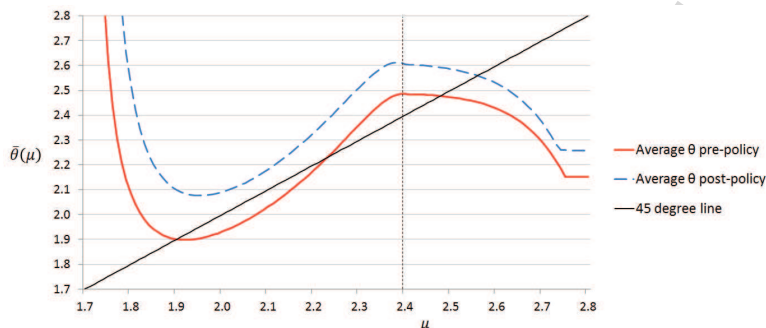
their products.²³ We focus on the situation in which the economy is initially in a LQE.

If there is no cost of implementing the standard, then from a global welfare point of view, it is optimal to set $\theta_{MQ} = \theta^*$. In other words, it is socially inefficient to let firms of quality below θ^* enter the export market. However, not all firms benefit from the standard. Let us call μ_{init} the initial steady-state reputation, and define $\theta_{L,\text{init}}$ and $\theta_{H,\text{init}}$ as the corresponding thresholds. Firms with quality $\theta < \theta_{L,\text{init}}$ lose from the policy: they were making positive profits from a fly-by-night strategy before the introduction of the standard, but are shut out of the market afterwards. The gains from introducing a MQS are reaped by firms with quality $\theta^* < \theta < \theta_{H,\text{init}}$, which were not exporting in the initial equilibrium, and by firms with quality $\theta > \theta_{H,\text{init}}$, which were already exporting but receive extra profits brought about by a higher reputation (see Appendix A.6 for details).

Suppose now that the cost of implementing the standard is $c(\theta_{MQ})$ and is strictly positive for any $\theta_{MQ} > \theta_m$. If the implementation cost does not depend on the threshold, i.e. if $c'(\theta_{MQ}) = 0$, the welfare-maximizing threshold conditional on the existence of the policy is still θ^* . However, if the cost of certifying quality is higher than the welfare gain from the resulting improvement in reputation, then it is too costly for the government to support the perfect information equilibrium and the optimal policy is to set no standard at all.

The most interesting case is one in which the cost of inspection increases with the threshold, i.e. $c'(\theta_{MQ}) > 0$. Then even if $c(\theta^*)$ is high enough that it is not worth implementing the perfect information outcome, setting the standard at a lower level can still be welfare-enhancing. While the non-linearity of quality and reputation precludes us from deriving a closed-form solution for the optimal MQS, we can show that even a small standard can support a HQE. Figure 6 illustrates such a case. In this example, a low MQS is implemented. It results in an upwards shift in the average quality function, which eliminates the LQEs of this economy. Starting from the stable LQE, the economy transitions towards the unique post-policy HQE. It is noteworthy that, first, the standard did not need to be set at a high level to induce this transition; and second, such a policy also improves the steady-state reputation when the economy starts in a HQE, contrary to

²³For the sake of simplicity, we assume that the inspection probability is always equal to 100%. A controlled non-compliant firm has to exit the export market after it incurs the production cost $w\theta + k$ and before it can sell its good. Controlled firms of quality $\theta < \theta_{MQ}$ thus make losses, and in equilibrium never enter.



Notes: The initial parameter values are identical to subfigure 3a. The perfect information threshold is $\theta^* = 2.4$. The minimum quality standard truncates the distribution of active firms at $\theta_{MQ} = 1.05$.

Figure 6: Equilibria before and after the introduction of a low MQS

export subsidies.

Lastly, it may be too costly for the government to set a minimum quality standard high enough to support a HQE. In such cases, if the cost of certifying quality is too high and the government is unable to pre-commit to a path of future taxes, the best policy may be to supplement the MQS with export subsidies.

6. Conclusion

We have shown that when consumers are not fully informed about the quality of what they buy, national reputation matters for exporters. The inability to reveal quality to consumers before purchase distorts the incentives to enter export markets for new firms. Low-quality firms rely on the national brand, while high-quality firms suffer from it. More broadly, unobservable quality tilts the long-run quality composition of an export-oriented industry towards its low end, all the more so as the exporting economy has a poor reputation for quality. In that respect, reputation has self-perpetuating features since future national reputation adjusts to past exports quality. These issues are particularly relevant for developing countries trying to grow into exporting increasingly sophisticated goods. National reputations create history dependence in the range of goods a country can successfully export. A damaged national reputation is a barrier to entry for companies that develop more expensive high-quality products, threatening the success of such a growth strategy. In those cases, we have examined several possible policy responses designed to enhance the quality reputation and welfare of an

exporting country.

This analysis suggests several avenues for future empirical research. In our model, the rational expectations steady-state can fall into one of two categories. In a high-quality equilibrium, all firms are able to export: low-quality firms enter initially as fly-by-nights and eventually exit after a given number of periods; high-quality firms enter and keep exporting. In a low-quality equilibrium, an intermediate range of middle-quality firms never become active exporters. This difference in the sorting of firms into exporters and non exporters may be useful for future empirical research to identify low-quality traps. In addition, case studies have already provided evidence of the benefits of a reputation for quality in terms of brand premia (Imbs et al., 2010) and image spillovers across products of the same brand (Sullivan, 1990). We develop these insights further by considering the demand for imports, where initial priors depend on country-of-origin and reputations are built not only for specific firms but also for exporting countries as a whole. Our findings could lay the ground for future research analyzing empirically the extent to which country reputations matter for trade flows.

Going further, our analysis provides a framework for a richer understanding of firms' sourcing decisions through the lens of a strategic use of "made in" rules. Exporters can find it optimal to resort to original equipment manufacturers or depart from the cost-minimizing way of splitting the production process across locations, in order to obtain a favorable country-of-origin denomination. The location of manufacturing and assembly will be decided not only according to cost considerations, but also depending on the regulations surrounding rules of origin, consumer sensitivity to quality, and the degree of asymmetric information in the industry. An extension of our model along these lines would generate testable predictions at the firm level. These topics will be investigated in future research.

Appendix A. Proofs

Appendix A.1. Proof of Proposition 1: Sorting of Firms into Exports

Appendix A.1.1. High-Quality Equilibrium

Assume there is a steady state equilibrium where $\mu > \frac{k}{1-w}$. First, consider firms with $\theta < \theta^*$ born at date t . Since their expected profits are decreasing with time, they are active in the first period if and only if $E_t \pi_{t+1}(\theta) = \mu - w\theta - k > 0$, which requires $\theta \leq \frac{\mu-k}{w}$. Since $\mu > \frac{k}{1-w}$, it is straightforward that $\frac{\mu-k}{w} > \mu > \theta^*$ which ensures $E_t \pi_{t+1}(\theta) > 0$ for all firms born at t which have quality $\theta < \theta^*$. Hence all such firms enter initially. Also, $\theta < \frac{k}{1-w}$ and $\rho' > 0$ imply that $E_t \pi_{t+s}$ in (4) is decreasing in s and $\lim_{s \rightarrow \infty} E_t \pi_{t+s} = (1-w)\theta - k < 0$ so all firms below θ^* expect to exit in finite time when their profits turn negative. The expected number of periods a firm θ born at t is active is $T(\theta)$ given by $[1 - \rho(T(\theta - 1))] \mu > [w - \rho(T(\theta) - 1)] \theta + k$ and $[1 - \rho(T(\theta))] \mu < [w - \rho(T(\theta))] \theta + k$.

The highest quality type θ_T that exits after selling for T periods (or the lowest quality type that exits after selling for $T + 1$ periods) is defined by $E_t \pi_{t+T+1}(\theta_T) = 0$, hence

$$\theta_T = \max \left\{ \frac{k - [1 - \rho(T)] \mu}{\rho(T) - w}, \theta_m \right\} \quad (\text{A.1})$$

and θ_T is increasing with T : $\frac{\partial \theta_T}{\partial T} \propto \rho'(T) (\mu(1-w) - k) > 0$ as $\rho' > 0$ and $\mu > \theta^*$

Second, consider firms with $\theta^* < \theta < \mu$. These firms expect positive profits at all periods: they have $E_t \pi_{t+s}(\theta)$ monotonically decreasing, from $\pi_{t+1}(\theta) = \mu - w\theta - k > \mu(1-w) - k > 0$ since $\theta < \mu$, to $\lim_{s \rightarrow \infty} E_t \pi_{t+s}(\theta) = \theta(1-w) - k > 0$ since $\theta > \theta^*$. Hence firms with $\theta^* < \theta < \mu$ always enter the market and stay until they are exogenously forced to exit.

Finally, consider the highest quality firms with $\theta > \mu$. Such firms have increasing expected profits over time. They enter the market if and only if their expected intertemporal profits are positive, which requires: $E_t \left(\sum_{s=1}^{\infty} \delta^{s-1} \pi_{t+s}(\theta) \right) = \sum_{s=0}^{\infty} \delta^s [(\rho(s) - w)\theta + (1 - \rho(s))\mu] - \frac{k}{1-\delta} > 0$ or equivalently:

$$\theta > \frac{k - \mu(1-\delta) \sum_{s=0}^{\infty} \delta^s (1 - \rho(s))}{(1-\delta) \sum_{s=0}^{\infty} \delta^s \rho(s) - w} \equiv \theta_H \quad (\text{A.2})$$

Let us show that $\theta_H < \mu$. Rearranging:

$$\theta_H < \mu \text{ iff } \mu [(1-\delta) \sum_{s=0}^{\infty} \delta^s (1 - \rho(s)) + (1-\delta) \sum_{s=0}^{\infty} \delta^s \rho(s)] > w\mu + k$$

or equivalently iff $\mu > \frac{k}{1-w}$ which holds by assumption in the high reputation case. Hence all firms with $\theta > \mu$ always export until they are hit by the exogenous shock.

Appendix A.1.2. Low-Quality Equilibrium

Assume there is a steady state equilibrium with $\mu < \frac{k}{1-w}$. First, consider firms with $\theta < \mu$ born at date t . Since their expected profits are decreasing with time, they are active in the first period if and only if $E_t\pi_{t+1}(\theta) = \mu - w\theta - k > 0$, which requires $\theta \leq \frac{\mu-k}{w} \equiv \theta_L$. We can immediately check that $\mu < \frac{k}{1-w} \Leftrightarrow \theta_L < \mu$. Expected second-period profits are $E_t\pi_{t+2}(\theta) = (\rho(1-w)\theta + (1-\rho(1))\mu - k) < (1-w)\mu - k < 0$ since $\theta < \mu$ and $\rho(1) > w$. Hence among firms with $\theta < \mu$, those with $\theta < \theta_L$ are active in the first period and exit afterwards, and those with $\theta_L \leq \theta < \mu$ are never active.

Second, consider firms with $\mu \leq \theta < \theta^*$. These firms have $E_t\pi_{t+1}(\theta) < 0$ since $\theta > \theta_L$, $E_t\pi_{t+s}(\theta)$ monotonically increasing in s since $\theta \geq \mu$, and $\lim_{s \rightarrow \infty} E_t\pi_{t+s}(\theta) < 0$ since $\theta < \theta^*$. Thus their expected profits are negative in all periods and they optimally exit after drawing their quality parameter.

Third, consider firms with $\theta > \theta^*$. These firms have $E_t\pi_{t+s}(\theta)$ monotonically increasing in s since $\theta > \mu$, and $\lim_{s \rightarrow \infty} E_t\pi_{t+s}(\theta) > 0$ since $\theta > \theta^*$. If they decide to be active in the first period, they expect to remain in the market as long as they survive the exogenous shock. However given $\theta > \theta_L$ they incur a loss in the initial periods. The condition for a firm of type $\theta > \theta^*$ to be active is for intertemporal expected profits to be positive, which requires $\theta > \frac{k-(1-A_\rho)\mu}{A_\rho-w} \equiv \theta_H$ as derived in (A.2), where we define $A_\rho \equiv (1-\delta)\sum_{s=0}^{\infty} \delta^s \rho(s)$. Finally, $\theta_H > \theta^*$ iff $\frac{k-(1-A_\rho)\mu}{A_\rho-w} > \frac{k}{1-w}$ which simplifies to $\frac{k}{1-w} > \mu$ and holds by definition in the low reputation case. Hence firms with $\theta^* \leq \theta \leq \theta_H$ are never active and firms with $\theta > \theta_H$ enter the export market and stay active.

Lastly, the equilibrium reputation must satisfy $\mu > k + w\theta_m$. Suppose μ is a steady state reputation and $\mu < k + w\theta_m$. The first period price μ does not cover the production cost of the lowest quality firm, hence no firm below θ^* finds it profitable to enter. It follows that no rational expectations equilibrium can have $\mu < \theta^*$, so there is no LQE with $\mu < k + w\theta_m$. Similarly, if $\theta^* < \mu < k + w\theta_m$, then firms with $\theta < \mu$ would have negative profits in all periods. Hence such firms are never active, and μ cannot be a HQE reputation under rational expectations.

Appendix A.2. Proof of Proposition 2: Existence Conditions

Proposition 2 establishes the existence of at least one steady state equilibrium and the possibility of multiple equilibria. We investigate how $\bar{\theta}(\mu)$ varies with μ on $[\theta_m, \infty)$. Then, we compute the fixed points of the function $\bar{\theta}(\mu)$, and show that there can be multiple fixed points.

Appendix A.2.1. Average Quality Function

Given Proposition 1 and equation (6), the average quality $\bar{\theta}$ of active firms in a steady state as a function of the country reputation μ is, on $[\theta_m, \theta^*]$:

$$\bar{\theta}(\mu) = \mu_0 \left(\frac{(1-\delta) \left(1 - \left(\frac{\theta_m}{\theta_L} \right)^{\alpha-1} \right) + \left(\frac{\theta_m}{\theta_H} \right)^{\alpha-1}}{(1-\delta) \left(1 - \left(\frac{\theta_m}{\theta_L} \right)^\alpha \right) + \left(\frac{\theta_m}{\theta_H} \right)^\alpha} \right) \quad (\text{A.3})$$

and on $[\theta^*, \infty)$:

$$\begin{aligned} \bar{\theta}(\mu) &= \mu_0 \left(\frac{1 - \sum_{T=0}^{\infty} \delta^{T+1} \left(\left(\frac{\theta_m}{\theta_T} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{T+1}} \right)^{\alpha-1} \right)}{1 - \sum_{T=0}^{\infty} \delta^{T+1} \left(\left(\frac{\theta_m}{\theta_T} \right)^\alpha - \left(\frac{\theta_m}{\theta_{T+1}} \right)^\alpha \right)} \right) \\ &= \mu_0 \left(\frac{1 + \sum_{T=\tilde{T}}^{\infty} \delta^T \left(\frac{\theta_m}{\theta_T} \right)^{\alpha-1}}{1 + \sum_{T=\tilde{T}}^{\infty} \delta^T \left(\frac{\theta_m}{\theta_T} \right)^\alpha} \right) \end{aligned} \quad (\text{A.4})$$

where \tilde{T} is the lowest value of T such that $\theta_T > \theta_m$ in equation (A.1). At $\mu = \theta^*$, $\theta_L = \theta_H = \theta^*$ and $\theta_T = \theta^*$ for all T . It follows that the function $\bar{\theta}(\mu)$ is continuous at θ^* since both equations above yield:

$$\bar{\theta}(\theta^*) = \mu_0 \left(\frac{1 - \delta + \delta \left(\frac{\theta_m}{\theta^*} \right)^{\alpha-1}}{1 - \delta + \delta \left(\frac{\theta_m}{\theta^*} \right)^\alpha} \right) \quad (\text{A.5})$$

Appendix A.2.2. Equilibrium Existence

A steady state equilibrium is a fixed point of the function $\bar{\theta}(\mu)$ defined in Appendix A.2.1. We have already established that the function is continuous on $[\theta_m, \infty)$. Let us show in addition that $\bar{\theta}(\theta_m) > \theta_m$ and $\lim_{\mu \rightarrow \infty} \frac{\bar{\theta}(\mu)}{\mu} < 1$.

If $\mu = \theta_m$, no firm below θ^* finds it profitable to export, as national reputation imposes a first-period loss on all firms. Some firms with high

enough θ have a positive NPV of future profits and enter. So since θ_m is the lower bound of the prior quality distribution, $\bar{\theta}(\theta_m) > \theta^* > \theta_m$.

As $\mu \rightarrow \infty$, it remains profitable for all firms to stay active, so firms of all qualities continue exporting until hit by the exogenous shock: $T(\theta) \rightarrow \infty$ for all θ . Therefore, $\lim_{\mu \rightarrow \infty} \bar{\theta}(\mu) = \mu_0$ which is finite, so $\lim_{\mu \rightarrow \infty} \frac{\bar{\theta}(\mu)}{\mu} = 0 < 1$.

By the fixed point theorem, we have established that $\bar{\theta}(\cdot)$ has at least one fixed point on $[\theta_m, \infty)$.

Appendix A.2.3. Existence Condition for a High Quality Equilibrium

A HQE is a steady state equilibrium with equilibrium reputation above θ^* . Therefore a sufficient condition for the existence of a HQE is $\bar{\theta}(\theta^*) > \theta^*$ in (A.5). We then prove that this is also a necessary condition and that if there exists at least one HQE, there is only one HQE.

Let us show that $\bar{\theta}(\mu)$ is strictly decreasing in μ on $[\theta^*, \infty)$. We can rewrite (A.4) as: $\bar{\theta}(\mu) = \mu_0 \left(\frac{1+K(\alpha-1)}{1+K(\alpha)} \right)$ where $K(\alpha) \equiv \sum_{T=\bar{T}}^{\infty} \delta^T \left(\frac{\theta_m}{\theta_T} \right)^\alpha$. It follows that $\frac{\partial K(\alpha)}{\partial \alpha} = \sum_{T=\bar{T}}^{\infty} \delta^T \ln(\theta_m/\theta_T) \left(\frac{\theta_m}{\theta_T} \right)^\alpha < 0$.

Consider a positive change in one of the thresholds, θ_S , leaving unchanged all other thresholds. Then all else equal, average quality rises:

$$\begin{aligned} \frac{\partial \bar{\theta}}{\partial \theta_S} &= \frac{\delta^S}{\theta_S} \left(\frac{\theta_m}{\theta_S} \right)^{\alpha-1} \left[\alpha \left(\frac{\theta_m}{\theta_S} \right) (1 + K(\alpha - 1)) - (\alpha - 1) (1 + K(\alpha)) \right] \\ &= \frac{\delta^S}{\theta_S} \left(\frac{\theta_m}{\theta_S} \right)^{\alpha-1} (1 + K(\alpha)) \left[\alpha \left(\frac{\theta_m}{\theta_S} \right) \frac{\bar{\theta}}{\mu_0} - (\alpha - 1) \right] \\ &= \frac{\delta^S}{\theta_S} \left(\frac{\theta_m}{\theta_S} \right)^{\alpha-1} (1 + K(\alpha)) (\alpha - 1) \left[\left(\frac{\bar{\theta}}{\theta_S} \right) - 1 \right] > 0 \end{aligned}$$

where the positive sign derives from $\bar{\theta} > \theta^* > \theta_S$ for all S in a HQE. An increase in μ lowers all θ_T given Assumptions 1 and 2 and differentiating: $\frac{\partial \theta_T}{\partial \mu} = -\frac{1-\rho(T)}{\rho(T)-w} < 0$. Thus, in a HQE, $\bar{\theta}(\mu)$ is a decreasing function:

$$\frac{\partial \bar{\theta}}{\partial \mu} = \sum_{T=\bar{T}}^{\infty} \frac{\partial \bar{\theta}}{\partial \theta_T} \frac{\partial \theta_T}{\partial \mu} < 0$$

We have proved that $\bar{\theta}(\cdot)$ is strictly and continuously decreasing in μ on $[\theta^*, \infty)$. It follows that $\bar{\theta}(\cdot)$ has at most one fixed point on $[\theta^*, \infty)$. Hence $\bar{\theta}(\theta^*) > \theta^*$ is a necessary and sufficient condition for the existence of a HQE, and if this condition is satisfied, there is only one HQE.

Lastly, we express the condition for $\bar{\theta}(\theta^*) > \theta^*$ as a function of fundamental parameters. At $\mu = \theta^*$, $\pi_t(\theta) < 0$ for all $t > 1$ and $\theta < \theta^*$. Then:
$$\bar{\theta}(\theta^*) = \frac{\int_{\theta_m}^{\theta^*} \theta dG(\theta) + \frac{1}{1-\delta} \int_{\theta^*}^{\infty} \theta dG(\theta)}{\int_{\theta_m}^{\theta^*} dG(\theta) + \frac{1}{1-\delta} \int_{\theta^*}^{\infty} dG(\theta)} = \mu_0 \left(\frac{1-\delta+\delta\left(\frac{\theta_m}{\theta^*}\right)^{\alpha-1}}{1-\delta+\delta\left(\frac{\theta_m}{\theta^*}\right)^{\alpha}} \right).$$
 So $\bar{\theta}(\theta^*) > \theta^*$ is equivalent to $\mu_0 \left(\frac{1-\delta+\delta\left(\frac{\theta_m(1-w)}{k}\right)^{\alpha-1}}{1-\delta+\delta\left(\frac{\theta_m(1-w)}{k}\right)^{\alpha}} \right) > \frac{k}{1-w}$, which after substituting for the value of μ_0 and rearranging yields the following condition for the existence of a HQE:

$$\alpha \left(\frac{\theta_m(1-w)}{k} \right) + \frac{\delta}{1-\delta} \left(\frac{\theta_m(1-w)}{k} \right)^{\alpha} > \alpha - 1$$

Conversely, there is no HQE and there is at least one LQE iff:

$$\alpha \left(\frac{\theta_m(1-w)}{k} \right) + \frac{\delta}{1-\delta} \left(\frac{\theta_m(1-w)}{k} \right)^{\alpha} < \alpha - 1$$

Appendix A.2.4. Multiple Equilibria

While there cannot be more than one HQE, we show that the non-monotonicity of $\bar{\theta}(\cdot)$ on $[\theta_m, \theta^*]$ creates the possibility of multiple equilibria. A LQE is a fixed point of $\bar{\theta}(\mu)$ given by (A.3) on $[\theta_m, \theta^*]$. On this interval, we can show that the sign of $\frac{\partial \bar{\theta}(\mu)}{\partial \mu}$ is indeterminate. Differentiating with respect to each threshold:

$$\begin{aligned} \frac{\partial \bar{\theta}}{\partial \theta_L} &= \frac{\mu_o(\alpha-1)}{1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha} + \frac{1}{1-\delta} \left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \left(\frac{1}{\theta_L}\right) \left(\frac{\theta_m}{\theta_L}\right)^{\alpha-1} \left[1 - \frac{\bar{\theta}}{\theta_L}\right] < 0 \\ \frac{\partial \bar{\theta}}{\partial \theta_H} &= \frac{\frac{1}{1-\delta} \mu_o(\alpha-1)}{1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha} + \frac{1}{1-\delta} \left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \left(\frac{1}{\theta_H}\right) \left(\frac{\theta_m}{\theta_H}\right)^{\alpha-1} \left[\frac{\bar{\theta}}{\theta_H} - 1\right] < 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \bar{\theta}}{\partial \mu} &= \frac{\partial \bar{\theta}}{\partial \theta_L} \frac{\partial \theta_L}{\partial \mu} + \frac{\partial \bar{\theta}}{\partial \theta_H} \frac{\partial \theta_H}{\partial \mu} = \frac{\mu_o(\alpha-1)}{1 - \left(\frac{\theta_m}{\theta_L}\right)^{\alpha} + \frac{1}{1-\delta} \left(\frac{\theta_m}{\theta_H}\right)^{\alpha}} \times \dots \\ &\dots \times \left[\left(\frac{\theta_m^{\alpha-1}}{\theta_L^{\alpha}}\right) \left[\frac{\bar{\theta}}{\theta_L} - 1\right] \left(\frac{1}{w}\right) - \frac{1}{1-\delta} \left(\frac{\theta_m^{\alpha-1}}{\theta_H^{\alpha}}\right) \left[1 - \frac{\bar{\theta}}{\theta_H}\right] \left(\frac{1-A_{\rho}}{A_{\rho}-w}\right) \right] \\ \frac{\partial \bar{\theta}}{\partial \mu} &< 0 \text{ iff } \frac{1}{1-\delta} \left(\frac{1}{\theta_H}\right)^{\alpha} \left[1 - \frac{\bar{\theta}}{\theta_H}\right] \left(\frac{1-A_{\rho}}{A_{\rho}-w}\right) > \left(\frac{1}{\theta_L}\right)^{\alpha} \left[\frac{\bar{\theta}}{\theta_L} - 1\right] \left(\frac{1}{w}\right) \end{aligned}$$

This condition can be rewritten as: $\delta > 1 - \left(\frac{\theta_L}{\theta_H}\right)^{\alpha+1} \left(\frac{\theta_H - \bar{\theta}}{\bar{\theta} - \theta_L}\right) \left(\frac{(1-A_{\rho})w}{A_{\rho}-w}\right)$.

Then note that the bracketed terms are $\frac{\theta_L}{\theta_H} = \frac{(\mu-k)(A_\rho-w)}{w(k-(1-A_\rho)\mu)}$ and $\frac{\theta_H-\bar{\theta}}{\bar{\theta}-\theta_L} = \frac{\frac{k-(1-A_\rho)\mu}{A_\rho-w}-\bar{\theta}}{\bar{\theta}-\frac{\mu-k}{w}} = \frac{w}{A_\rho-w} \left(1 - \frac{A_\rho(\bar{\theta}-\mu)}{k-\mu+w\bar{\theta}} \right)$. Therefore $\bar{\theta}(\mu)$ decreases in μ when:

$$\delta > 1 - \left(\frac{\mu-k}{k-(1-A_\rho)\mu} \right)^{\alpha+1} \left(\frac{A_\rho-w}{w} \right)^{\alpha-1} \left(1 - \frac{A_\rho(\bar{\theta}-\mu)}{k-\mu+w\bar{\theta}} \right)$$

and increases in μ otherwise. The reason why $\bar{\theta}(\mu)$ needs not be monotonic over $[\theta_m, \theta^*]$ is that μ has opposite effects on $\bar{\theta}$ coming from θ_L and θ_H . Which effect dominates depends on the position of μ as well as the shape parameter α and the survival parameter δ . This non-monotonicity is what gives rises to the possibility of multiple equilibria.

To sum up, we have shown that if condition (13) holds, there is one HQE (and there may be zero or a positive number of LQEs). If condition (13) does not hold, there is no HQE and there is at least one LQE.

Appendix A.3. Proof of Proposition 3: Equilibrium Stability

Suppose μ_S is a steady-state LQE such that $\frac{\partial \bar{\theta}(\mu)/\mu}{\partial \mu} < 0$ at μ_S . We will show that μ_S is a stable equilibrium for $\eta < 1$. Let us define $\theta_{L,S} \equiv \frac{\mu_S-k}{w}$ and $\theta_{H,S} \equiv \frac{k-(1-A_\rho)\mu_S}{A_\rho-w}$. At time $t-1$ the economy is in an initial steady-state where $\mu_S = \bar{\theta}(\mu_S)$ as given by (12).

Assume there is a perturbation at time t such that $\mu_t = \mu_S + \varepsilon$, $\varepsilon > 0$ and ε is small. For all $\mu \in (\mu_S, \mu_S + \varepsilon)$, $\bar{\theta}(\mu) < \mu$. The entry thresholds at t are:

$$\begin{aligned} \theta_{L,t} &= \frac{\mu_t - k}{w} = \frac{\mu_S + \varepsilon - k}{w} \\ \theta_{H,t} &= \frac{k - (1 - \delta) \sum_{u=0}^{\infty} (1 - \rho(u)) \mu_{t+u}}{A_\rho - w} \end{aligned}$$

where $\theta_{H,t}$ is determined by the zero intertemporal profits condition

$$\sum_{u=0}^{\infty} \delta^u [(\rho(u) - w) \theta_{H,t} + (1 - \rho(u)) E_t \mu_{t+u} - k]$$

and the absence of aggregate uncertainty allows us to remove the expectations operator.

Let us conjecture, to be verified, that $\mu_S \leq \mu_{t+u+1} < \mu_{t+u} < \mu_S + \varepsilon$ for

all $u \geq 1$. Then for all $u \geq 1$:

$$\begin{aligned}\theta_{L,S} &\leq \theta_{L,t+u+1} < \theta_{L,t+u} < \theta_{L,t} \\ \theta_{H,S} &\geq \theta_{H,t+u+1} > \theta_{H,t+u} > \theta_{H,t}\end{aligned}$$

The average quality of exports is determined by the θ_L and θ_H thresholds in the periods after the shock in the following manner: for $u \geq 0$,

$$\begin{aligned}\bar{\theta}_{t+u} &= \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_{L,t+u}}\right)^{\alpha-1} + \sum_{l=0}^u \delta^{u-l} \left(\frac{\theta_m}{\theta_{H,t+l}}\right)^{\alpha-1} + \sum_{l=u+1}^{\infty} \delta^l \left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_{L,t+u}}\right)^{\alpha} + \sum_{l=0}^u \delta^{u-l} \left(\frac{\theta_m}{\theta_{H,t+l}}\right)^{\alpha} + \sum_{l=u+1}^{\infty} \delta^l \left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha}} \right) \\ \theta_{L,t+u} &= \frac{\mu_{t+u} - k}{w} \\ \theta_{H,t+u} &= \frac{k - (1 - \delta) \sum_{l=0}^{\infty} (1 - \rho(l)) \mu_{t+u+l}}{A_{\rho} - w}\end{aligned}$$

At time t , let us define $\bar{\theta}_t^{perm}$ as the average quality that would prevail if firms expected the shock to be permanent, i.e. if $E_t \mu_{t+u} = \mu_t$ for all $u \geq 0$. We calculate:

$$\begin{aligned}\bar{\theta}_t &= \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_{L,t}}\right)^{\alpha-1} + \left(\frac{\theta_m}{\theta_{H,t}}\right)^{\alpha-1} + \frac{\delta}{1-\delta} \left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_{L,t}}\right)^{\alpha} + \left(\frac{\theta_m}{\theta_{H,t}}\right)^{\alpha} + \frac{\delta}{1-\delta} \left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha}} \right) \\ \bar{\theta}_t < \bar{\theta}_t^{perm} &= \mu_0 \left(\frac{1 - \left(\frac{\theta_m}{\theta_{L,t}}\right)^{\alpha-1} + \left(\frac{\theta_m}{\theta_{H,t}^{perm}}\right)^{\alpha-1} + \frac{\delta}{1-\delta} \left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha-1}}{1 - \left(\frac{\theta_m}{\theta_{L,t}}\right)^{\alpha} + \left(\frac{\theta_m}{\theta_{H,t}^{perm}}\right)^{\alpha} + \frac{\delta}{1-\delta} \left(\frac{\theta_m}{\theta_{H,S}}\right)^{\alpha}} \right)\end{aligned}$$

as $\theta_{H,t}^{perm} = \frac{k - (1 - A_{\rho})(\mu_S + \varepsilon)}{A_{\rho} - w} < \theta_{H,t}$ from the conjecture $\mu_S \leq \mu_{t+u+1} < \mu_{t+u} < \mu_S + \varepsilon$ for all $u \geq 1$. Also, we know that $\bar{\theta}_t^{perm} < \bar{\theta}(\mu_S + \varepsilon) < \mu_t$. The first inequality results from $\theta_{H,S} > \theta_{H,t}^{perm}$. The second inequality comes from $\bar{\theta}(\mu) < \mu$ for $\mu \in (\mu_S, \mu_S + \varepsilon)$. Hence $\bar{\theta}_t < \mu_t$ and therefore $\mu_{t+1} = \mu_t + \eta(\bar{\theta}_t - \mu_t) < \mu_t$. Additionally as long as η is not too close to 1, $\mu_{t+1} > \mu_S$.

We can show, similarly, that in all subsequent periods, $\bar{\theta}_{t+u} < \mu_{t+u}$ as long as $\mu_{t+u} > \mu_S$. Thus $\mu_{t+u+1} < \mu_{t+u}$ for all u and the conjecture that μ_{t+u} follows a decreasing path from $\mu_S + \varepsilon$ to μ_S is verified. In case of a negative shock to μ at time t starting from a steady state where $\frac{\partial \bar{\theta}(\mu)/\mu}{\partial \mu} < 0$,

the proof is identical with opposite signs. It follows that if μ_S is a steady-state LQE reputation and $\frac{\partial \bar{\theta}(\mu)/\mu}{\partial \mu} < 0$ at μ_S , then μ_S is stable.

By the same reasoning, if μ_U is a steady-state LQE and $\frac{\partial \bar{\theta}(\mu)/\mu}{\partial \mu} > 0$ at μ_U , then μ_U is unstable. Suppose there is a negative shock to μ starting from $\mu_U = \bar{\theta}(\mu_U)$. Let us denote by μ_S the equilibrium of rank immediately preceding μ_U . Further define $\theta_{L,U} \equiv \frac{\mu_U - k}{w}$, $\theta_{H,U} \equiv \frac{k - (1 - A_\rho)\mu_U}{A_\rho - w}$, $\theta_{L,S} \equiv \frac{\mu_S - k}{w}$ and $\theta_{H,S} \equiv \frac{k - (1 - A_\rho)\mu_S}{A_\rho - w}$. At time t , $\mu_t = \mu_U - \varepsilon$, where $\varepsilon > 0$ and ε small. We conjecture $\mu_S \leq \mu_{t+u+1} < \mu_{t+u} < \mu_U - \varepsilon$, which implies $\theta_{L,S} \leq \theta_{L,t+u+1} < \theta_{L,t+u} < \theta_{L,U}$ and $\theta_{H,S} \geq \theta_{H,t+u+1} > \theta_{H,t+u} > \theta_{H,U}$ for all $u \geq 0$. Then $\bar{\theta}_{t+u} < \mu_{t+u}$ and thus $\mu_{t+u+1} < \mu_{t+u}$ for all $u \geq 0$ as long as $\mu_{t+u} > \mu_S$. It follows that the economy converges to the steady-state equilibrium of immediately lower (or higher in the case of a positive shock) rank where $\bar{\theta}(\mu)$ crosses the 45-degree line from above.

Lastly, the stability of a HQE is derived from a similar proof. Suppose that μ_Q is the initial steady-state HQE. We know from Appendix A.2.4 that $\frac{\partial \bar{\theta}(\mu)/\mu}{\partial \mu} < 0$ at μ_Q . Let us define $\theta_{T,Q} \equiv \max \left\{ \frac{k - [1 - \rho(T)]\mu_Q}{\rho(T) - w}, \theta_m \right\}$ for all $T \geq 1$. Assume there is a perturbation at time t such that $\mu_t = \mu_Q + \varepsilon$, where $\varepsilon > 0$ and ε small.

We conjecture that $\mu_Q \leq \mu_{t+u+1} < \mu_{t+u} < \mu_Q + \varepsilon$. Then at time $t + u$, firms having already exported for T periods exit if their quality parameter is below $\theta_{T,t+u}$ given by $\theta_{T,t+u} \equiv \max \left\{ \frac{k - [1 - \rho(T)]\mu_{t+u}}{\rho(T) - w}, \theta_m \right\}$ for all $T \geq 1$. It follows that $\theta_{T,Q} \leq \theta_{T,t+u+1} < \theta_{T,t+u} < \theta_{T,t}$ for all T and $u \geq 1$. As in the stable LQE case, we can define $\bar{\theta}_t^{perm}$ as the average quality that would prevail at t if the shock was expected to be permanent. It is straightforward that $\bar{\theta}_t < \bar{\theta}_t^{perm} < \bar{\theta}(\mu_Q + \varepsilon) < \mu_t$, from which it follows that $\mu_{t+1} < \mu_t$. The same reasoning applies to show more generally that $\mu_{t+u+1} < \mu_{t+u}$ for all u as long as $\mu_{t+u} > \mu_Q$. Therefore μ_Q is stable.

Appendix A.4. Proof of Proposition 4: Export Subsidies

Appendix A.4.1. Low-Quality Equilibrium

In a LQE, average quality is given by (A.3). With $\frac{\partial \bar{\theta}}{\partial \theta_L}$ and $\frac{\partial \bar{\theta}}{\partial \theta_H}$ derived in Appendix A.2.4, we obtain:

$$\begin{aligned} \frac{\partial \bar{\theta}}{\partial k} &= \frac{\partial \bar{\theta}}{\partial \theta_L} \frac{\partial \theta_L}{\partial k} + \frac{\partial \bar{\theta}}{\partial \theta_H} \frac{\partial \theta_H}{\partial k} = \frac{\mu_o (\alpha - 1)}{1 - \left(\frac{\theta_m}{\theta_L}\right)^\alpha + \frac{1}{1-\delta} \left(\frac{\theta_m}{\theta_H}\right)^\alpha} \times \dots \\ &\times \left[\frac{1}{1-\delta} \left(\frac{1}{\theta_H}\right) \left(\frac{\theta_m}{\theta_H}\right)^{\alpha-1} \left(1 - \frac{\bar{\theta}}{\theta_H}\right) \left(\frac{1}{A_\rho - w}\right) - \left(\frac{1}{\theta_L}\right) \left(\frac{\theta_m}{\theta_L}\right)^{\alpha-1} \left(\frac{\bar{\theta}}{\theta_L} - 1\right) \left(\frac{1}{w}\right) \right] \\ \frac{\partial \bar{\theta}}{\partial k} &> 0 \text{ iff } \frac{1}{1-\delta} \left(\frac{1}{\theta_H}\right)^\alpha \left(1 - \frac{\bar{\theta}}{\theta_H}\right) \left(\frac{1}{A_\rho - w}\right) > \left(\frac{1}{\theta_L}\right)^\alpha \left(\frac{\bar{\theta}}{\theta_L} - 1\right) \left(\frac{1}{w}\right) \end{aligned}$$

This condition can be rewritten as $\delta > 1 - \left(\frac{\theta_L}{\theta_H}\right)^{\alpha+1} \left(\frac{\theta_H - \bar{\theta}}{\bar{\theta} - \theta_L}\right) \left(\frac{w}{A_\rho - w}\right)$. Note that, starting from a steady-state ($\bar{\theta} = \mu$), the bracketed terms are $\frac{\theta_L}{\theta_H} = \frac{(\mu - k)(A_\rho - w)}{w(k - (1 - A_\rho)\mu)}$ and $\frac{\theta_H - \bar{\theta}}{\bar{\theta} - \theta_L} = \frac{\frac{1}{A_\rho - w}(k - (1 - w)\mu)}{\frac{1}{w}(k - (1 - w)\mu)} = \frac{w}{A_\rho - w}$.

Therefore $\bar{\theta}$ decreases in k if and only if

$$\delta > 1 - \left(\frac{\mu - k}{k - [1 - A_\rho]\mu}\right)^{\alpha+1} \left(\frac{A_\rho - w}{w}\right)^{\alpha-1}$$

The RHS is decreasing in μ and α , so this holds for δ not too low, α not too high and an initial μ not too low. Then starting from a LQE, a decrease in k moves up the $\bar{\theta}(\mu)$ function left of the initial μ . The new steady-state equilibrium quality and reputation are necessarily higher. If the steady-state is unique, the new steady-state has higher μ . If there are multiple steady-states, ranked by increasing μ , either the new steady-state has the same rank and higher μ , or the new steady-state has higher rank and higher μ .

The welfare effect of a subsidy σ ($\sigma = -dk$) has three components. First, for firms with θ parameters such that they sell both without and with the subsidy, the policy adds to their profits the amount it costs to the government, plus the extra profits brought by a higher reputation $\mu' > \mu$. The total effect is unambiguously positive.

Second, for new exporters that enter around θ_L because of the policy

$(\theta_L < \theta < \theta_{L'})$, the net benefit NB_L of the subsidy is positive:

$$\begin{aligned}
 NB_L &= \int_{\theta_L}^{\theta_{L'}} (\mu' - w\theta - k + \sigma) dG(\theta) - \int_{\theta_L}^{\theta_{L'}} \sigma g(\theta) d\theta \\
 &= (\mu' - k) \int_{\theta_L}^{\theta_{L'}} g(\theta) d\theta - w \int_{\theta_L}^{\theta_{L'}} \theta dG(\theta) \\
 &= \left[(\mu' - k) \left(\left(\frac{\theta_m}{\theta_L} \right)^\alpha - \left(\frac{\theta_m}{\theta_{L'}} \right)^\alpha \right) - w \frac{\alpha}{\alpha-1} \theta_m \left(\left(\frac{\theta_m}{\theta_L} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{L'}} \right)^{\alpha-1} \right) \right] \\
 &= w\theta_m \left[\left(\frac{\theta_{L'}}{\theta_L} \left(\frac{\theta_m}{\theta_L} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{L'}} \right)^{\alpha-1} \right) - \frac{\alpha}{\alpha-1} \left(\left(\frac{\theta_m}{\theta_L} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{L'}} \right)^{\alpha-1} \right) \right] \\
 &> \frac{w\theta_m}{\alpha-1} \left(\left(\frac{\theta_m}{\theta_L} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{L'}} \right)^{\alpha-1} \right) > 0
 \end{aligned}$$

where we go from the second to the third line using $w\theta_{L'} = \mu' - k$.

Third, for new exporters that enter around θ_H because of the policy $(\theta_{H'} < \theta < \theta_H)$, the net benefit NB_H of the subsidy is also positive:

$$\begin{aligned}
 NB_H &= \int_{\theta_{H'}}^{\theta_H} \left(\sum_{t=0}^{\infty} \delta^t (\rho(t)\theta + (1-\rho(t))\mu' - w\theta - k + \sigma) \right) g(\theta) d\theta - \dots \\
 &\dots \frac{1}{1-\delta} \int_{\theta_{H'}}^{\theta_H} \sigma g(\theta) d\theta \\
 &= \frac{1}{1-\delta} \int_{\theta_{H'}}^{\theta_H} ((A_\rho - w)\theta + (1 - A_\rho)\mu' - k) g(\theta) d\theta \\
 &= \frac{1}{1-\delta} \left[- (k - (1 - A_\rho)\mu') \left(\left(\frac{\theta_m}{\theta_{H'}} \right)^\alpha - \left(\frac{\theta_m}{\theta_H} \right)^\alpha \right) + \frac{\alpha(A_\rho - w)}{\alpha-1} \theta_m \left(\left(\frac{\theta_m}{\theta_{H'}} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_H} \right)^{\alpha-1} \right) \right] \\
 &= \frac{1}{1-\delta} (A_\rho - w) \theta_m \left[\left(\left(\frac{\theta_m}{\theta_{H'}} \right)^{\alpha-1} - \frac{\theta_{H'}}{\theta_H} \left(\frac{\theta_m}{\theta_H} \right)^{\alpha-1} \right) - \frac{\alpha}{\alpha-1} \left(\left(\frac{\theta_m}{\theta_{H'}} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_H} \right)^{\alpha-1} \right) \right] \\
 &> \frac{1}{1-\delta} (A_\rho - w) \frac{\theta_m}{\alpha-1} \left(\left(\frac{\theta_m}{\theta_{H'}} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_H} \right)^{\alpha-1} \right) > 0
 \end{aligned}$$

So the overall welfare gain is positive.

Appendix A.4.2. High-Quality Equilibrium

In a HQE, average quality is given by (A.4). Using the derivations in the proof of Proposition 2, we obtain:

$$\begin{aligned}\frac{\partial \theta_T}{\partial k} &= \frac{1}{\rho(T) - w} > 0 \quad \text{for all } T > \tilde{T} \\ \frac{\partial \bar{\theta}}{\partial k} &= \sum_{T=\tilde{T}}^{\infty} \frac{\partial \bar{\theta}}{\partial \theta_T} \frac{\partial \theta_T}{\partial k} > 0\end{aligned}$$

Hence a subsidy that lowers k shifts down the $\bar{\theta}(\mu)$ function. As $\bar{\theta}$ is decreasing in μ in the HQE region, the new steady-state equilibrium defined by $\bar{\theta}(\mu) = \mu$ necessarily has lower μ . So average quality and national reputation are higher in the HQE steady-state without subsidies than with subsidies.

Appendix A.5. Proof of Proposition 5: Tax/Subsidy Scheme

In this section, we show that the age-dependent tax/subsidy in Proposition 5 sustains a steady-state equilibrium which replicates the perfect information entry and exit patterns by quality level. First, all per-period profits, and therefore all entry or exit decisions, depend on μ , which is itself affected by the policy. If the tax/subsidy scheme induces decisions identical to the perfect information setting, then firms below θ^* never enter and firms above θ^* always enter and stay. The resulting average quality and steady-state reputation would therefore be: $\bar{\theta}(\mu) = \mu = \int_{\theta^*}^{\infty} \theta dG(\theta) = \frac{\alpha}{\alpha-1} \theta^*$.

Second, let us show that with $\mu = \frac{\alpha}{\alpha-1} \theta^*$ and taxes and subsidies set as in Proposition 5, firms below θ^* choose never to enter. In the first export period, profits are decreasing in θ . To show that no firm below θ^* can profitably enter as fly-by-night and exit after one period, it suffices that a firm of quality θ_m cannot do so. For firm θ_m , first-period profits are $\pi_{t+1}(\theta_m) = \mu - k - w\theta_m - \tau_1 = \frac{\alpha}{\alpha-1} \theta^* - k - w\theta_m - \left[\frac{\alpha}{\alpha-1} \left(\frac{k}{1-w} \right) - k - w\theta_m \right] = 0$.

Hence no firm can realize strictly positive first period profits. However, if subsidies are offered in later periods, it could still be the case that some firms below θ^* have an incentive to enter and pay the initial tax in order to benefit from subsidies in later periods. The tax/subsidy combination for $s \geq 2$ ensures that this is not the case. The expected per-period profit of a continuing exporter is, for $s \geq 2$:

$$E_t \pi_{t+s}(\theta) = [\rho(s-1) - w] \theta + [1 - \rho(s-1)] \frac{\alpha}{\alpha-1} \theta^* - k - \tau_s$$

Among low-quality firms, this expression is the highest for firms of quality just below θ^* . Then the definition of θ^* yields:

$$\begin{aligned} E_t \pi_{t+s}(\theta^*) &= \theta^* - k - w\theta^* + \left(\frac{1-\delta}{\delta}\right) \left(\tau_1 - \frac{1}{\alpha-1} \frac{k}{1-w}\right) \\ &= \left(\frac{1-\delta}{\delta}\right) \left(\tau_1 - \frac{1}{\alpha-1} \theta^*\right) = \left(\frac{1-\delta}{\delta}\right) (\theta^* - w\theta_m) > 0 \end{aligned}$$

Hence if firm θ^* enters, it incurs a loss in the first period and positive profits thereafter. It follows that it never exits voluntarily and its intertemporal expected profits are:

$$\begin{aligned} E_t \sum_{s=1}^{\infty} \delta^{s-1} \pi_{t+s}(\theta^*) &= \pi_{t+1}(\theta^*) + \sum_{s=2}^{\infty} \delta^{s-1} \left(\frac{1-\delta}{\delta}\right) \left(\tau_1 - \frac{1}{\alpha-1} \theta^*\right) \\ &= \frac{\alpha}{\alpha-1} \theta^* - k - w\theta^* - \tau_1 + \tau_1 - \frac{1}{\alpha-1} \theta^* = \theta^* - k - w\theta^* = 0 \end{aligned}$$

Since the after-tax, after-subsidy profits of continuing exporters are increasing in θ , it follows that firms below θ^* realize negative intertemporal profits regardless of their exit date and never enter.

Third, by the same token, all firms above θ^* have initially negative profits but a positive net present value of their profit stream, and have no incentive to exit voluntarily. Finally, note that τ_s can be rewritten as $\tau_s = \left(\frac{1-\rho^{(s-1)}}{\alpha-1}\right) \theta^* - \frac{1-\delta}{\delta} (\theta^* - k - w\theta_m)$ where the second term is negative. τ_1 is positive and given Assumption 1, τ_s is decreasing in s , τ_2 may be positive or negative, and τ_s negative for s large enough.

Appendix A.6. Minimum Quality Standard

Let us assume a minimum quality standard $\theta_{QS} = \theta^*$, such that all the firms with $\theta < \theta^*$ choose not to enter the export market. Thanks to this minimum quality standard, an economy initially in a stable LQE will move to a HQE. In this HQE, all the firms of quality $\theta > \theta^*$ will export. Then the average quality of exports across quality levels and cohorts of firms is $\bar{\theta}_t = \frac{\int_{\theta^*}^{\infty} \theta N_t(\theta) d\theta}{\int_{\theta^*}^{\infty} N_t(\theta) d\theta}$, where the total number of active firms of quality θ is given by $N_t(\theta) = \frac{1}{1-\delta} g(\theta)$. Simple calculations give us $\bar{\theta}(\mu) = \frac{\alpha}{\alpha-1} \theta^*$.

Let us call μ_{init} the initial steady-state reputation μ (before the minimum quality standard), and define $\theta_{L,\text{init}} \equiv \frac{\mu_{\text{init}} - k}{w}$ and $\theta_{H,S} \equiv \frac{k - (1-A\rho)\mu_{\text{init}}}{A\rho - w}$. The welfare effect of the minimum quality standard for the exporting country has three components. First, for firms with quality $\theta < \theta_{L,\text{init}}$, the effect is

unambiguously negative: these firms were exporting as fly-by-night before the introduction of the minimum standard making positive profits. With the minimum quality standard, they cannot enter the export market. In the initial LQE, the total number of active firms of quality $\theta < \theta_{L,init}$ was $g(\theta)$ and given that these firms exported just for one period, their profit was $\mu_{init} - w\theta - k$. Hence the lost profits due to the minimum quality standard are given by:

$$\begin{aligned}
 \text{Loss } \theta < \theta_{L,init} &= \int_{\theta_m}^{\theta_{L,init}} (\mu_{init} - w\theta - k) dG(\theta) \\
 &= (\mu_{init} - k) \int_{\theta_m}^{\theta_{L,init}} g(\theta) d\theta - w \int_{\theta_m}^{\theta_{L,init}} \theta dG(\theta) \\
 &= (\mu_{init} - k) \left(1 - \left(\frac{\theta_m}{\theta_{L,init}} \right)^\alpha \right) - w \frac{\alpha}{\alpha - 1} \theta_m \left(1 - \left(\frac{\theta_m}{\theta_{L,init}} \right)^{\alpha-1} \right) \\
 &= w\theta_m \left[\frac{\theta_{L,init}}{\theta_m} - \frac{\alpha}{\alpha - 1} + \frac{1}{\alpha - 1} \left(\frac{\theta_m}{\theta_{L,init}} \right)^{\alpha-1} \right]
 \end{aligned}$$

Second, for firms with quality $\theta^* < \theta < \theta_{H,init}$, the effect is unambiguously positive: these firms were not exporting before the introduction of the minimum standard. With the minimum quality standard, they now enter the export market and export and stay until they are hit by the exit shock. The value of their profit stream is positive and the gain is given by:

$$\begin{aligned}
 \text{Gain } \theta^* < \theta < \theta_{H,init} &= \int_{\theta^*}^{\theta_{H,init}} \left(\sum_{t=0}^{\infty} \delta^t \left(\rho(t)\theta + (1 - \rho(t)) \left(\frac{\alpha}{\alpha-1} \theta^* \right) - w\theta - k \right) \right) g(\theta) d\theta \\
 &= \frac{1}{1-\delta} \int_{\theta^*}^{\theta_{H,init}} \left((A_\rho - w)\theta + (1 - A_\rho) \left(\frac{\alpha}{\alpha-1} \theta^* \right) - k \right) g(\theta) d\theta \\
 &= \frac{1}{1-\delta} \left[- \left(k - (1 - A_\rho) \frac{\alpha}{\alpha-1} \theta^* \right) \left(\left(\frac{\theta_m}{\theta^*} \right)^\alpha - \left(\frac{\theta_m}{\theta_{H,init}} \right)^\alpha \right) + \frac{\alpha(A_\rho - w)}{\alpha-1} \theta_m \left(\left(\frac{\theta_m}{\theta^*} \right)^{\alpha-1} - \left(\frac{\theta_m}{\theta_{H,init}} \right)^{\alpha-1} \right) \right] \\
 &= \frac{\theta_m}{(1-\delta)(\alpha-1)} \left[(1 - w) \left(\frac{\theta_m}{\theta^*} \right)^{\alpha-1} - \frac{\alpha}{\theta_{H,init}} \left(\frac{k}{\alpha} + (1 - A_\rho) (\theta^* - \mu_{init}) \right) \left(\frac{\theta_m}{\theta_{H,init}} \right)^{\alpha-1} \right] \\
 &= \frac{1}{(1-\delta)(\alpha-1)} \left[k \left(\frac{\theta_m}{\theta^*} \right)^\alpha - (k + \alpha(1 - A_\rho) (\theta^* - \mu_{init})) \left(\frac{\theta_m}{\theta_{H,init}} \right)^\alpha \right]
 \end{aligned}$$

Third, for firms with quality $\theta > \theta_{H,init}$, the effect is also unambiguously positive: these firms receive extra profits brought by a higher reputation (from μ_{init} to $\frac{\alpha}{\alpha-1} \theta^*$). For a given reputation μ , the aggregate profit of firms

with quality $\theta > \theta_H$ is given by:

$$\begin{aligned}
 & \int_{\theta_H}^{\infty} \left(\sum_{t=0}^{\infty} \delta^t (\rho(t) \theta + (1 - \rho(t)) \mu - w\theta - k) \right) g(\theta) d\theta \\
 &= \frac{1}{1-\delta} \int_{\theta_H}^{\infty} ((A_\rho - w) \theta + (1 - A_\rho) \mu - k) g(\theta) d\theta \\
 &= \frac{1}{1-\delta} \left[- (k - (1 - A_\rho) \mu) \left(\frac{\theta_m}{\theta_H} \right)^\alpha + \frac{\alpha(A_\rho - w)}{\alpha-1} \theta_m \left(\frac{\theta_m}{\theta_H} \right)^{\alpha-1} \right] \\
 &= \frac{1}{1-\delta} \left[-k + (1 - A_\rho) \mu + \frac{\alpha(A_\rho - w)}{\alpha-1} \theta_H \right] \left(\frac{\theta_m}{\theta_H} \right)^\alpha
 \end{aligned}$$

Hence the gain from the higher reputation is given by:

$$\text{Gain}_{\theta > \theta_{H,\text{init}}} = \left(\frac{\alpha}{\alpha-1} \theta^* - \mu_{\text{init}} \right) (1 - A_\rho) \left(\frac{\theta_m}{\theta_{H,\text{init}}} \right)^\alpha$$

Overall, the total welfare effect of a minimum quality standard is:

$$\begin{aligned}
 & \left(\frac{\alpha}{\alpha-1} \theta^* - \mu_{\text{init}} \right) (1 - A_\rho) \left(\frac{\theta_m}{\theta_{H,\text{init}}} \right)^\alpha \\
 & + \frac{1}{(1-\delta)(\alpha-1)} \left[k \left(\frac{\theta_m}{\theta^*} \right)^\alpha - (k + \alpha(1 - A_\rho)(\theta^* - \mu_{\text{init}})) \left(\frac{\theta_m}{\theta_{H,\text{init}}} \right)^\alpha \right] \\
 & - w\theta_m \left[\frac{\theta_{L,\text{init}}}{\theta_m} - \frac{\alpha}{\alpha-1} + \frac{1}{\alpha-1} \left(\frac{\theta_m}{\theta_{L,\text{init}}} \right)^{\alpha-1} \right]
 \end{aligned}$$

From simple computations (derivatives of the lost profits terms $\text{Loss}_{\theta < \theta_{L,\text{init}}}$ with respect to various parameters), we obtain that the lower α (the shape parameter of the distribution), the lower the loss from the minimum quality standard. Intuitively, a low α means that there are more firms at the right tail of the distribution, and so relatively less firms that lose from the standard. The lost profits due to the minimum quality standard also decrease with the costs (w and k) (remember that $\theta_{L,\text{init}} = \frac{\mu_{\text{init}} - k}{w}$). On the contrary, they increase with the initial reputation (before the standard); the higher the reputation, the higher the profits the fly-by-night firms were able to extract when they exported for one period.

Appendix B. Informed and Uninformed Buyers

Suppose the population of importers is divided into N equal-sized groups. There is perfect information diffusion within groups but no information diffusion across groups. Thus, if any individual in group n has previously

consumed the output of firm j , then all buyers in group n are informed about good j . When firm j is matched with buyer i , i is informed if there exists $i^{prime} \in n$ such that i' has been matched with j in the past, and $i \in \text{Uninformed}$ if there is no $i' \in n$ such that i' has been matched with j in the past. Further assume that the firm observes in any period whether its buyer is informed or not, but not which group the buyer belongs to; hence it does not know the exact proportion of informed buyers in any period but only its expectation.

It follows immediately from this setup that $\rho(0) = 0$. After the firm has exported for one period, one group is informed, so $\rho(1) = \frac{1}{N}$. For each subsequent period, if the fraction of informed buyers after s export periods is $\rho(s)$, then with probability $\rho(s)$, the firm is matched with a buyer in an informed group, and the proportion of informed importers stays at $\rho(s)$ for the next period. With probability $1 - \rho(s)$, the firm is matched with a buyer in an uninformed group; then the fraction of informed importers next period is $\rho(s) + \frac{1}{N}$.

Therefore, the expected fraction of informed buyers is given by the following path: for $s \geq 0$,

$$\begin{aligned}\rho(0) &= 0 \\ \rho(s+1) &= \rho(s)^2 + (1 - \rho(s)) \left(\rho(s) + \frac{1}{N} \right) = \rho(s) \left(\frac{N-1}{N} \right) + \frac{1}{N}\end{aligned}$$

We can check that this function satisfies Assumption 1.

$$\begin{aligned}\rho(s+1) - \rho(s) &= \frac{1}{N} (1 - \rho(s)) > 0 \\ \frac{\rho(s+1) - \rho(s)}{\rho(s)} &= \frac{1}{N} \left(\frac{1}{\rho(s)} - 1 \right) \text{ is decreasing in } s \\ \lim_{s \rightarrow \infty} \rho(s) &= \frac{1}{N} \left(1 - \frac{N-1}{N} \right)^{-1} = 1\end{aligned}$$

So $\rho(s)$ is increasing in s , rises with s at a falling rate, and converges to 1.

Appendix C. Path of Prices for a Given Cohort of Firms

In this Appendix, we characterize the path of prices for a given cohort of firms. At the firm level, there is a “brand premium” for high-quality firms both in a HQE and in a LQE: the price charged increases over time for a given good provided that its quality is higher than the country average.

Result 1 establishes that on average, incumbents receive higher prices than entrants, and the average price among a cohort of firms is higher, the longer the cohort has been active on export markets. This result follows from the fact that over time, an increasing fraction of prices reflect firms' true quality parameters, and the average quality of a cohort of firms weakly increases over time as the lowest quality firms exit.

Result 1 (Unit Prices).

In a steady-state low-quality equilibrium, the average unit price charged at $t + s$ by firms born at date t is strictly increasing in s .

In a steady-state high-quality equilibrium, the average unit price charged at $t + s$ by firms born at date t is strictly increasing in s for all s if $\mu > \frac{\alpha}{\alpha-1}\theta_1$ and for $s \geq T(\frac{\alpha-1}{\alpha}\mu)$ otherwise.

Proof. In a LQE, the set of continuing firms is $[\theta_H, \infty)$ from the second period onwards, so the average price $\bar{p}_{t,t+s}^{lqe}$ of cohort t at time $t + s$ is:

$$\bar{p}_{t,t+s}^{lqe}(\bar{\theta}) = \begin{cases} \bar{\theta} & \text{if } s = 1 \\ \bar{\theta} + \rho(s) \left(\frac{\alpha}{\alpha-1}\theta_H - \bar{\theta} \right) & \text{if } s > 1 \end{cases}$$

As $\bar{\theta} < \theta_H$ in a LQE and $\rho(s)$ increases in s , it immediately follows that $\bar{p}_{t,t+s}^{lqe}$ increases with s .

In a HQE, the set of active firms of cohort t at time $t + s$ is $[\theta_{s-1}, \infty)$, and their average price is:

$$\bar{p}_{t,t+s}^{hqe}(\bar{\theta}) = \begin{cases} \bar{\theta} & \text{if } s = 1 \\ \bar{\theta} + \rho(s) \left(\frac{\alpha}{\alpha-1}\theta_{s-1} - \bar{\theta} \right) & \text{if } s > 1 \end{cases}$$

$\rho(s)$ and θ_{s-1} increase with s . Immediately following the entry of cohort t , $\bar{p}_{t,t+s}^{hqe}$ may fall with s if the distribution of θ has low variance (α high), such that $\frac{\alpha}{\alpha-1}\theta_1 > \mu$. In this case, there is initially a large mass of firms at the bottom of the distribution of continuing firms and their prices are falling. However, since $\mu < \frac{\alpha}{\alpha-1}\theta^*$, there is some finite s' such that for all $s \geq s'$, $\bar{p}_{t,t+s+1}^{hqe}(\bar{\theta}) > \bar{p}_{t,t+s}^{hqe}(\bar{\theta})$ and thus at each given point in time, the average unit price is higher for older cohorts of firms.

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An online Appendix with additional figures is available at:
<http://scholar.harvard.edu/cage/publications>.

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Under asymmetric information about quality, country reputation affects demand for imported goods.

We obtain two types of steady states: high-quality and low-quality equilibria.

With endogenous reputations, countries may be stuck in low-quality traps (e.g. China).

Export subsidies improve welfare in countries with bad reputation, and decrease it otherwise.

There is a tax/subsidy path based on export experience that replicates the perfect information outcome.