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# Monetary policy with heterogeneous agents and borrowing constraints<sup>☆</sup>

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## ABSTRACT

We show that the long-run neutrality of inflation on capital accumulation obtained in complete market models no longer holds when households face binding credit constraints. Borrowing-constrained households are not able to rebalance their financial portfolio when inflation varies, and thus adjust their money holdings differently compared to unconstrained households. This heterogeneity leads to a new precautionary savings motive, which implies that inflation increases capital accumulation. We quantify the importance of this new channel in an incomplete market model where the traditional redistributive effects of inflation are also introduced. We show that this model provides a quantitative rationale for the observed hump-shaped relationship between inflation and capital accumulation.

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## 1. Introduction

The long-run relationship between inflation, capital accumulation and growth, is one of the most celebrated issue in modern macroeconomics. This tradition dates back at least to the classic monetary neutrality result of Sidrauski (1967), who showed that money has no long-run effect on capital accumulation and output in the neoclassical growth model. This result has been challenged by recent empirical studies which conclude on the existence of a hump-shaped relationship between long-run inflation and capital accumulation. At low level of inflation, there is a positive relationship between inflation and capital accumulation as shown by Fischer (1993), Loayza et al. (2000) or Khan et al. (2006). But this relationship becomes negative at higher rate of inflation, as stressed by Bullard and Keating (1995) and Barro (1995).

These evidences on the non-neutrality of money have fueled many theoretical contributions. The literature has provided a rationale for the non-neutrality of money by including frictions such as distorting capital taxes (Phelps, 1973 and Chari et al., 1996 among others) and search frictions (Shi, 1999). The literature has also looked at redistributive issues of the seigniorage rents across households (Grandmont and Younès, 1973; Kehoe et al., 1992; and Erosa and Ventura, 2002) or across generations (Weiss, 1980; Weil, 1991). But all these studies have maintained the convenient assumption of the absence of binding borrowing constraints. They have thus abstracted from the possibility that saving decisions in capital and

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money, following changes in monetary policy could depend on the extent of binding borrowing constraints and incomplete markets.

The aim of this paper is to contribute towards filling this gap by investigating the role of binding borrowing constraints and heterogeneity in standard macroeconomic monetary models. The focus on borrowing constraint is motivated by two main considerations. First, this friction is empirically relevant. The tightness of borrowing constraints is a well-established empirical fact (Jappelli, 1990) and it is thus important to understand to what extent it interacts with monetary policy. Second, this friction challenges some key predictions of the standard neoclassical monetary growth model. In particular we show that the existence of borrowing constraints and incomplete markets can provide a rationale not only for the non-neutrality of money but also for the hump-shaped relationship between inflation and capital. This result is the product of two opposite effects of inflation that are only due to borrowing constraint and incomplete markets.

The positive effect of inflation on capital accumulation stems from the heterogeneity in household portfolio adjustment depending on their capacity to borrow. If households can use both money and capital to partially self-insure against idiosyncratic shocks, they substitute away money for capital if inflation rises and the real return on money falls. This is the traditional portfolio substitution effect that Tobin (1965) has shown in a world without credit constraint. But in presence of financial market imperfections, borrowing-constrained households are not able to undertake such portfolio adjustment and they adjust their money holdings differently compared to unconstrained households. Inflation triggers endogenous heterogeneity in money holdings in presence of borrowing constraints. This heterogeneity provides incentives for unconstrained households with positive income shocks to increase their precautionary savings in financial assets. As a consequence, inflation directly affects the aggregate stock of capital and output in the long-run. This new effect due to borrowing constraint has been ignored so far in the literature.

The negative effect of inflation on capital accumulation is the result of the redistributive impact of the inflation tax. Lump-sum monetary transfers from the inflation tax provide additional insurance when markets are incomplete. The redistribution of the inflation tax can thus decrease precautionary savings and the aggregate capital stock in the long-run. This redistributive effect is not new and was already assessed by Scheinkman and Weiss (1986), Kehoe et al. (1992) and more recently by Erosa and Ventura (2002). But these papers do not show that incomplete markets can provide a rationale for a hump-shaped relationship between inflation and capital accumulation since they do not investigate the so-called Tobin effect in presence of binding credit constraints.

The first contribution of this paper is theoretical. To the best of our knowledge, we provide a new channel for the non-neutrality of money on capital accumulation and output only due to borrowing constraints. In an economy with deterministic income shocks à la Woodford (1990), we show that inflation has a long-run effect as long as borrowing constraints are binding. Inflation affects in a different way the real balances demanded by constrained and unconstrained households. This leads to higher precautionary savings and increase capital accumulation and output. This real effect occurs even in the absence of any other potential channels proposed so far in the literature, such as tax distortions or leisure-labor supply distortions. Importantly, this non-neutrality result also shows up when we shut down the redistributive effect of the seigniorage rent which could provide insurance against idiosyncratic risks and could lead to a real effect of inflation.

The second contribution of this paper is quantitative. We show that the interplay between this new positive effect of inflation on capital accumulation and the negative redistributive effect of the inflation tax can match the hump-shaped relationship between inflation and capital. We run this quantitative analysis in an incomplete markets production economy à la Aiyagari (1994). *Ex-ante* identical infinitely-lived agents can accumulate interest-bearing financial assets in the form of capital to partially insure against individual income risks, but they face borrowing constraints. In this framework we embed money in the utility function (MIU). Money is valued both for its liquidity service and as a store of value which provides additional insurance against labor-market risks. Assuming money in the utility function is a reduced form to provide motives for money demand. But it has the key advantage to introduce simple departures, namely incomplete market and borrowing constraints, from the textbook MIU model in which money is neutral absent frictions. The analysis is carried on in an economy in which the wealth distribution and the fraction of borrowing-constrained households closely resemble that of the United States.

The benchmark model predicts a hump-shaped response of capital accumulation to inflation. At low values of the inflation rate, the model predicts that the Tobin effect will offset the redistributive effect. For higher values of inflation, the redistributive effect dominates as the lump-sum monetary transfers providing insurance increase with inflation. The maximum is reached at an annual inflation rate around 6 percent. This value is close to the empirical result of Bullard and Keating (1995) who find that the level of inflation maximizing the capital stock ranges between 3 percent and 6 percent in cross-country comparison. Moreover, an increase in the annual inflation rate from 0 percent to 6 percent raises the aggregate capital stock by 1 percent. In absolute terms, this result is quantitatively modest. But relatively to complete markets, this effect is significant since inflation would be neutral in this latter case.

We also investigate the quantitative importance of the different channels at work by looking at various redistributive schemes of the inflation tax. We first shut down the redistributive effect of the inflation tax to assess the quantitative impact of the Tobin effect. We find that the capital stock increases continuously with inflation. An increase in the annual inflation rate from 0 percent to 6 percent is associated with a rise in the capital stock by 0.9 percent. We also assume that the inflation tax might be used to decrease the distortionary taxes rather than being redistributed by lump-sum transfers. This possibility was suggested in particular by Phelps (1973). In this case, the capital stock increases with inflation even in the complete market environment. But we find that incomplete markets and credit constraints amplify the Phelps effect. An

increase in the annual inflation rate from 0 percent to 6 percent is associated to a rise of the capital stock by 0.6 percent in the complete market economy and by 2.8 percent in the incomplete market framework.

The literature on inflation under incomplete markets starts with the theoretical contributions of Bewley (1980, 1983), Scheinkman and Weiss (1986), Kehoe et al. (1992) and Imrohroglu (1992) analyze the effect of inflation under incomplete markets, but in economies where money is the only asset that can be used to self-insure against idiosyncratic shocks. Akyol (2004) analysis the effect of inflation in an endowment economy where both financial assets and money coexist because of the assumed sequence of market opening. In this environment, he considers various policies with both different public debt and inflation rate, which makes it difficult the identification of the various effects. Moreover, the author is silent about the effect of inflation on capital accumulation since the analysis is run in an endowment economy. The closer paper to ours is Erosa and Ventura (2002). They analyze the redistributive effect of inflation in an incomplete markets production economy, but without binding borrowing constraints. The authors introduce a transaction technology with scale effects to get real effect of inflation. But in this context, the aggregate effect of inflation is similar under complete markets and incomplete markets as shown by the author themselves since their mechanism hinges on the transaction technology rather than on the market environment. In contrast, we show that heterogeneity and binding borrowing constraints yield a significant departure from the representative agent economy. Our result holds in a textbook macroeconomic model of money without any scale effect in the money demand.

Our paper is organized as follows. Section 2 provides a simple model with deterministic individual shocks to show analytically the non-neutrality of money that is only due to the existence of borrowing constraints. Section 3 lays out the full model with stochastic uninsurable individual shocks. Section 4 quantifies the real effect of inflation and reports sensitivity analysis.

## 2. A simple model

In this section, we provide a theoretical model to show that inflation is no longer neutral in a production economy with binding borrowing constraints. To obtain closed-form solutions, we set out a simplified version of the fully-fledged model used in the next quantitative section. In particular, we shut down the traditional redistributive effect of the inflation tax in order to stress a new Tobin effect with borrowing constraint.

### 2.1. The model

The model draws upon a standard heterogeneous agent production economy à la Aiyagari (1994) in which agents face individual income fluctuations and borrowing constraints. But we make the key assumption that households alternate *deterministically* between the different labor market states. This liquidity-constrained model has been used, for instance, by Woodford (1990) to study the effect of public debt and by Kehoe and Levine (2001) to characterize the equilibrium interest rate. We extend this framework to monetary policy issues by taking account of the value of money in the utility function.<sup>1</sup> We show analytically that Sidrauski's neutrality result no longer holds when borrowing constraints are binding in this framework. Inflation affects the long-run interest rate, even when seigniorage revenue is redistributed in the most neutral way, and regardless of any other potential frictions.

#### 2.1.1. Preferences and technology

Households are infinitely-lived and have identical preferences. Each household can be in two states,  $H$  or  $L$ . In state  $H$  (resp.  $L$ ), households have a high labor endowment  $e^H$  (resp.  $e^L$ ). For the sake of simplicity we assume that  $e^H = 1$  and  $e^L = 0$ . Households alternate deterministically between state  $H$  and  $L$  at each period. At the initial date, there is a unit mass of the two household types. Type 1 households are in state  $H$  at date 1, type 2 households are in state  $L$  at date 1. Consequently, type 1 (resp. 2) households are in state  $H$  (resp.  $L$ ) every odd period and in state  $L$  (resp.  $H$ ) every even period. Type  $i$  ( $i = 1, 2$ ) households seek to maximize an infinite-horizon utility function over consumption  $c^i$  and real money balances  $m^i$  which provide liquidity services. The period utility function  $u$  of these households is assumed to have the simple form

$$u(c_t^i, m_t^i) = \phi \ln c_t^i + (1 - \phi) \ln m_t^i$$

where  $1 > \phi > 0$  weights the marginal utility of consumption and money. For the sake of simplicity we use a log-linear utility function in this section. The result holds for general utility functions as shown in a technical appendix of the paper available upon request.

At each period  $t \geq 1$ , a type  $i$  household can use her revenue for three different purposes. She can first buy an amount  $c_t^i$  of final goods. We denote by  $P_t$  the price of the final good in period  $t$ , and  $\Pi_{t+1}$  is the gross inflation rate between

<sup>1</sup> An alternative would be to use a cash-in-advance (CIA) constraint. The motivations for using the money-in-the-utility function (MIU) are twofold. Theoretically, the MIU assumption is more general and flexible. Naturally, one can show that the non-neutrality result still holds under the CIA hypothesis. Furthermore, the MIU representation directly refers to the standard Sidrauski's neoclassical monetary model, and allows a direct assessment of the role of borrowing constraints under this framework. The second reason has to do with the quantitative evaluation. We will calibrate the model on a quarterly basis and it seems too extreme to assume that households must choose their money holdings one quarter in advance.

period  $t$  and period  $t + 1$ , that is  $\Pi_{t+1} = P_{t+1}/P_t$ . She also saves an amount  $a_{t+1}^i$  of financial assets yielding a return of  $(1 + r_{t+1})a_{t+1}^i$  in period  $t + 1$ , where  $1 + r_{t+1}$  is the gross real interest rate between period  $t$  and period  $t + 1$ . A borrowing constraint is introduced in its simplest form, in that we assume that no household is able to borrow:  $a_{t+1}^i \geq 0$ . Finally, type  $i$  household buys a nominal quantity of money  $M_t^i$ , which corresponds to a level of real balances  $m_t^i = M_t^i/P_t$ . This yields revenue  $m_t^i/\Pi_{t+1}$  in period  $t + 1$ . In addition to labor income and to the return on her assets, each household receives by helicopter drop a monetary transfer from the State, denoted  $\mu_t^i$  in real terms.

The problem of the type  $i$  household,  $i = 1, 2$ , is given by

$$\max_{\{c_t^i, m_t^i, a_{t+1}^i\}_{t=1 \dots \infty}} \sum_{t=1}^{\infty} \beta^t u(c_t^i, m_t^i) \quad \text{with } 0 < \beta < 1 \quad (1)$$

$$\text{s.t. } c_t^i + m_t^i + a_{t+1}^i = (1 + r_t)a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i + \mu_t^i \quad \text{with } a_t^i, c_t^i, m_t^i \geq 0 \quad (2)$$

where  $\beta$  stands for the discount factor,  $a_1^i$  and  $M_0^i = P_0 m_0^i$  are given, and  $a_t^i$  and  $m_t^i$  are subject to the standard transversality conditions.

The production function of the representative firm has a simple Cobb–Douglas form  $K^\alpha L^{1-\alpha}$  where  $L$  stands for total labor supply and  $K$  is the amount of total capital which fully depreciates in production. Profit maximization is given by  $\max_{K_t, L_t} F(K_t, L_t) - (1 + r_t)K_t - w_t L_t$ , and yields the standard first-order conditions

$$1 + r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}, \quad w_t = (1 - \alpha) K_t^\alpha L_t^{-\alpha} \quad (3)$$

In period  $t \geq 1$ , financial market equilibrium is given by  $K_{t+1} = a_{t+1}^1 + a_{t+1}^2$ . Labor market equilibrium is  $L_t = e_t^1 + e_t^2 = 1$ . Goods market equilibrium implies  $F(K_t, L_t) = K_{t+1} + c_t^1 + c_t^2$ .

### 2.1.2. Monetary policy with neutral redistribution

Let  $\bar{M}_t$  denotes the nominal quantity of money in circulation and  $\Omega_t = \bar{M}_t/P_t$  the real quantity of money in circulation at the end of period  $t$ . Money market equilibrium implies  $m_t^1 + m_t^2 = \Omega_t$  in real terms and  $M_t^1 + M_t^2 = \bar{M}_t$  in nominal terms.

Let  $\pi$  denotes the growth rate of money. Monetary authorities provide a new nominal quantity of money in period  $t$ , which is proportional to the nominal quantity of money in circulation at the end of period  $t - 1$ . As a result,  $\mu_t^1 + \mu_t^2 = \pi \bar{M}_{t-1}/P_t$ , where the initial nominal quantity of money,  $\bar{M}_0 = M_0^1 + M_0^2$ , is given. The law of motion of the nominal quantity of money is thus

$$\bar{M}_t = (1 + \pi) \bar{M}_{t-1} \quad (4)$$

In order to focus on the specific role of borrowing constraints on the non-neutrality of inflation, it is assumed that monetary authorities follow the “most” neutral rule, which is to distribute by lump-sum transfer the exact amount of resources paid by private agents due to the inflation tax. Obviously this assumption is unrealistic and its only aim is to stress the specific role of borrowing constraints regardless of any redistributive effects. As a consequence, new money is distributed proportionally to the level of beginning-of-period money balances. In period  $t$ , type  $i$  agents have a beginning-of-period quantity of money  $M_{t-1}^i$ . Hence, we assume that  $\mu_t^i = \pi M_{t-1}^i/P_t$ , and the real transfer is

$$\mu_t^i = \frac{\pi}{\Pi_t} m_{t-1}^i \quad (5)$$

## 2.2. Stationary equilibrium

Given the initial conditions  $a_1^1, a_1^2, M_0^1, M_0^2$ , and given  $\pi$ , an equilibrium in this economy is a sequence  $\{c_t^1, c_t^2, m_t^1, m_t^2, a_{t+1}^1, a_{t+1}^2, P_t, r_t, w_t\}_{t=1 \dots \infty}$  which satisfies the households' problem (1), the first-order condition of the firms' problem (3), and the different market equilibria. More precisely, we focus on symmetric stationary equilibria, where all variables are constant, and where all agents in each state  $H$  and  $L$ , denoted  $H$  and  $L$  households, have the same consumption and savings levels. The variables describing agents in state  $H$  will be denoted  $m^H, c^H, a^H$ , and those for in state  $L$  will be described by  $m^L, c^L, a^L$ . As a consequence, since the real quantity of money in circulation  $\Omega = \bar{M}_t/P_t$  is constant in a stationary equilibrium, Eq. (4) implies that the price of the final good grows at rate  $\pi$ , and hence  $\Pi = 1 + \pi$ .

Note that under our assumption of a neutral redistributive monetary policy, we can use the budget constraint (2), and the amount  $\mu_t^i/P_t$  given by (5), to obtain the budget constraints of  $H$  and  $L$  households at the stationary equilibrium

$$H \text{ households: } c^H + m^H + a^H = (1 + r)a^L + m^L + w \quad (6)$$

$$L \text{ households: } c^L + m^L + a^L = (1 + r)a^H + m^H \quad (7)$$

The inflation rate does not appear in these equations since the creation of new money does not imply any transfer between the two types of households. The redistributive effects of the seigniorage rent analyzed for instance by Kehoe et al. (1992) are canceled out.

Using standard dynamic programming arguments, the households' problem can be solved easily. This is done in Appendix A.

For  $H$  households, we have the following optimality conditions

$$u_c(c^H, m^H) = \beta(1+r)u_c(c^L, m^L) \quad (8)$$

$$u_c(c^H, m^H) - u_m(c^H, m^H) = \frac{\beta}{\Pi}u_c(c^L, m^L) \quad (9)$$

Eq. (8) is the Euler equation for  $H$  households, who can smooth their utility thanks to positive savings.  $H$  households are high-income and are never borrowing constrained. The second equation is the arbitrage equation, which determines the demand for real money balances.  $H$  households equate the marginal cost of holding money in the current period (i.e. the left-hand side of Eq. (9)), to the marginal gain of transferring one unit of money to the following period when they are in state  $L$  (i.e. the right-hand side of Eq. (9)). The marginal utility of money shows up here as a decrease in the opportunity cost of holding money. And the gain from money holdings takes into account the real return  $1/\Pi$  of cash.

The solution of the program of  $L$  households depends on whether borrowing constraints are binding or not. If borrowing constraints are binding, the solution is  $a^L = 0$  and

$$u_c(c^L, m^L) > \beta(1+r)u_c(c^H, m^H) \quad (10)$$

$$u_c(c^L, m^L) - u_m(c^L, m^L) = \frac{\beta}{\Pi}u_c(c^H, m^H) \quad (11)$$

The first inequality shows that  $L$  households would be better off if they could transfer some income from the next period to the current period. The second equation involves the same trade-off as that for  $H$  households discussed above. Finally, if borrowing constraints are not binding for  $L$  households, inequality (10) becomes an equality and  $a^L > 0$ .

Using expression (8) together with condition (10), we find that borrowing constraints are binding if and only if  $1+r < 1/\beta$ . If borrowing constraints are not binding, Eq. (8) and the relationship (10) taken with equality imply  $1+r = 1/\beta$ . The following proposition<sup>2</sup> summarizes this standard result.

**Proposition 1.** *Borrowing constraints are binding for  $L$  households if and only if  $1+r < 1/\beta$ . If borrowing constraints are not binding then  $1+r = 1/\beta$ .*

When borrowing constraints are binding, the gross real interest rate  $1+r$  is lower than the inverse of the discount factor. As a result, there is always capital over-accumulation due to the precautionary saving motive, which is a standard result in this type of liquidity-constrained model (see Woodford, 1990; Kehoe and Levine, 2001, amongst others). The next section establishes sufficient conditions for borrowing constraints to be binding in this simple framework.

### 2.3. Monetary policy with binding borrowing constraints

#### 2.3.1. Perfect financial markets

As a starting point, we present the conditions required to produce Sidrauski's neutrality result in this simple framework. If markets were complete and borrowing constraints were not binding, the Euler equation would hold with equality whatever the state of the labor market. In this case, money demand would be identical across households of types  $H$  and  $L$ . Using a log utility specification and taking the Euler equation with equality, we can rewrite money demand as follows

$$\frac{m^H}{c^H} = \frac{m^L}{c^L} = \frac{1-\phi}{\phi} \frac{1}{1 - \frac{1}{\Pi} \frac{1}{1+r}} \quad (12)$$

In this case, whatever the current state and the history of the labor market, the ratio of money over consumption is determined only by the preference parameters and the opportunity cost of holding money. To see this, assume that  $r$  and  $\pi$  are small, so that  $1 - 1/(1+\pi)(1+r) \simeq r - (-\pi)$ , which is the difference between the real net return on financial titles and the real net return on money or, in other words, the nominal interest rate.

In this case, inflation has no real effect on savings since households adjust their money demand in exactly the same proportion following a rise in inflation. Inflation does not then bring about any intra-period heterogeneity between household  $H$  and  $L$ ; it therefore has no effect on saving patterns for inter-period smoothing motives, or on the equilibrium interest rate. This is the traditional Sidrauski result regarding the long-run neutrality of money.

<sup>2</sup> Note that  $1+r$  cannot be lower than  $1/\Pi$ , otherwise financial markets cannot clear. As such, an equilibrium with binding credit constraints can exist only if  $1/\Pi < 1/\beta$ . Moreover, we assume that the surplus left for consumption is positive at the Friedman rule, which implies  $\alpha < 1/\Pi$ .

### 2.3.2. Binding borrowing constraints

This long-run neutrality result no longer holds in this simple framework when borrowing constraints are binding.

Since  $H$  households are never borrowing-constrained and profit from their good employment state to accumulate a buffer financial stock, their Euler equation always holds with equality. The money demand of  $H$  households is therefore still only determined by the opportunity cost of holding money

$$\frac{m^H}{c^H} = \frac{1 - \phi}{\phi} \frac{1}{1 - \frac{1}{\Pi} \frac{1}{1+r}} \quad (13)$$

By contrast, the money demand of  $L$  households might be affected, depending on whether borrowing constraints are binding since the Euler equation no longer holds with equality. When borrowing constraints are binding, that is when  $1 + r < \frac{1}{\beta}$ , we have the following money demand equation from (8) and (11):

$$\frac{m^L}{c^L} = \frac{1 - \phi}{\phi} \frac{1}{1 - \frac{\beta^2}{\Pi}(1+r)} \quad (14)$$

The equilibrium ratio for  $L$  households is not simply determined by the opportunity cost of holding money, but by the difference between consumption in the current period and the return on money holdings two periods hence. The ratio  $\beta^2(1+r)/\Pi$  is the discounted value of one unit of money held in state  $L$ , transferred to state  $H$ , and then saved via financial market on to the next period, where the household is in state  $L$  again. As this ratio rises,  $L$  households increase the ratio of their money holdings over their consumption.  $L$  households then increase the relative demand for money as the real interest increases, contrary to  $H$  households. The real interest rate appears here as the remuneration of future savings and not as the opportunity cost of holding money. The following proposition summarizes this key property of the model. The proof can be found in Appendix A.

**Proposition 2.** *If  $\alpha < 1/(2 + \beta)$ , there exists a unique equilibrium with binding borrowing constraints. In such an equilibrium, the real interest rate falls as inflation rises.<sup>3</sup>*

When borrowing constraints are binding, a rise in inflation triggers a heterogeneous response in money demand across households.  $L$  households decrease their money holdings  $m^L$  proportionately less than do  $H$  households, because money is their only available store of value. As a result,  $H$  households have more resources since their budget constraint is  $c^H + a^H = w + m^L - m^H$ . An increase in inflation thus provides an incentive for  $H$  households to save more in order to smooth consumption between periods. Thus in this simple framework with binding borrowing constraints, inflation unambiguously favors capital accumulation and output, in line with the traditional result of Tobin (1965).

This simple model has shown that imperfections on financial markets give rise to heterogeneity in money demand, which is at the core of the non-neutrality of inflation. Finally, the well-known distributional effect of the inflation tax (Scheinkman and Weiss, 1986; Kehoe et al., 1992) has been canceled in this simple model where money is distributed proportionally to the beginning-of-period money holdings. In the general model below the redistribution of the inflation is likely to play an additional role. Note that this last effect depends only on incomplete markets and not on credit constraints being actually binding. The next section provides a quantitative evaluation of these channels.

## 3. The general model

We now describe a fully-fledged model including more general assumptions about idiosyncratic risks, endogenous labor supply and distorting taxes in order to investigate quantitatively the role of inflation. The economy considered here is based on the traditional heterogeneous agent framework à la Aiyagari (1994). However, we embed money in the utility function in this framework. This section presents the most general model. Different specifications of this model will be used in the simulation exercise to disentangle the various channels through which inflation affects the real economy.

### 3.1. Agents

#### 3.1.1. Households

The economy consists of a unit mass of *ex ante* identical and infinitely-lived households. They maximize expected discounted utility from consumption  $c$ , from leisure and real balances  $m = \frac{M}{P}$ . Labor endowment per period is normalized to 1, working time is  $l$  and thus leisure is  $1 - l$ . For the sake of generality, we follow the literature which directly introduces money  $m$  in the utility function of private agents to capture its liquidity services. For the benchmark version of the model,

<sup>3</sup> Condition holds for  $\alpha = 1/3$ ,  $\beta < 1$  and  $\Pi > 1$ , for instance.

we assume that the utility function has a general CES specification, following Chari et al. (2000). The utility of agent  $i$  is given by

$$u(c_i, m_i, l_i) = \frac{1}{1-\sigma} \left[ \left( \omega c_i^{\frac{\eta-1}{\eta}} + (1-\omega) m_i^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} (1-l_i)^\psi \right]^{1-\sigma} \quad (15)$$

where  $\omega$  is the share parameter,  $\eta$  is the interest elasticity of the demand for real balances,  $\psi$  is the weight of leisure and  $\sigma$  is risk aversion.

Individuals are subject to idiosyncratic shocks on their labor productivity  $e_t$ . We assume that  $e_t$  follows a three-state Markov process over time with  $e_t \in E = \{e^h, e^m, e^l\}$ , where  $e^h$  stands for high productivity,  $e^m$  for medium productivity, and  $e^l$  for low productivity. The productivity process follows a  $3 \times 3$  transition matrix<sup>4</sup>  $Q$ . The probability distribution across productivity is represented by a vector  $n_t = \{n_t^h, n_t^m, n_t^l\}$ :  $n_t \geq 0$  and  $n_t^h + n_t^m + n_t^l = 1$ . Under technical conditions, that we assume to be fulfilled, the transition matrix has a unique vector  $n^* = \{n^h, n^m, n^l\}$  such that  $n^* = n^*Q$ . Hence,  $n_t$  converges toward  $n^*$  in the long run.  $n^*$  is distribution of the population in each state. For instance,  $n^h$  is the proportion of the population with high labor productivity. In the general model, there is endogenous labor supply for each productivity level.

Markets are incomplete and no borrowing is allowed. In line with Aiyagari (1994), households can self-insure against employment risks by accumulating a riskless asset  $a$  which yields a return  $r$ . But they can also accumulate real money assets  $m = M/P$ , which introduces a new channel compared to the previous heterogeneous agent literature. With the price level of the final good at period  $t$  being denoted  $P_t$ , the gross inflation rate between period  $t-1$  and period  $t$  is  $\Pi_t = \frac{P_t}{P_{t-1}}$ . If a household holds a real amount  $m_{t-1}$  of money at the end of period  $t-1$ , the real value of her money balances at period  $t$  is  $\frac{m_{t-1}}{\Pi_t}$ . As long as  $\Pi_t > \frac{1}{1+r_t}$ , money is a strictly-dominated asset, but which will nonetheless be in demand for its liquidity services. As before,  $\mu_t^i$  denote the lump sum transfers from the government received by agent  $i$ , expressed in real terms. Households are not allowed to borrow and cannot issue any money.

The budget constraint of household  $i$  at period  $t$  is given by

$$c_t^i + m_t^i + a_{t+1}^i = (1+r_t)a_t^i + \frac{m_{t-1}^i}{\Pi_t} + w_t e_t^i l_t^i + \mu_t^i \quad t = 0, 1, \dots \quad (16)$$

where  $(1+r_0)a_0^i$  and  $m_{-1}^i$  are given. The sequence of constraints on the choice variables is

$$a_{t+1}^i \geq 0, \quad 1 \geq l_t^i \geq 0, \quad c_t^i \geq 0, \quad m_t^i \geq 0 \quad t = 0, 1, \dots \quad (17)$$

The value  $r_t$  is the after-tax return on financial assets,  $e_t^i$  is the productivity level of the worker in period  $t$ , and  $w_t$  is after-tax labor income per efficient unit.

For the sake of realism, we assume that there is a linear tax on private income. The tax rate on both capital and labor income at period  $t$  is denoted  $\chi_t$ . Letting  $\tilde{r}_t$  and  $\tilde{w}_t$  denote capital cost and labor cost per efficient unit, the returns for households then satisfy the following relationships

$$r_t = \tilde{r}_t(1 - \chi_t)$$

$$w_t = \tilde{w}_t(1 - \chi_t)$$

Let  $q_t^i$  denote total wealth in period  $t$

$$q_t^i = (1+r_t)a_t^i + \frac{m_{t-1}^i}{\Pi_t}$$

With this definition, the program of agent  $i$  can be written in the following recursive form

$$\begin{aligned} v(q_t^i, e_t^i) &= \max_{\{c_t^i, m_t^i, l_t^i, a_{t+1}^i\}} u(c_t^i, m_t^i, l_t^i) + \beta E[v(q_{t+1}^i, e_{t+1}^i)] \\ \text{s.t.} \quad c_t^i + m_t^i + a_{t+1}^i &= q_t^i + w_t e_t^i l_t^i + \mu_t^i \end{aligned}$$

with the sequence of constraints on the choice variables in (17) and the transition probabilities for labor productivity given by the matrix  $Q$ .

Since the effect of inflation on individual behavior depends heavily on whether borrowing constraints are binding, we distinguish two cases.

<sup>4</sup> This assumption is based on Domeij and Heathcote (2004), who found that at least three employment states are needed to fit crucial empirical features of the employment process and wealth distribution. See the section devoted to the specification of the model parameters.



- *Non-binding borrowing constraints*

In this case, the first-order conditions of agent  $i$  are as follows

$$u_c(c_t^i, m_t^i, l_t^i) = \beta(1 + r_{t+1})E[v_1(q_{t+1}^i, e_{t+1}^i)] \quad (18)$$

$$u_c(c_t^i, m_t^i, l_t^i) - u_m(c_t^i, m_t^i, l_t^i) = \frac{\beta}{\Pi_{t+1}}E[v_1(q_{t+1}^i, e_{t+1}^i)] \quad (19)$$

$$u_l(c_t^i, m_t^i, l_t^i) = -w_t e_t u_c(c_t^i, m_t^i, l_t^i) \quad (20)$$

Eq. (20) only holds if the solution satisfies  $l_t^i \in [0; 1]$ . Otherwise,  $l_t^i$  takes on a corner value, and the solution is given by (18) and (19).

Let  $\gamma_{t+1}$  denote the real cost of money holdings

$$\gamma_{t+1} \equiv 1 - \frac{1}{\Pi_{t+1}} \frac{1}{(1 + r_{t+1})}$$

This indicator measures the opportunity cost of holding money. When the after-tax nominal interest rate  $r_{t+1}^n$ , defined by  $1 + r_{t+1}^n = \Pi_{t+1}(1 + r_{t+1})$ , is small enough, then  $\gamma_{t+1} \simeq r_{t+1}^n$ . With this notation and the expression of the utility function given in (15) above, the first-order conditions (18) and (19) yield

$$m_t^i = \left( \frac{1 - \omega}{\omega} \frac{1}{\gamma_{t+1}} \right)^\eta c_t^i$$

This equation shows that the money demand of unconstrained households is only affected by the substitution effect, which depends on the opportunity cost of holding money.

- *Binding borrowing constraints*

When the household problem yields a negative value for financial savings, borrowing constraints are binding,  $a_{t+1} = 0$ , and the first-order condition yields the inequality

$$u_c(c_t^i, m_t^i, l_t^i) > \beta(1 + r_{t+1})E[v_1(q_{t+1}^i, e_{t+1}^i)]$$

The first-order conditions of the constrained problem are given by

$$u_c(c_t^i, m_t^i, l_t^i) - u_m(c_t^i, m_t^i, l_t^i) = \frac{1}{\Pi_{t+1}} \beta E \left[ v_1 \left( \frac{m_t^i}{\Pi_{t+1}}, e_{t+1}^i \right) \right] \quad (21)$$

$$u_l(c_t^i, m_t^i, l_t^i) = -w_t e_t u_c(c_t^i, m_t^i, l_t^i) \quad (22)$$

There is no simple expression for money demand in the case of binding constraints. The static trade-off between money demand and consumption demand appears on the left-hand side of (21). Were money not to be a store of value, this expression would be equal to 0. However, as money allows individuals to transfer income to the next period, this introduces an additional motive for holding money.

The right-hand side of Eq. (21) makes clear that inflation has two opposing effects on the demand for money by borrowing-constrained households. On the one hand, inflation induces a substitution effect which serves to decrease money demand as inflation rises (represented by the term  $1/\Pi_{t+1}$ ); on the other hand, as inflation enters the value function via a revenue effect, there might be an increase in money demand as inflation increases.

The core reason for this result is that money is the only store of value which can be adjusted if households are borrowing-constrained. If the function  $v$  is very concave, and for realistic parameter values, this second effect may dominate, and the demand for money can increase with inflation. We will show in the quantitative analysis that this result holds for the poorest agents. As a consequence, this case proves that the change in money demand resulting from inflation, the so-called *Tobin effect*, can be decomposed into a revenue effect and a substitution effect for borrowing-constrained households.

Finally, working hours are determined by Eq. (22). If the value of  $l_t$  from (22) is negative, then  $l_t = 0$  and the first-order condition (22) holds with inequality.

The solution of the households' program provides a sequence of functions which yield at each date the policy rules for consumption, financial savings, money balances and leisure as a function of the level of labor productivity and wealth:

$$\left. \begin{aligned} c_t(\cdot, \cdot) : E \times \mathbb{R}^+ &\longrightarrow \mathbb{R}^+ \\ a_{t+1}(\cdot, \cdot) : E \times \mathbb{R}^+ &\longrightarrow \mathbb{R}^+ \\ m_t(\cdot, \cdot) : E \times \mathbb{R}^+ &\longrightarrow \mathbb{R}^+ \\ l_t(\cdot, \cdot) : E \times \mathbb{R}^+ &\longrightarrow \mathbb{R}^+ \end{aligned} \right\} t = 0, 1, \dots$$

### 3.1.2. Firms

We assume that all markets are competitive and that the only good consumed is produced by a representative firm with aggregate Cobb–Douglas technology. Let  $K_t$  and  $L_t$  stand for aggregate capital and aggregate effective labor used in production respectively. It is assumed that capital depreciates at a constant rate  $\delta$  and is installed one period ahead of production. Since there is no aggregate uncertainty, aggregate employment and, more generally, aggregate variables are constant at the stationary equilibrium

Output is given by

$$Y_t = F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha} \quad 0 < \alpha < 1$$

Effective labor supply is equal to  $L_t = L_t^h e^h + L_t^m e^m + L_t^l e^l$ , where  $L_t^h$ ,  $L_t^m$  and  $L_t^l$  are the aggregate demand for each type of labor. Prices are set competitively:

$$\tilde{w}_t = (1 - \alpha)(K_t/L_t)^\alpha \quad (23)$$

$$\tilde{r}_t + \delta = \alpha(K_t/L_t)^{\alpha-1} \quad (24)$$

### 3.1.3. Government

The government levies taxes to finance a public good, which costs  $G$  units of final goods in each period. Taxes are proportional to the revenue of capital and labor, with coefficients  $\chi_t$  in period  $t$ . In addition, the government receives the revenue of the new money created at period  $t$ , denoted  $\tau_t^{\text{tot}}$  in real terms. It is assumed that the government does not issue any debt. Finally, the total transfer to households is the sum of the lump sum transfer over the whole population,  $\int_0^1 \mu_t^i di$ . The government budget constraint is given by

$$\int_0^1 \mu_t^i di + G = \chi_t \tilde{r}_t K_t + \chi_t (L_t^h e^h + L_t^l e^l + L_t^m e^m) \tilde{w}_t + \tau_t^{\text{tot}} \quad (25)$$

### 3.1.4. Monetary policy

Monetary policy is assumed to follow a simple rule. In each period, the monetary authorities create an amount of money that is proportional, with factor  $\pi$ , to the nominal quantity of money in circulation,  $P_t \Omega_t = P_{t-1} \Omega_{t-1} + \pi P_{t-1} \Omega_{t-1}$  where  $\Omega_t$  stands for aggregate real money and  $M_t = \Omega_t P_t$  stands for nominal money. As it is standard in the monetary literature, we assume that the State receives all the revenue from the inflation tax.<sup>5</sup> As a result the real quantity of money in circulation at period  $t$  is

$$\Omega_t = \frac{\Omega_{t-1}}{\Pi_t} + \pi \frac{\Omega_{t-1}}{\Pi_t} \quad (26)$$

The real value of the inflation tax in period  $t$  is

$$\tau_t^{\text{tot}} = \pi \frac{\Omega_{t-1}}{\Pi_t} \quad (27)$$

Note that if the real quantity of money in circulation is constant (which is the case in equilibrium), Eq. (26) implies that  $\Pi = 1 + \pi$ , and hence  $\tau^{\text{tot}} = \frac{\pi}{1+\pi} \Omega$ , which is the standard expression for the inflation tax.

## 3.2. Equilibrium

### 3.2.1. Market equilibria

Let  $\lambda_t : E \times \mathbb{R}^+ \rightarrow [0, 1]$  denote the joint distribution of agents over productivity and wealth. Aggregate consumption  $C_t$ , aggregate real money holdings  $M_t/P_t$ , aggregate effective labor  $L_t^S$  and aggregate financial savings  $A_{t+1}$  are respectively given by

$$C_t = \iint c_t(e^k, q) d\lambda_t(e^k, q)$$

$$M_t/P_t = \iint m_t(e^k, q) d\lambda_t(e^k, q)$$

$$L_t^S = e^h \int l_t(e^h, q) \lambda_t(e^h, q) dq + e^l \int l_t(e^l, q) \lambda_t(e^l, q) dq + e^m \int l_t(e^m, q) \lambda_t(e^m, q) dq$$

$$A_{t+1} = \iint a_{t+1}(e^k, q) d\lambda_t(e^k, q)$$

<sup>5</sup> In practice, the profits of Central Banks are redistributed to the State and are not used for specific purposes.

**Table 1**  
Benchmark calibration.

Parameters	$\beta$	$\alpha$	$\delta$	$\omega$	$\eta$	$\psi$	$\sigma$
Values	0.99	0.36	0.025	0.988	0.5	2	1

Equilibrium in the final good market implies

$$C_t + K_{t+1} + G_t = Y_t + (1 - \delta)K_t \quad (28)$$

Equilibrium in the labor market is

$$L_t = L_t^s$$

Equilibrium in the financial market implies

$$K_{t+1} = A_{t+1} \quad (29)$$

Last, money-market equilibrium is defined by

$$M_t/P_t = \Omega_t \quad (30)$$

where  $\Omega_t$  is the real quantity of money in circulation at period  $t$ .

### 3.2.2. Competitive equilibrium

A stationary competitive equilibrium for this economy consists of constant decision rules  $c(e, q)$ ,  $m(e, q)$ ,  $l(e, q)$  and  $a(e, q)$  for consumption, real balances, leisure and capital holdings respectively, the steady state joint distribution over wealth and productivity  $\lambda(e, q)$ , a constant real return on financial assets  $r$ , a constant real wage  $w$ , the real return on real balances  $1/\Pi$ , and taxes  $\chi$ , consistent with the exogenous supply of money  $\pi$  and government public spending  $G$  such that

1. The long-run distribution of productivity is given by a constant vector  $n^*$ .
2. The functions  $a(\cdot, \cdot)$ ,  $c(\cdot, \cdot)$ ,  $m(\cdot, \cdot)$ ,  $l(\cdot, \cdot)$  solve the households' problem.
3. The joint distribution  $\lambda$  over productivity and wealth is time invariant.
4. Factor prices are competitively determined by Eqs. (23)–(24).
5. Markets clear: Eqs. (28)–(30).
6. The quantity of money in circulation follows the law of motion (26).
7. The tax rate  $\chi$  is constant and defined to balance the budget of the State (25), where the seigniorage rent from the inflation tax  $\tau^{\text{tot}}$  is given by (27).

Note that equilibrium on the money market and stationary of the joint distribution imply that the real quantity of money in circulation is constant.

### 3.3. Parameterization

We parameterize the model by using data from the U.S. economy. The first critical point of the parametrization is the choice of the model period to generate a reasonable inflation tax base. Since real balances consist of liquid assets, we choose a model period equals to one quarter rather than one year, consistently with the previous quantitative literature on the inflation tax (see Erosa and Ventura, 2002, or Cooley and Hansen, 1989, among others). The key targets of this parametrization are the wealth distribution, including the share of borrowing-constrained individuals, the individual process of income fluctuations, the interest-elasticity of money demand and the key ratio of  $M1/Y$  and  $K/Y$ . In what follows we focus on the benchmark incomplete market economy described above with endogenous prices, proportional taxes, and endogenous labor. We calibrate the model for an initial quarterly rate of inflation  $\pi$  of 0.75 percent. This initial inflation rate corresponds to an inflation rate of 3 percent per year, which corresponds to the average inflation rate over the period 1980–2006. In the benchmark case, we set the level of lump-sum transfers to be 0 when inflation is equal to 0.75 percent.

#### 3.3.1. Technology and preferences

Table 1 shows the parameters for preferences and technology. Parameter values for the utility function (15) are chosen as follows.

The first key parameter is the interest elasticity of money. We follow the estimates of Hoffman et al. (1995) who find an interest elasticity of money demand close to 0.5 in the United States. We thus set  $\eta$  equal to 0.5 in the benchmark parameterization. However, the interest elasticity of money demand seems to be estimated imprecisely in the literature. Holman (1998) provides an estimate of  $\eta$  close to 1 by directly estimating the parameters of the utility function on a long-run period in the U.S. from 1989 to 1991. This result suggests that in the long-run the data fails to reject the Cobb–Douglas assumption. In contrast, Chari et al. (2000) estimate a much lower value of the interest elasticity. We will thus check the robustness of our results in the sensitivity analysis by running different experiments with  $\eta = 1$  and  $\eta = 0.25$ .

**Table 2**  
Steady-state values.

	$\pi$	$\bar{r}$	$\chi$	$w$	$K$	$Y$	$\Omega$
Benchmark	0.75%	0.39%	0.29	2.64	17.04	1.37	0.72

**Table 3**  
Wealth distribution.

	U.S. data	Benchmark model
Gini	0.82	0.72
Percentage of households borrowing constrained	[2%, 19%]	6%
Cumulative distribution of wealth held by:		
1st quintile (bottom 20%)	0.09	2.62
2nd quintile (bottom 40%)	0.2	4.14
3rd quintile (bottom 60%)	4.0	5.33
4th quintile (bottom 80%)	15.3	21.56
80–100%	84.7	78.44
99–100%	34.3	7.3

Next, we pin down the value of the share parameter  $\omega$  to match the ratio of M1 over output. For the period 1980–2006, the ratio M1/GNP reached on average 13.2 percent at the annual level for an average yearly inflation rate of 3 percent. We thus set  $\omega = 0.988$  to match the corresponding ratio M1/GNP at a quarterly frequency. The weight on leisure  $\psi$  is set to reproduce a steady state fraction of labor of 33 percent of total time endowment. The risk aversion is set at a standard value of  $\sigma = 1$  as in Chari et al. (2000) baseline case. The parameters relating to the production technology and the capital's share also take on their standard values:  $\alpha$  is set equal to 0.36 and the capital depreciation rate is 0.025. The value of the discounting factor is then set equal to  $\beta = 0.99$  so as to reproduce a capital/output ratio of 12.5 at the quarterly level (Cooley, 1995). Eventually, we set  $G = 0.28$  to reproduce a share of  $G$  over GDP of 20 percent. Given the value of  $G$ , we find that the budget of the government is balanced for an average tax rate on labor and capital equal to  $\chi = 0.29$ , which is close to the values observed for that period (Domeij and Heathcote, 2004). Table 2 reports the corresponding steady-state values of the main aggregates of the model.

### 3.3.2. Employment process

An important aspect of the parameterization is to find a stylized process for wages which is both empirically relevant and able to replicate the U.S. wealth distribution such as the Gini coefficient and the share of people who are borrowing-constrained. We follow the traditional quantitative macroeconomic literature by assuming a first-order autoregressive process for wages or earnings. Various authors have estimated these parameters by using PSID data, and found a coefficient of autocorrelation close to 0.9 and a standard deviation of innovation in the range 0.12 and 0.25 (see Hubbard et al., 1995; Heathcote et al., 2005). However the stochastic process of relevance in our framework pertains to wages since hours are endogenous. We thus draw on Floden and Lindé (2001) who estimated, at the annual level, a model with a labor supply choice and thus focused on a process for wage rather than earnings. We use their findings by imposing that the quarterly Markov process can reproduce a coefficient of autocorrelation equal to 0.91 and a standard deviation in the innovation term equal to 0.22 at the annual level.

The second issue is to find a process able to match the observed Gini coefficient of wealth and the extent of households who are borrowing constrained. To that respect we follow the current literature (see among others Domeij and Heathcote, 2004) which shows that a Markov chain with three states and nil probabilities to transit between extreme states could make a good job in matching the Gini index. We thus assume a set of employment states represented by  $E = \{e^h, e^m, e^l\}$  where  $e^h$  stands for high productivity,  $e^m$  for medium productivity, and  $e^l$  for low productivity. And we assume that  $p_{h,l} = p_{l,h} = 0$ . This leaves us with four restrictions to identify the Markov process. Two restrictions are given by the previous autocorrelation process and the standard error of the innovation in the wage process. The two other parameters are chosen so that to obtain a realistic Gini index for wealth equals to 0.72 and a share of borrowing constraint households equals to 6 percent. This calibration procedure delivers parameters values that satisfy all four criteria. The implied probability of transitions are  $p_{l,l} = p_{h,h} = 0.9750$  and  $p_{m,m} = 0.9925$ , and the ratio for productivity values are  $e_1/e_2 = 4.64$  and  $e_2/e_3 = 5.65$ .

Table 3 summarizes the relevant variables for the wealth distribution (net worth) based on the Survey of Consumer Finance 2004. We use the estimates of the U.S. wealth distribution given by Wolff (2007). The model predicts a fairly high Gini index, equal to 0.72, but still underestimates the Gini found in the data, equal to 0.82. The model does a reasonably good job in matching the lower tail of the U.S. wealth distribution. It reproduces a positive share of borrowing constrained households. It also predicts a very low share of wealth held by the bottom 20 percent, 40 percent and 60 percent, even if these shares are overestimated compared to that found in the SCF 2004. The model also matches reasonably well the upper quintile of the distribution. It predicts that the top 20 percent hold 78.44 percent of the wealth while they hold

84.7 percent in the data. However, our model fails in matching the share of the top 1 percent of the wealth distribution. This category holds 34.7 percent of the wealth in the data against 7.3 percent in the model. This flaw is common to the different models which use only idiosyncratic income risks to reproduce the wealth distribution (see Heathcote, 2005). Taking into account entrepreneurship or stochastic discounting factor might be necessary to improve the model along this line, but this is beyond the scope of the paper (see Quadrini, 2000 and Krusell and Smith, 1998, for seminal attempts).

The Gini coefficient in consumption is 0.30, consistent with Krueger and Perri (in press). The empirical measure of the share of borrowing constrained households heavily depends on the choice of the indicator chosen. By using information on the number of borrowing requests which were rejected in the Survey of Consumer Finance (SCF), Jappelli showed that up to 19 percent of families are liquidity constrained. But by using updated SCF data, Budria Rodriguez et al. (2002) reported that only 2.5 percent of household have zero wealth, which might correspond to our theoretical borrowing limit in the model. Obviously this figure does not mean that these households are liquidity-constrained. In particular, Budria Rodriguez et al. (2002) also report that 6 percent of households have delayed their debt repayments for two months or more, which could be used as another proxy for liquidity constraints. To this extent, our measure of 6 percent of liquidity-constrained individuals in our model can be considered as an intermediate value, which prevents us from over-estimating the effect of borrowing constraints on the non-neutrality of inflation.<sup>6</sup>

We will study other environment where public spending are set to 0. In this case, we change the parameter of the utility function  $\beta$  and  $\omega$  to start from the same steady states for  $K/Y$  and  $M1/Y$  when the quarterly inflation rate is equal to 0.75 percent. The other parameters are unchanged.

## 4. Results

### 4.1. Individual policy rules

We start by discussing the impact of inflation on individual policy rules in the benchmark economy with endogenous hours and taxes.

Fig. 1 illustrates the main policy rules in the benchmark economy with an inflation rate of 0.75 percent.<sup>7</sup> Consumption, real balances and financial assets are an increasing function of labor productivity and current total wealth  $q$ , made up of financial assets and cash. But due to the presence of borrowing constraints, the value functions and the implied policy rules for consumption and money demand are concave at the low values of wealth and productivity. Moreover the policy rule for financial assets held by medium- and low-productivity workers displays kinks at low levels of wealth, indicating that these two types of workers are net-dissavers. By contrast high productivity workers are net-savers in order to smooth consumption across less favorable productivity states.

Fig. 2 shows the impact of a permanent variation in inflation from  $\pi = 0.5\%$  to  $\pi = 0.75\%$ , on next-period asset holdings and money balances as a function of total beginning of period wealth. The focus is on policy rules around the kink where the main non-linearity lies. We focus on the high productivity state and the low productivity state, as households in the medium state have similar policy rules than low-productivity households. For the high value of productivity, an increase in inflation provides more incentives to save via financial assets at the expense of real money balances whose value has been slashed by inflation. This behavior stands in sharp contrast with that of households in lower productivity states. These households are borrowing-constrained on asset holdings at the low level of total wealth. In this case they have no alternative but to carry over higher level of money balances following a rise in inflation in order to smooth their consumption. Money is used as a store of value, and the revenue effect dominates the substitution effect when wealth is low, as explained in the discussion of Eq. (21). Their level of real-money balances decreases only at the higher level of total wealth for which borrowing constraints on financial assets are no longer binding and households can thus use their capital as a buffer stock. This contrasting effect suggests that the impact of inflation on the real economy and welfare crucially depends on borrowing constraints. Moreover, these policy rules show that wealth-poor households hold a higher fraction of their wealth in real balances compared to wealth-rich households. This endogenous outcome is consistent with the data (see Erosa and Ventura, 2002).

### 4.2. Aggregate results

We quantify the effects of inflation under different assumptions regarding the redistribution of the seigniorage rent, the tax structure and the adjustment of labor supply. The quantitative analysis proceeds as follows.

<sup>6</sup> Our model with capital and real balances yields quite naturally a positive number of people who are liquidity-constrained with the employment process at stake. This result is due to the introduction of real balances in the traditional Aiyagari model. The previous literature generally uses stochastic discounting factors to fit this dimension (Krusell and Smith, 1998; Carroll, 2000). We do not follow this strategy since the goal of this paper is to look at the specific role of credit constraints and incomplete markets in the non-neutrality of money regardless of any additional heterogeneity, in particular with respect to preferences.

<sup>7</sup> We calculate the policy rules with value function iteration. We use a grid of 8000 points and an increment of 0.015 on the interval for wealth  $[0, 120]$ . The standard Euler equation error, averaged over the grid points and the productivity status, reaches 0.004 percent.

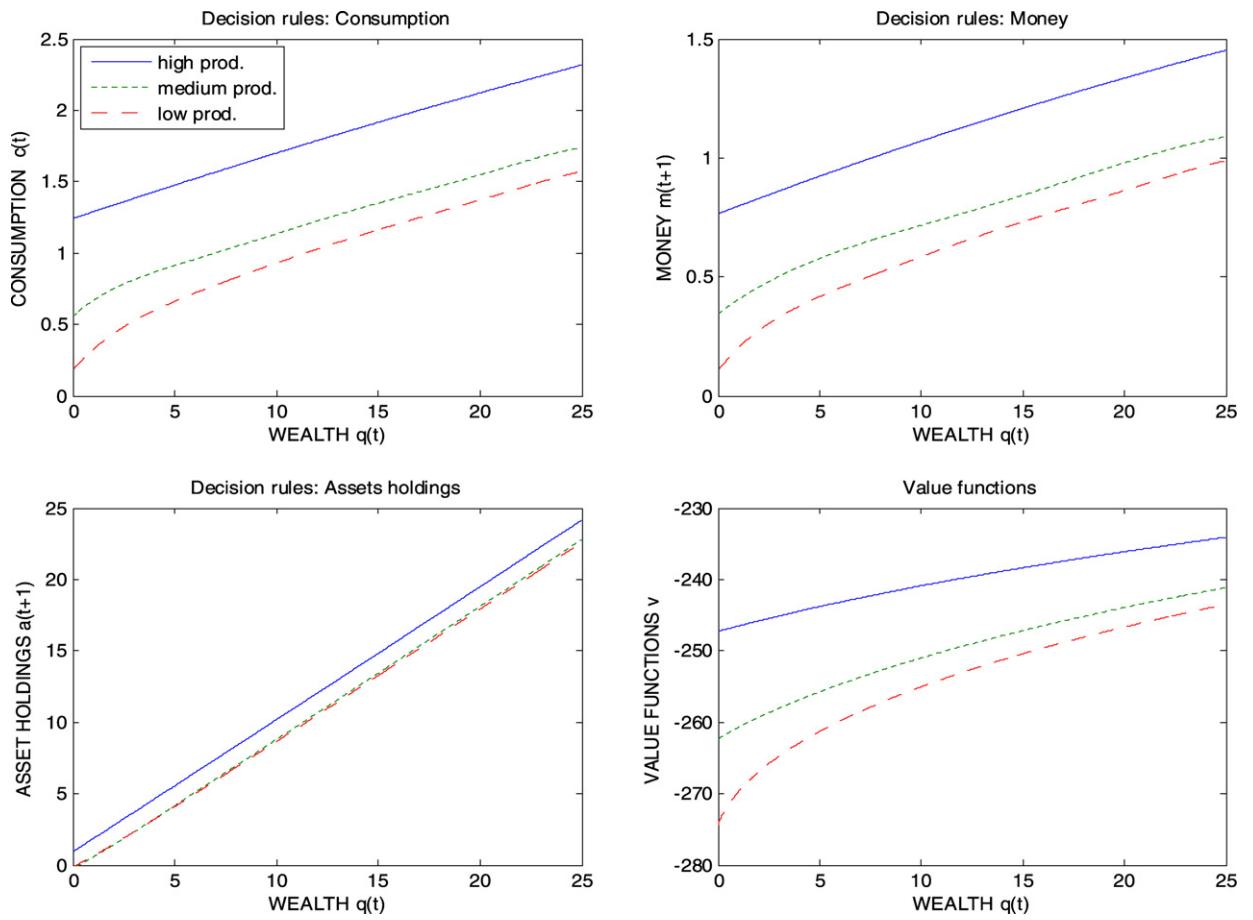


Fig. 1. Individual policy rules.

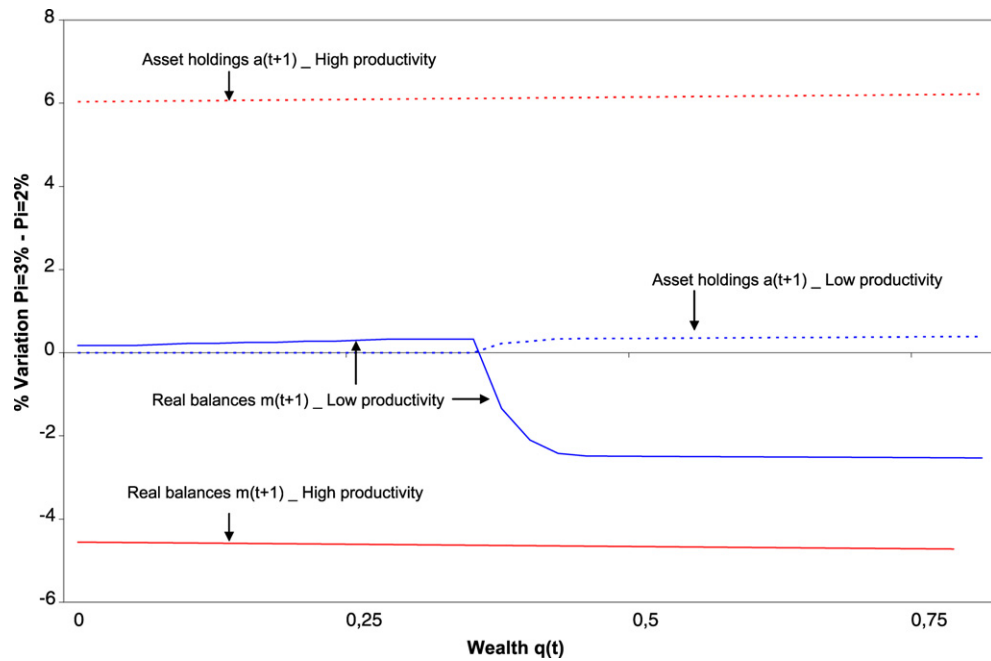
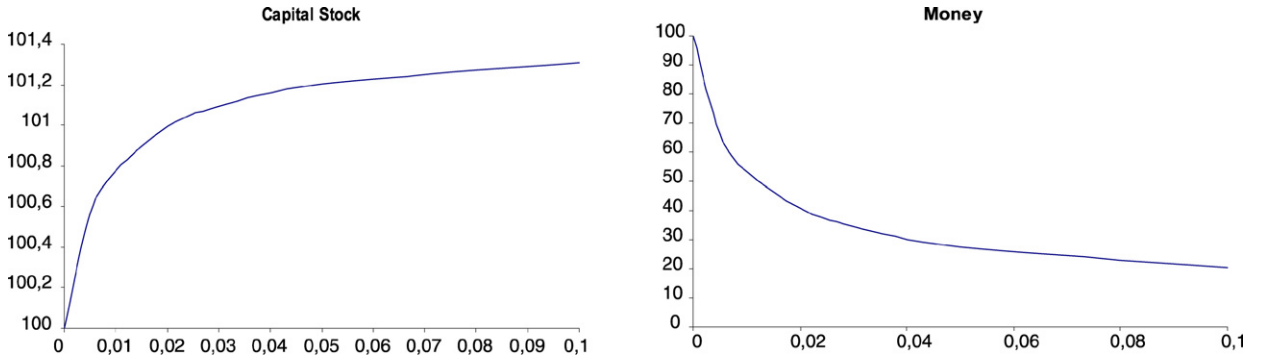


Fig. 2. Effect of inflation on individual policy rules.



**Fig. 3.** Capital and money as a function of the quarterly inflation rate, exogenous labor, lump-sum monetary transfers proportional to beginning of period real balances.

First we consider a version of the model in which hours are exogenous and money creation is redistributed proportionally to the beginning-of-period real balances of households. Households regard these transfers as lump-sum ones. We thus abstract from any redistributive and distortionary issues discussed in the previous literature. Consistent with our theoretical results in Section 2, this set-up allows us to quantify the non-neutrality of monetary policy which transmits through borrowing constraints only. This framework is thus mainly illustrative because if markets were complete the neutrality of money would apply under these assumptions.

Second, we take into account the traditional redistributive effect of inflation and assess its interaction with borrowing constraints. Money creation is made by helicopter drops and distributed equally across households. In this case, money creation and inflation consist in a transfer from cash-rich households to cash-poor households. Money creation provides thus an additional insurance and interact with incomplete markets and borrowing constraints. Labor supply is still assumed to be exogenous.

Third, we consider the benchmark model described above in which labor supply is endogenous and taxes are distortionary. We quantify the effect of inflation under two assumptions. We start by assuming that the newly money created, that is the inflation tax, is redistributed equally to all households through an helicopter drop. Next, we assess the impact of the inflation tax on other distorting taxes: The increase in the inflation tax is assumed to decrease the distortion on capital tax and labor tax. This effect is often called the Phelps effect, as Phelps (1973) introduces this effect to justify a positive inflation rate.

The calibration given above corresponds to the cases three and four. In each other cases, we adjust the discounting factor  $\beta$  and parameter of the utility function  $\omega$  are adjusted to start from the same initial capital-output ratio and money-output ratio.

#### 4.2.1. Proportional lump-sum transfers and exogenous labor

We assume that each household supplies inelastically  $l = \bar{l}$  hours of labor. We set  $\bar{l} = 0.33$ , which corresponds to the steady state value of labor with endogenous labor supply. In this specification, government spending is set to zero  $G = 0$ , there are no distortionary taxes  $\chi = 0$ , and all the monetary transfers are lump-sum.<sup>8</sup> The government redistributes the new money proportionally to the beginning of period level of real balances held by each household. In consequence, the real transfer to households  $i$  in period  $t$  is simply  $\frac{\pi}{H_t} m_{t-1}^i$ . This environment corresponds to the simple model presented in Section 2 but with a more general labor income process. With this assumptions, the budget constraint (16) is  $c_t^i + a_{t+1}^i + m_t^i = (1 + \tilde{r}_t) a_t^i + \tilde{w}_t e_t^i \bar{l} + m_{t-1}^i$ . Here inflation no longer appears in the individual budget constraint. But since the seigniorage tax is redistributed ex-post, the inflation rate is still taken into account by households as the anticipated inflation rate affects the arbitrage conditions to hold money.

Fig. 3 shows the effect of inflation on aggregate capital and the demand for real balances in this set-up. Consistently with the theoretical findings, inflation is non-neutral in this environment and triggers an increase in precautionary saving motives and the aggregate capital stock. This is the new Tobin effect in an environment with incomplete markets and binding borrowing constraints. Aggregate output mimics the evolution pattern of the capital stock since labor supply is exogenous in this set-up. The relation between inflation and capital accumulation is monotonic but the marginal effect of inflation is decreasing ( $\delta K / \delta \pi > 0$  and  $\delta^2 K / \delta \pi^2 < 0$ ). The increase in the capital stock reaches 1.3 percent when the quarterly inflation rate increase from 0 percent to 10 percent. The real money demand decreases continuously with inflation. The implied interest elasticity of money demand is of 0.5 due to the calibration of the model.

<sup>8</sup> The fact that the amount of transfers is taken as given by the households can be justified by the following argument. The government is able to solve the money demand of households as a function of the history of their idiosyncratic shocks. The transfer is made conditionally on the history of the income shocks and not on the choice variables of the households. The households consider thus this transfer as lump-sum.

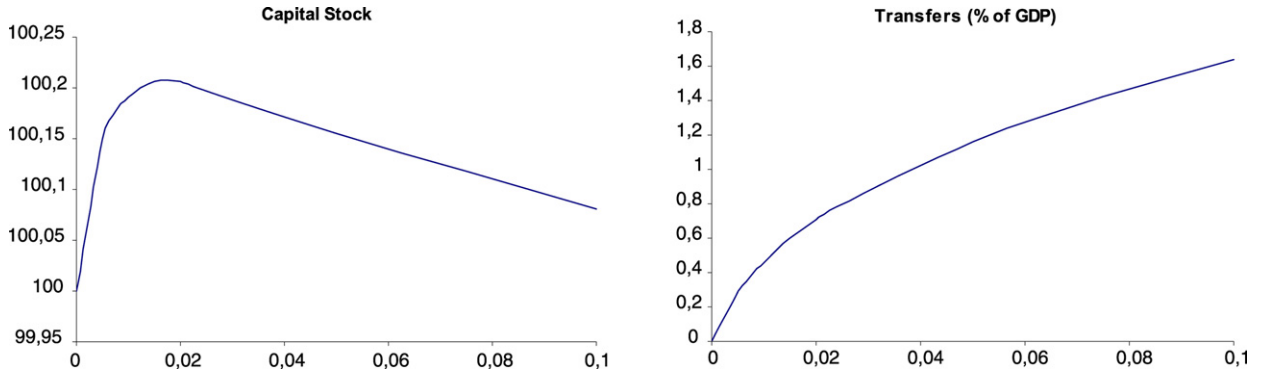


Fig. 4. Capital and transfers as a function of the quarterly inflation rate, exogenous labor, symmetric lump-sum monetary transfers.

Under complete markets and non-distorting taxes, the impact of inflation on capital is nil. Inflation has no real effect on the stationary values of the real aggregate variables. Individuals adjust their real balances and financial assets in the same proportions, leading to a neutral effect of inflation on real variables, such as consumption, capital and output.

#### 4.2.2. Redistributive effects of the inflation tax

We now quantify the interaction between incomplete markets and the redistribution of the inflation tax. We assume that all agents receive the same fraction of the inflation tax from the monetary authorities  $\mu_t^i = \pi \frac{\Omega_t - 1}{H_t}$ . This experiment refers basically to the helicopter drop model with symmetric transfers. As the monetary transfers to all households increase with the inflation level, this experiment introduces an additional insurance effect of inflation, which goes in the opposite direction to the previous Tobin effect. The incentives for self-insurance through capital accumulation thus decrease. We still assume exogenous labor supply and zero government spending  $G = 0$ . The budget constraint of households is given by (16) with  $\mu_t^i = \pi \frac{\Omega_t - 1}{H_t}$ ,  $l_t^i = \bar{l}$ .

Fig. 4 shows the impact of inflation in this environment. The left panel reports the evolution of aggregate capital as a function of inflation. The evolution pattern is now hump-shaped, reaching a peak for a quarterly inflation rate of 1.5 percent. The Tobin channel offsets the insurance channel at low levels of inflation. But the insurance effect, brought about by the monetary transfers of the inflation tax, dominates at higher levels of inflation.

The right panel of Fig. 4 highlights the insurance channel by reporting the relationship between inflation and the real transfers  $\mu$ , expressed as a percentage of GDP. Although the real quantity of money decreases, the monetary transfers increase with inflation since the interest elasticity of money demand is lower than 1. As the insurance brought about by the monetary transfers increases with inflation, the self-insurance motives decrease. Importantly enough, the insurance effect is also only due to the existence of incomplete markets and borrowing constraints. Inflation would still have no real effect in this environment if markets were to be complete.

#### 4.2.3. Endogenous labor, distorting taxes and redistribution of inflation taxes

We now consider the benchmark model with distorting taxes and endogenous labor supply. We compare two redistributive schemes of the inflation tax. First, as in the previous helicopter drop experiment, we assume that the inflation tax is redistributed equally to all households. In the second environment, the additional inflation tax is used to decrease distorting taxes on labor and capital.

**4.2.3.1. Symmetric lump-sum redistribution of the inflation tax** In line with the calibration of the benchmark model presented above, we assume that public spending  $G$  is positive and financed through distortionary taxes. But the inflation tax provides additional revenue to be redistributed. All agents receive the same monetary transfers. Importantly, the inflation tax is not used by the government to modify the distortionary taxes, which remain fixed at their steady-state values  $\bar{\chi}$ . To start from the same steady state, we consider that the monetary transfers from the inflation tax are nil when the inflation rate is equal to 0.75 percent. As before, the budget constraint of households is given by (16) when  $\mu_t^i = \pi \frac{\Omega_t - 1}{H_t}$ ,  $G > 0$ ,  $\chi_t = \bar{\chi}$  and  $l_t^i$  endogenous.

Fig. 5 shows the evolution pattern of the main aggregates as a function of the inflation rate. The solid lines correspond to the case with equal lump-sum redistribution of the inflation tax. Consistently with the above experiment, the capital stock follows a hump-shaped evolution. The new effect here only comes from the endogenization of the labor supply. Fig. 5 – left panel – shows that labor supply decreases monotonically with inflation. This evolution is the result of the insurance effect of the inflation tax. The increase in the monetary transfers provide additional insurance and lower the incentives to work for self-insurance motives. This effect always dominates the substitution effect linked to the rise in the capital stock and in labor productivity at low level of inflation. For high level of inflation, both the insurance and the substitution effect go in the same direction and lead to a sharper decrease in labor supply.



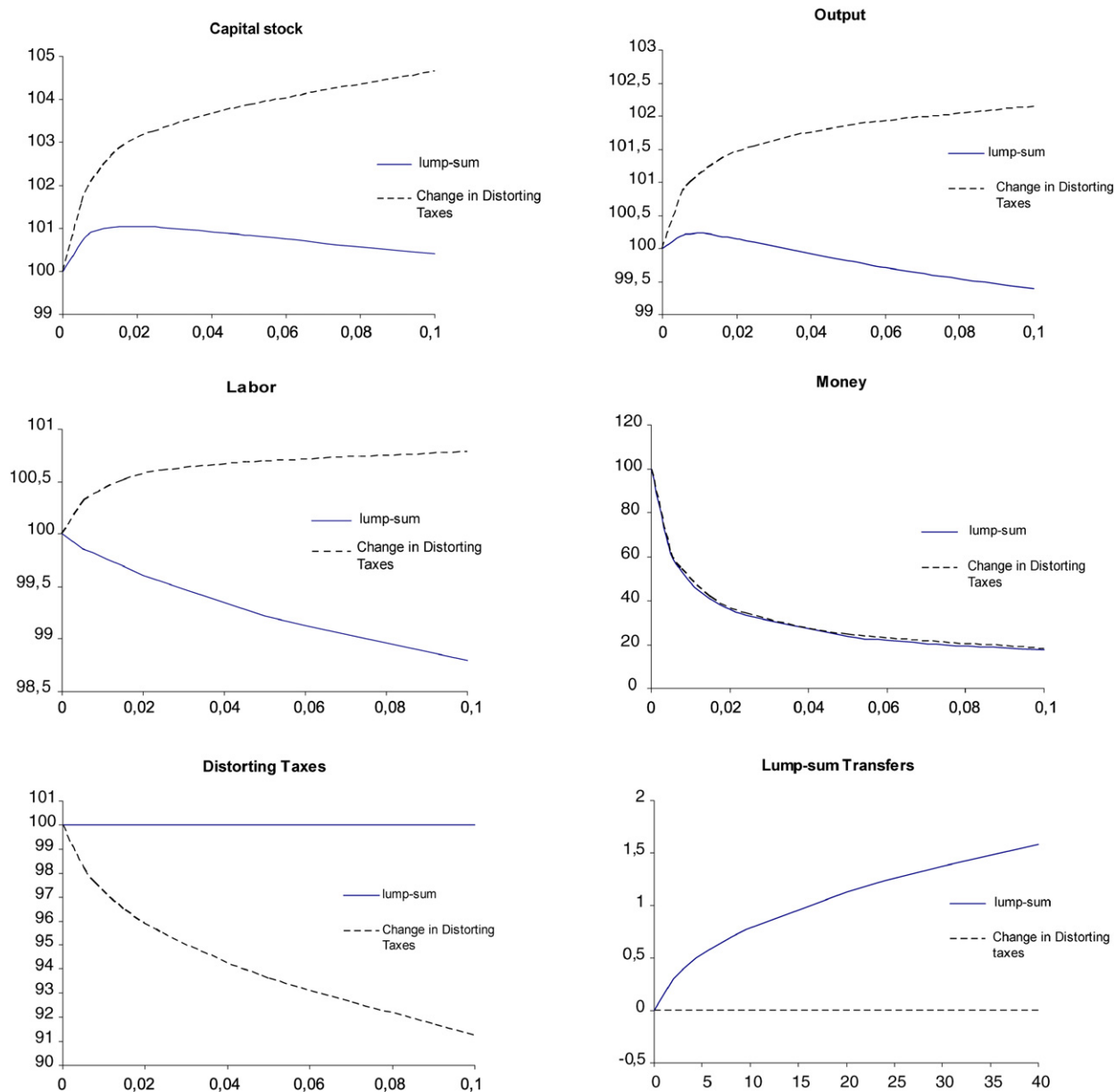


Fig. 5. Benchmark model: symmetric monetary transfers versus changes in distorting taxes.

As a consequence of the evolution pattern of capital and labor, output follows a non-monotonic evolution pattern. The rise in the capital stock at low levels of inflation offsets the decline in labor supply, and leads to an increase in output. In contrast, output declines for higher levels of the inflation rate.

To provide a quantitative sense of the results, Table 4 – line 1 shows the variation in the aggregate variables when the quarterly inflation rate increases from 0.5 percent to 0.75 percent. The capital stock increases by 0.139 percent and the money stock decreases by 12.79 percent. The increase in lump-sum monetary transfers provide some insurance that leads to a decrease in labor supply by 0.03 percent. The net effect on output is an increase by 0.029 percent. Table 4 – line 2 shows that the real effect of inflation would be nil if markets were to be complete.

This environment with lump-sum monetary transfers is likely to be the most relevant for the analysis of the long-run effect of inflation. There is little evidence that distorting taxes decrease when inflation increases. But as a theoretical investigation, we nevertheless study this case below.

**Table 4**

Aggregate impact of inflation – benchmark model.

	Percentage change following a rise in quarterly inflation $\pi = 0.5\% \rightarrow 0.75\%$							
	Redistribution of the inflation tax: Lump-sum transfers <sup>a</sup>							
	$Y$	$K$	$M/P$	$C$	$L$	$\chi$	$\tilde{r}$	$\tilde{w}$
Incomplete markets	0.029	0.139	-12.79	-0.015	-0.030	0	-0.83	0.06
Complete markets	0	0	-12.80	0	0	0	0	0
	Redistribution of the inflation tax: Change in distorting taxes <sup>b</sup>							
Incomplete markets	0.182	0.380	-12.73	0.121	0.062	-0.540	-1.454	0.112
Complete markets	0.056	0.085	-12.79	0.063	0.041	-0.467	-0.029	0.016

<sup>a</sup> The inflation tax is redistributed by lump-sum monetary transfers to all agents.

<sup>b</sup> The inflation tax is used to decrease the distortionary taxes  $\chi$  on labor and capital.

#### 4.2.4. Change in distorting taxes

In this environment, the government uses the inflation tax to decrease the distorting taxes on labor and capital. We thus set  $\mu_t = 0$  in the individual budget constraint and government budget constraint. The inflation tax  $\pi \frac{\partial L}{\partial \pi}$  in the government budget constraint directly affects the distorting taxes  $\chi_t$ .

In this experiment, the previous insurance effect of inflation through the equal redistribution of the inflation tax is shut down. In addition to the Tobin effect, still present in this set-up, inflation might now affect the economy through a new channel. Since distortionary taxes on capital decrease with the rise in the inflation tax, inflation has an additional positive effect on capital accumulation. Distortionary taxes on labor supply also decrease and the gap across wages net of taxes now widens. This effect might drive a rise in labor supply.

Fig. 5 represents in dashed lines the evolution pattern of the aggregate variables as a function of inflation. The upper-left panel shows that the capital stock is now monotonically increasing with inflation. Since the insurance channel is shut down, there is no longer any force to counteract the positive Tobin effect of inflation on capital accumulation. On the contrary, the Phelps effect of inflation, linked to the decrease in the capital tax, magnifies the positive influence of inflation on the capital stock. The lower left panel of Fig. 5 documents the drop in distortionary taxes. Moving from an inflation rate of 0 percent to 0.75 percent leads to a drop in capital tax by 3.5 percent.

Table 4 – line 3 shows the quantitative effect on aggregate variables of an increase in inflation from 0.5 percent to 0.75 percent. The rise in inflation has a first order effect on distorting taxes, which decrease by 0.5 percent. The capital stock increases now by 0.38 percent, a rise more than twice as high as the one observed in the environment with lump-sum monetary transfers. Labor supply increases by 0.062 percent, leading to an overall increase in output by 0.182 percent.

Table 4 – line 4 shows the results in the complete market case. The change in distorting taxes leads to an increase in labor and capital, consistently with the Phelps effect in complete markets. But the overall effect of inflation on capital and output under incomplete markets is more than three times as high as the one under complete markets. The incomplete markets environment magnifies the effect of monetary policy on fiscal policy.

#### 4.3. Sensitivity analysis

This section runs a sensitivity analysis on the different parameters driving the real effect of inflation on capital accumulation. We focus on three main parameters: the elasticity of money demand  $\eta$ , the risk aversion  $\sigma$ , and the borrowing constraint. The experiments are run under the benchmark model with symmetric redistribution of the inflation tax, endogenous labor supply and constant distorting taxes.

##### 4.3.1. Interest elasticity of money demand

We start by looking at the value of the elasticity of substitution between goods and money, which drives the interest elasticity of money demand. As mentioned in the calibration section, the elasticity is imprecisely estimated in the literature. The estimated values range in most cases from 0.2 to 1. We thus look at two polar cases with  $\eta = 0.25$  and  $\eta = 1$ . The higher the elasticity, the higher the substitution in favor of financial assets will occur after a hike in inflation. One should thus expect a stronger positive effect of inflation on capital accumulation as the elasticity of money demand becomes higher. To make the comparison meaningful, we adjust the value of  $\omega$  in each case to start from the same steady state value for the ratio  $(M/P)/Y$ . The steady states value of the capital-output ratio  $K/Y$  are also identical across the different experiments.

Fig. 6 shows the evolution-pattern of the main aggregates as a function of inflation for the three values of  $\eta$ . The upper panel shows that the drop in money demand and the rise in capital accumulation become sharper as  $\eta$  gets higher. When  $\eta = 1$ , the Tobin effect of substitution between financial assets and real balances always outweighs the insurance effect provided by the equal redistribution of the inflation tax. The capital stock thus increases monotonically with inflation, contrary to the cases with  $\eta = 0.25$  and  $\eta = 0.5$ . Two reasons for that. First, the Tobin effect gets stronger as mentioned above. Second the insurance effect becomes lower as  $\eta$  gets higher. The reason is that the money stock, on which the inflation tax is based, decreases with  $\eta$ . The lower-left panel documents this effect of  $\eta$  on the lump-sum monetary transfers.

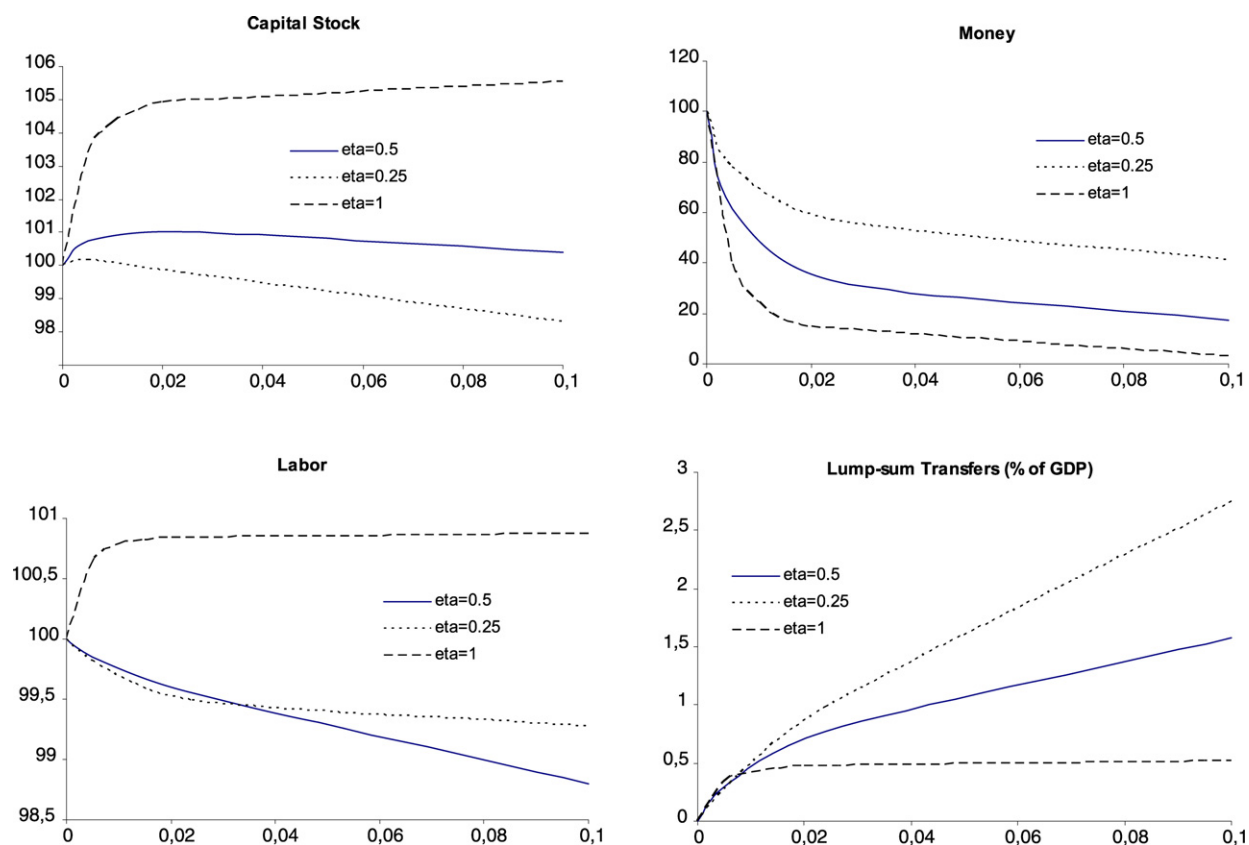


Fig. 6. Sensitivity analysis – elasticity of money demand.

In contrast, when  $\eta = 0.25$ , the capital reaches a peak for  $\pi$  equal to 0.4 percent and then decreases. The insurance effect dominates the Tobin effect for lower level of inflation as the lump-sum monetary transfers becomes much higher.

Fig. 6 – lower right panel – shows that the evolution pattern of the labor supply also depends on the values of  $\eta$ . Two effects alter the labor supply in this framework. First, the increase in the capital stock raises labor productivity and real wages, favoring an increase in labor supply. Second, the distribution of the inflation tax provides some insurance that decreases the incentive to work for self-insure motives. The interplay between these two effects yield different results according to the value of  $\eta$ . For low values of  $\eta$  the distribution of the inflation tax tends to decrease labor supply. For high value of  $\eta$ , this effect is lower and the rise in real wage increases labor supply.

**4.3.1.1. Risk aversion** We consider an increase in the coefficient of relative risk aversion from 1 to 2. The environment corresponds to the benchmark model with an elasticity of money demand  $\eta = 0.5$ . We adjust the discounting factor  $\beta$  to start from the same steady-state value of the capital-output ratio.

Fig. 7 compares the evolution pattern of the main aggregate variables for  $\sigma = 1$  and  $\sigma = 2$ . For a given level of inflation, the demand for money increases with  $\sigma$  as money can be used as a buffer stock against idiosyncratic shocks. As a matter of fact, the aggregate money stock and the lump-sums transfers are higher in an economy with  $\sigma = 2$  compared to the initial case  $\sigma = 1$ . The insurance effect coming from the redistribution of the inflation tax thus plays a higher role when  $\sigma = 2$ , which decreases the incentive to accumulate financial assets. The capital stock decreases faster with inflation as  $\sigma$  increases.

**4.3.1.2. Credit constraints** We end-up this section by exploring the sensitivity of the quantitative results with regard to the strictness of the credit limit. As shown by Huggett (1993), the stringency of credit constraints can affect the equilibrium interest rate. This effect could be all the more important in our context that the effect of inflation directly interact with borrowing constraints. As a robustness check, we allow for borrowing up to 25 percent of the average annual after-tax earnings to match the proportion of 15 percent of agents with negative wealth in the data (Wolff, 2000).

Fig. 8 compares the evolution pattern when borrowing is allowed ( $a < 0$ ) and the benchmark model without any borrowing. The hump-shaped relationship between capital and inflation still holds with  $a < 0$  since both the Tobin effect and the insurance effect are still at work. However the quantitative impact of inflation on capital accumulation is smaller when individuals can borrow. A hike in quarterly inflation from 0 percent to 0.75 percent is associated with an increase by 0.83 percent of the stock of capital in an environment where individual are allowed to borrow. The rise in capital reaches

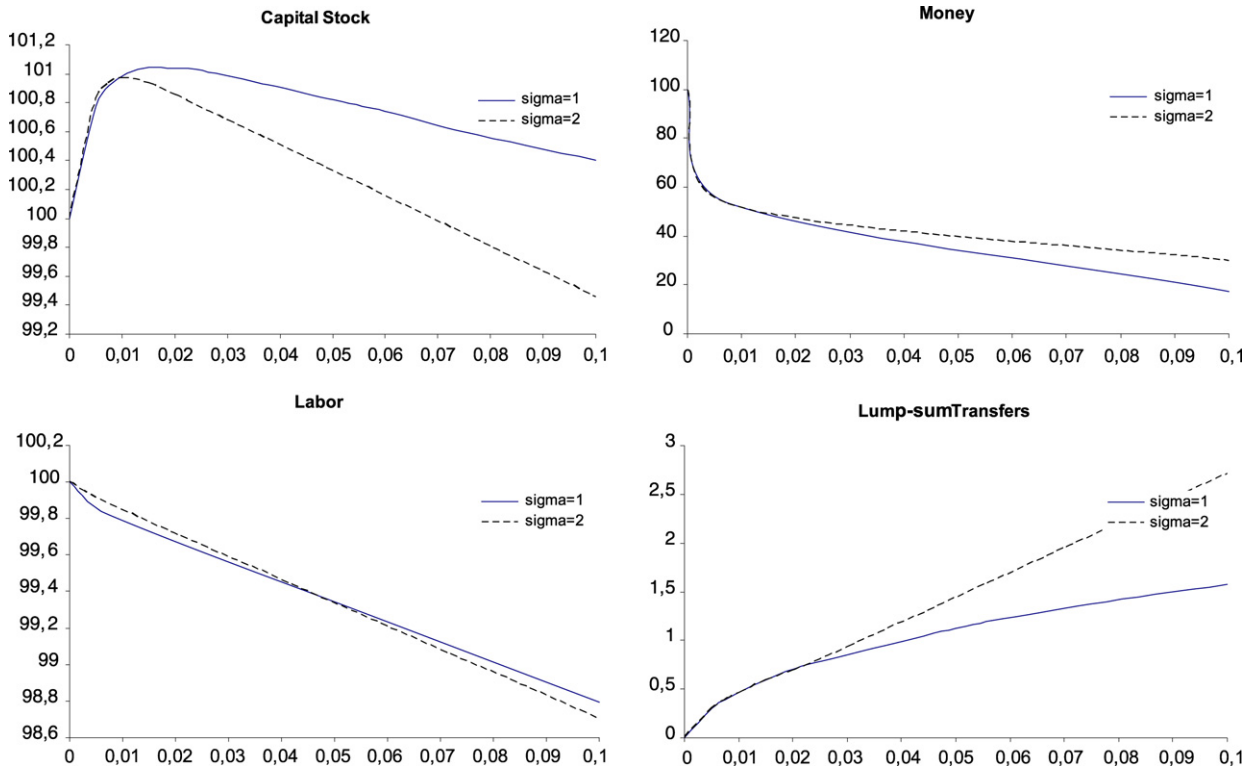


Fig. 7. Sensitivity analysis – risk aversion.

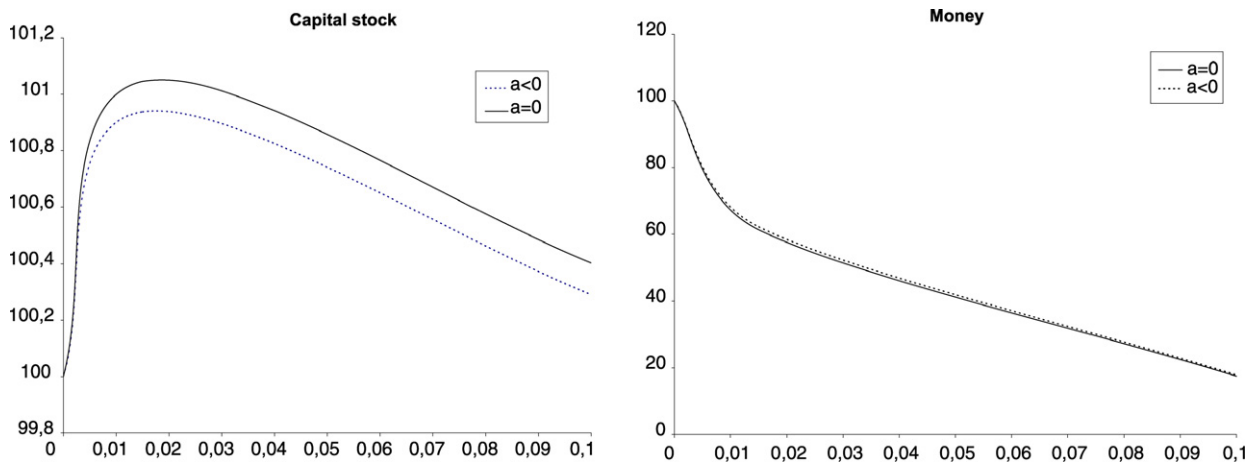


Fig. 8. Sensitivity analysis – credit constraints.

0.91 percent in an environment without borrowing. The reason for this result is that the precautionary savings motive is less important in the former environment since households can smooth consumption by holding debts. The wealth-poor can adjust the hike in inflation by borrowing more financial assets.

## 5. Conclusion

This paper studies the real effect of inflation in an incomplete-market economy where agents face idiosyncratic shocks. It has been shown that inflation has a hump-shaped effect on output and on the aggregate capital stock due to borrowing constraint and market incompleteness. First, inflation affects in a different way borrowing constrained households and unconstrained households, since the former cannot adjust their financial portfolio. This first effect yields an increase in precautionary savings and thus increases the capital stock. Second, the redistribution of the inflation tax provides some insurance to households, who decrease in consequence their precautionary savings. The first effect dominates for low level

of inflation, whereas the second effect dominates for higher level of inflation. Not only do incomplete markets and borrowing constraints have a real quantitative impact on their own, they also amplify significantly the other potential distorting channels of inflation compared to complete market frameworks.

This paper has focused on the long-run properties of money with borrowing constraints. A promising route for future research would be to analyze the short-run effects of monetary shocks in this type of incomplete market economy with borrowing constraints and idiosyncratic shocks (see a first attempt by Heer and Maussner, 2007 in a heterogeneous agents model but without idiosyncratic shocks). This paper also abstracts from growth analysis. Kormendi and Meguire (1985), Fisher (1991), and Roubini and Sala-i-Martin (1995) have identified the negative impact of the inflation rate on economic growth in alternative frameworks. The analysis of the relationship between inflation and growth under incomplete markets is left for future research.

## Appendix A. Solution to the households' problem

Using the Bellman equations, the households' problem can be written in recursive form. Stationary solutions satisfy, of course, the usual transversality conditions. As a consequence, we can focus on the first-order condition of the households' problem. This is given by the program

$$V(q_t^i, e_t^i) = \max_{\{c_t^i, m_t^i, a_{t+1}^i\}} u(c_t^i, m_t^i) + \beta V(q_{t+1}^i, e_{t+1}^i)$$

$$c_t^i + m_t^i + a_{t+1}^i = q_t^i + w_t e_t^i + \frac{\mu_t^i}{P_t} \quad (31)$$

$$q_{t+1}^i = R_{t+1} a_{t+1}^i + \frac{m_t^i}{\Pi_{t+1}} \quad (32)$$

$$c_t^i, m_t^i, a_{t+1}^i \geq 0 \quad (33)$$

where  $q_1^1, q_1^2$  are given,  $R_{t+1} = 1 + r_{t+1}$ , and income shocks are deterministic:  $e_{t+1}^i = 0$  if  $e_t^i = 1$ , and  $e_{t+1}^i = 1$  if  $e_t^i = 0$ . Using (31) and (32) to substitute for  $c_t^i$  and  $q_{t+1}^i$ , we can maximize only over  $a_{t+1}^i$  and  $m_t^i$ . Using the first-order conditions, together with the envelope theorem (which yields in all cases  $V'(q_t^i, e_{t+1}^i) = u'_c(c_t^i, m_t^i)$ ), we have

$$u'_c(c_t^i, m_t^i) = \beta R_{t+1} u'_c(c_{t+1}^i, m_{t+1}^i) \quad (34)$$

$$u'_c(c_t^i, m_t^i) - u'_m(c_t^i, m_t^i) = \frac{\beta}{\Pi_{t+1}} u'_c(c_{t+1}^i, m_{t+1}^i) \quad (35)$$

If the equations above yield a quantity  $a_{t+1}^i < 0$ , then the borrowing constraint is binding and the solution is given by  $a_{t+1}^i = 0$  and  $u'_c(c_t^i, m_t^i) > \beta R_{t+1} u'_c(c_{t+1}^i, m_{t+1}^i)$ , together with (35). In a stationary equilibrium, all  $H$  agents become  $L$  agents the next period and *vice versa*. Since  $H$  agents are in the good state, they always take the opportunity to save for precautionary motives and their borrowing constraints are never binding (see next section). We can rewrite the previous equations using the state of the households instead of their type. With the logarithm utility function, this yields the expressions given in Section 2.

## Appendix B. Proof of Proposition 2 on binding borrowing constraints and the non-neutrality of money

In this proof, we assume as a first step that borrowing constraints are binding for  $L$  households to derive the equilibrium interest rate. In a second step, we check that borrowing constraints are actually binding for  $L$  agents but not for  $H$  agents. By using Proposition 1, it will suffice to check that the equilibrium interest rate satisfies  $1 + r < \frac{1}{\beta}$ .

First, by using the first-order condition (8), we obtain  $\frac{c^L}{c^H} = \beta(1 + r)$ . Equilibrium on the goods market implies that  $c^H + c^L = K^\alpha - K$ , and the first-order conditions of the firm imply that  $1 + r = \alpha K^{\alpha-1}$  and  $w = (1 - \alpha)K^\alpha$ . Substituting for  $c^H$ ,  $w$  and  $K$  we obtain

$$c^L = \beta \frac{1 + r - \alpha}{\beta(1 + r) + 1} \left( \frac{\alpha}{1 + r} \right)^{\frac{\alpha}{1-\alpha}}$$

The budget constraint of  $L$  agents, given by (7), yields

$$\frac{m^L}{c^L} - \frac{m^H}{c^H} \frac{c^H}{c^L} = \frac{a^H(1 + r) - c^L}{c^L}$$

Using the value of the ratio  $\frac{c^L}{c^H} = \beta(1 + r)$  and the expressions (13) and (14), one finds

$$f(r) = g(r, \Pi) \quad (36)$$

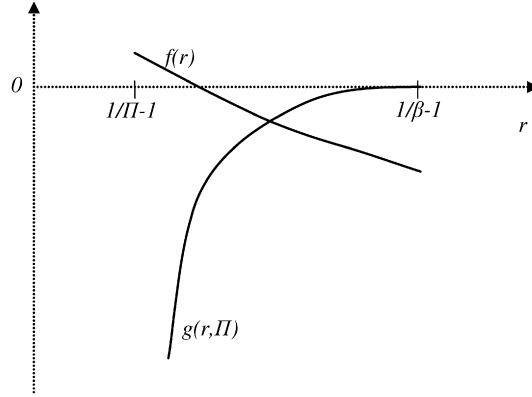


Fig. 9. Existence of an equilibrium with  $1+r < 1/\beta$ .

with

$$f(r) \equiv \frac{\phi}{1-\phi} \left( \alpha \frac{\beta(1+r)+1}{1+r-\alpha} - \beta \right) \quad \text{and} \quad g(r, \Pi) \equiv \frac{\beta}{1-\frac{\beta^2}{\Pi}(1+r)} - \frac{1}{1+r-\frac{1}{\Pi}}$$

Eq. (36) determines the equilibrium interest rate as a function of the parameters of the model and  $\Pi$ . We now have to prove the existence and uniqueness of the equilibrium.

### B.1. Existence of a solution with binding borrowing constraints

Recall that we assume that  $\alpha < 1/\Pi < 1/\beta$ . We then look for the existence of a solution  $r^*$  such that  $1+r^* \in (1/\Pi; 1/\beta)$ . If such a solution exists, borrowing constraints are binding and both money and financial titles are held in equilibrium.

Note that  $f(r)$  is continuous in  $r$ , for  $1+r \in (\frac{1}{\Pi}; \frac{1}{\beta})$  and  $f$  takes finite values at the boundaries  $\frac{1}{\Pi}$  and  $\frac{1}{\beta}$ . For a given value of  $\Pi$ ,  $g(r, \Pi)$  is continuous in  $r$  for  $1+r \in (\frac{1}{\Pi}; \frac{1}{\beta})$ . However,  $g(r, \Pi) \rightarrow -\infty$  when  $1+r \in (\frac{1}{\Pi}; \frac{1}{\beta})$  and  $1+r \rightarrow 1/\Pi$ . And  $g(\frac{1}{\beta}-1, \Pi) = 0$ . As a result, a sufficient condition for an equilibrium to exist is  $f(\frac{1}{\beta}-1) < 0$ . This condition is equivalent to  $\alpha < 1/(2+\beta)$ . Hence, if  $\alpha < 1/(2+\beta)$ , there exists an equilibrium interest rate  $r^*$  such that  $1+r^* \in (1/\Pi; 1/\beta)$ . From Proposition 1, borrowing constraints are binding in such an equilibrium.  $\square$

### B.2. Uniqueness and variations

Note that  $f(r)$  is decreasing in  $r$  when  $1+r \in (\frac{1}{\Pi}; \frac{1}{\beta})$  as  $\alpha < 1/\Pi$  (a simple derivative of  $f$ ). We can show that  $g(r, \Pi)$  is increasing in  $r$ . As a result, the solution is unique, for continuity reasons. Finally, we can show that  $g(r, \Pi)$  is increasing in  $\Pi$ . Define a function  $h$  such that

$$h(y) = \frac{y^3(1+r)^3}{(1+r-\frac{y^2}{\Pi}(1+r)^2)^2} \quad (37)$$

The function  $h$  is positive and increasing in  $y$ . Now, the derivative  $g'_\Pi(r, \Pi)$  can be written as  $g'_\Pi(r, \Pi) = \frac{1}{\Pi^2} (h(\frac{1}{1+r}) - h(\beta))$ . At the equilibrium  $1/(1+r^*) > \beta$ , and hence we have  $g'_\Pi(r^*, \Pi) > 0$ .

Consequently, by the implicit function theorem  $f(r^*) = g(r^*, \Pi)$  defines implicitly  $r^*$  as a decreasing function of  $\Pi$ . Fig. 9 illustrates the existence and the uniqueness of the equilibrium with binding borrowing constraints.  $\square$

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