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## Excellence for all? Heterogeneity in high-schools' value-added

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# Excellence for all? Heterogeneity in high-schools' value-added

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## Abstract

This paper presents a new method that goes beyond the measurement of average value-added of schools by measuring whether schools mitigate or intensify grades dispersion among initially similar students. In practice, school value-added is estimated at different levels of final achievements' distribution by quantile regressions with school specific fixed effects. This method is applied using exhaustive data of the 2015 French high-school diploma and controlling for initial achievements and socio-economic background. Results suggest that almost one-sixth of the high schools significantly reduce, or on the contrary increase, the dispersion in final grades which were expected given the initial characteristics of their intake.



# Introduction

Assessing the performance of schools has become standard in most developed countries. A first objective is to provide tools for public action, for example by evaluating the effectiveness of resource allocation between schools. These evaluations may also aim at providing useful information to families when they are in position to choose the school where to enrol their children. Several performance metrics on school effectiveness have been suggested in the literature. Most of them however focus on the *average* school impact which may mask significant disparities in achievement outcomes across students. For instance, a positive mean value-added may be due to a combination of an impressive academic progress for a few students with a stagnation of results for the other. In this paper, we propose to go beyond the classic *average* school value-added by modelling whether a school contributes to decrease or on the contrary increase the inequalities in academic outcomes of its intake. One of the objectives would be to identify the high schools which help *all* of their students to achieve a high level of performance.

Evaluating school performance raises a number of challenges around the definition, the estimation and the interpretation of the estimators of “school effects”. As stated by Reardon and Raudenbush [2009], holding a school or a teacher accountable for students’ final achievements is only possible under a set of assumptions. The main issue arises from selective matching between schools and students. Students are usually not randomly assigned to schools, and differences in school raw performances are largely due to composition effects. For instance, some high schools may skim the students with the highest academic potentials - and achieve outstanding final outcomes thanks to this selective admission policy (rather than effective teaching practices). In order to disentangle school effects from selection effects, it is common to rely on the so-called value-added models (Koedel et al. [2015], Raudenbush and Willms [1995]). In practice, value-added are derived by regressing the individual schooling performance on school fixed effects, controlling for students observable characteristics (such as socio-economic background or past scores), assuming that selection happens only on observable characteristics. These models are usually estimated under a homogeneity assumption. Measuring the academic pay-off of students’ characteristics over the entire population indicates whether the average performance in a specific school exceeds or conversely falls short of what is expected, given the school enrollment. Relaxing the focus on the average is the purpose of this paper, by building on value-added models.

However, even in case of random allocation of students, it is generally impossible to isolate in the “school effects” what pertains to school specific actions (for instance, teachers practices and resource allocation) from what is due to the social and schooling composition of its intake (for instance, because of peer effects or because school immediate neighbourhood influences achievements). Nevertheless, all these components of school effects are *in fine* impacting achievements and therefore of interest for students and families. Because peer effects are usually prevalent in education (for a survey see for instance Sacerdote [2011] or Epple and Romano [2011]), the school performance of one student is usually affected by the performances of his or her classmates - regardless of the teaching practices or school investment. Raudenbush and Willms [1995] emphasized that if these measures may not be appropriate for authorities to evaluate school practices, they may be valuable for parents when choosing a school for their children. Value-added may measure the difference between a pupil’s actual performance and the performance that would have been expected if he or she had attended an average school - as parents may not be interested in distinguishing what pertains to the quality of the teaching practices from the composition of the high school intake. In the French context, the Ministry of Education publicly published every year an estimation of the French high schools’ value added Evain and Evrard, 2017, notably based on the success rate at the high school exam (French *Baccalaureat*).

However, the information provided by value-added estimates to families may be more or less relevant depending on the extent of school effects heterogeneity around its average. A higher average achievement would not be interpreted the same way if it reflects a homogeneous improve-

ment of achievements, an increase in the lowest achievements or an increase in the polarisation at the top of the distribution of achievements.

This paper proposes new indicators of the impact of high schools that capture, per school, the impact both on performance of the enrolled students and on the dispersion of these performances within school. These estimates are derived from conditional quantile regressions with high-school-specific effects, estimated at several quantiles of the distribution of final schooling achievements. As in value-added models, we control for students' characteristics, including previous scores in order to reduce the bias due to sorting of students across high schools depending on their observable abilities. High schools may be described along two dimensions: first, whether they tend to shift the distribution of achievements, and second whether they tend to spread or on the contrary gather the achievements. A typology of high schools is then derived, that relies on their egalitarian (or inequalitarian) effect. Comparing this typology with the median value-added of the high school may for instance help to identify high schools that make all their students attain a high level of performance, without increasing within school inequalities.

We use an exhaustive database on the French non vocational high schools (*lycées*, corresponding to secondary education for children between the ages of 15 and 18) for the year 2015. Empirical estimates are based on the results at a national, standardized evaluation corresponding to the final exam of secondary school (the French *baccalauréat*, see for instance Duclos and Murat [2014]). These data provide detailed grades but also information on the individual characteristics of the pupils, such as familial background and gender. First and foremost, the dataset contains a detailed measure of the schooling level of pupils at the entry of high school, the grades obtained at the final exam (*brevet des collèges*) that is taken by French pupils at the end of middle school (year 10/ninth grade).

By comparing the school specific effects at the top and the bottom of the final grade distribution, we discriminate between “egalitarian” high schools in the sense that they tend to reduce performance gaps between students compared to what would be expected given their enrollment, and “unequal” high schools, which on the contrary tend to increase achievement differences. We observe that these two categories account for almost a third of high schools, in an almost equal proportion. The other high schools have a “homogeneous” effect on all their students, meaning that they have an impact, positive or negative, that is not statistically different at the top of the score distribution than at its bottom. When accounting for multiple hypothesis testing, one-sixth of schools is found to have an heterogeneous impact, either leading to increase the dispersion of the results between students, either to reduce it.

Finally, we study how our estimates would be affected if there were a high-school-specific eviction based on students performance. In this case, estimates of dispersion may be downward biased if some high schools are able to exclude the least promising students. We use both data on mobilities of students between schools and simulations to assess the magnitude of the bias. In private schools (which are more able to select students), our empirical findings are consistent with this type of selection, in the sense that higher students' mobilities are correlated with a reduced dispersion. As a robustness check, we verify that our typology is robust to student mobility, which may partly reflect strategic high-school behaviour. Overall, we conclude that if selection most likely exists in some schools, its impact on our classification estimates is at most mild.

The following section presents an overview of the institutional context in France and the data. The second section discusses the econometric model, detailing in particular fixed-effect quantile estimation and their interpretation. The third section details the results obtained and their robustness to selection of students during the high-school years, and then details how school features are related to the high-school fixed effects.

# 1 Institutional context and data

## 1.1 French High schools, curriculum and assignment to schools

This analysis focuses on the French *lycées* (senior high schools) which constitute the second part of the French secondary education. These high schools enroll students from age 15 to 18, after the *collège* (junior high school) that enroll pupils from age 11 to 15. At the end of junior high school, some students may be tracked to vocational education and be enrolled in dedicated vocational high schools. Other students follow for one year the same curriculum and are then assigned to one of two tracks (see Figure 1): general or technological tracks. Tracking is made in theory according to student preferences, but in practice it depends mostly on the schooling achievement, the general track being considered as the most selective and the vocational track the least one. Before 2019, students had to choose within each tracks between different streams, called *séries*. For instance, in the general track, pupils should choose between three main streams.<sup>1</sup> The technological track includes six main streams.<sup>2</sup> The vocational track includes more than eighty streams and is offered only in dedicated high schools. On the other hand, many high schools offer both a general and technological curriculum. The curricula of all the streams and tracks are defined by the French Ministry of Education. In the following, we do not consider vocational high schools, as they offer very heterogeneous and numerous streams. This makes the comparison between achievements difficult (within each stream, the number of students may be rather small). We analyze separately the general and technological *baccalauréats*, as they are different exams whose enrollment differs.

Both junior and senior high schools end with a national mandatory examination, respectively the *diplôme national du brevet* and the *baccalauréat*. Concerning the former, the final score is partly based on three written parts on three main domains (Mathematics, French and History, Geography and Civic Education), with a national examination test.<sup>3</sup> The grades obtained at this written examination provide a standardized proxy of pupil level before entering high school. At the end of high school, students sit for the *baccalauréat*, a national examination. The type of *baccalauréat* depends on the schooling track: it distinguishes general, technological and vocational ones. It relies on standardized tests that occur at the end of the final grade. In theory, the *baccalauréat* diploma qualifies for university entrance (with some restrictions depending on the type of *baccalauréat*). It is, inter alia, a standardized measure of pupils' achievement at the end of high school. The French *baccalauréat* is a national institution whose results are each year highly publicized. Newspapers regularly provide a ranking of high schools depending on the proportion of their students who pass the exam.

Since 2007, a school choice procedure was introduced in France at the level of the school district ("*académie*", meaning in 2015 30 large administrative regions) for the assignment to public high school, that was before mainly residence-based. The assignment uses a variant of the school-proposing deferred acceptance algorithm (so-called "Boston mechanism"). Students submit their preference lists to a central authority at the district level. The assignment is made following an algorithm that tries to match as many students to their stated preferred schools as possible, subject to pre-specified priorities of students. Priority is given in a similar way for all high schools in the district and is based on geographic (for instance home address located within the district), academic (prior school achievement) and social (priority to students from low-income family) criteria. The exact scheme - and notably the weight on previous achievements - depends on the district. So far, private schools are not integrated in the school choice procedure.

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<sup>1</sup>sciences, humanities, economics and social sciences

<sup>2</sup>sciences and technologies in health and social field, hotel and restaurants management, industrial science and technologies and sustainable development, laboratory science and management technologies.

<sup>3</sup>The final result at the exam also takes into account grades obtained at school (and since 2017 a measure of the "skills" in the main domains), but these measures are not standardized - some schools may have different academic standards resulting in more or less generous grading policies.





social position index is that of Rocher [2016], who derives a continuous index from measures of the occupation of both parents.<sup>5</sup>

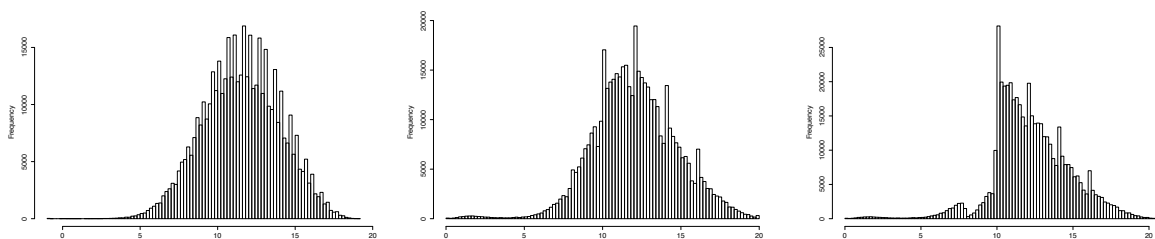
The school of each student in the year of the *baccalauréat* is recorded in both data sets, while the FAERE files allow us to recover the past school history of students. Finally, the APAE<sup>6</sup> database contains information on the high school: sector, staff, teaching options, etc.

Table 1: Characteristics of students applying for the *baccalauréat* 2015

Track	General		Technological	
	Mean	sd	Mean	sd
DNB grade (past score, over 20)	12.3	2.3	9.7	2.0
Social position index	122.4	35.0	104.6	33.6
Repeaters	5.7%		18.3%	
Girls	56.5%		50.0%	

Source: FAERE and APAE databases, Author calculations.  
DNB corresponds to the pre high-school examination.

At the end of the first session of the *baccalauréat* exam, the candidate gets a grade, that results from the first set of tests which are graded by teachers. In order to guarantee uniform marks, marking teachers attend a standardisation jury. These jury may increase the score obtained at the first session in order to allow a candidate to obtain 10 out of 20, the pass mark, or 12, 14 or 16, the thresholds for particular distinctions. When a student gets an average mark below 10 out of 20 but above 8 out of 20, he or she may resit the exam in a second session. The final mark (after the second session) obtained at the *baccalauréat* shows a very strong threshold effect at 10, due to a large extent to the students who barely pass. We choose to work on the grade at the first session, i.e. the average of the scores on the tests with their respective weightings, which represents a relatively homogeneous test for the whole sample. The raw grades, before standardisation, are not available in the data. The grades are centered and standardized in what follows.



(a) DNB grades (raw prior score) (b) Baccalauréat grades (first session) (c) Baccalauréat grades (after second session, i.e. resit)

Figure 2: Empirical grades distributions (first session and final grades after resit) Source: FAERE database (2015 baccalauréat), author calculations.

<sup>5</sup>This social position index has been constructed using survey data on social, economic and cultural characteristics of parents. The social position index corresponds to linear combination of level of diploma, earnings of the parents as well as cultural capital (number of books at home for instance) that are correlated with future performance. It takes into account the occupation of both the mother and father.

<sup>6</sup>*Aide au Pilotage et à l'Auto-évaluation des Établissements*, the school entry on this database is made available to the head of school as administrative information and performance indicators.

Each stream (*série*) has a different weighting for subjects and students choose two years before taking the *baccalauréat* which exam path they will prepare. Empirically, at given initial level (DNB score) and equal characteristics, a student taking the scientific *série* (S) will on average have a lower *baccalauréat* score than a student taking the humanities *série* (L). This observation is partly due to the fact that students are graded and therefore compared between students within the same path. Students taking the scientific *série* have more often than those taking the humanities one characteristics related to academic success. In practice, when the grading is partly standardized, scientific students compete with students who are better at school and their results are on average worse compared to students in humanities with equal characteristics. To account for the peculiarities in the grading in each *série* (stream) of the *baccalauréat* exam, a fixed “*série*” effect is introduced in the empirical analysis.

## 2 High school value-added at different levels of the distribution

### 2.1 Value-Added and Student growth percentile models

The identification of school (or teacher) effectiveness is challenging because of selection effects. When enrollment is selective based on past schooling performances, schools’ success mostly reflects the enrollment of high-performing students.

With observational data, value-added models (“VA”) are the prominent way of addressing these selection effects. In their simplest form, these models rely on a linear specification of the final grade of students as a function of their observable characteristics in addition to a school specific effect, aimed at characterizing the action of the school. In practice, these school effects are modeled either by fixed or random effects. This class of models is in general used for modeling the impact on the average test-score level, but may be used for distinct parameters. For instance, in the French context, the ministry of Education provides estimates of the value added of high schools corresponding to the passing rate at the final exam, or the proportion of students who had obtained the baccalaureat with merit Evain and Evrard, 2017,<sup>7</sup>. In this case, the outcomes of interest are dummies measuring whether a student had obtained an average grade at the final exam above the passing threshold (10 out of 20), or the threshold for merit (12 out of 20).<sup>8</sup> More generally, the estimates may be extended to analyze the impact on the entire distribution of test scores. Page et al. [2016], exploring school distributional impacts as well, define a statistical VA model per quantile based on schools’ random effects. Random effects estimators are more effective (especially when few observations are available). However, they may be biased and inconsistent when pupil characteristics are correlated with school effects (which is most likely the case in our setting).

On the other hand, fixed-effects estimators are consistent even when selection on observable occurs. When only a few observations are available by cluster, they may be not correctly estimated. As opposed to survey data, sampling only a few observations per school, it is less likely to be an issue with exhaustive data.

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<sup>7</sup>Estimates are also provided for the retention rate (probability to be enrolled in final year of high school for those enrolled in first year of high school for instance)

<sup>8</sup>This indicator has been chosen to overcome the fact that the passing rate of the baccalaureat has steadily increased since the 1990s and is now close to 90%. In technical terms, this means that the dispersion between the average passing rate between high school is quite low, with a no-null accumulation at the top level of 100% of passing rate. In addition, this means that the information provided by high school final exam completion is quite low, notably regarding selection in higher education.

The Student Growth Percentiles (SGP) are another popular tool to assess schools’ or teachers’ effectiveness (see Betebenner [2009]). SGP rely on repeated observations of test scores, and compares one student’s performance with those of other students with similar prior test scores. Each student is assigned to a rank in the conditional final test score distribution.<sup>9</sup> In practice, the calculation of the SGP-based school effectiveness relies on several steps. First, quantile regressions of the final test score on prior test scores are performed. Quantile regressions rely on a local linear approximation of conditional quantiles, which formally writes:

$$q_\tau(Y_i|X_i) = m(\tau) + X_i\beta(\tau) \quad (1)$$

where  $Y_i$  stands for the final test score of student  $i$  and  $X_i$  individual characteristics - here the student’s past test scores. These regressions can be run for several levels of quantiles  $\tau$  in order to approximate the test score distribution conditional on previous achievement. The final score of a student may thus be located in the corresponding distribution conditional on his or her prior test scores. In practice, the SGP is the student’s grade percentile in the distribution of grades conditional on his or her past scores. In other words, it is a rank among peers comparable in terms of prior grades. Formally, the rank of student  $i$  is computed from

$$\widehat{SGP}_i = \max\{\tau : \widehat{q}_\tau(Y|X_i) \leq Y_i\}$$

School (or teacher) performance is usually defined by aggregating the SGPs of the students attending the school. The effectiveness of the school  $j$  is then calculated as the average of its students’ SGP, for instance  $\hat{\tau}_j = \langle \widehat{SGP}_i \rangle_{i \in j}$ . The SGP rationale can be simply summarized as a way to answer the question: “How well did a student perform compared to those who had the same grades than him or her?”. If this student performed better than say 80% of the others, the school effectiveness indicator is increased by  $\frac{0.80}{n}$  where  $n$  is the number of students in the school  $j$ . As far as we know, this methodology has not been used to measure the school effectiveness beyond the average effect, while it is by construction heterogeneous (as a per student measure).

The SGP models provide an intuitive way of measuring school or teacher effectiveness. However, controlling solely for previous test scores may provide biased estimates. First, as emphasized by Walsh and Isenberg [2013] and Guarino et al. [2014], some other student’s characteristics (social background for instance) may be correlated with both the past and final test scores. As students are not randomly assigned to school, one may wrongly attribute to the school action the simple consequence of a favorable enrollment. This is all the more serious if these student characteristics are also correlated with the expected high school effectiveness. For instance, if students from a privileged background have easier access to the “best” high schools or are assigned to the “best” teachers. Using simulations, Guarino et al. [2014] observe that SGP model may be biased when this selective matching operates. Specifically, they compare the ability of both SGP model and value-added model (with fixed-effects) to correctly assess the effectiveness of teachers, depending on the way students have been allocated in classes. When students are assigned to teachers based on their prior grade (dynamic grouping), SGP models perform in a similar way than a value-added model that controls for past score. However, in case of selective matching (teachers allocated to particular classes depending on their effectiveness), the

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<sup>9</sup> Barlevy and Neal [2012] rationalize the advantage of measures based only on ordinal ranking, in particular see SGP as a possible implementation of their “pay per percentile” method. In school systems where teachers’ careers or earnings depend directly on the measures of their performances, they may be tempted to manipulate the measures - by focusing their teaching on a particular set of skills in order to increase student performance on the mandated test. Measures such SGP that rely only on ranking - and not score - allow the assessing agency to use completely new assessments at each testing date, reducing the possibility to “teach to the test”.

SGP models underperform fixed effects models and are not able to recover the true teachers' effectiveness ranking.

## 2.2 Estimation of high school value-added per quantile

We suggest indicators which offer a more complete picture of school action. Specifically, we want to model simultaneously whether a high school mitigates the differences in school achievements across students (equality) and whether it increases achievements (effectiveness). To that end, we rely on quantile regressions to model the high school value-added at different points of the distribution of final test scores. We thus model both the median school effect (close to the classic analysis on the average effect) and a measure of the inequalities within schools (defined by the difference in extreme quintiles).

A simplified data generating process of student test scores helps to illustrate these measures. Very generally, the academic performance of students at the end of high school depends on the characteristics of the students (schooling ability as measured by past examination, gender, socio-economic status for instance), but also on the high school where he or she is enrolled. Introducing heterogeneous effects is achieved in a straightforward way with a "location-scale" model (see for instance Koenker [2005]).<sup>10</sup> The determinant of the outcomes may have an heterogeneous impact, and this is reflected by allowing, on top of a shift on the average outcome (location effect), a spread or a shrinkage, in its dispersion (scale effect). For instance, we observe a positive relationship between prior performance and current performance - but also the range of final performance among the lowest achievers at the beginning of the high school is much lower than the one of the best achievers (see Figure 10 in the Appendix).

Formally, the location-scale model states that the performance at the final exam, the test score  $Y_i$  of the student  $i$ , can be approximated by:

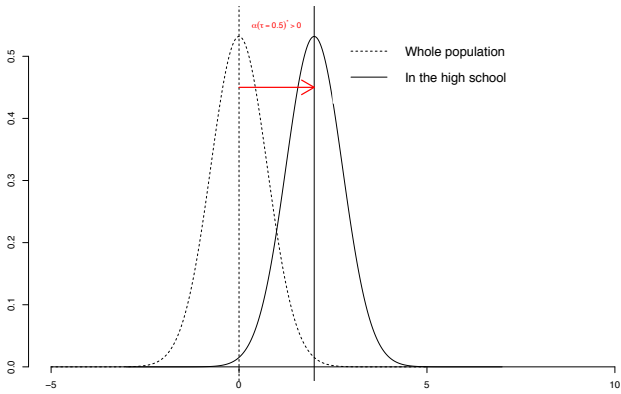
$$Y_i = \gamma_0 + \gamma_j + X_i' \gamma_X + (\lambda_0 + \lambda_j + X_i' \lambda_X) \epsilon_i \quad (2)$$

where our main parameters of interest are  $\gamma_j$  and  $\lambda_j$ , which correspond to the impacts of being enrolled in the high school  $j$  on both the expected outcome and its dispersion (with  $\sum_j \lambda_j = 0$  and  $\sum_j \gamma_j = 0$ ). As discussed in introduction, these impacts may be due to specific school practices (for instance grouping by abilities, innovative teaching practices, maintaining a good disciplinary climate within the school, etc.) but also the characteristics of its enrollment (which the high school may not directly control).  $X_i$  stands for the observable characteristics of the students such as gender, socio-economic background and his or her academic level (past test score) as measured just before the entry in high school. The individual term  $\epsilon_i$  corresponds to characteristics that are not observable in the data: for instance the student's motivation or ability, that is not captured by past test score - but also potential different sensitivity to teaching practices or to the competitive pressure of their peers. Using an interaction between the high school and the unobservable  $\epsilon_i$  captures this idea that the same context or practices may vary from one student to another.<sup>11</sup>

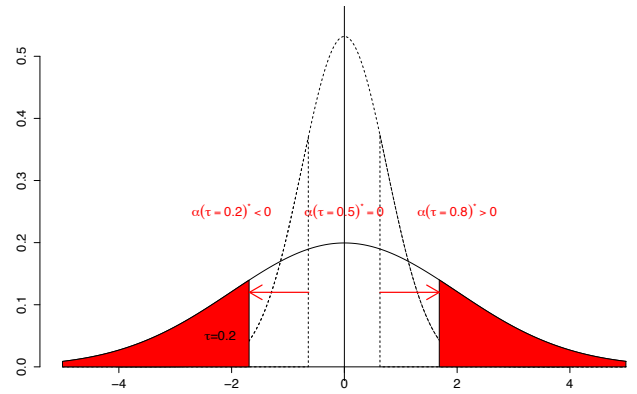
The Figure 3 exemplifies the distinction between these two effects. The three panels represent archetypal situations where the high school has either an homogeneous impact on all the outcomes of students - or on the contrary has only an effect on the dispersion of the outcomes. When the

<sup>10</sup>The location-scale model is typically used to model Engel curves which model how households' expenditures vary with households' incomes.

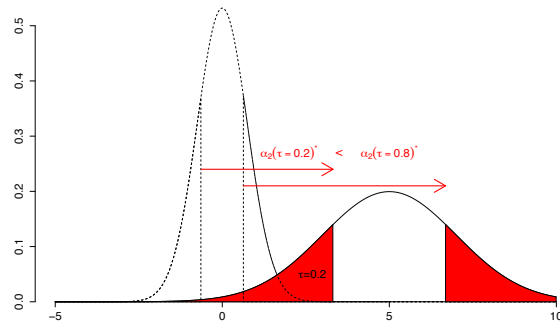
<sup>11</sup>The actual impact of an high school on every of its students can not be identified - as it would require to observe simultaneously the very same student in other high schools.



(a) Homogeneous and positive effect



(b) Heterogeneous effect



(c) Heterogeneous and positive effect

Figure 3: Location-scale model: illustration on fictitious type of school effects on the expected grade distribution

high school has the same impact on all students, whatever their characteristics (including the unobservables), one expects to observe a simple translation of the conditional distribution of the outcome (as illustrated in panel (a) of Figure 3). On the contrary, panel (b) of Figure 3 illustrates the case of a high school that would have no average impact on outcomes, but where these outcomes are more scattered than expected. As illustrated in the panel (c), both impact may be combined: a high school may intensify inequalities in outcomes *and* increase the observed results at every level of the final test score distribution. Conversely, a high school may have a negative impact on the score (compared to what is expected) *and* reduce homogeneity. Both dimensions, that we respectively relate to effectiveness and equality, are analyzed here.

The location-scale model provides a very simple framework to introduce heterogeneity - as it allows us to estimate both the shift in the distribution (location) and the impact in dispersion (scale) that a high school may have on the distribution of outcomes. For estimation, we assume a slightly more complex modelization that reflects the fact that this heterogeneous effect may be different at the bottom and the top of the distribution.

We estimate the following quantile regression equation, which is conditional on both observable characteristics of the students and on high schools' enrollment:

$$q_\tau(Y_{ij}|X_{ij}, 1\{i \in j\}) = \underbrace{m(\tau) + X_{ij}\beta(\tau)}_{\text{Expectation}} + \underbrace{\alpha_j^*(\tau)}_{\text{School effect}} \quad (3)$$

where  $Y_{ij}$  corresponds to the grades obtained at the final test *baccalauréat* (standardized) and  $X_{ij}$  are the observable covariates (initial standardized test score and its square, social position index, gender and a repeating dummy).  $\alpha_j^*(\tau)$  is the fixed effect specific to high school  $j$  at quantile  $\tau \in [0, 1]$  of the distribution of score conditional on observables. No assumption is made about the distribution of the school specific effects  $\alpha_j^*(\tau)$  among the whole set of  $J$  schools. For each quantile, the school effects  $\alpha_j^*(\tau)$  are normalized across the set of schools, with  $\sum_j \alpha_j^*(\tau) = 0$ .<sup>12</sup> This specification is close to the one proposed by Page et al. [2016]. However, while they assume a Gaussian distribution for the high school effects, we do not have *a priori* on this distribution. Specifically, we do not assume that these effects are not correlated with other observable covariates. This is important in the French context, as one may assume that endogenous selection may occur in high school enrollment. For instance, some high schools may be tempted to skim the best students if they have the possibility to do so (this is especially the case for private high schools that have more latitude than public schools to select their students based on their previous school records). However, we still rely on the strong assumption that students selection within high school is based on observable characteristics (in the underlying model above, that the unobservable is independent of covariates used in the model), in absence of experimental or quasi-experimental data.

Koenker [2005] has shown that estimates can be obtained as the solution of a convex optimization program.<sup>13</sup> Estimations are run for three quantiles: the lowest quintile ( $\tau = 0.2$ ), the median ( $\tau = 0.5$ ) and the highest quintile ( $\tau = 0.8$ ). In practice, we thus estimate each school effects, thus three coefficients for every  $J$  high schools. Quantile regression on grouped data (students within high schools) can be related to recent contributions to the econometric literature on quantile regressions on panel data (see for instance Koenker [2004], Canay [2011], Kato et al. [2012]). While in panel data fixed effects are not interesting *per se*, high school effects are the main parameters of interest here. Usually, we observe a higher number of observations per cluster (high school) than in standard panel data and thus do not face “small T” issues (see also Ponomareva [2011]). In some rare cases, some high schools enroll a very small number of students. As this may lead to very imprecise estimates of the fixed effects for these schools, we choose to exclude the smallest high schools. In practice, we restrict the sample to high schools with more than 65 students in general tracks or 25 students in the technological tracks. These thresholds are set in order to keep 95% of students in each curricula and robustness to this choice is then assessed. They correspond to a trade-off between a sufficient number of observations per high school and keeping the sample representative of the entire population of students. We test the robustness of the estimates and the classification to the choice of the threshold and to the inclusion of very small schools. In practice, the estimation sample corresponds to respectively around 318,000 students enrolled in the general track in 1,759 high schools, and 123,000 students enrolled in the technological track in 1,549 high schools. This comes to exclude 15% of high schools from the initial sample (see Table 8 in the Appendix).

<sup>12</sup>In practice we estimate  $q_\tau(Y_{ij}|X_{ij}, 1\{i \in j\}) = \mu(\tau) + X_{ij}\beta(\tau) + \alpha_j(\tau)$ , with  $\alpha_1(\tau) = 0$  and recover the  $(\alpha_j^*(\tau))_{j=1\dots J}$  by  $\alpha_j^*(\tau) = \alpha_j(\tau) - \frac{1}{|J|} \sum_l \alpha_l(\tau)$  and  $m(\tau) = \frac{1}{|J|} \sum_l \alpha_l(\tau) + \mu(\tau)$

<sup>13</sup>Formally,

$$(\{\hat{\alpha}_j(\tau)\}_{1 \leq j \leq J}, \hat{\beta}(\tau)) = \operatorname{argmin} \sum_{i,j} \rho_\tau(Y_{ij} - \mu(\tau) - \alpha_j(\tau) - X_{ij}\beta(\tau)) \quad (4)$$

where  $\rho_\tau(u) = |u|\{\tau 1\{u \geq 0\} + (1 - \tau)1\{u < 0\}\}$  and  $\alpha_1(\tau) = 0$ .

As emphasized by Angrist and Pischke [2008], quantile regression deals with distribution and not individuals, and should be interpreted cautiously. If  $\alpha_j^*(0.8) < 0$ , the highest achievers tend to perform worse in high school  $j$  than the highest performers in other high schools.<sup>14</sup> We can not infer that the highest achievers in the high school  $j$  would have performed better in another high school without additional assumptions. We would have to require that the achievement ranking among students in  $j$  would have been the same in another high school. This assumption of rank invariance is strong. For instance, Dong and Shen [2018] and Frandsen and Lefgren [2018] provide recent evidences of educational programs which result in significant shift in individual ranks - and thus dissuade an individual interpretation of quantile treatment effects.

Moreover, when we condition on past achievements, we implicitly model the distribution of gains in test scores (this is the intuition behind the model of “pay per percentile” of Barlevy and Neal [2012] for instance). Quantile regression conditional on the initial test scores inform for instance on the school impact on the highest gains observed in the high school. These gains are not necessarily obtained by the students who were initially the highest achievers (in the distribution of initial test score). The highest gain may be achieved for instance by students in the bottom of the distribution of initial test score (see Powell [2010] or Fort [2012] for a discussion).

That being said, the specification can be modified to analyze how the impact of high school varies across students. Alongside our baseline specification, we estimate a model that interacts the high school fixed effects with observable students’ characteristics. For instance, these estimates point out whether already low-achieving or disadvantaged students benefit more in terms of effectiveness and equality from a particular school. We define different types of students depending on their initial achievement at the beginning of high school and on their social background (measured by the social position index). These types are defined by the quartile of the distribution of past score (respectively the distribution of the social position index) they belong to. Formally, we estimate:

$$q_\tau(Y_{ij}|X_{ij}, 1\{i \in j\}) = m(\tau) + X_{ij}\beta(\tau) + \gamma_q(\tau) + \underbrace{\alpha_{jq}^*(\tau)}_{\text{quartile-specific school effect}} \quad (5)$$

as before,  $X_{ij}$  corresponds to student observable characteristics.  $q \in \{1, 4\}$  stands for the position of the student  $i$  in the distribution of prior test score (respectively social position index) and  $\gamma_q(\tau)$  is a dummy for belonging to the type  $q$  ( $q = 1$  when the student belongs to the lowest quartile of test score, or respectively social position index).  $\alpha_{jq}^*(\tau)$  is a school and student-quartile specific effect. This specification is more demanding, as we estimate high school effects for student types. In order to have “enough” observations for the estimation of empirical distribution within clusters, for these specifications we keep only high schools which enroll at least 10 students in each quartile of past achievements in the general tracks (in the technological track, high schools usually enroll fewer students and this condition is very stringent). In these specifications, we keep around 290,000 students enrolled in the general track in around 1,500 high schools (see Table 8 in the appendix).

The numerical estimation of a large number of coefficients can be computationally complex. We use the adaptation of the Frish-Newton algorithm for sparse matrices proposed by Koenker and Ng [2005]. Estimates of the precision are made by bootstrap. The bootstrap sample is

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<sup>14</sup>Here, the highest achievers are defined as those above the quantile  $\tau = 0.8$  of their conditional distribution: they are the top performers among initially similar students (but may not be however the top performers at the final exam, *unconditionally*, as for instance being among the top performers among initially low-achieving students do not insure to outperform the bottom performers among initially high-achieving students).

stratified per high school, using within high school sampling with replacement. The sampled unit is the student, i.e. the final test score and his individual characteristics. Significance criteria are based on the  $B = 500$  bootstrap  $b$  draws.<sup>15</sup>

## 2.3 Effective and unequal high schools: definitions

The school effect quantifies whether the performance reached by a given proportion of students outperforms or underperforms what is expected given the observable characteristics of students enrolled in this school. For example, a positive value of  $\alpha_j^*(\tau = 0.2)$  means that the grade exceeded by 80% of students of this school is higher than what was expected given its enrollment. The lowest achievers in such a high school perform better than the lowest achievers in a high school with similar enrollment. By convention, the effectiveness of a high school  $j$  is measured by the fixed effect at the median  $\hat{\alpha}_j^*(M)$ .<sup>16</sup> In addition, we define the within-group dispersion in each high school  $j$  by the difference between the estimated school effects at the highest and lowest quintiles  $\hat{\alpha}_j^*(\tau = 0.8) - \hat{\alpha}_j^*(\tau = 0.2) := \hat{\alpha}_j^*(H) - \hat{\alpha}_j^*(L)$ . In the following, high schools for which this difference is significantly different from zero are referred as “heterogeneous”. These high schools tend to either widen or reduce inequalities in outcomes from what would be expected. In the former case, we refer to the school as “unequal” (since inequalities in outcomes are higher at the exit of the high school than expected), and in the latter case as “egalitarian”.

If the empirical probability of order conservation across bootstrap  $b$ ,  $\mathbb{P}_b(\text{order}(\alpha^{*(b)}(L), \alpha^{*(b)}(H)) = \text{order}(\hat{\alpha}^*(L), \hat{\alpha}^*(H)))$ , is higher than 90%, the high school is assigned to the category of the corresponding heterogeneous effect, “unequal” or “egalitarian”, otherwise, the effect is assumed homogeneous. As we perform thousands of tests, we should account for multiple hypothesis testing (MHT). Under the full null hypothesis (no heterogeneous effect in our sample), 10% of false positives are expected. The first type of corrections that have been proposed in the literature concentrates on the type one error (Family-Wise Error Rate, False Discovery Rate). It implies to compare the p-values to a threshold corrected by a factor inversely proportional to the number of tests  $N_H$ , hence  $\frac{0.1}{N_H}$  for a 10% significance. However, these tests are very conservative as soon as the number of tests is high (Carvajal-Rodríguez et al. [2009]).<sup>17</sup> Several alternatives have been proposed in order to have tests with enough statistical power (for a discussion, see de Uña-Alvarez [2011] and Castro-Conde and de Uña-Álvarez [2015]). We follow Carvajal-Rodríguez et al. [2009] and de Uña-Alvarez [2012] and use their so-called Sequential-Goodness-of-Fit (hereafter *SGoF*) method which has proven to provide enough power in cases where, as here, thousands of tests are performed. This test is based on the difference between the observed proportion of p-values below the chosen significance threshold and the expected proportion under the complete null (no heterogeneous effects).

We also check that the typology is robust to technical choices made for estimations. We analyze in the Figures 12 and 13 of the Appendix the sensitivity of the estimations to the restriction of the sample to the “larger” schools. The high school effects appear less precisely estimated when using smallest high schools, as expected, but the sample restriction does not

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<sup>15</sup>The 95% confidence intervals computed with  $[q_{0.025}(\{\alpha_\tau^{*(b)}\}_{b \in B}), q_{0.975}(\{\alpha_\tau^{*(b)}\}_{b \in B})]$ . There is no closed-form estimates of the precision of the estimators for quantile regressions except beyond strong assumptions on the underlying generating model. To our knowledge there are no results for inference on fixed effects by quantile.

<sup>16</sup>In practice, we observe that the median value added and the mean value added of high schools are very close - without large dispersion - see Figure 11 in Appendix.

<sup>17</sup>In practice it requires a very large number of bootstrap samples so that  $\frac{1}{B}$ , the minimal scale of precision, is at least smaller than  $\frac{0.1}{N_H}$ . Otherwise, we will accept that order is conserved only if it is conserved on *all* the bootstrap draws.



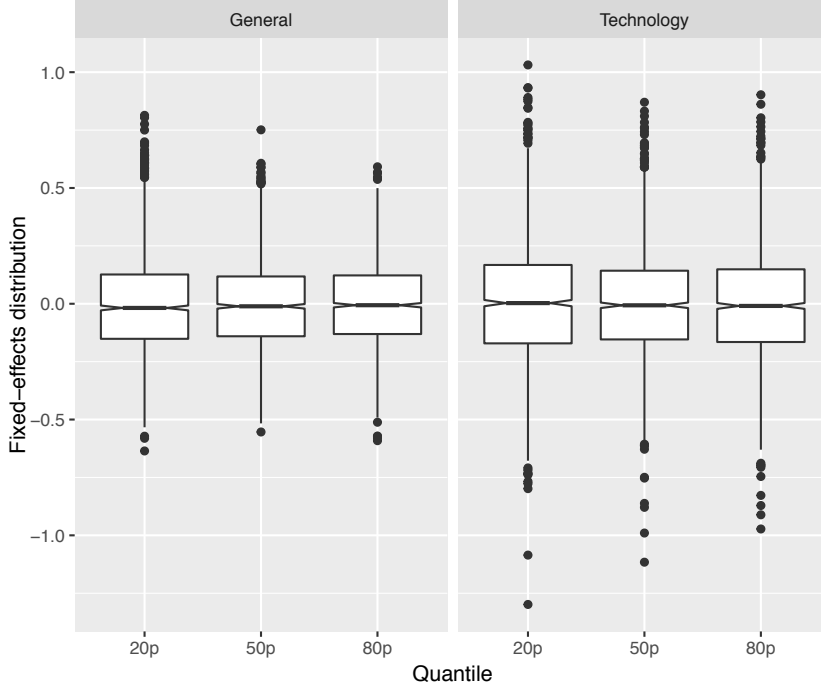


Figure 4: Characteristics of the distributions of the high school specific effects

appear to change significantly the results.<sup>18</sup>

### 3 Results

#### 3.1 Estimation of quantile high school effects and high school classification

In our main specification, we estimate three quantile regressions with high school fixed effects (first quintile, median and last quintile). The analysis is conducted separately for the general and technological tracks. For each regression (quantile), we derive a distribution of school-specific effects. The estimation requires an identification constraint, thus each distribution is normalized to zero-mean for interpretation (see Figure 4). We observe large dispersions amongst the school-specific estimates. This dispersion is higher in the technological track than in the general one, and in both tracks it is higher for the lower quintile than the higher.

For each high school, we compare the estimates obtained at the lowest and highest quintiles in order to evaluate whether it has a “heterogeneous” effect, meaning whether it tends to modify the distribution in student outcomes compared to model prediction based on student observable characteristics. As explained above, a high school  $j$  is defined as unequal (respectively egalitarian) if it verifies  $\alpha_j^*(L) < \alpha_j^*(H)$  (resp.  $\alpha_j^*(L) > \alpha_j^*(H)$ ). After correction for multiple

<sup>18</sup>For instance, this could happen if excluding some small high schools would bias the estimations of the students’ observable characteristics coefficients - and also those of fixed effects. However, when comparing the classification obtained when using smallest high schools, we do not observe large differences. Unequal high school are never classified as egalitarian (or reciprocally) when using less restrictive sample selection. The proportion of high school that may be identified as unequal rather than homogeneous (or inversely) and as egalitarian rather than homogeneous (or inversely) are always below 10%.

hypothesis testing, we find heterogeneous effects in 16.7 % (13.6%) of high schools in the general (technological) track, with unequal and egalitarian schools equally represented (see Table 2).<sup>19</sup>

Table 2: Presence of heterogeneous effects: tests results, accounting for multiple hypothesis tests (MHT)

	Proportion of high schools (%)		
	<u>Unequal</u>	<u>Egalitarian</u>	<u>Heterogenous</u>
<i>General track</i>			
No correction	13.6	15.2	28.8
MHT correction: SGoF	8.4	8.9	17.3
MHT correction: BBSGoF	8.2	8.5	16.7
<i>Technological track</i>			
No correction	12.5	14.5	27
MHT correction: SGoF	6.5	8.6	15.1
MHT correction: BBSGoF	6	7.6	13.6

*Note:* SGoF corrects for the excess proportion of false positive (under the full null hypothesis).

Based on the same principle, BBSGoF accounts for the dependence between tests.

Regarding the estimates corresponding to other covariates, they are in line with previous results in related literature (see Table 3). The three analyzed quantiles of the final grades mainly depend on the student past scores. The conditional distribution of final grades of students who had repeated a year during their past schooling is also strongly lower than the one of non-repeaters, the gap being wider at the bottom of the distribution. Girls usually have better and less dispersed final results than boys, especially in the technology track. Higher social position index is correlated with better final grade.

To gauge the extent of selection within schools based on observable characteristics, estimates without high school fixed effects are also shown. Most of the estimates vary between the two specifications (even if it does not correspond to a formal statistical test), especially the social position index and the prior score. This is consistent with a selective matching between the most favored students and the best schools. As suggested by Guarino et al. [2014], neglecting these correlations would result in biased estimates.

Figure 5 illustrates the cases of two high schools with similar median effect but classified in distinct categories. The figure represents (a) the estimated coefficients for the three quantiles and (b) the empirical distributions of the final grade  $Y_{ij}$  as a function of the prior grade  $y_{ij}$  as observed in two high schools  $j_1$  and  $j_2$  and compares it with the expected conditional quantiles in these high schools for  $\tau \in \{0.2, 0.5, 0.8\}$  (from the estimates of equation (3)). The dotted lines represent the prediction without the high school effect while the solid lines take them into account. The shape of the conditional quantiles is driven by the quadratic dependency in prior grades. For the high school where the triplet  $\alpha_j^*(\tau)$ ,  $\tau \in \{0.2, 0.5, 0.8\}$  is in descending order (left panel) - and thus classified as egalitarian, we expect a smaller dispersion of final grades than the one that would have been expected from the observable characteristics of these pupils. On the contrary, in the high school where the triplet is in ascending sequence (right panel in Figure 5)

<sup>19</sup>When we do not correct for multiple testing, these proportions are 28.8% and 27% in respectively general and technology tracks. Under the absence of heterogeneity hypothesis ( $\alpha_j^*(L) = \alpha_j^*(H)$  for all  $j$ ), without correction for multiple testing, 10% of heterogeneous effects are expected.

Table 3: Quantile regression estimates - main covariates

Quantile	With fixed effects						Without fixed effects					
	20%		50%		80%		20%		50%		80%	
General track												
Intercept	-0.705	(0.059)	-0.097	(0.082)	0.683	(0.143)	-0.586	(0.004)	-0.004	(0.004)	0.607	(0.004)
Prior score	0.593	(0.002)	0.632	(0.002)	0.646	(0.002)	0.622	(0.002)	0.655	(0.002)	0.661	(0.002)
Prior score squared	0.107	(0.001)	0.105	(0.001)	0.082	(0.001)	0.108	(0.001)	0.105	(0.001)	0.079	(0.001)
<b>Social position</b>	0.079	(0.002)	0.079	(0.002)	0.079	(0.002)	0.108	(0.002)	0.106	(0.002)	0.102	(0.002)
Repeaters	-0.271	(0.008)	-0.245	(0.007)	-0.193	(0.008)	-0.284	(0.008)	-0.253	(0.008)	-0.201	(0.008)
Girls	0.080	(0.004)	0.052	(0.003)	0.032	(0.004)	0.080	(0.004)	0.051	(0.003)	0.035	(0.004)
L (réf ES)	0.074	(0.005)	0.086	(0.005)	0.088	(0.006)	0.065	(0.005)	0.077	(0.005)	0.070	(0.005)
S	-0.194	(0.004)	-0.172	(0.004)	-0.147	(0.004)	-0.214	(0.004)	-0.184	(0.004)	-0.162	(0.004)
Technology track												
Intercept	-0.518	(0.171)	-0.139	(0.089)	0.329	(0.138)	-1.004	(0.022)	-0.423	(0.019)	0.176	(0.024)
Prior score	0.358	(0.004)	0.392	(0.003)	0.408	(0.004)	0.388	(0.003)	0.404	(0.003)	0.409	(0.003)
Prior score squared	0.018	(0.003)	0.025	(0.002)	0.034	(0.002)	0.009	(0.002)	0.021	(0.002)	0.029	(0.002)
<b>Social position</b>	0.034	(0.004)	0.027	(0.003)	0.027	(0.003)	0.060	(0.003)	0.049	(0.003)	0.043	(0.003)
Repeaters	-0.285	(0.009)	-0.258	(0.007)	-0.228	(0.009)	-0.299	(0.009)	-0.268	(0.007)	-0.236	(0.009)
Girls	0.242	(0.007)	0.211	(0.006)	0.189	(0.007)	0.235	(0.008)	0.206	(0.006)	0.189	(0.008)
ST2S (réf.HOT)	0.205	(0.057)	0.229	(0.039)	0.291	(0.056)	0.218	(0.023)	0.235	(0.020)	0.234	(0.025)
STD2A	0.362	(0.066)	0.385	(0.049)	0.511	(0.060)	0.339	(0.030)	0.368	(0.026)	0.451	(0.030)
STI2D	0.371	(0.059)	0.464	(0.039)	0.624	(0.054)	0.329	(0.023)	0.428	(0.020)	0.547	(0.025)
STL	0.500	(0.061)	0.604	(0.039)	0.717	(0.055)	0.450	(0.027)	0.579	(0.022)	0.637	(0.028)
STMG	0.360	(0.057)	0.397	(0.038)	0.456	(0.055)	0.354	(0.022)	0.407	(0.019)	0.430	(0.024)

Notes: Estimation of the  $\hat{\beta}(\tau)$  in the equation 3. The variable of interest (final high school grade), as well as continuous covariates are normalized and standardized. *Séries*: L: Humanities, ES: Economics and Social Sciences, S: Sciences. ST2S: Sciences and Technologies in Health and Social, HOT: Hotel and restaurants management, STD2A: Sciences and Technologies in Design and Applied Arts, STI2D: Industrial Science and Technologies and sustainable development, STL: Laboratory Science and Technologies, STMG: Management Sciences and Technologies. Restriction to high school with headcount  $\geq 65$  (resp. 25) in the general (technology) track.

- and thus classifies as unequal- the empirical dispersion of the conditional distribution of final grade is higher than that would have been observed in general in other schools.<sup>20</sup>

These effects will not be apparent by considering only the average effect. For the high school in the left panel of Figure 5, the classification as egalitarian comes from the fact that it performs worse than average at the top of the final grade distribution: there is a deficit of top performances.

The fact that a high school reduces the inequalities in academic level among its students may be an ambiguous indicator, though. For instance, in the previous example (Figure 5) for both the unequal and the egalitarian high schools, the median effect is not significantly different from zero. Egalitarian results could be obtained by lowering the academic standards within the schools (race-to-the-bottom phenomena). We thus investigate the correlation between equality, as defined by the comparison between the first and last quintiles and a measure of performance. For the latter, we use the estimate of the high school specific effect at the median. A “performing” high school is one where the median estimate is significantly positive (and on the contrary a non-performing is a high school whose median value-added is negative). When correlating these two classifications, we observe a - not so expected - positive correlation between belonging to the performing class and to the egalitarian one. The egalitarian schools are less often under-performing and have more often than other categories positive value-added (see Table 4). If most positive value-added schools do not display heterogeneous effects, they are more often egalitarian than unequal (29 % vs 8% in general track) while the opposite holds for negative value-added schools. The Figure 6 suggests indeed a slight negative relation between the median efficiency of the high school (as measured by  $\alpha_j^*(M)$ ) and the dispersion of the results (as measured by  $\alpha_j^*(H) - \alpha_j^*(L)$ ). The slope of this relationship is steeper in the general track than in the technological track, although the dispersion appears very high.

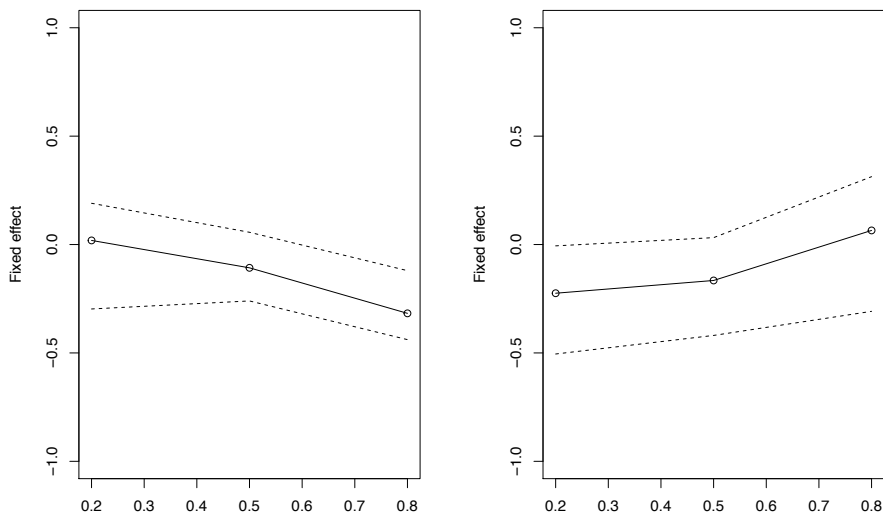
As detailed in the Section 2, we also use a more complex specification which interacts the high school effects with the position of the students in the distribution of either prior grades or social position index (as defined by the Equation 5). We restrict the sample to the general track in order to have enough observations in each high schools. The main parameters of the obtained empirical distribution of estimated fixed effects are of the same extent whatever the quartile (see Figure 15 in the Appendix). We obtain rather similar proportion of unequal or egalitarian high schools when considering separately every types of students (see Table 5). We observe that around one-third of the high schools are classified as heterogeneous for at least one type of students. However, when a high school is classified as heterogeneous for one type of students, it is rarely heterogeneous on another. We also observe that the classification independent of student type is loosely consistent with the classification obtained by type of students, in the sense that in half of the cases, the global classification matches what is found on a sub-population, and in the other half, a high school classified as heterogeneous for one quartile is classified as homogeneous when considering the whole sample (see Table 10 in the Appendix).

### 3.2 High school characteristics and high school value-added

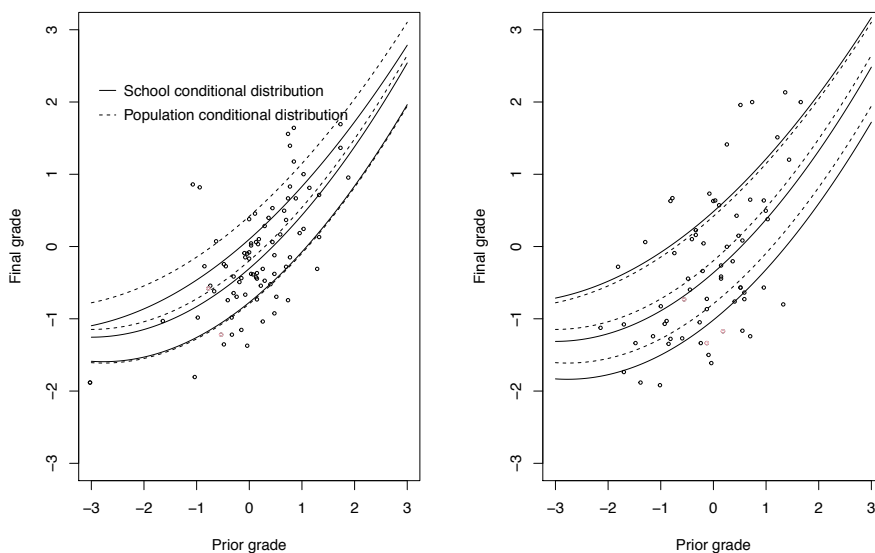
In order to better characterize “egalitarian” or “unequal” high schools, we then correlate the high school effects with observable school characteristics: the private or public status, high school

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<sup>20</sup>Moreover, the high school have usually asymmetric impact on the distribution of performance - the bottom and the top tails of the distribution are not equally affected by the high school effect. As illustrated by the Figure 14 in the Appendix, the difference  $\alpha_j^*(M) - \alpha_j^*(L)$  is not consistently closed to  $\alpha_j^*(H) - \alpha_j^*(M)$ . This may be due to either the skewness of the distribution of individual heterogeneities or an heterogeneous impact at the top and the bottom of the distribution. This is an additional motivation for using quantile regressions for the estimation - as they do not rely on a symmetrical effect of the high school.



(a) High-school specific effects (left: egalitarian/right: unequal)



(b) Predicted distributions of final grades conditional on prior grade (quantile 20%,50%, 80%) before and after adding specific high school effect

Figure 5: Empirical and predicted conditional distribution in two high schools, and specific high school effects

Table 4: Comparison of high schools in terms of dispersion in value-added vs median value-added (in %)

	Unequal	No het. effect	Egalitarian	
<b>General</b>				
Negative median value added	15.6	76.1	8.3	100
	<i>30.9</i>	<i>29.2</i>	<i>15.6</i>	
Non significant median value-added	15.5	73.1	11.3	100
	<i>55.1</i>	<i>50.5</i>	<i>38.3</i>	
Positive median value added	8.4	62.6	29.1	100
	<i>14</i>	<i>20.3</i>	<i>46.1</i>	
	<i>100</i>	<i>100</i>	<i>100</i>	
<b>Technological</b>				
Negative median value added	16.4	72.5	11.1	100
	25.8	18.9	16.2	
Non significant median value added	11.8	75.8	12.4	100
	60.5	64.5	59.3	
Positive median value added	9.8	71.4	18.8	100
	13.7	16.6	24.5	
	<i>100</i>	<i>100</i>	<i>100</i>	

Source: FAERE and APAE database, author calculations.

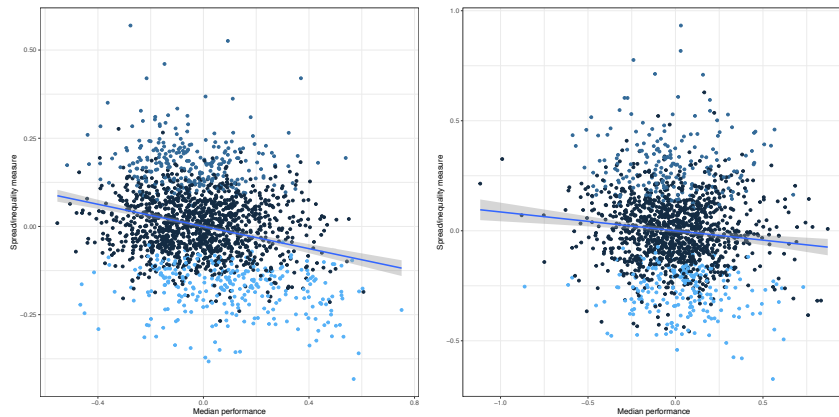


Figure 6: Dispersion in results  $\alpha_j^*(H) - \alpha_j^*(L)$  vs median high school effect  $\alpha_j^*(M)$  in the general and technological track

Table 5: Proportion of egalitarian and unequal high schools by quartile (general track)

	Overall	On at least		On quartile			
		a quartile	two quartile	Q1	Q2	Q3	Q4
Unequal	0.148	0.145	0.014	0.041	0.042	0.038	0.039
Egalitarian	0.141	0.191	0.023	0.055	0.052	0.052	0.058

Source: FAERE and APAE database, baccalaureat 2015, authors' calculation. Note: Restriction to high school with headcount  $\geq 10$  in each quartile. A high school  $j$  is defined as egalitarian (resp. unequal) when  $\alpha_{0.20}^j > \alpha_{0.80}^j$  (resp.  $\alpha_{0.20}^j < \alpha_{0.80}^j$ )

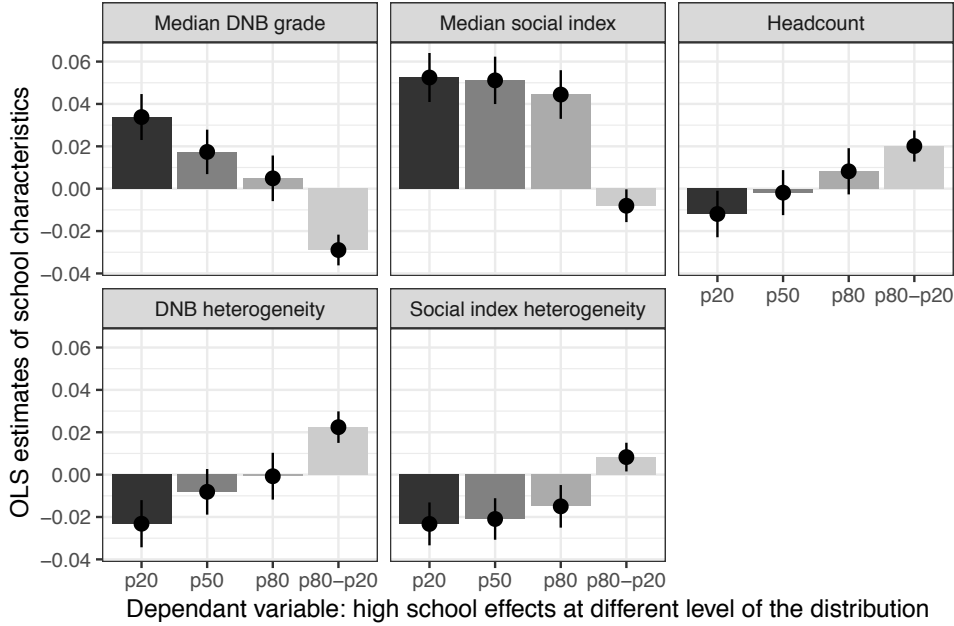


Figure 7: Correlation between estimated high school effects and high school observable characteristics (continuous variables, standardized) - General track.

size, the median initial academic level and the median social position index of its enrollment, academic and social disparities as measured by inter-quartile of these variables within school.

Specifically, we simply relate the estimated high school fixed effects at various quintiles with high school-level characteristics, using a simple linear regression at the school level :

$$\hat{\alpha}_j^*(\tau) = Z_j' \theta_\tau + \xi_j^\tau$$

Three regressions are run, for  $\tau \in \{L, M, H\}$ , as well as a regression with the dispersion inter-quintiles  $\hat{\alpha}_j^*(H) - \hat{\alpha}_j^*(L)$  as dependant variable (which could also be recovered directly from the two previous estimations). These correlations remain descriptive and cannot be interpreted in a causal way.

The estimated coefficients  $\theta_\tau$  are represented in Figures 7 for the school-level continuous variables, that are standardized in order to compare the coefficients. The fourth bar in Figure 7 plots the coefficient when the dependant variable is  $\hat{\alpha}_j^*(H) - \hat{\alpha}_j^*(L)$ , and therefore indicates whether the school characteristic under consideration is significantly related to dispersion (positive coefficient) or shrinkage (negative coefficient) of within-group achievements.

The high school effect at the bottom of the distribution has systematically a higher correlation with school characteristics than the high school effects at other positions in the distribution, and the  $R^2$  decreases as we try to explain the variability of the median and the highest quintile high school effects. This suggests that school characteristics carry more information about school effectiveness relative to the bottom of the distribution of achievements.<sup>21</sup> The high school-specific gains at the top of distribution are less grasped by observable school characteristics, and may be driven by other sources than school means and composition. As a result, many school-level characteristics are correlated with reduced or increased dispersion in achievements, by being correlated positively or negatively but differently with the lowest quintile and the highest quintile high school effects.

<sup>21</sup> Precisely, the bottom of the conditional distributions. The bottom of the distribution is not specific to initially low-achieving students but to any student at the bottom of similar-students achievements distribution.

One exception is the social composition, which is a favorable school characteristic at all levels of the distribution, and for all groups of students, to almost the same extent. Being enrolled in a high school with a more affluent enrollment is positively linked with the distribution of final grades at all considered quartiles. A potential reason under this positive correlation is that high schools with a more advantaged intake are usually located in the wealthiest neighborhoods and are more attractive for the most qualified or experienced teachers. The most educated parents may also be highly involved in the schooling of their offspring, are able to help on homework and to cooperate more efficiently with the teachers, or simply to obtain additional resources for the school or for the child's education. A favored composition is more often a characteristic of schools reducing dispersion, but the association is marginally significant. A favorable academic profile within the school is most strongly associated with the high school effect at the bottom of the distribution (Figure 7). This impact is decreasing along the distribution of outcomes - a favorable academic enrollment is uncorrelated with outcomes at the top. Schools with a favorable academic enrollment tend to be more equal, in the sense that the high school effect is strongest at the bottom of outcome distribution.

We next separate between school effects by prior achievement groups (Figure 16 in the appendix). For the initially struggling students (below the median of prior achievements), a favorable academic composition is associated with a high school performance higher than expected at the bottom of the outcome distribution, as it was the case for all students. On the contrary, for the initially best students, the high school effect at the top of the distribution of outcomes is negatively associated with a favorable academic enrollment. In terms of initial academic profiles, generally speaking the average previous performance of their peers has no impact on the top of the distribution, whatever the student type. However, the situation is more contrasted at the bottom quintile of the distribution. The bottom of the distribution of the grade distribution is higher for initially low performers in high schools with a favorable academic background, while on the contrary it is higher for high achievers.

We observe a similar duality for academic *heterogeneity* (based on prior grades): it is negatively correlated with high school outcomes at the bottom of the distribution, in particular for the initially lowest-achievers, and positively at the top of within-high school performances, specifically for initially higher achievers. Academic heterogeneity is therefore a predictor of unequal schools in the sense of our classification (a high-school effect higher at the top than at the bottom of outcomes). This echoes recent evidence on peer effects (Sacerdote, 2011). Empirical conclusions point out the overall negative impact of a high concentration of low achievers and positive influence of the concentration of high achievers. However, these conclusions may be tempered or even reversed by group of students (see for instance Hoxby and Weingarth [2005]). From the same data on French *baccalauréat*, Boutchenik and Maillard [2018] observe a strong heterogeneity in the impact of class composition on student final achievements. According to their results, homogeneous classes with a large proportion of high-ability peers benefit to low-ability students (ability being defined with DNB prior grades) while the high-ability students benefit from within-class academic heterogeneity and their final achievement may even be weakened when they are in presence of a large proportion of high-ability students.

To a lesser extent but consistently across groups of students, a smaller school size (headcount) is associated with lower inequalities within schools: high school effect at the lowest quintile is negatively correlated with high school size. The larger the high school, the more unequal are the final results in the high school. Potential effort granted to each student may decrease with size, as may unifying social interactions. This can be compared to a large literature on the link between school size and students' achievements. In a recent literature survey on this issue, [Scheerens et al., 2014] (see also [Leithwood and Jantzi, 2009]) conclude that if school size has probably no impact on cognitive outcomes, it may improve equity (as disadvantaged students are doing better in smaller schools).<sup>22</sup> Teachers and principals are much more likely to interact

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<sup>22</sup>For France, Afsa [2014] (in French) notices that, once controlled for the composition of schools,



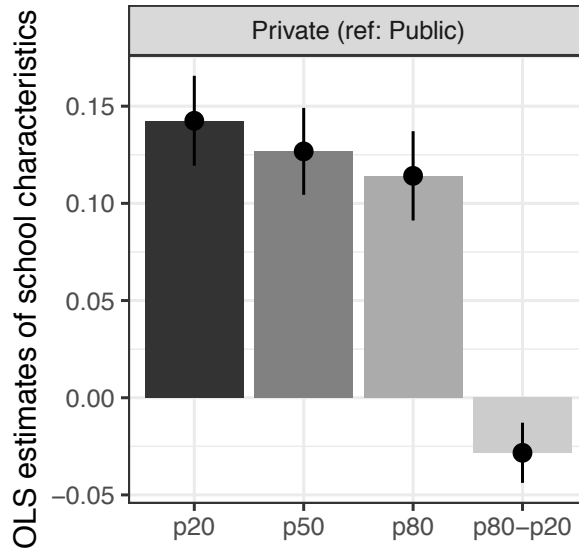


Figure 8: Correlation between estimated specific high school effects and public/private status - general track.

about and to be concerned with specific cases in small schools. As teacher effects have been found quite significant but varying significantly within-school (Rivkin et al. [2005], Thiemann [2017]), small schools limit the scope for this variation. In addition, social interaction between a limited number of students in a particular context may foster more homogeneous results (as postulated by the “endogenous” peer effects, reviewed for example in Yeung and Nguyen-Hoang [2016]). Still, it is worth emphasizing that these correlations should not be extrapolated far beyond what observational data can tell. If unequal schools are more often large schools, this does not mean that smaller schools are more equal *because* enrolling fewer students. For instance, small schools might be more subject to endogenous selection on unobservable social traits that translate in more homogeneous peers, even after controlling for the main determinant of academic achievements.

Finally, one striking results is the positive association between performance (at all levels of the distribution and all types of students) and private status of schools (when compared to public school, see Figure 8). This is at least partly due to a sorting effect, as discussed in the following subsection.

### 3.3 Selection of students and high school classification

Private high schools are more often classified as “egalitarian” than public (state-run) high schools - and more generally seem to perform better. This may be due to a more individualized pedagogy that reduces inequalities. However, this may also be due to sorting effects - indeed, while the enrollment in public high schools is strictly regulated and depends mostly on home address of students, private schools have a greater degree of freedom over student selection. While we control for the academic level of students just before high school, some characteristics of students may be observed during the recruitment process by the private high school but not measured in the data (for instance parent involvement). Moreover, students may be selected during the high school years. As the raw percentage of high school students passing the “baccalauréat” is highly publicized and scrutinized by parents, high schools may be tempted to push aside promising students who eventually obtained disappointing achievements on the first or second year of high school. Entering only the best students at the final examination would lead to a better publicity

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smaller medium schools perform better, especially for students from disadvantaged backgrounds.

for the school. If public high schools are in principle not allowed to choose their students as private high schools are, indirect selection may also occur. For instance, by directing students after the first year of high school to a stream that is not available in the school (as the headmaster of the school has the final decision regarding academic direction taken in secondary school).

Because of this rather common sorting effects in high schools, the French ministry of education publicizes not only the average value-added on success rate at the final examination, but also the “access rate” within high school. This value added in access rate measures the retention rate, compared to what would have been expected from the enrollment composition. The retention rate corresponds to the ratio of students that are still in the high schools from one year to the next (they are either enrolled in the next level, or have repeated the same grade *within* the same school). As retention rate may also reflect voluntary moves (because they are not satisfied with the school for instance, or simply because they are moving too far away), the indicator compares the observed retention rate to the one that would have been expected given the enrollment. We observe that the distributions of this indicator vary depending on whether we consider private or public high schools (see Figure 9). In private high schools, the actual access rate falls short of expectations much more often, while in the public sector we observe a symmetric distribution, centered around zero.

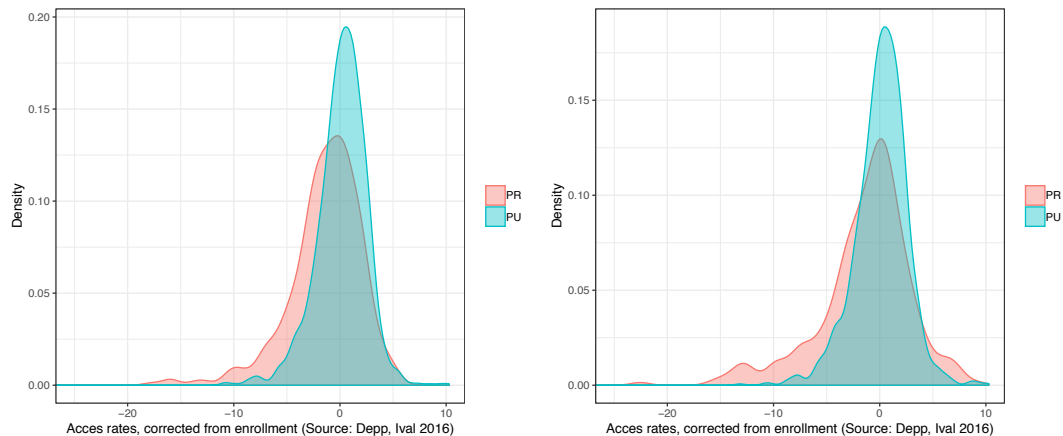


Figure 9: Access rate to last grade, from previous grade, in public and private schools (Left panel: in-sample-high schools with a General track, right panel: in-sample-high schools with a Technological track). Source: Depp, Ival 2016. Access rates are high school specific.

This selection process may affect the classification of high schools. For instance, if a high school chooses to get rid of disappointing students in order to avoid being held accountable of their failure at the final exam, the estimated high school effect at the lower quintile will be artificially high. In such a case of “cream skimming”, the high school may be classified as homogeneous or even egalitarian instead of unequal, or instead of homogeneous as egalitarian. We study in Appendix D how such a high-school-specific selection affects our estimates. For all scenarii considered, a higher selection leads to lower inequality measure.

To evaluate how selection of students affect our estimates in practice, we replicate the previous analysis but we reallocate the students: we estimate the high school fixed effect based on students enrolled in high schools the year before the final exam (even if they are not actually enrolled in the same high school the year of the exam). The estimated high school fixed effect are thus partially biased (as they possibly mix several high school effects) but expected to be less sensitive to the selection effects (even if a high school leaves behind the lowest achievers for the year of the exam they will be re-attributed to this high school in the estimation). We thus confront the classification obtained with the reallocation using the enrollment in 2014, with the baseline obtained using the enrollment in 2015. Both classifications appear highly correlated (see Table 6). Comfortingly, we never observe a high divergence in classification (from unequal

to egalitarian), suggesting that the endogenous selection described above is not of sufficient magnitude to radically change the classification. Between both classification we observe some movement between the extreme classes (unequal, egalitarian) and the homogeneous class.

To link these movements to the churning within the school, we consider the spreading measure in the baseline,  $\Delta\alpha = \hat{\alpha}_j^*(H) - \hat{\alpha}_j^*(L)$ , and in the estimation with students reallocation, denoted  $\Delta\alpha_{N-1}$ . We consider here the continuous variable, while the classification relies only on its sign. The higher this variable, the more unequal the high school is. We test whether reallocation of students between year N and N-1 (forced or chosen) allows a change in this indicator of within-high school dispersion. Table 7 shows how the difference between the dispersion indicator  $\Delta\alpha$  and the indicator which would have been derived based on  $N - 1$  population,  $\Delta\alpha_{N-1}$ , is related to access rates. We indeed observe that in the general track, the reallocation that generates the difference between  $\Delta\alpha_{N-1}$  and  $\Delta\alpha_N$  is beneficial to high schools with low access rate, as their dispersion measure is lower after reallocation. The results suggest that high schools with low access rate (selective or suffering from chosen departure) indeed decrease dispersion in outcomes by welcoming fewer or distinct students. On the contrary, the dispersion measure increases with reallocation for high schools with high access rate (that are able to keep their students). As this correlation is strongly borne by the private sector, it points out plausibly to controlled selection, in line with Appendix D predictions. This phenomenon is absent in the technological track. However, the extent of this selection process only marginally affects the classification.

All in all, this suggests that even though we find evidence of private-sector cream-skimming effects in the general track, the impact on the inequality measure is most likely weak.

Table 6: Changes in high school classification when taking into account the mobility of students

General track				
Students 2015/Students 2014	U	H	E	
U	227	33	0	
H	24	1146	36	
E	0	40	230	
Technological track				
Students 2015/Students 2014	U	H	E	
U	149	37	0	
H	34	1057	38	
E	0	39	166	

Sources: FAERE and APAE database (baccalaurat 2014 and 2015), authors' calculation. Note: U stands for unequal, H for homogenous and E for egalitarian.

Table 7: Inequality measures depending on access rate within the high school and sector.

	General				Technological			
	$\Delta\alpha$		$\Delta\alpha - \Delta\alpha_{N-1}$		$\Delta\alpha$		$\Delta\alpha - \Delta\alpha_{N-1}$	
$\Delta\alpha_{N-1}$	0.949*** (0.007)	0.948*** (0.007)			0.912*** (0.011)	0.913*** (0.011)		
Access rate to final grade	0.001*** (0.0002)		0.001*** (0.0002)		-0.0005 (0.0004)		-0.001 (0.0005)	
Access rate to final grade <i>Private</i>		0.002*** (0.0005)		0.002*** (0.0005)		-0.001 (0.001)		-0.001 (0.001)
Access rate to final grade <i>Public</i>		0.001* (0.0003)		0.0005* (0.0003)		-0.0003 (0.001)		-0.0005 (0.001)
Constant	0.0005 (0.001)	0.001 (0.001)	0.0005 (0.001)	0.001 (0.001)	0.001 (0.002)	0.001 (0.002)	0.001 (0.002)	0.001 (0.002)
Observations	1,734	1,734	1,734	1,734	1,500	1,500	1,500	1,500
R <sup>2</sup>	0.918	0.919	0.006	0.009	0.834	0.834	0.002	0.002
Adjusted R <sup>2</sup>	0.918	0.919	0.005	0.007	0.834	0.834	0.001	0.001

Note:

Source: FAERE and APAE database, authors' calculation.

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

## 4 Conclusion

In this paper we propose new indicators that complement the existing measures of high-school performance. We focus on the way high schools reduce or increase the inequalities in academic performance of their students. According to our results, a non negligible proportion of French high schools have significant within-school distributional effects, either toward more homogeneity in final achievements (egalitarian schools) or toward spreading achievements of students (unequal schools). We do not find evidence that a more equal effect is obtained through a “race-to-the-bottom” phenomena whereby high-achieving students would be harmed, as the fact of being classified as egalitarian (respectively unequal) is also positively (respectively negatively) correlated with higher performance at the median. In other words, some high schools appear to enable progress of all of their students, and excellence does not come at the price of some students being “left behind”.

The methodology proposed here can be viewed as a synthesis between two popular models, SGP and mean value-added. It provides more detailed information on the value-added of the high schools, detailing the dispersion within school. We address a potential concern about selection effects that arises with SGP model by controlling for various observable covariates (such as academic level before enrollment but also socio-economic background). The estimation of the high-school effects does not require independence of these effects and those covariates.

These indicators should be used with caution, though. As pointed out by Raudenbush and Willms [1995], since students are not randomly distributed among schools it is not possible to isolate the impact attributable to educational practices provided by the school from those due to the social and schooling composition of its enrollment. Specifically, even when controlling for observable individual characteristics, the causal impact of the high school cannot be distinguished from some unobservable characteristics of its enrollment (for instance parental involvement in the student schooling) and second from the general composition of the enrollment that may impact student achievements through peer effects. The estimated high-school effects measured here thus corresponds to the type “B” school effect in the typology proposed by Raudenbush and Willms

[1995]: it measures how a pupil actual performance differs from the one that would have been expected if he or she had attended another school.<sup>23</sup> The type of indicators proposed here are thus still useful for families confronted to a choice between several high schools and in need of indicators more informative than the “raw” characteristics (as provided by the rate of success at the final exam for instance).

In the same vein, the observed correlations between homogeneous performance and school characteristics correspond only to descriptive evidence and do not inform on potential causal relationship between the high school specific features and its performance regarding inequalities in student performances. Providing causal empirical evidence supporting these correlations would require specific and detailed analysis which are much beyond the scope of the paper. The easily observable characteristics can however be used by families as indicators that one or another high school would tend to reduce inequalities among its student performances.

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<sup>23</sup>The type “A” being the causal impact of the high school once controlling entirely from enrollment effect and thus relates to the required information for public authorities that would evaluate the high-school performance.

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# Appendix

## A Additional figures

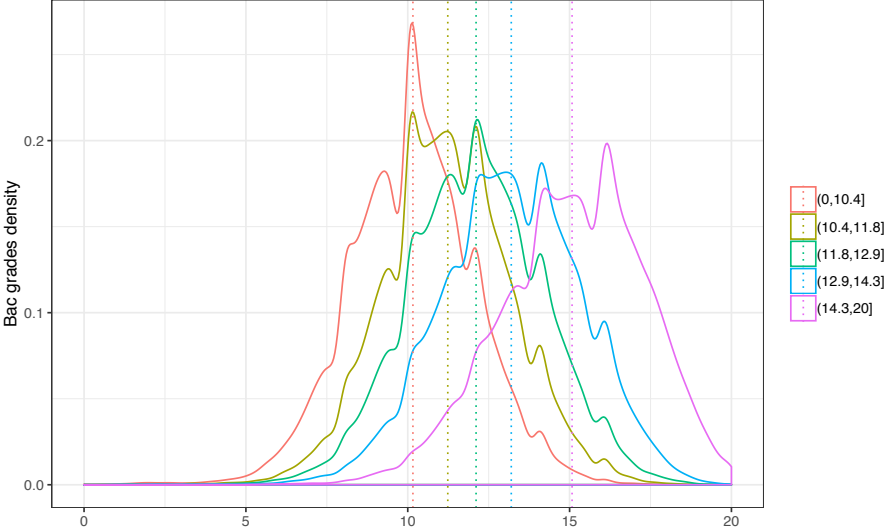


Figure 10: Bac grades density by prior academic level (5 groups of prior grades at DNB)

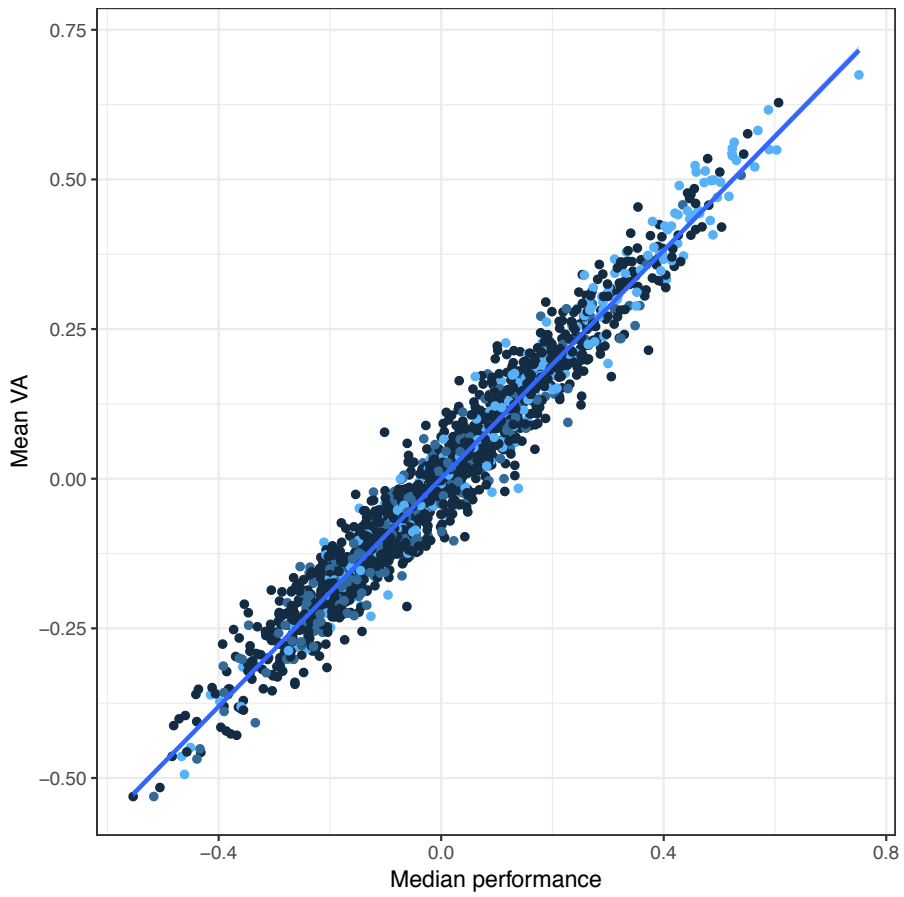


Figure 11: Mean Value-added estimates and Median value-added estimates

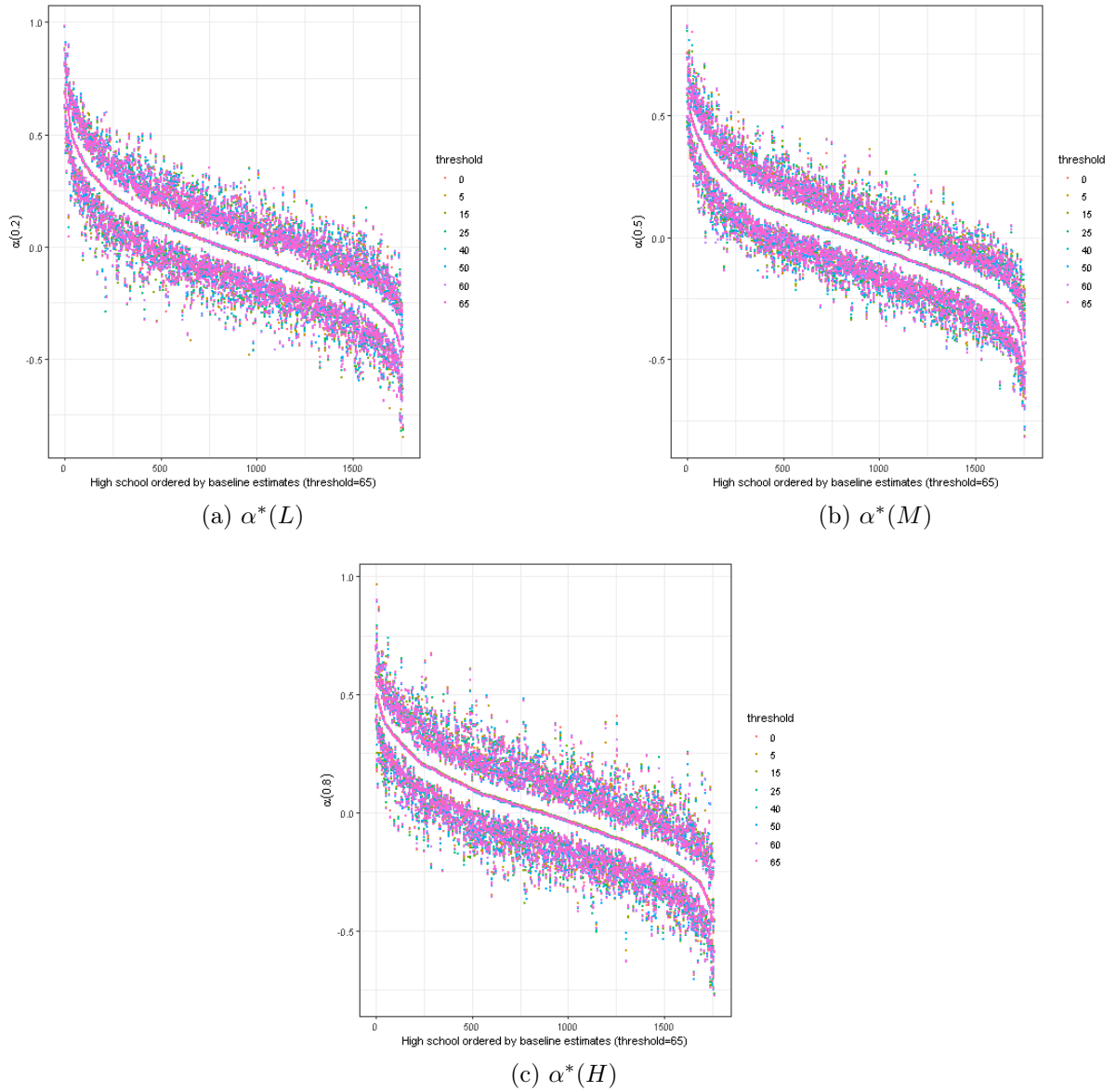


Figure 12: Estimates of high-school fixed effects (lowest quintile, median, highest quintile) and their confidence intervals when adding successively smaller schools (decreasing thresholds on school size for sample inclusion) , General track

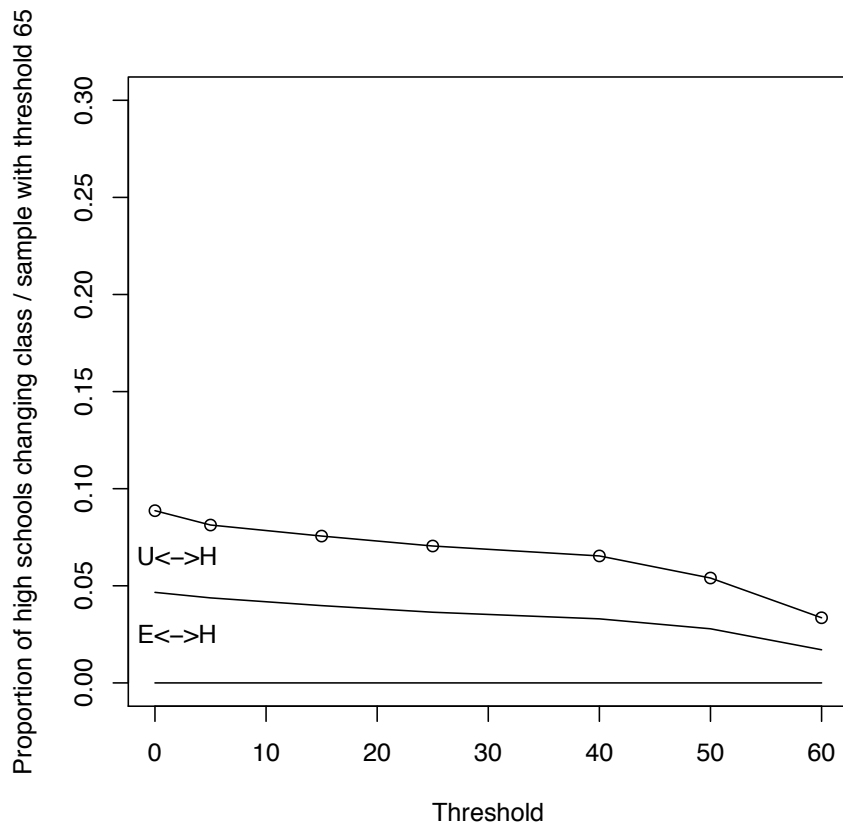


Figure 13: Change in classification when adding succesively smaller schools (decreasing thresholds on school size for sample inclusion), General track. Only changes with the adjacent category are observed (between egalitarian and homogenous schools,  $E \leftrightarrow H$  and between unequal and homogeneous schools  $U \leftrightarrow H$ ). When all schools are considered, the classification change for less than 10% of high schools compared to estimations conducted on schools with more than 65 students. About half of the schools changing class change between  $U \leftrightarrow H$ , the other half between  $E \leftrightarrow H$ .

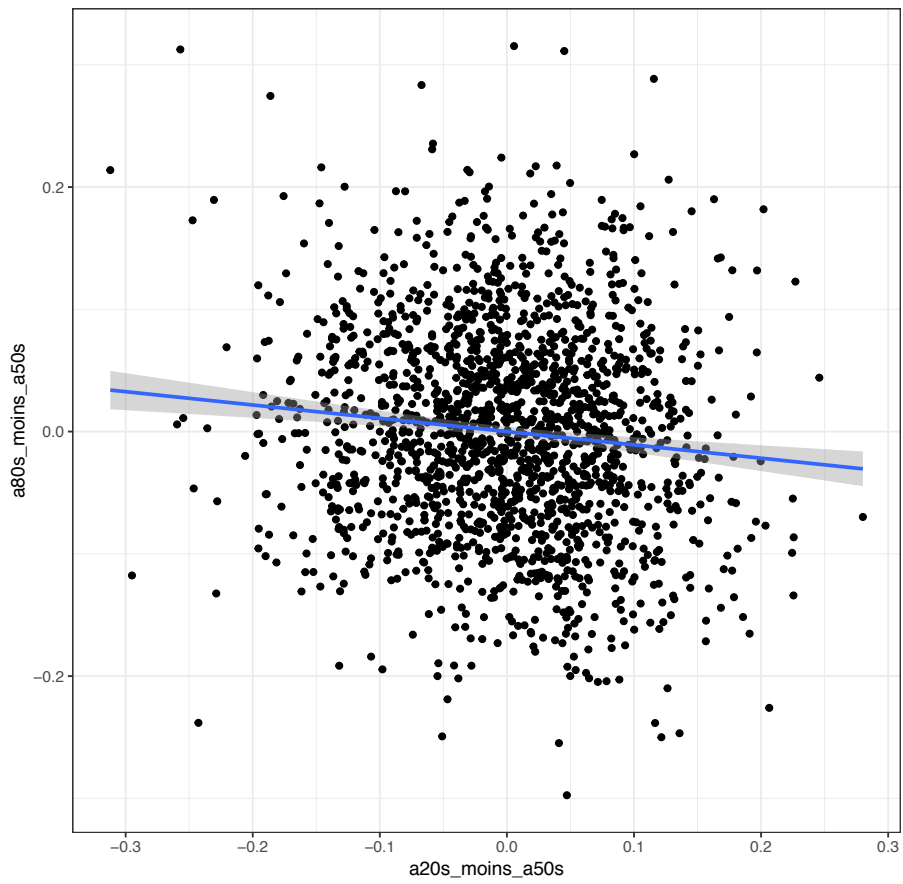


Figure 14: Asymetry of high-school effects at bottom and top tails:  $\alpha_j^*(M) - \alpha_j^*(L)$  and  $\alpha_j^*(H) - \alpha_j^*(M)$

## B Sample restriction

Table 8: Students, high schools and sample restriction

	High schools	Students	Mean	Sd	Students per school Median
Candidates at the baccalauréat 2015					
General tracks	2248	340469	151.5	95.9	140
technological tracks	1895	130887	69.1	49.3	58
All	2521	471356	187.0	125.7	169
Sample (A) restricted to headcounts $\geq 65$ (25) in general (technology)					
General tracks	1759	318222	180.9	82.3	166
technological tracks	1549	122286	78.9	46.3	67
All	2113	440508	208.5	118.2	195
Sample (B): (A) with more than 10 students within each quartile of prior grade					
General tracks	1534	290617	189.5	82.0	178
			Students per school per quartile		
			47.4	26.3	42
Sample (C): (A) + with more than 10 students within each quartile of social position index					
	1518	291400	226.3	86.4	218
			Students per school per quartile		
			48	29.1	42

## C Per quantile high school fixed effects

Table 9: Quantile fixed effect per quartile of prior grade, in terms of standard deviation of the baccalaureat grade

Per quartile of prior grade												
Quantile	Q1			Q2			Q3			Q4		
	20%	50%	80%	20%	50%	80%	20%	50%	80%	20%	50%	80%
Min.	-0.901	-0.745	-0.790	-0.827	-0.735	-0.786	-0.728	-0.787	-0.758	-1.310	-1.014	-0.768
1st Qu.	-0.177	-0.153	-0.154	-0.158	-0.154	-0.160	-0.172	-0.165	-0.173	-0.173	-0.147	-0.152
Median	-0.020	-0.017	-0.021	-0.010	-0.015	-0.016	-0.008	-0.013	-0.003	-0.006	0	0.002
3rd Qu.	0.154	0.130	0.135	0.141	0.138	0.150	0.165	0.166	0.168	0.175	0.161	0.163
Max.	0.994	0.912	0.946	0.966	0.962	1.001	0.797	0.781	0.931	1.088	0.844	0.840
Mean	0	0	0	0	0	0	0	0	0	0	0	0
Per quartile of social position index												
Min.	-1.045	-0.711	-0.769	-0.744	-0.663	-0.916	-1.219	-1.020	-0.918	-1.119	-0.902	-1.046
1st Qu.	-0.170	-0.151	-0.152	-0.159	-0.149	-0.148	-0.166	-0.150	-0.157	-0.153	-0.144	-0.138
Median	-0.018	-0.011	-0.016	-0.017	-0.015	-0.013	0.009	-0.002	-0.007	0.007	-0.002	0.009
3rd Qu.	0.148	0.136	0.152	0.139	0.138	0.138	0.170	0.154	0.162	0.160	0.149	0.151
Max.	1.150	0.907	0.901	0.849	0.925	0.886	0.871	0.658	0.736	0.965	0.770	0.826
Mean	0	0	0	0	0	0	0	0	0	0	0	0

Source: FAERE and APAE database, author calculations.

Table 10: Proportion of egalitarian and unequal high-schools by quartile (general track)

Heterogeneous on at least a quartile	0.326
Among which,	
<i>no overall heterogeneous effect</i>	0.470
<i>consistent with overall effect</i>	0.530
	1

Source: FAERE and APAE database, baccalaureat 2015, authors' calculation. Note: Restriction to high-school with headcount  $\geq 10$  in each quartiles. A high-school  $j$  is defined as egalitarian (resp. unequal) when  $\alpha_{0.20}^j > \alpha_{0.80}^j$  (resp.  $\alpha_{0.20}^j < \alpha_{0.80}^j$ )

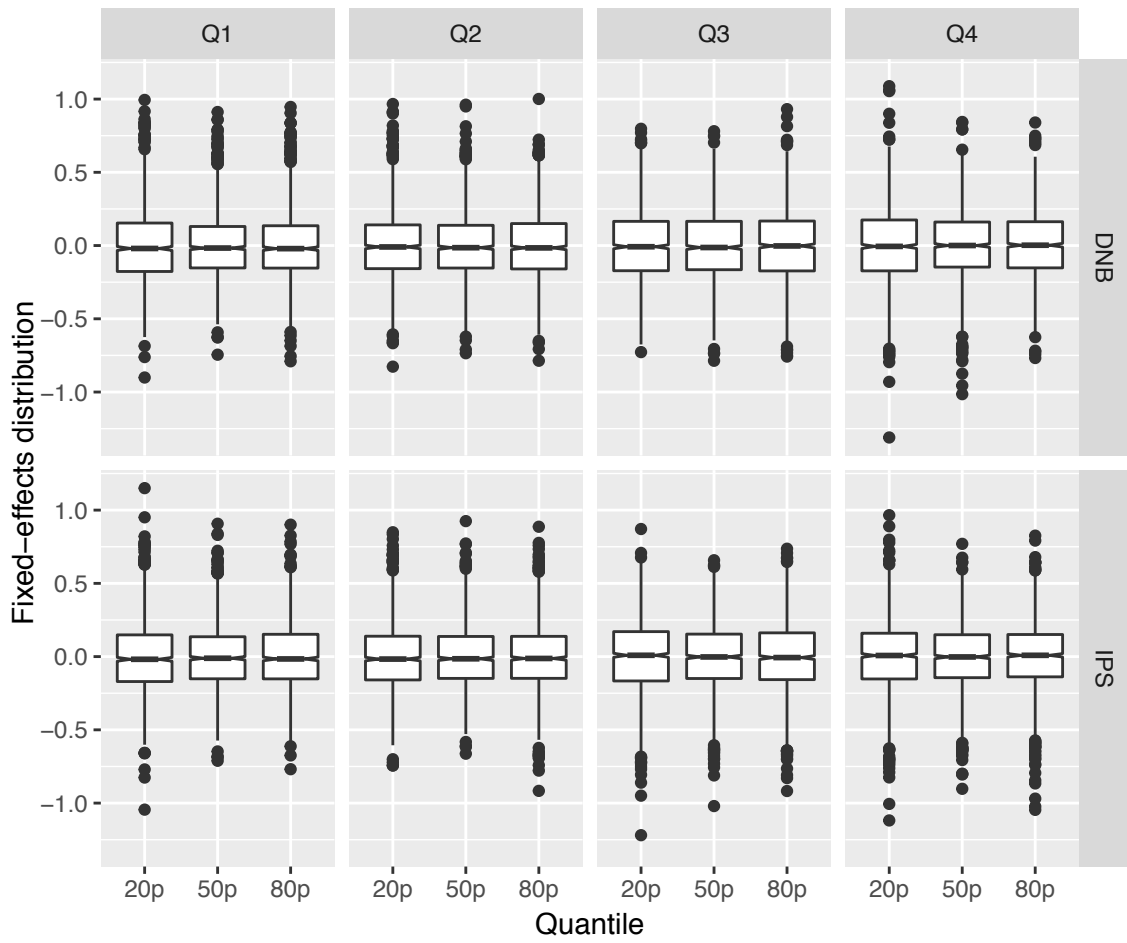


Figure 15: Distribution of quantile fixed effect per quartile of prior grade, scaled in terms of standard deviation of the baccalaureat grade



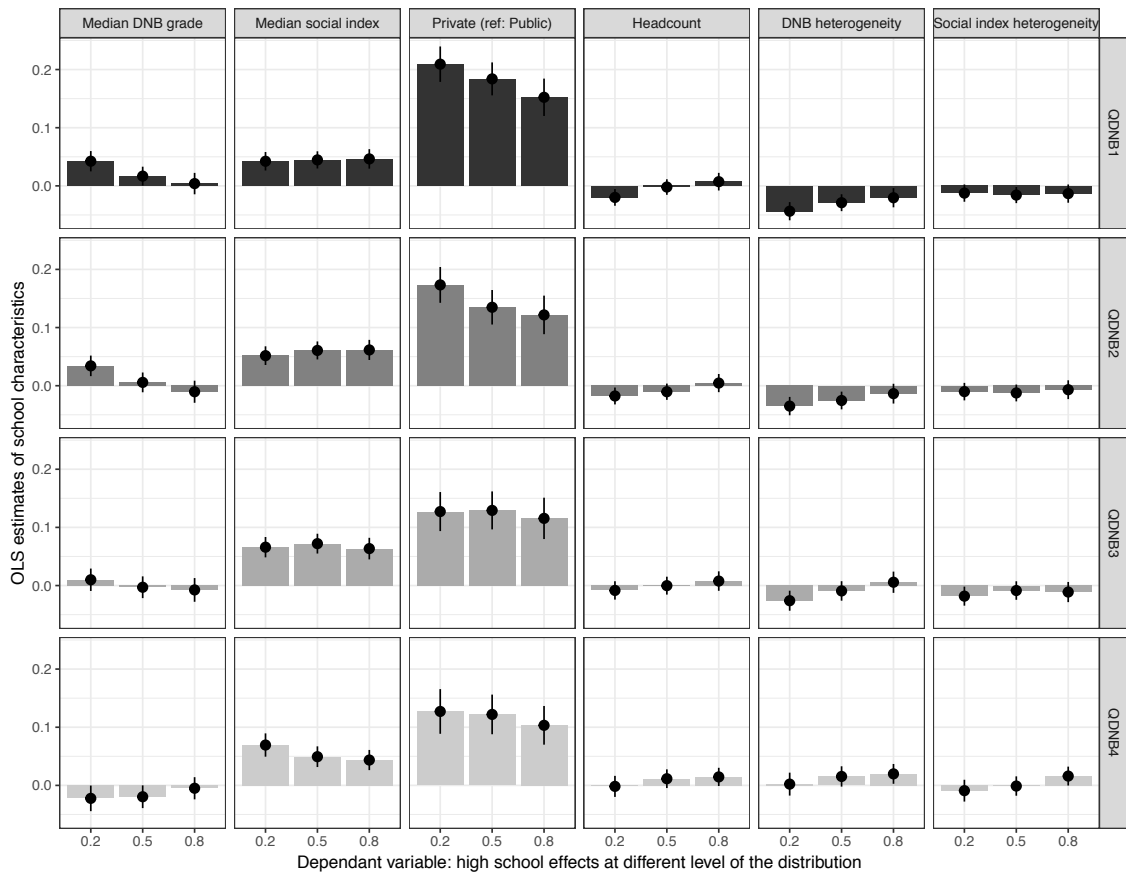


Figure 16: Descriptive regressions at high-school level, by quartile of prior grade

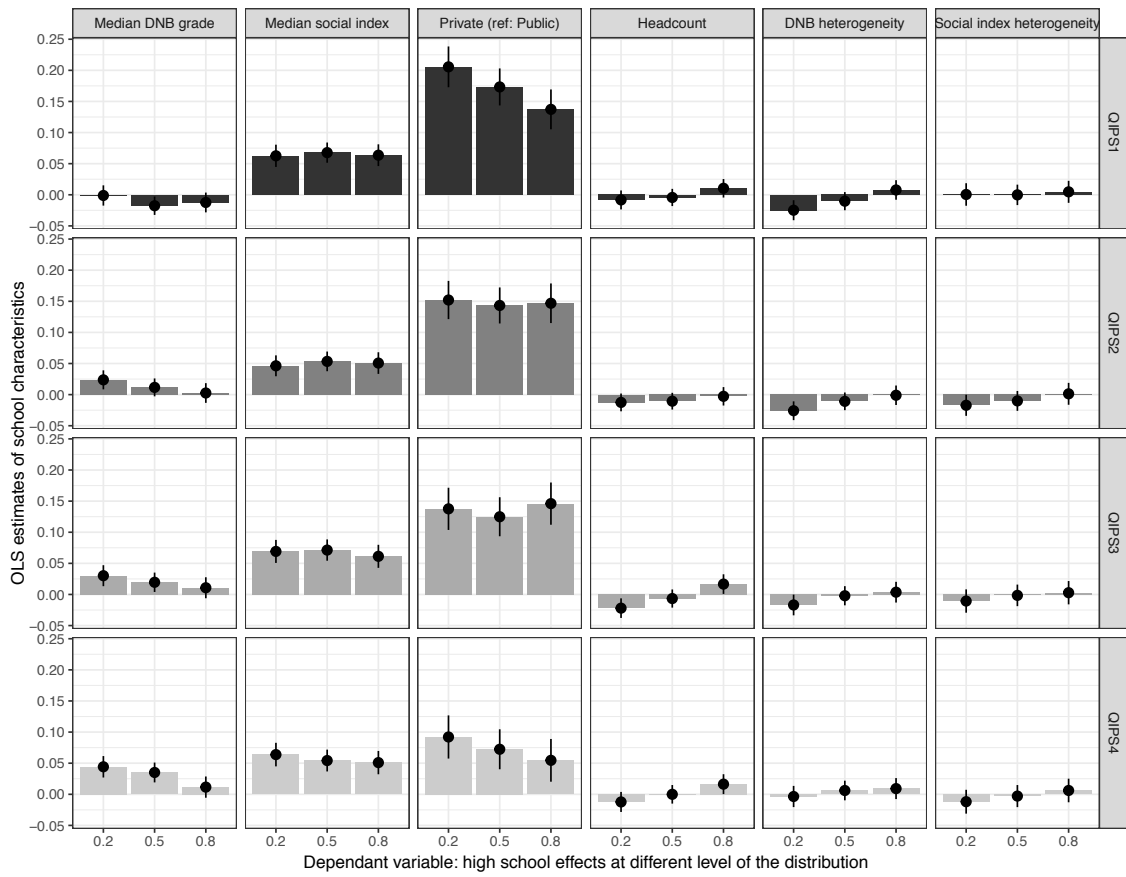


Figure 17: Descriptive regressions at high-school level, by quartile of social position index

## D Consequences of selection during schooling

In this appendix, we explore the consequences of a form of selection which could bias our estimates: school-student pairs could separate based on the expected results of the student within the school.

Assume that the potential outcomes for student  $i$  in school  $j$  is

$$Y_i^* = \gamma_0 + \gamma_j + X_i' \gamma_X + (\lambda_0 + \lambda_j + X_i' \lambda_X) \epsilon_i$$

with the same notations that in section 2.2. The allocation of students in high schools is assumed independent of  $\epsilon_i$ , while they may be functions of  $X_i, \gamma_i, \lambda_i$ . Identification requires  $\sum_j \lambda_j = 0$ ,  $\sum_j \gamma_j = 0$ .

Selection takes the following form: we only observe  $Y_i = Y_i^*$  when  $Y_i^* \geq \eta_j$ . Selection is therefore school specific. If  $\epsilon_i$  has a cumulative distribution function  $F$  which is invertible, the quantile of the within-school distribution of outcomes may be written:

$$q_\tau(Y_i | X_i, \gamma_j, \lambda_j, \eta_j, Y_i^* \geq \eta_j) = \gamma_0 + \gamma_j + X_i' \gamma_X + (\lambda_0 + \lambda_j + X_i' \lambda_X) \times F^{-1}(p_j + (1 - p_j)\tau)$$

where  $p_j = p_j(X_i) = F\left(\frac{\eta_j - (\gamma_0 + \gamma_j + X_i' \gamma_X)}{\lambda_0 + \lambda_j + X_i' \lambda_X}\right)$ . The case without selection (which we consider for our main estimates) is given by  $\eta_j \rightarrow -\infty$ ,  $p_j \rightarrow 0$ .  $p_j$  increasing in  $\eta_j$  (by design), decreasing in high school/students quality  $\gamma_0 + \gamma_j + X_i' \gamma_X$ , decreasing in grades dispersion  $(\lambda_0 + \lambda_j + X_i' \lambda_X)$ .

In practice, a high school can not exclude many students so we may consider the approximation  $p_j \approx 0$ . In the case without covariates, our parameters of interest can be written

$$\alpha_{j,\tau}^* = \gamma_j + \lambda_j \times F^{-1}(p_j + (1 - p_j)\tau) - \frac{1}{J} \sum_k \lambda_k F^{-1}(p_k + (1 - p_k)\tau)$$

Therefore, our continuous measure for inequalities writes, for  $\tau > 0.5$

$$\alpha_{j,\tau}^* - \alpha_{j,1-\tau}^* = \lambda_j \times \underbrace{\{F^{-1}(p_j + (1 - p_j)\tau) - F^{-1}(p_j + (1 - \tau)(1 - p_j))\}}_{f_j} - \frac{1}{J} \sum_k f_k$$

In the absence of selection,

$$\alpha_{j,\tau}^* - \alpha_{j,1-\tau}^* = \lambda_j \times (F^{-1}(\tau) - F^{-1}(1 - \tau))$$

Part of the bias arising from selection applies to all schools ( $\sum_k f_k \neq 0$ ), and part of it is high-school specific. The multiplicative bias specific to each high school may be written:

$$B(p_j) = \frac{F^{-1}(p_j + \tau(1 - p_j)) - F^{-1}(p_j + (1 - \tau)(1 - p_j))}{F^{-1}(\tau) - F^{-1}(1 - \tau)}$$

Figure 18 shows for two choices of  $F$  than this function is decreasing in  $p_j$ . When  $p_j$  is close to zero, we can approximate at first order this bias for a more general  $F$  ( $f = F'$  symmetric to simplify the expression):

$$1 - p_j \times \frac{1}{f(F^{-1}(\tau))} \frac{2\tau - 1}{F^{-1}(\tau) - F^{-1}(1 - \tau)}$$

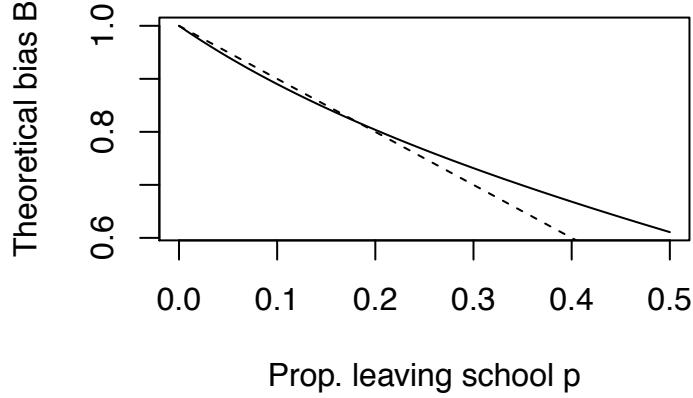


Figure 18: Bias under  $F$  normal or uniform, and  $\tau = 0.8$

Thus, the estimated dispersion (inequality measure) is expected to be downward biased in the high schools that select the most their students.

### A simulation set up

We verify this claim with simulations allowing for reallocation of students, and with various scenarii on  $\eta_j$ , the school selection threshold. The extent of overall selection is indexed by  $p$  which is the average of  $p_j$ .

- **Scenario 1:** All schools exclude their bottom  $p_j = p\%$  students
- **Scenario 2 and 2bis:**  $\eta_j$  is positively/negatively correlated with  $\gamma_j$  (elistism vs higher pressure on low-quality schools).  $p_j = p \times (1 + \gamma_j / \max |\gamma_j|)$  and  $p_j = p \times (1 - \gamma_j / \max |\gamma_j|)$
- **Scenario 3:** Two types of schools, public/private, with a proportion of private school  $p_P$ . Private schools are the only schools able to perform selection, and for them,  $p_j = p/p_P\%$ .  $p_j = 0$  in public schools.

Each scenario is considered with three distinct reallocation schemes:

- **Reallocation “none”:** students below threshold leave (do not present the exam)
- **Reallocation “random”:** students are reallocated to a random school. The later do not modify their outcome.
- **Reallocation “where accepted”:** students are reallocated to a random school among schools where they would have been above the threshold. The later do not modify their outcome.

### Parameters

There are  $J = 1000$  schools, drawing their headcount in  $[[50, 250]]$ .  $\gamma_0 = 10$ ,  $\lambda_0 = 2$ . Overall qualities  $\gamma_j$  are drawn in  $\text{Unif}[-2, 2]$ . Dispersion can be correlated with overall quality (parameter  $c$ ):  $\lambda_j = (1 - c^2) \times \text{Unif}[-1, 1] + c^2 \times \text{sign}(c) \gamma_j$ , and then constrained to be of sum zero. Finally,

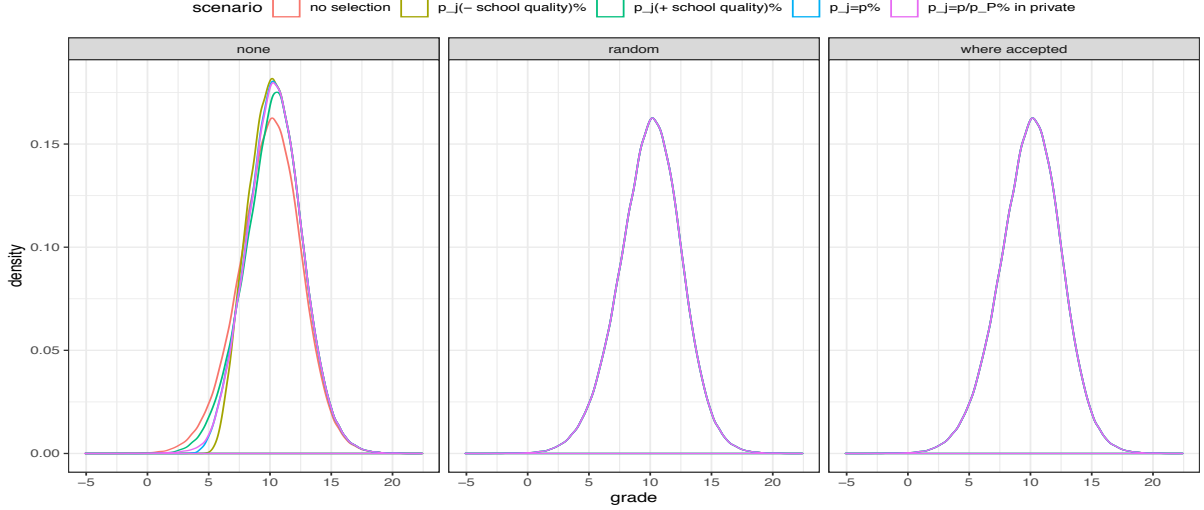


Figure 19: Overall grade densities by reallocation schemes

$X_i \sim \mathcal{N}(0, 1)$ ,  $\gamma_X = 1$ ,  $\lambda_X = 0.1$ ,  $\epsilon \sim \mathcal{N}(0, 1)$ ,  $p_P = 0.2$ . These parameters generate outcomes close to real data (Fig 19 for  $p = 0.1$  and  $c = 0$ ).

### Comparison between quantile estimates and true parameters

For each combination of  $(p, c)$ , we draw a sample of  $J$  schools and apply our estimation procedure: we estimate by quantile regression  $\{\hat{\gamma}_j\}, \{\hat{\lambda}_j\}$ , and deduce the category of the school with bootstrap resampling. We first confirm the previous derivation by plotting quartiles of  $\frac{\hat{\alpha}_{j,\tau}^* - \hat{\alpha}_{j,1-\tau}^*}{\lambda_j}$  as a function of  $p$ . Regardless of the scenario considered or the value for  $c$ , when  $p > 0$  there is a downward bias in the dispersion estimation (Figure 20), all the more pronounced than  $p$  is high. Reallocation worsen the downward bias all the more that  $c$  is negative (performance is already and before selection associated with small dispersion of grades).

In Figure 21, we assess the error due to ignoring selection when it happens by quantifying the proportion of misclassified schools (compared to the classification we would have derived absent selection). The overwhelming majority of errors are transitions with the “no heterogenous effect” category. Transitions toward either “Equal” or “Unequal” categories are much fewer than departures from these categories. Transitions between extreme categories (“Equal” to “Unequal” or conversely) are seldom. These transitions lead to an increase of the “no heterogenous effect” category overall, to the detriment of the unequal category first, and then the equal category (consistently across scenarii). Reallocations increase the error rate, in particular it seems to increase the predominance of transitions to the “no heterogenous effect” category. Across scenarii, error rates are rather similar.

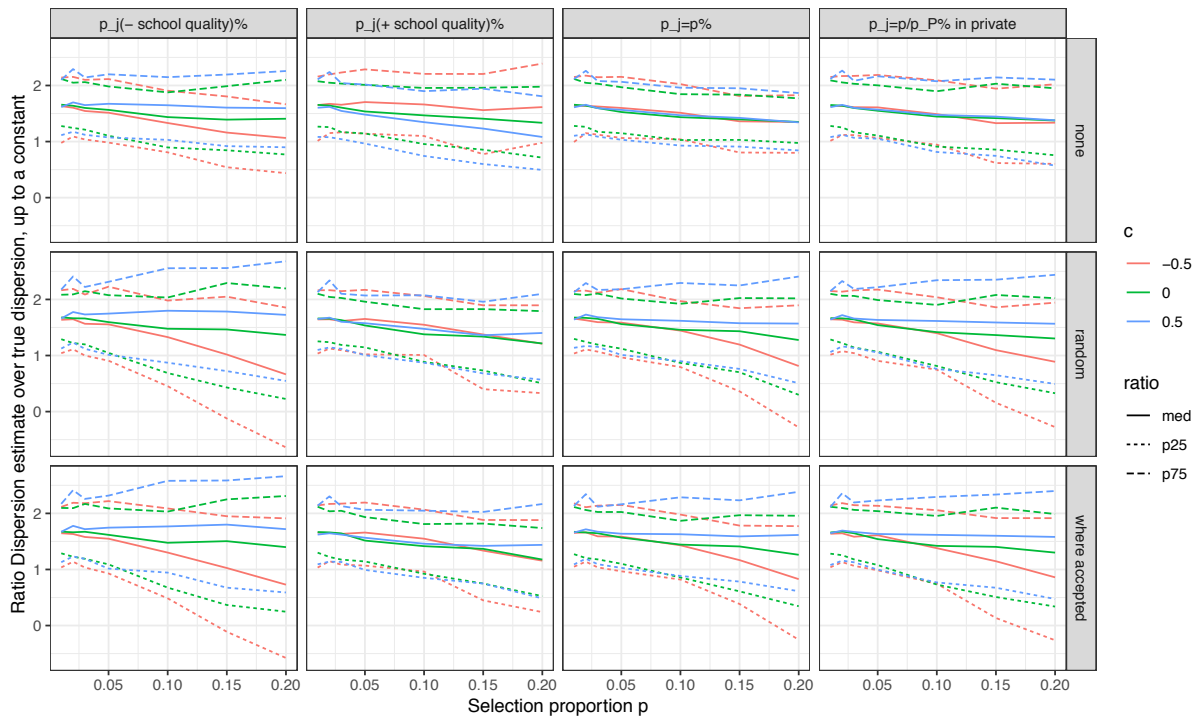


Figure 20: Quartiles of  $\frac{\hat{\alpha}_{j,\tau}^* - \hat{\alpha}_{j,1-\tau}^*}{\lambda_j}$  as a function of  $p$  and  $c$ . Column: scenario, Row: reallocation scheme.

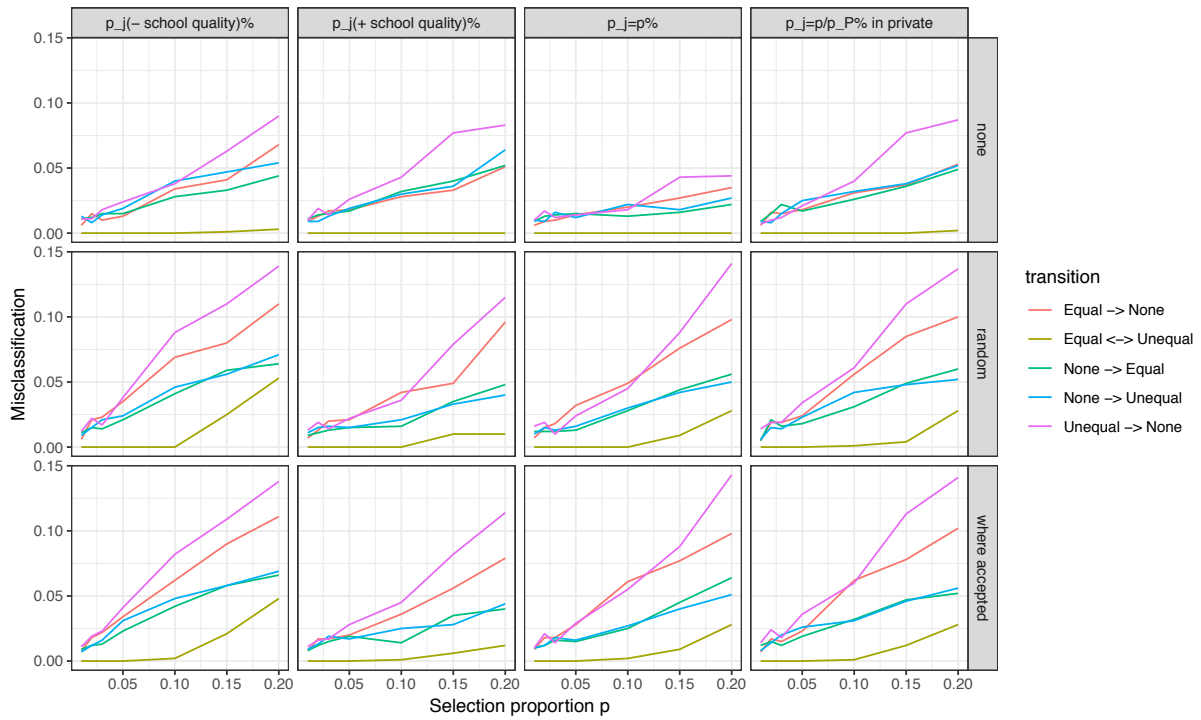


Figure 21: Misclassification of schools as a function of  $p$ , when  $c = 0$ . Column: scenario, Row: reallocation scheme. *None*  $\rightarrow$  *Equal*: schools which should be classified as having no significant relative dispersion which are classified as equal because of selection.



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