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## The Macrodynamics of Sorting between Workers and Firms<sup>†</sup>

By JEREMY LISE AND JEAN-MARC ROBIN\*

*We develop an equilibrium model of on-the-job search with ex ante heterogeneous workers and firms, aggregate uncertainty, and vacancy creation. The model produces rich dynamics in which the distributions of unemployed workers, vacancies, and worker-firm matches evolve stochastically over time. We prove that the surplus function, which fully characterizes the match value and the mobility decision of workers, does not depend on these distributions. This result means the model is tractable and can be estimated. We illustrate the quantitative implications of the model by fitting to US aggregate labor market data from 1951–2012. The model has rich implications for the cyclical dynamics of the distribution of skills of the unemployed, the distribution of types of vacancies posted, and sorting between heterogeneous workers and firms. (JEL E24, E32, J24, J63, J64)*

How does the distribution of skills among the unemployed vary with the business cycle? How does the quality of matches for workers transiting from unemployment vary with the aggregate state? Similarly, how is the reallocation of currently employed workers to more appropriate matches related to the business cycle? To answer these questions and understand the interactions requires a model of the labor market which incorporates both worker- and firm-level heterogeneity and aggregate uncertainty. Existing equilibrium search models of the labor market with heterogeneous workers and firms, such as Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002) are steady-state models. They rely heavily on stationarity for tractability, which means the equilibrium distributions can effectively be treated as parameters rather than state variables.

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In this paper, we develop a stochastic model of random search on the job, with *ex ante* heterogeneous workers and firms and aggregate productivity shocks in which firms make state-contingent offers and counteroffers to workers. The model extends the work of Postel-Vinay and Robin (2002), incorporating aggregate productivity shocks and nonstationary distributions of unemployed workers and worker-firm matches, and Robin (2011), introducing firm heterogeneity, vacancy creation, and a meeting function à la Pissarides (1985). We obtain tractability by working with the function that defines the joint surplus of a worker-firm match, rather than the individual value functions of the worker and firm separately. The model delivers rich dynamics in which the distributions of unemployed workers, posted vacancies, and worker-firm matches evolve stochastically over time. We prove that the surplus function, which defines the value of a match and fully characterizes the mobility decision of workers, does not depend either on the current wage or on the current distributions of unemployed workers, posted vacancies, or worker-firm matches. The implication is that the fixed point defining the surplus can be solved for independently from the current distributions of vacancies, unemployed workers, and worker-firm matches. The evolution of these distributions in the stochastic economy can then be solved for exactly, given the initial conditions. In addition, decisions about matching and separations are entirely independent of the current wage contract (but not of the rules governing rent-sharing).

In the quantitative section of the paper we illustrate the mechanisms of the model fitted to the facts about the relative volatility of output, unemployment, vacancies, and transitions rates (see Shimer 2005; Hall 2005; and Hagedorn and Manovskii 2008, among others). We fit the model to moments on the level and volatility of output, unemployment, vacancies, transition rates, as well as moments on unemployment duration and the cross-sectional standard deviation of value added per job. The ability of the model to reproduce the observed dynamics of unemployment at different unemployment durations is very good. We then analyze the interaction between two-sided heterogeneity and aggregate shocks with respect to the employment/unemployment stocks and flows. The estimated matching set is definitely cone-shaped and sorting is strongly positive. Therefore, the business-cycle affects two margins of the match distribution: low-type worker/high-type firm, and high-type worker/low-type firms. We find that when the economy recovers from a recession, employment expands as the result of improving employment opportunities for the low-type workers and expanded hiring by low-type firms. Moreover, when the economy enters a recession, low-type workers are fired, particularly those matched with high-type firms. At the same time, low-type firms hire less and medium/high-type firms hire relatively more medium/high-type unemployed workers.<sup>1</sup>

Our work is related to several recent developments in directed search and wage-posting models. In the directed-search, wage-posting model of Shi (2009); Menzio and Shi (2010a, b, 2011); and Rudanko (2011), the equilibrium is also block-recursive: i.e., such that agents' value and policy functions are independent of the endogenous distribution of workers across employment states.<sup>2</sup> We have a

<sup>1</sup> See related work by Barlevy (2002) and Sahin et al. (2014) among others.

<sup>2</sup> Kaas and Kircher (2015) extends Menzio and Shi's model to allow for firm size. Schaal (2012) presents an application of Menzio and Shi's directed search model with firm size, shocks to the first- and second-order moments of aggregate productivity, and with idiosyncratic productivity shocks.

closely related result. The advantages of our model with respect to the directed search model are that two-sided worker-firm heterogeneity is easily introduced; search frictions generate imperfect sorting (mismatch) at the equilibrium; workers search on the job and employers counter outside offers; and decisions about wages and matching are naturally separated. Dynamic versions of Burdett and Mortensen's (1998) wage-posting model have been proposed by Moscarini and Postel-Vinay (2010, 2013) and Coles and Mortensen (2016).<sup>3</sup> Wage-posting models with large firms, with random or directed search, allow one to analyze firm size growth from entry to steady-state equilibrium, as well as the response of firm size to aggregate shocks. They also link firm size to posted wages. They do not extend well to describing how different workers match with different firms, and the interaction between worker and firm heterogeneity and aggregate shocks.

Unlike wage-posting models, our model does not have a natural definition of firm size because we adopt the common approach of one-worker-one-firm matches. Yet, it does produce an equilibrium distribution of workers by firm type that moves along with the business cycle. Given that all the aforementioned wage-posting models assume constant returns to scale with respect to firm size and rule out within-firm worker externalities, it is not clear that grouping matches by firm type and calling this firm size would produce very different predictions. In addition, because search is not directed, our model naturally produces imperfect matching in equilibrium (i.e., workers are willing to accept a range of job types and firms are willing to hire a range of worker types). For this reason business-cycle fluctuations of mismatch will be our main empirical application. As far as we are aware this is the first empirical application of a tractable model of equilibrium search with two-sided *ex ante* heterogeneity and aggregate shocks.

Our model adds many important innovations to the prototype model of Robin (2011). Both workers and firms are *ex ante* heterogeneous, which allows us to analyze the cyclical patterns of sorting (or mismatch) between workers and firms. We also have an explicit vacancy creation decision which means the contact rate between workers and firms changes endogenously with the aggregate state, as does the distribution of types of vacancies that are posted. Lastly, we allow for production technologies in which workers may not agree on the ranking of firms; that the best job for worker A may not be the best job for worker B is a natural consequence of two-sided heterogeneity. Identification and estimation is also an important source of differences. Robin (2011) identified the distribution of worker types from unemployment volatility since in that model identical firms have a threshold worker type that varies with the aggregate state; all workers below this threshold are unemployable. We identify worker heterogeneity from unemployment duration dependence (some worker types will be in greater demand than others due to firm heterogeneity). We identify firm heterogeneity from cross-sectional dispersion of firms' value added. Moreover, we identify the vacancy cost function from time-series variation

<sup>3</sup>Barlevy (2002) presents a model with worker and firm heterogeneity and aggregate shocks. However workers (firms) are identical in terms of potential outcomes and the heterogeneity is effectively match-specific. Additionally, the full stochastic model is not tractable and is solved by replacing the dynamic bargaining with a fixed piece rate wage.

in the covariance between vacancies and output, and the cross-sectional distributions of firm productivities.

The rest of the paper is organized as follows. In Section I, we present the model and our main theoretical result. In Section II, we estimate the model to illustrate the quantitative ability to produce reasonable-looking business-cycle dynamics. We study the cyclical behavior of mismatch between worker- and firm-types in Section III. We conclude in Section IV.

## I. The Model

### A. Heterogeneous Agents and Aggregate Shocks

The economy is populated by a continuum of infinitely lived workers indexed by ability  $x$ , and a continuum of firms indexed by technology  $y$ . The total measures of workers and firms are fixed and normalized to 1. The distribution of  $x$  across workers is denoted  $\ell(x)$  and is exogenous. The distribution of  $y$  across firms is uniform. So  $y$  is just a way of ranking firms. The distribution of workers per job type is endogenous, and determined by firms' recruiting decisions and workers' mobility decisions. The cost of posting  $v$  job opportunities  $c(v)$  is exogenous. The aggregate state of the economy is indexed by  $z_t$ . At the beginning of each period the aggregate state changes from  $z$  to  $z'$  according to the Markov transition probability  $\pi(z, z')$ .

### B. The Meeting Technology

At the beginning of period  $t$ , a measure  $u_t(x)$  of unemployed workers of type  $x$  and a measure  $h_t(x, y)$  of workers of type  $x$  employed at firms of type  $y$  are inherited from period  $t - 1$ , with

$$u_t(x) + \int h_t(x, y) dy = \ell(x).$$

Then, the aggregate state changes from  $z_{t-1}$  to  $z_t$ . For simplicity, we assume that separations and meetings occur sequentially after the realization of the aggregate productivity shock: separations first, then the unemployed and the surviving employees get a chance to draw a new offer. Throughout we assume that match formation and separation is efficient.

Let  $u_{t+}(x)$  denote the stock of unemployed workers of type  $x$  immediately after the realization of  $z_t$  (at time  $t+$ ) and the ensuing job destructions, and let  $h_{t+}(x, y)$  be the stock of matches of type  $(x, y)$  that survive the destruction shocks. Together they produce effective search effort

$$L_t = \int u_{t+}(x) dx + s \iint h_{t+}(x, y) dx dy,$$

where the search effort of unemployed workers has been normalized to 1 and  $s$  is thus the relative search effort of employed workers. Let  $v_t(y)$  denote the measure of type  $y$  job opportunities chosen by firm  $y$  (see Section IE for details). Let  $V_t = \int v_t(y) dy$  denote the aggregate number of job opportunities. The total measure of meetings at time  $t$  is given by  $M_t = M(L_t, V_t)$ . Define  $\lambda_t = M_t/L_t$  as the

probability an unemployed searcher contacts a vacancy, and  $s\lambda_t$  is the probability an employed searcher contacts a vacancy in period  $t$ . Let  $q_t = M_t/V_t$  be the probability per unit of recruiting effort  $v_t(y)$  that a firm contacts any searching worker.

### C. The Value of Unemployment

Let  $B_t(x)$  be the value of unemployment to a type  $x$  worker at  $t$ . Consider a worker of type  $x$  who is unemployed for the whole period  $t$ . During that period she earns  $b(x, z_t)$ , which depends on her own type and the current aggregate productivity of the economy. She anticipates that at the beginning of period  $t + 1$ , after revelation of the new aggregate state, she will meet a vacancy of type  $y$  with probability  $\lambda_{t+1} \frac{v_{t+1}(y)}{V_{t+1}}$ . Let  $W_{0,t}(x, y)$  be the value to a type  $x$  worker who is hired from unemployment by a firm of type  $y$ . We assume that unemployed workers have zero bargaining power and are offered their reservation value,  $W_{0,t}(x, y) = B_t(x)$ . The value to this unemployed worker is then

$$\begin{aligned} (1) \quad B_t(x) &= b(x, z_t) \\ &+ \frac{1}{1+r} E_t \left[ (1 - \lambda_{t+1}) B_{t+1}(x) + \lambda_{t+1} \int W_{0,t+1}(x, y) \frac{v_{t+1}(y)}{V_{t+1}} dy \right] \\ &= b(x, z_t) + \frac{1}{1+r} E_t B_{t+1}(x), \end{aligned}$$

where  $r$  is the discount rate and  $E_t$  is the expectation operator with respect to future aggregate states given the information set at time  $t$ . The function  $B_t(x)$  is independent of any worker-specific history as long as home production  $b(x, z)$  is such.

### D. The Value and Surplus of a Match

Firms have access to a production technology, defined at the match level, that combines the skills of a worker and the technology of a firm with aggregate productivity to create value added  $p(x, y, z)$ . We will allow for the possibility that positive value added may require a threshold level of inputs. For example, a given production function may require workers of sufficient skill before value added is positive. We allow for complementarities between worker and firm types:  $p_{xy} \neq 0$ . However, since the production technology is defined at the level of the match, there is no complementarity across workers within a firm. Any correlations in output between workers at the same firms are attributed to the common firm component.

Let  $P_t(x, y)$  denote the present value of a match  $(x, y)$ , including the continuation values to the worker and firm upon separation, given the aggregate state of the economy at time  $t$ . A match  $(x, y)$  produces  $p(x, y, z_t)$  in period  $t$ , and the continuation value depends on whether the match is destroyed at the beginning of period  $t + 1$ .

Assuming zero fixed investment in job creation, any vacancy generated by job destruction is lost and has zero continuation value. There is no severance payment or experience rating. Hence, after revelation of the new aggregate shock  $z_{t+1}$ , the worker and the firm are better off separated than staying together if and only

if  $P_{t+1}(x, y) < B_{t+1}(x)$ . In addition, we allow for a source of idiosyncratic job destruction shocks  $\delta$ . The match is therefore destroyed with probability

$$\mathbf{1}\{P_{t+1}(x, y) < B_{t+1}(x)\} + \delta \times \mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\},$$

and if the job is destroyed the continuation value of the match is the value of unemployment  $B_{t+1}(x)$ .

The current match continues in period  $t + 1$  with probability  $(1 - \delta)\mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\}$ . Then the worker draws an alternative offer from a firm of type  $y'$  with probability  $s\lambda_{t+1}\frac{v_{t+1}(y')}{V_{t+1}}$ . Let  $W_{1,t}(x, y, y')$  be the value offered at time  $t$  by a firm of type  $y$  to a worker of type  $x$  who has an alternative employment opportunity of type  $y'$ . We adopt the sequential auction framework of Postel-Vinay and Robin (2002). The incumbent employer can make new wage offers to their existing workers in an attempt to retain those with outside offers. Incumbent and poaching firms engage in Bertrand competition which grants the worker a value equal to the second highest bid. Specifically, either  $P_{t+1}(x, y') > P_{t+1}(x, y)$  and the worker moves to firm  $y'$  and receives the incumbent employer's reservation value  $W_{1,t+1}(x, y', y) = P_{t+1}(x, y)$  as continuation value; or  $P_{t+1}(x, y') \leq P_{t+1}(x, y)$  and the worker stays with her current employer with continuation value  $W_{1,t+1}(x, y, y') = P_{t+1}(x, y)$ .<sup>4</sup> Hence, Bertrand competition makes the continuation value of the match independent of whether the employee is poached:

$$(2) P_t(x, y) = p(x, y, z_t)$$

$$\begin{aligned} & + \frac{1}{1+r} E_t \left[ \left( 1 - (1 - \delta)\mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\} \right) B_{t+1}(x) \right. \\ & \quad + (1 - \delta)\mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\} \left( (1 - s\lambda_{t+1}) P_{t+1}(x, y) \right. \\ & \quad \left. \left. + s\lambda_{t+1} \int \max\{P_{t+1}(x, y), W_{1,t+1}(x, y', y)\} \frac{v_{t+1}(y')}{V_{t+1}} dy' \right) \right] \\ & = p(x, y, z_t) \\ & \quad + \frac{1}{1+r} E_t \left[ \left( 1 - (1 - \delta)\mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\} \right) B_{t+1}(x) \right. \\ & \quad \left. + (1 - \delta)\mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\} P_{t+1}(x, y) \right]. \end{aligned}$$

<sup>4</sup> In the latter case, the aggregate shock may still force the employer and employee to renegotiate if  $P_{t+1} \geq B_{t+1}$  but the existing contract would imply either  $W_{t+1} < B_{t+1}$  or  $W_{t+1} > P_{t+1}$ , as in Hall (2005) and Postel-Vinay and Turon (2010).

Finally, defining match surplus as  $S_t(x, y) = P_t(x, y) - B_t(x)$ , and combining equations (1) and (2), we have

$$\begin{aligned} P_t(x, y) - B_t(x) &= p(x, y, z_t) - b(x, z_t) \\ &\quad + \frac{1}{1+r} E_t \left[ (1 - (1 - \delta) \mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\}) B_{t+1}(x) \right. \\ &\quad \left. + (1 - \delta) \mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\} P_{t+1}(x, y) - B_{t+1}(x) \right] \\ &= p(x, y, z_t) - b(x, z_t) \\ &\quad + \frac{1-\delta}{1+r} E_t \left[ \mathbf{1}\{P_{t+1}(x, y) \geq B_{t+1}(x)\} [P_{t+1}(x, y) - B_{t+1}(x)] \right]. \end{aligned}$$

In other words,

$$S_t(x, y) = p(x, y, z_t) - b(x, z_t) + \frac{1-\delta}{1+r} E_t \max\{S_{t+1}(x, y), 0\}.$$

We are now in a position to state our main result.

**PROPOSITION 1:** *The surplus from an  $(x, y)$  match at time  $t$  depends on time only through the current aggregate productivity shock  $z$  and does not depend on the distributions of vacancies, unemployed workers, or worker-firm matches. Specifically,  $S_t(x, y) \equiv S(x, y, z)$  such that*

$$(3) \quad S(x, y, z) = s(x, y, z) + \frac{1-\delta}{1+r} \int S(x, y, z')^+ \pi(z, z') dz',$$

where  $s(x, y, z) = p(x, y, z) - b(x, z)$  and we denote  $x^+ = \max\{x, 0\}$ .

Outside offers do not change the size of the match surplus, only how it is shared between the worker and the firm. When a firm counters an outside offer to retain the worker this is done by transferring more of the match surplus to the worker, but has no impact on the total surplus in the match. Similarly, when a worker is poached by another firm, Bertrand competition ensures that the value to the worker of moving to the new match is exactly the total surplus at the previous match. The previous firm is then left with zero since vacancies do not have a continuation value.

We can now explicitly express the stocks  $u_{t+}(x)$  and  $h_{t+}(x, y)$  as

$$(4) \quad u_{t+}(x) = u_t(x) + \int [\mathbf{1}\{S_t(x, y) < 0\} + \delta \mathbf{1}\{S_t(x, y) \geq 0\}] h_t(x, y) dy$$

and

$$(5) \quad h_{t+}(x, y) = (1 - \delta) \mathbf{1}\{S_t(x, y) \geq 0\} h_t(x, y).$$

### E. Vacancy Creation

Each period firms can buy the advertising of  $v$  job opportunities from job placement agencies at a price  $c(v) \geq 0$  that is assumed independent of the firm's type,

increasing, and convex. In equilibrium, the number of advertised job opportunities is determined by equating the marginal cost to the expected value of a job opening,

$$(6) \quad c'[v_t(y)] = q_t J_t(y),$$

where  $J_t(y)$  denotes the expected value of a contact by a vacancy of type  $y$ , and  $q_t$  is the probability, per unit of recruiting effort, that a firm contacts a searching worker. The assumption that  $c(\cdot)$  is increasing and convex guarantees a nondegenerate distribution of vacancies  $v_t(y)$ .

Any job opportunity that does not deliver a contact with a worker in the period is lost and generates no continuation value. Any contact that does not end up in an employment contract is lost and has zero value.<sup>5</sup>

The expected value of a contact is calculated as

$$(7) \quad J_t(y) = \int \frac{u_{t+}(x)}{L_t} S(x, y, z)^+ dx \\ + \iint \frac{sh_{t+}(x, y')}{L_t} [S(x, y, z) - S(x, y', z)]^+ dx dy'.$$

The contact is with an unemployed worker of type  $x$  with probability  $\frac{u_{t+}(x)}{L_t}$  and a match is formed if the match surplus is positive, in which case it is entirely appropriated by the employer. The contact is with a worker of type  $x$  that is currently employed at a firm of type  $y'$  with complementary probability  $\frac{sh_{t+}(x, y')}{L_t}$ . Poaching is successful if  $S(x, y, z) > S(x, y', z)$  and Bertrand competition grants the poacher a value  $S(x, y, z) - S(x, y', z) = P(x, y, z) - P(x, y', z)$ .

#### F. Labor Market Flows

The law of motion for unemployment resulting from meetings between unemployed workers and vacant jobs is therefore

$$(8) \quad u_{t+1}(x) = u_{t+}(x) \left[ 1 - \int \lambda_t \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x, y) \geq 0\} dy \right],$$

and for employment

$$(9) \quad h_{t+1}(x, y) = h_{t+}(x, y) \left[ 1 - \int s \lambda_t \frac{v_t(y')}{V_t} \mathbf{1}\{S_t(x, y') > S_t(x, y)\} dy' \right] \\ + \int h_{t+}(x, y') s \lambda_t \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x, y) > S_t(x, y')\} dy' \\ + u_{t+}(x) \lambda_t \frac{v_t(y)}{V_t} \mathbf{1}\{S_t(x, y) \geq 0\},$$

subtracting those lost to more productive poachers, and adding the  $(x, y)$ -jobs created by poaching from less productive firms and hiring from unemployment.

<sup>5</sup> A more sophisticated job creation process could be envisioned, in which fixed initial investments would give value to job creation above and beyond the service provided by placement agencies.

### G. Computation of the Stochastic Search Equilibrium

Following directly from the results presented above, we can solve for the stochastic equilibrium in this environment in two stages:

- (i) For given home production and value-added technologies  $b(x, z)$  and  $p(x, y, z)$ ; discount rate  $r$ ; exogenous job destruction rate  $\delta$ ; and stochastic process for the aggregate state  $\pi(z, z')$  the surplus function  $S(x, y, z)$  is sufficient to determine all decisions regarding worker mobility and is defined by the unique solution to the functional equation (3).
- (ii) For a given distribution of worker ability  $\ell(x)$ , vacancy cost function  $c(v)$ , and meeting technology  $M(L, V)$ ; and for any given initial distributions of unemployed workers  $u_0(x)$  and worker-firm matches  $h_0(x, y)$ , a sequence of aggregate productivity shocks  $\{z_t\}_{t=0}^T$  implies a unique sequence of distributions of vacancies, unemployed workers, and worker-firm matches

$$\{v_t(y), u_{t+1}(x), h_{t+1}(x, y)\}_{t=0}^T.$$

This sequence of distributions can be calculated by using the surplus function  $S(x, y, z)$  and iterating on equations (4) to (9) starting from

$$\{z_0, u_0(x), h_0(x, y)\}.$$

It is quite remarkable that the dynamics of the distributions of unemployed worker types and worker-firm type matches can be solved independently of any reference to wages. This property simplifies equilibrium computation substantially, but we also think that it makes sense in the current context. It results from the agreement between the worker and the firm that the match surplus represents the present value of output in the current match plus the continuation value upon separation. The fact that its size does not depend on the way it is going to be shared is a property of a wide class of models. Moreover, the assumption that match formation and match destruction are efficient does not seem very strong. It implies that workers and firms never walk away from mutually beneficial matches. If it is better for a worker to take a wage cut than become unemployed then the worker will agree to this. If it is better for the firm to grant a wage increase than terminate the match then this will occur. The value of the surplus, not the wage, is at the heart of the decision to continue or terminate the match. However, intuition is rarely straightforwardly confirmed by formal models. Some strong assumptions are necessary for the surplus not to be a function of the wage or the distributions. Specifically, transferability of present values requires linear utility, and our specified offer and counteroffer mechanism (which relies on observable states and actions) is needed in order to remove the current wage and distributions from the state space.

Before turning to the quantitative application, we note that it is straightforward to extend our results to accommodate aggregate productivity growth, idiosyncratic shocks to worker and firm productivity, shocks to the meeting technology and/or the cost of vacancy creation, and a birth-death process in which the distribution

of skills for new labor market entrants may differ from previous generations. We briefly sketch some of these extensions in Appendix A.

## II. Heterogeneity and Unemployment Volatility

To illustrate the mechanics of the model, we fit the model to moments of standard US time series data for the period 1951:I to 2012:IV. We first study the implications for the business-cycle fluctuations of job creation, job destruction, employment transitions, and unemployment. Then, in the following section, we study the implications for the dynamics of heterogeneous matching between workers and firms.

In estimation we target the means and standard deviations of the unemployment rate, the number unemployed more than 5, 15, and 27 weeks, unemployment-to-employment transitions, employment-to-unemployment transitions, job-to-job transitions, the vacancy-to-unemployment ratio, and the cross-sectional standard deviation of value added per match. We also target the standard deviation of vacancies, the standard deviation and autocorrelation of value added, the correlation between vacancies and unemployment, the correlation between unemployment-to-employment and job-to-job transitions. Lastly, we target the correlations between aggregate value added, on one hand, and unemployment, vacancies, unemployment-to-employment transitions, employment-to-unemployment transitions, and the cross-sectional standard deviation of value added per match, on the other hand.<sup>6</sup> The values of the data moments we target in estimation, along with their estimated standard errors calculated using Newey and West's heteroskedasticity and autocorrelation consistent estimator, are listed in Table 1.

These moments are all standard moments used in the literature except for moments related to unemployment rates by unemployment duration and to cross-sectional standard deviations of firm total factor productivity (TFP). Unemployment duration dependence aims at identifying the parts of the model that relate to worker heterogeneity. The cross-sectional dispersion of firm TFP aims at identifying the parts of the model that relate to firm heterogeneity.

We solve and simulate data from the model at a weekly frequency. We then aggregate the weekly simulated series exactly as in the data to obtain simulated series at the quarterly or annual frequency. Details pertaining to the construction of data moments, the method of simulated moments estimator, construction of standard errors, and specific implementation details are provided in Appendix B.

### A. Parametrization

We choose the following parametrization of the model. The distribution of workers is assumed to be beta with parameters  $\beta_1$  and  $\beta_2$  to be estimated. We approximate the continuous heterogeneity by a grid of linearly spaced points  $x_1, x_2, \dots, x_{N_x}$  on  $[0, 1]$ . We specify the set of potential job types,  $y_1, y_2, \dots, y_{N_y}$  on  $[0, 1]$ . The distributional assumption on  $x$  has no effect on the value of the surplus function  $S(x, y, z)$ . Similarly,

<sup>6</sup>The firm-level statistics on value added per worker data (more exactly total factor productivity) are taken from Bloom et al. (2014, Table A1) and from <http://www.stanford.edu/~nbloom/RUBC.zip>. These statistics are based on a subset of large US firms and, as such, should be thought of as a lower bound for productivity dispersion.

TABLE 1—DATA MOMENTS AND MODEL SIMULATED MOMENTS

Fitted moments	Data	Model	Fitted moments	Data	Model
$E[U]$	0.058 (0.003)	0.059	$sd[U]$	0.191 (0.018)	0.203
$E[U^{5p}]$	0.035 (0.003)	0.032	$sd[U^{5p}]$	0.281 (0.027)	0.315
$E[U^{15p}]$	0.018 (0.002)	0.018	$sd[U^{15p}]$	0.395 (0.038)	0.413
$E[U^{27p}]$	0.010 (0.002)	0.011	$sd[U^{27p}]$	0.478 (0.045)	0.439
$E[UE]$	0.421 (0.020)	0.468	$sd[UE]$	0.127 (0.011)	0.127
$E[EU]$	0.025 (0.001)	0.028	$sd[EU]$	0.100 (0.011)	0.095
$E[EE]$	0.025 (0.002)	0.025	$sd[EE]$	0.095 (0.005)	0.112
$E[V/U]$	0.634 (0.001)	0.744	$sd[V/U]$	0.381 (0.029)	0.306
$E[sd \text{ labor prod}]$	0.494 (0.009)	0.505	$sd[sd \text{ labor prod}]$	0.039 (0.005)	0.038
$sd[V]$	0.206 (0.015)	0.105	$corr[V, VA]$	0.721 (0.149)	0.996
$sd[VA]$	0.033 (0.003)	0.034	$corr[UE, VA]$	0.878 (0.122)	0.978
$autocorr[VA]$	0.932 (0.132)	0.991	$corr[EU, VA]$	−0.716 (0.133)	−0.910
$corr[V, U]$	−0.846 (0.119)	−0.975	$corr[UE, EE]$	0.695 (0.108)	0.977
$corr[U, VA]$	−0.860 (0.124)	−0.983	$corr[sd \text{ labor prod}, VA]$	−0.366 (0.260)	−0.365
Model prediction					
$corr(x, y)$		0.479		mean	0.700
Mean firm share of			$b(x)/p(x, y^*, z)$	min	0.561
initial match surplus		0.884		max	0.873

Notes: Standard errors for the data moments, corrected for serial correlation, are presented in parentheses (see Appendix BB3 for details). The data used to construct the moments is BLS 1951:I–2012:IV, with the exception of the moments involving EE transitions which uses CPS 1994:I–2011:III, and the moments involving sd labor prod., which use Bloom et al.’s (2014) 1972–2009 annual Compustat data.  $y^*(x, z) = \arg \max_y S(x, y, z)$  is the best match for worker  $x$  when the economy is in state  $z$ .

we specify a grid of linearly spaced points  $a_1, a_2, \dots, a_{N_z}$  on  $[\varepsilon, 1 - \varepsilon] \subset (0, 1)$ , used to define the aggregate productivity shocks  $z_i$ . The aggregate productivity shock is given by  $z_i = F^{-1}(a_i)$ , and the transition probability  $\pi(z_i, z_j) \propto C(a_i, a_j)$ , where  $C$  is a copula density, and we normalize  $\sum_j \pi(a_i, a_j) = 1$ . Specifically, we set  $N_x = 21$ ,  $N_y = 21$ ,  $N_z = 51$ ,  $F$  is log-normal with parameters zero and  $\sigma$ , and  $C$  is a Gaussian copula density with parameter governing dependence  $\rho$ .<sup>7</sup> We set the

<sup>7</sup>This specification is therefore consistent with the AR(1) model:

$$\log z_t = \rho \log z_{t-1} + \sigma \sqrt{1 - \rho^2} \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, 1).$$

Using a copula representation allows us to change the specification of the marginal distribution independently of that of the copula, which controls the dynamics of ranks. It also straightforwardly delivers a discrete version of the stochastic process.

length of a period in the model to be one week. The discount rate is set to 5 percent annually.

We approximate value added at the match level by a second order Taylor series in worker and firm type, assuming proportionality to the aggregate shock  $z$ :

$$p(x, y, z) = z(p_1 + p_2x + p_3y + p_4x^2 + p_5y^2 + p_6xy).$$

We do not place any restrictions on the polynomial coefficients, which are to be estimated.<sup>8</sup>

We set the flow value of home production to be a fixed fraction of market production in the absence of mismatch or aggregate shocks:

$$b(x) = 0.7 \times p(x, y^*(x, 1), 1),$$

where  $y^*(x, 1) = \arg \max_y S(x, y, 1)$  is the firm type that maximizes surplus for a worker of type  $x$  when the aggregate shock is equal to 1. The factor 0.7 is based on Hall (2005).

For the meeting function we assume a Cobb-Douglas form

$$M_t = M(L_t, V_t) = \min\{\alpha L_t^\omega V_t^{1-\omega}, L_t, V_t\},$$

with  $\alpha > 0$  and  $\omega \in [0, 1]$ . With this meeting function and a recruiting cost function of the form

$$c(v) = \frac{c_0 v^{1+c_1}}{1 + c_1},$$

equilibrium vacancy creation by firm-type is given by the expression

$$v_t(y) = \left( \frac{\alpha J_t(y)}{\theta_t^\omega c_0} \right)^{\frac{1}{c_1}},$$

where aggregating over  $v_t(y)$  and substituting for  $q_t$  gives equilibrium market tightness

$$\theta_t = \left( \frac{1}{L_t} \int \left( \frac{\alpha J_t(y)}{c_0} \right)^{\frac{1}{c_1}} dy \right)^{\frac{c_1}{c_1 + \omega}}.$$

<sup>8</sup> Recall that we are modeling value added not total output. We want to allow the possibility that higher- $y$  firms may operate with more costly nonlabor inputs. In this case only workers with skill above a particular threshold would produce enough to cover the nonlabor costs and hence deliver positive profit to the firm. For example, suppose  $p_1 = p_2 = p_3 = p_4 = 0$ ,  $p_5 = -1$ , and  $p_6 = 1$ , then only matches in which  $x > y$  will produce positive value added. Alternatively, suppose  $p_1 = 1$ ,  $p_2 = p_3 = 0$ ,  $p_4 = p_5 = -1$ , and  $p_6 = 2$ , then value added is maximized when  $x = y$  and decreases as  $x$  and  $y$  differ ( $x$  is not well matched to  $y$ ).

### B. Fit

The empirical and simulated moments are presented in Table 1. Overall the model fits the moments very well, the exceptions being the volatility of vacancies, which is only one-half as volatile in the model as in the data; the autocorrelation of output, which is too high; and the correlations between output and unemployment, vacancies, and the job-finding rate, which are somewhat stronger than in the data.

Table 1 shows that if we draw a long series of aggregate shocks from the estimated Markov process (700 years at weekly frequency, discarding the first 100 years) and simulate labor market outcomes we obtain series that behave like the observed ones in level, volatility, and correlations. We then illustrate the ability of the model to propagate the single productivity shock to unemployment, vacancies, and transition rates in Figure 1. Figure 1 displays the one-period-ahead predictions of the measurement series. We first filter out aggregate productivity shocks by exactly fitting the series of aggregate output. Then, we simulate the change in the other variables using this series.<sup>9</sup> The fit of the unemployment series is simply spectacular. We slightly overestimate the employment-state transition rates (*UE* and *EU*) but the dynamics are very well reproduced. Job to job transitions, on the limited period of observation that we have, are predicted very well. The volatility of simulated job opportunities is smaller than in the help wanted series, but vacancy data are notoriously difficult to gather. One interpretation of the difference is that it reflects the discrepancy between the measurement of job openings in the data and the model concept of aggregate recruiting intensity.

### C. Parameter Estimates

We now briefly discuss the parameter estimates in Table 2 before turning to the implications for the mechanics of the model. Overall the parameter estimates are quite precise, indicating that they are well identified by the data, at least locally. It is only  $p_4$ , the loading on  $x^2$  in the production function, that has a large and imprecise coefficient. Aggregate matching efficiency is estimated to be 0.497, with search on the job 0.027 times as effective as unemployed search.<sup>10</sup> The cost of creating a vacancy is mildly convex with the power term estimated to be 1.08. Jobs exogenously separate with a weekly probability of 0.013, or a quarterly probability of 0.17. The unconditional standard deviation of  $\log z$  is estimated to be 0.071, with Gaussian copula parameter 0.999. Note that the copula parameter governs the variance of  $z_t$  conditional on  $z_{t-1}$  at weekly frequency, making the variance of the change in  $z$  very small (see footnote 9).

<sup>9</sup>The time-series properties of our filtered shock match the model estimates very well. Regressing the filtered series for  $\log z_t$  on  $\log z_{t-1}$  we obtain an autocorrelation coefficient equal to 0.9967 and a standard deviation of the residual of 0.001698. Our model estimates imply corresponding values equal to 0.9997 and 0.001638 (see the expression in footnote 7).

<sup>10</sup>Typical estimates of the relative efficiency of search on versus off the job are in the range of 0.1 to 0.2. Our estimate is lower due to treating workers who transit from employment to unemployment and then back to employment within the same period as having made a job-to-job transition. One implication is that a substantial fraction of job-to-job moves in the model would result in a wage cut; an implication that is consistent with empirical evidence (see Jolivet, Postel-Vinay, and Robin 2006).

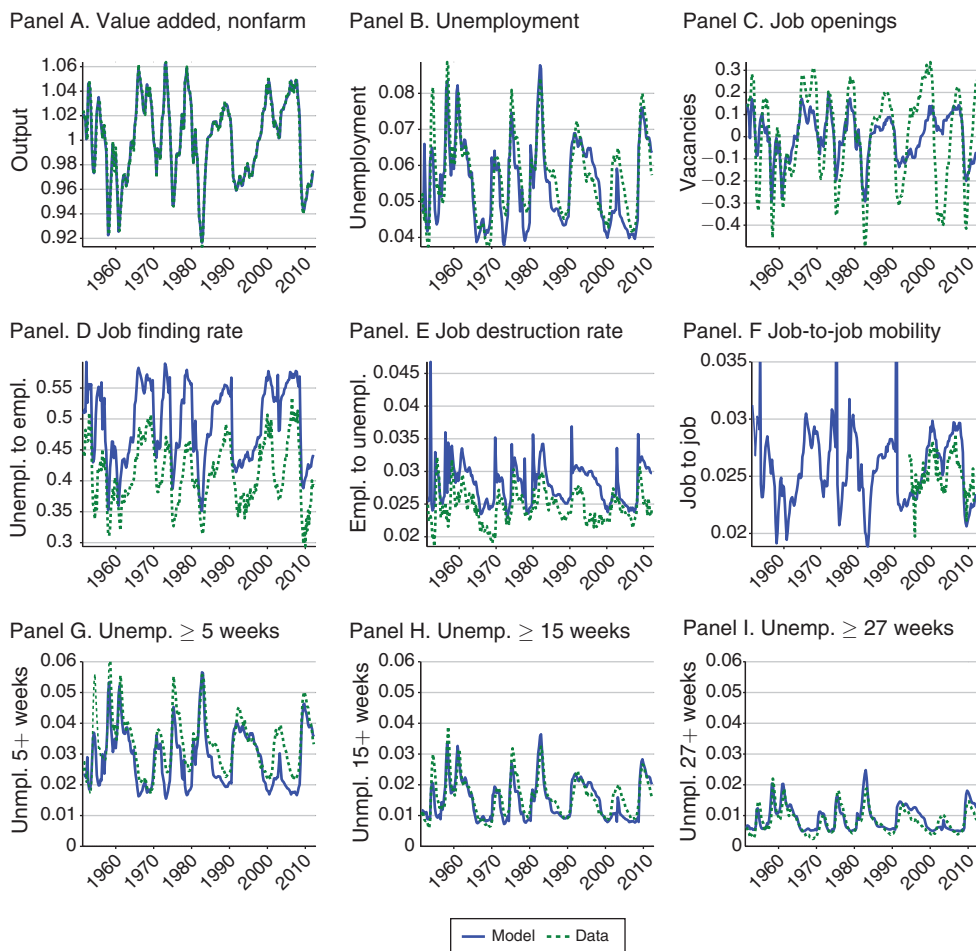


FIGURE 1. DATA AND MODEL PREDICTED TIME SERIES

Notes: The dashed line is the HP-filtered data while the solid line is the model simulated data. The simulated data is produced by choosing the latent shock process  $z_t$  and the scale of output  $p_0$  to match the time-series of output and the mean unemployment rate for 1951–2012.

The estimated distribution of worker types is presented graphically in panel A of Figure 2, where we also present the average distribution of worker types in unemployment (averaged over time). As expected, the skill distribution in unemployment is skewed toward low-skilled workers relative to the population distribution.

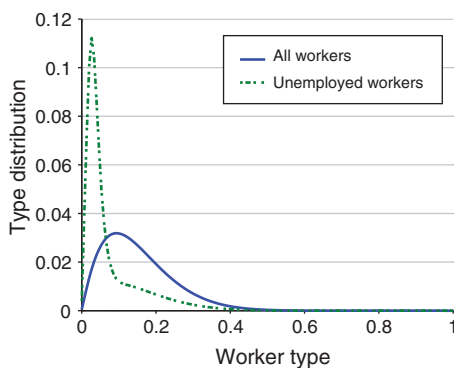
Market production,  $p(x, y, z)$ , is increasing in worker-type conditional on firm-type  $y$ . We can think of worker type and worker productivity as having a one-to-one mapping. At the same time market production is nonmonotonic in firm type  $y$ , conditional on worker type. There is positive complementarity between worker skills and firm technology which is stronger in booms than in recessions:  $\partial^2 p(x, y, z) / \partial x \partial y = 6.6z$ . In panel B of Figure 2 we plot the contour lines of  $p(x, y) \equiv p(x, y, z) / z$ . The contour lines represent increasing output moving from left to right. The output-maximizing firm-type is (weakly) increasing in worker type. The fact that the output maximizing firm-type is estimated to be increasing

TABLE 2—PARAMETER ESTIMATES

Matching $\alpha \sqrt{LV}$	$\alpha$	0.497 (0.083)	Worker heterogeneity Beta( $\beta_1, \beta_2$ )	$\beta_1$	2.148 (0.192)
Search intensity	$s$	0.027 (0.007)		$\beta_2$	12.001 (1.951)
Vacancy costs $\frac{c_0}{1+c_1} v(y)^{1+c_1}$	$c_0$	0.028 (0.014)	Value added $p(x, y, z) = z(p_1 + p_2x + p_3y$ $+ p_4x^2 + p_5y^2$ $+ p_6xy)$	$p_1$	0.003 (0.006)
	$1 + c_1$	1.084 (0.040)		$p_2$	2.053 (0.684)
Exogenous separation	$\delta$	0.013 (0.001)		$p_3$	-0.140 (0.504)
Productivity shocks Gaussian copula ( $\sigma, \rho$ )	$\sigma$	0.071 (0.009)		$p_4$	8.035 (5.422)
	$\rho$	0.999 (0.001)		$p_5$	-1.907 (0.355)
				$p_6$	6.596 (0.835)

Notes: Standard errors, corrected for serial correlation in the data, are presented in parentheses (see Appendix BB3 for details). Home production is set to 70 percent of maximum output when  $z = 1$ , i.e.,  $b(x) = 0.7p(x, y^*(x, 1), 1)$ , where  $y^*(x, 1) = \arg \max_y S(x, y, 1)$ . The interest rate  $r$  is fixed at 0.05 annually.

Panel A. Distribution of worker types



Panel B. Production function

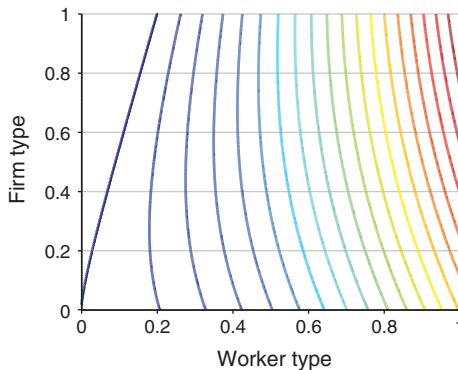


FIGURE 2. ESTIMATED DISTRIBUTION OF WORKER TYPES AND THE PRODUCTION FUNCTION

Notes: In panel A we also show the estimated distribution of worker types among the unemployed (averaged over all aggregate states). In panel B the contour lines are increasing from left to right. Conditional on firm-type  $y$ ,  $p(x, y) \equiv p(x, y, z)/z$  is increasing in worker-type  $x$ . However  $p(x, y)$  is nonmonotonic in firm-type  $y$  for workers with  $x$ -type below 0.6.

in worker-type, and that it is interior for all worker types below 0.6 implies that the equilibrium will display positive sorting. Heuristically, we learn about sorting as follows. The pattern of unemployment rates by duration of unemployment tells us that workers leave unemployment at different rates (they do not have homogeneous exit rates). Within the structure of the model, this means that some workers can match with more jobs than others, illustrated by the cone-shaped matching sets in Figure 4 (discussed below). The cone-shaped matching sets are generated by a production function with positive complementarity (positive sorting). Finally, the fact that there is substantial variation in value added per match tells us the complementarity must

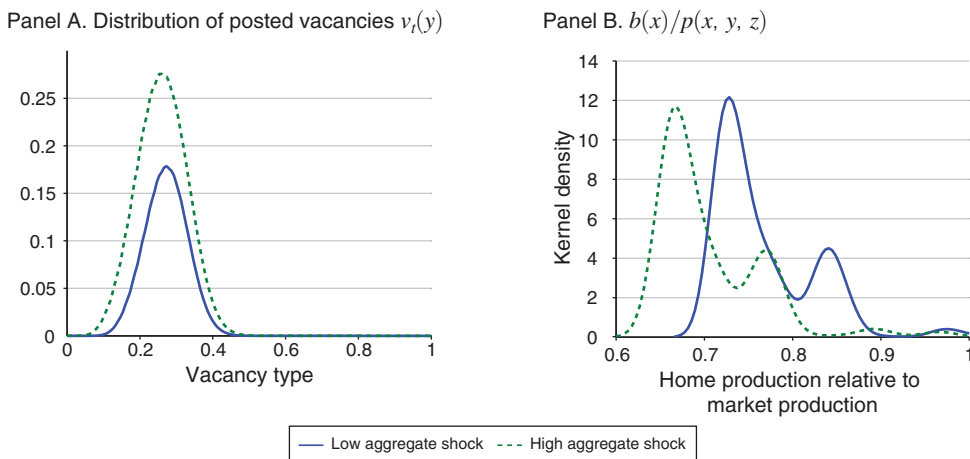


FIGURE 3. EQUILIBRIUM VACANCY CREATION AND HOME, RELATIVE TO MARKET PRODUCTIVITY

Note: Low (high) refers to periods when the aggregate state is in the bottom (top) decile of the simulation.

be strong enough to produce substantial differences in output between well-matched low-type workers and well-matched high-type workers. Identification of sorting clearly leans on the structure of the model. The parametric specification is chosen to be flexible yet parsimonious. The nonmonotonicity of output with respect to  $y$  is required to generate matching sets that are not identical for all worker types, and the resulting differential exit rates from unemployment by worker type. We consider this further, along with the cyclical patterns of worker-firm matches in Section III.

#### D. Model Implications for Aggregate Shocks, Job Creation, and Job Separation

What does the model imply for how the types of vacancies posted and the composition of unemployment flows change with the aggregate state of the economy? In panel A of Figure 3, we plot the model-predicted expected number of posted vacancies by firm type,  $v_i(y)$ , when the aggregate state is low or high (below the tenth percentile or above the ninetieth percentile of the simulation). Moving from a boom to a recession, the number of vacancies posted by firms contracts everywhere, with posting of low-type vacancies contracting proportionately more than higher-type vacancies.

Our assumption on contracts implies that workers are always offered their reservation value. The implication is that when a firm hires a worker out of unemployment it receives the entire match surplus. However, when it hires a worker who is already employed by another firm it only receives the share of the surplus in excess of what is necessary to poach the worker. Since there are many more employed than unemployed workers, firms expect to receive a share below 1 from newly formed matches. The estimated model implies that the initial share of surplus going to the firm is 0.88 on average (Table 1). Over time, as the match progresses, the worker receives outside offers which erodes the firms' share, eventually leaving the worker with the entire share and the firm with zero when competing with a poaching firm of the same type.

How much value do productive matches create relative to home production? In panel B of Figure 3 we plot the density of  $b(x)/p(x, y, z)$  for low and high aggregate productivity (below the tenth percentile or above the ninetieth percentile of the simulation). Recall that we fixed home production for each worker-type to be equal to 0.7 times their maximum market production in the absence of productivity shocks. Fluctuations in aggregate productivity will scale this ratio up and down, and the ratio will be higher for those workers who are not currently employed in their best match. The mode of the densities shifts down from 0.725 to 0.675 as we move from the low to the high aggregate state. In the low (high) aggregate state there is another mass at 0.84 (0.77), corresponding to newly hired workers who have not yet been poached and tend to still be some ways from their ideal match.

There is always a small mass of workers for whom home production is very close to market production in their current match, more so in the low than in the high state. These mismatched workers are the ones at risk of endogenous separation due to a negative aggregate productivity shock. We discuss further the composition of this group in Section III. What is worth noting here is that for the vast majority of matches, market production is substantially higher than home production, this is not a small surplus economy. This is especially true in the high aggregate state leading firms to respond by posting more vacancies. Fluctuations in the expected match surplus generates volatility in vacancy posting and hiring while fluctuations in the mass of mismatched workers generates volatility in job separations. The combination generates volatility in unemployment. Finally, worker and firm heterogeneity generates heterogeneity in job-finding rates which produces the observed pattern of unemployment rates by duration.

### III. Model Implications for Aggregate Shocks and Sorting (or Mismatch)

In this section, we study the implications of the model for the interaction of heterogeneity and aggregate shocks. In particular we look at which types of worker-firm matches are feasible in different aggregate states; which types of matches are most at risk of separating due to negative aggregate shocks; and the extent to which the source of new hires (hired from unemployment or poached from other firms) changes with the aggregate state.

#### A. Feasible Matches and Sorting

The mechanism for how the model combines two-sided heterogeneity and aggregate shocks to produce the cyclicalities summarized by the moments in Table 1 and the time-series in Figure 1 can be readily understood by examining the shape of the estimated surplus function. In Figure 4 we plot the set of feasible matches for the tenth, fiftieth, and ninetieth percentiles of the distribution of the aggregate shock. The shaded area bounded by the solid line represents all matches which are feasible when the aggregate shock is high (at the ninetieth percentile). The two solid lines inside the shaded area represent the boundaries of the feasible set of matches when the aggregate shock is at the fiftieth and tenth percentiles. If the aggregate state moved from the ninetieth to the tenth percentile, all matches outside the new bounds would immediately separate. Additionally, the set of jobs that are feasible

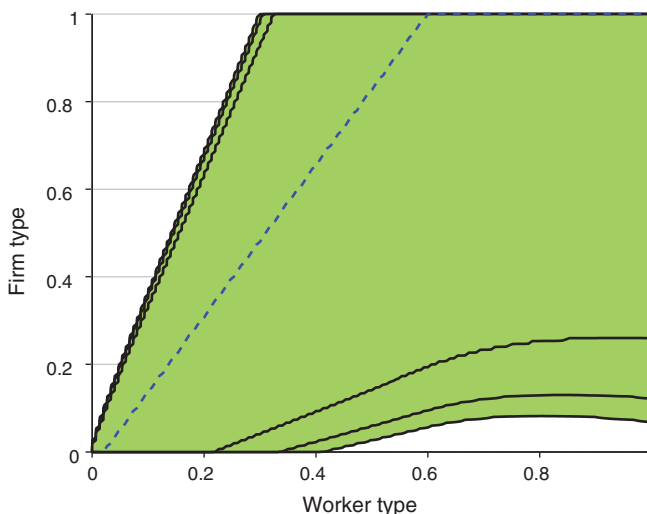


FIGURE 4. CYCLICALITY OF FEASIBLE MATCHES

Notes: The solid lines mark the boundaries for the set of feasible matches when the aggregate shock is at the ninetyth, fiftieth, and tenth percentile (moving from outer to inner lines). The dotted line plots the optimal firm type for each worker type,  $y^*(x, 1) = \arg \max_y S(x, y, 1)$ .

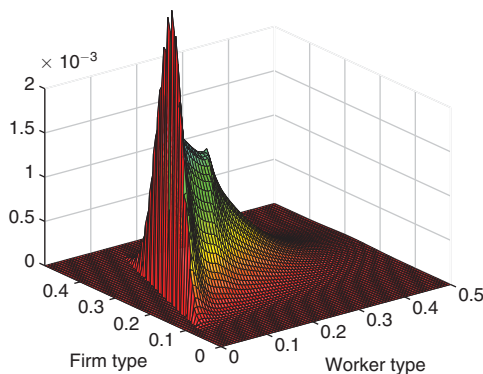
for unemployed workers shrinks, since any meetings between workers and firms in this region no longer result in a match. Contractions are characterized by both fewer contacts, since firms post fewer vacancies, and fewer accepted job offers conditional on a contact.

The dotted line represents the ideal firm-type for each worker-type and is calculated as  $y^*(x, 1) = \arg \max_y S(x, y, 1)$ .<sup>11</sup> Unemployed workers initially accept any job in the feasible set (conditional on the aggregate state), and through the process of on-the-job search gradually move toward the dotted line. As employed workers receive offers from other firms they will move toward their preferred firm, which will be further from the boundary of the feasible matching set than the initial job taken out of unemployment, and hence protected from the effect of aggregate shocks. Low-type workers have fewer employment opportunities than higher-type workers, and workers with short employment tenure are more cyclically sensitive than workers with long tenures who have enjoyed a sustained period of continuous employment. Note that the model requires both worker and firm heterogeneity to produce these implications.

The upper left boundary of the matching set in Figure 4 corresponds to the minimum worker type that is acceptable to a particular firm type, conditional on the aggregate state. The lower boundary similarly corresponds to the lowest firm type that is acceptable to a worker. Looking at the feasible sets by aggregate state, the firms' reservation worker type fluctuates substantially less than the workers' reservation firm type. The effect of the business cycle is therefore mostly located in the lower part of the distribution of firm types. The matches which are most at risk of

<sup>11</sup> It happens that  $y^*(x, z) = \arg \max_y S(x, y, z)$  varies little with  $z$  and is close to the value of  $y$  that maximizes  $p(x, y, z)$ , which is indeed independent of  $z$  because of the multiplicative specification.

Panel A. Low aggregate shock



Panel B. High aggregate shock

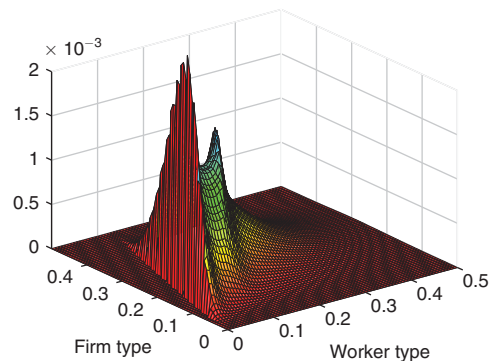


FIGURE 5. JOINT DISTRIBUTION OF WORKER-FIRM MATCHES BY AGGREGATE STATE

Notes: The low and high aggregate shocks are defined as a  $z$ -shock at the tenth and the ninetieth percentile of the distribution of aggregate shocks. These correspond to the inner and outer bound in Figure 4. Specifically, we plot  $E[h_t(x, y) / \iint h_t(x, y) dx dy | z_t = z_p]$ , where  $p$  indicates the tenth or the ninetieth percentile of the shock distribution. The range of  $x$  and  $y$  is restricted to  $[0, 0.5]$  as almost all of the mass in this region.

endogenous separations due to negative aggregate shocks seem to be the matches between low-type firms ( $y$  below 0.15) and high-type workers ( $x$  above 0.2). How this translates into aggregate volatility, however, depends on how many matches happen to be at these two margins of the matching set in equilibrium.

### B. Equilibrium Distribution of Worker-Firm Matches

In Figure 5 we plot the joint distribution of worker-firm matches when the aggregate shock is at the tenth and ninetieth percentile of the shock distribution. Specifically, we plot  $E[h_t(x, y) / \iint h_t(x, y) dx dy | z_t = z_p]$ , where  $p$  indicates the tenth or ninetieth percentile of the shock distribution.

From this figure it is clear that there is substantial mass along the boundary relating to the firms' reservation worker type. In the model, fluctuations in the aggregate state translate into fluctuations in the reservation worker type that is acceptable to a given firm type. Comparing panel A and B, we see that while there is substantial mass along this boundary in both the low and high aggregate states, the share of matches near the boundary is substantially lower in the high state. A smaller share of matches are susceptible to endogenous separations, leading to a countercyclical job loss rate. The reason there are fewer matches at the boundary in the good states is that workers move more quickly to their preferred matches through on-the-job search. The process of on-the-job search, results in the second ridge in the joint distribution, the center of which corresponds to the surplus maximizing job for each worker (the dotted line in Figure 4).

Despite the random meeting of workers and firms, the equilibrium joint distribution of worker-firm matches tends to feature positive sorting. This results both from the selection of workers hired out of unemployment (meetings between high-type firms and low-type workers or between low-type firms and high-type workers never

result in matches) and also from the tendency for workers to reallocate through on-the-job search. To quantify how much dispersion around the optimal matching line search frictions and idiosyncratic job destruction shocks are responsible for, we calculate the correlation between worker and firm type among productive matches holding the aggregate shock at the mean value. There is a moderate rank correlation of 0.48 (bottom panel of Table 1). Sorting is definitely positive yet far from perfect.

### C. The Business-Cycle Dynamics of Heterogeneous Matches

The comparison of equilibrium distributions of matches for two extreme points in the cycle does not allow us to precisely quantify the interaction of aggregate shocks and heterogeneity in terms of the types of matches that are formed and dissolved and the resulting employment shares. We concisely summarize this interaction in Figure 6.

Consider the nonstochastic stationary distribution of worker-firm pairs. This is the stationary distribution of matches that is implied by  $z_t = 1$  forever. Define  $Q_1(X)$  and  $Q_3(X)$  as the first and third quartile of workers in the marginal stationary distribution, and partition workers into the three groups:  $[0, Q_1(X))$ ,  $[Q_1(X), Q_3(X))$ , and  $[Q_3(X), 1]$ , which we refer to as low, medium and high worker types. Similarly partition firms into low, medium, and high firm types. In Figure 6, each row corresponds to a different labor market outcome. The three columns correspond to the low-, medium-, and high-type workers. The three lines in each subfigure correspond to the low-type firms (solid), medium-type firms (dashed), and high-type firms (dotted).

In the top row of Figure 6, we plot the employment share for all worker and firm types as a function of the aggregate state  $z \in [0.875, 1.125]$ . These figures reveal that the expansion of employment that accompanies a move from lowest to highest aggregate productivity state is largely the result of improving employment opportunities for the low-type workers and expanded hiring by low-type firms. The employment share for low-type workers increases monotonically with  $z$  (the first panel), as does the employment share for low-type firms (the solid line), with the largest gain coming from the increased share of low-/medium-type worker, low-type firm pairs.

In the second row we plot the job separation rate by type of match. As expected, separation rates from all types of matches are (weakly) declining with the aggregate state  $z$ . It is the low-type workers who are the most susceptible to job destruction induced by a contraction in  $z$ . This is particularly true for low-type workers matched with high-type firms. The job destruction rate for these *mismatched* workers rises by 20 percent when we move from the highest to lowest aggregate state. For low-type workers matched to low-type firms, the job destruction rate only increases by 2 percent. Among medium-type workers, only those who are matched with high-type firms have an increased separation rate in low- $z$  states, and this is quite mild. High-type workers are completely shielded from separations induced by the aggregate shocks.

Finally, in the last two rows we plot the share of hires into match-types separately for workers hired from unemployment and for workers who made a job-to-job transition. The share of all workers hired by low-type firms (the solid lines in each panel) is increasing in the aggregate state for all match types, both for hires out of unemployment and for hires from other firms. The hiring patterns for medium- and high-type firms are more nuanced. If we consider matches of type

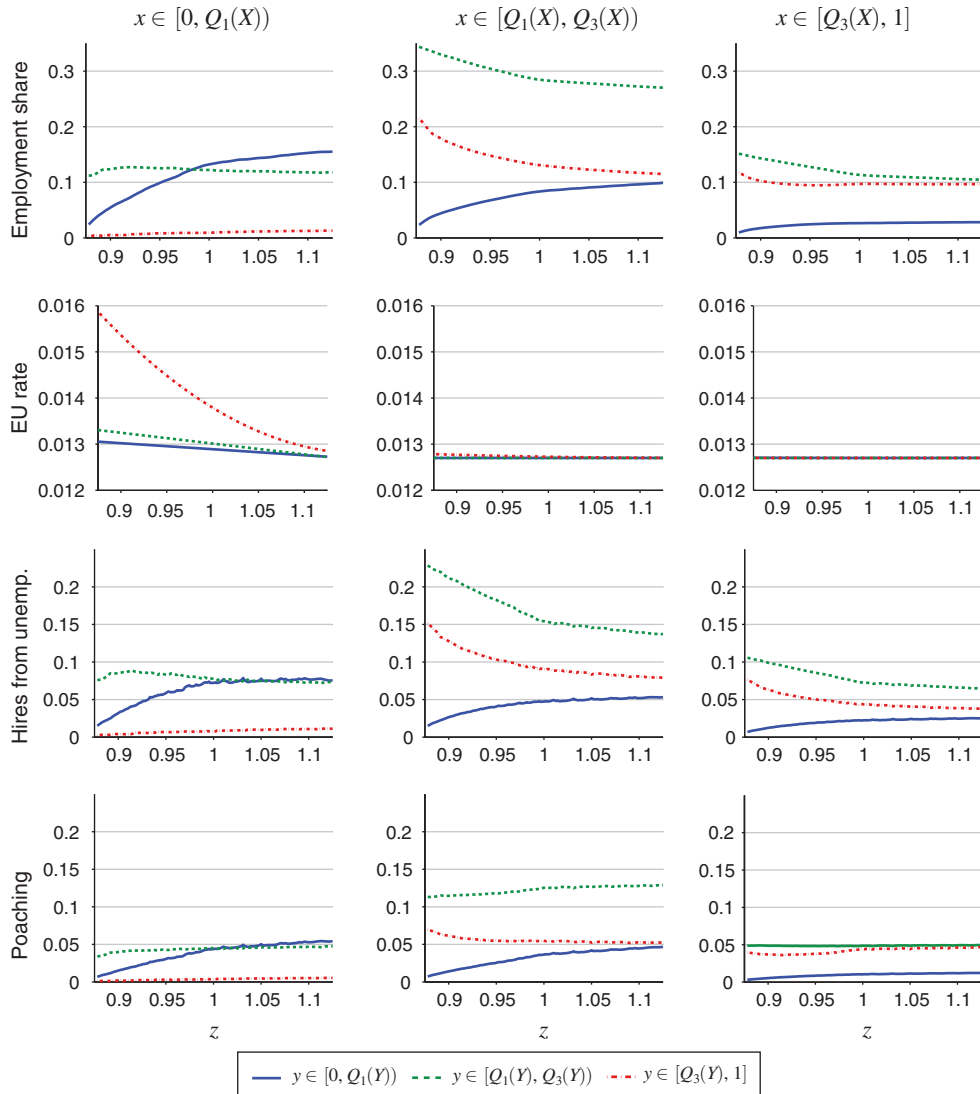


FIGURE 6. EMPLOYMENT SHARES, SEPARATION RATES, AND HIRING SHARES BY AGGREGATE SHOCKS

Notes: The columns refer to  $x \in [0, Q_1(X))$ ,  $x \in [Q_1(X), Q_3(X))$ , and  $x \in [Q_3(X), 1]$  respectively. The solid, dashed, and dotted lines correspond to  $y \in [0, Q_1(Y))$ ,  $y \in [Q_1(Y), Q_3(Y))$ , and  $y \in [Q_3(Y), 1]$  respectively. For example, the solid line in the top-left panel is equal to  $E\left[\int_{x \in [0, Q_1(X))} \int_{y \in [0, Q_1(Y))} h_t(x, y, z) dx dy / \iint h_t(x, y, z) dx dy | z\right]$ .

medium-medium (column 2, dashed line) and high-high (column 3, dotted line), we see that the share of hires into medium-medium and high-high matches who come from unemployment is decreasing with  $z$ . In contrast, the share of hires into these types of matches where the worker was previously employed is increasing in  $z$ . This reflects increased sorting from on-the-job reallocation, which speeds up when the aggregate state is high.

Summing up, when the economy enters a recession, low-type workers are fired, particularly those matched with high-type firms. At the same time, low-type firms

hire less and medium-/high-type firms hire relatively more medium-/high-type unemployed workers.

*D. The Composition of Unemployment by Unobserved Worker Types  
in the Model and by Education in the Data*

Our notion of worker-type in the model corresponds to permanent differences in productivity across workers. For the purposes of fitting the model in Section II, we treated worker-type as unobserved and estimated the distribution  $\ell(x)$ . A natural measure of permanent productivity in the data is education. Indeed we would expect the distribution of worker ability to differ by education level, with likely stochastic dominance when moving from lower to higher education levels. In panel A of Figure 7, we plot the education-specific unemployment rate from the CPS data as a function of the economy-wide unemployment rate. The unemployment rate for each education level is increasing with the overall unemployment rate. There is also a clear ordering of unemployment rates by education, with high school dropouts having the highest rates and steepest slope, high school graduates having lower rates and slope, and college graduates the lowest rates and slope.

In order to construct a similar figure from our model economy, we group workers in a way to mimic education groups. Specifically we create three broad groups with differing skill distributions. The low group is drawn from workers below the thirty-third percentile of  $\ell(x)$ ; the medium group is drawn from workers below the sixty-sixth percentile; and the high group is drawn from workers above the sixty-sixth percentile. In this way we have created groups of workers in which the distributions of skill stochastically dominate as we move from low to high. We plot the unemployment rates of our constructed groups against the overall unemployment rate in our model economy in panel B. Our group-specific unemployment rates line up very closely to the education-specific unemployment rates from the data.

Differences in the level of unemployment across education groups can result from differences in either the flows into unemployment, the flows out of unemployment, or both. In panels C and E we plot the education-specific flows from unemployment-to-employment ( $UE$ ) and employment-to-unemployment ( $EU$ ) as a function of the overall unemployment rate. For all education groups the  $UE$  rates fall and  $EU$  rates rise with the overall unemployment rate. Interestingly, while the  $UE$  rates do increase with education, the differences are quite small. The differences across education groups in the  $EU$  rate are large, with the  $EU$  rate strongly decreasing in education. We plot the corresponding  $UE$  and  $EU$  rates for our model groups in panels D and F. While our artificial grouping provides a good replication of the education differences in unemployment rates, it does less well in replicating the transitions. In particular, both the  $UE$  and the  $EU$  rates for the highest model group are too high; the model generates too much churning for this group relative to the transitions of college graduates in the data. Interestingly, we would obtain a very good approximation for all three groups if we simply multiplied the transition rates of the high group by 0.5, which would leave the unemployment rate unchanged. Of course then this group no longer corresponds to a subset of the skill distribution we estimated.

The distribution of worker heterogeneity in our model is identified using data on unemployment levels by duration of unemployment spells. We do not use wage

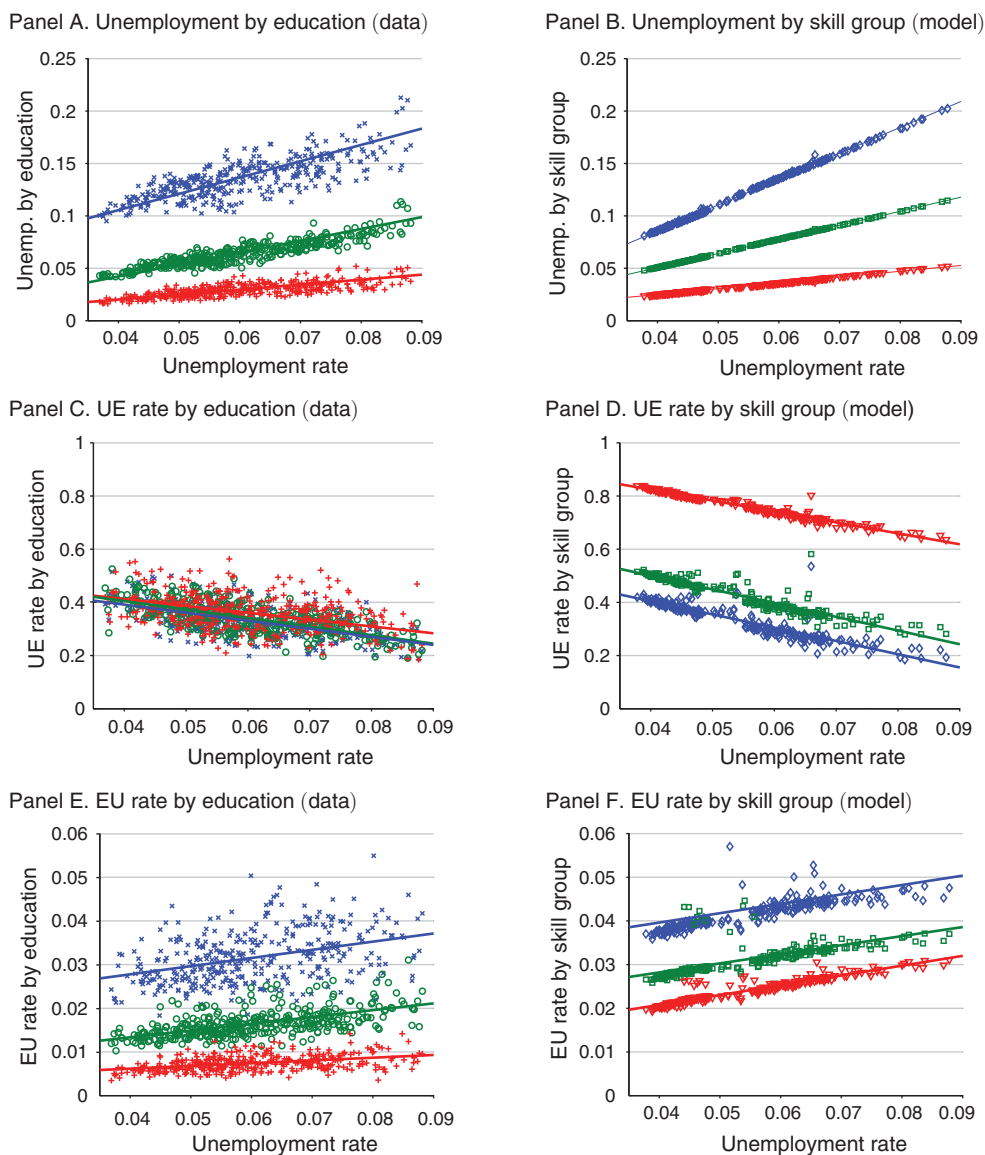


FIGURE 7. CYCLICALITY OF UNEMPLOYMENT AND TRANSITIONS

*Notes:* The three panels on the left plot the unemployment rate, the UE rate, and EU rate by education against the aggregate unemployment rate (source: CPS). The education groups are high school dropouts ( $\times$ ), high school graduates ( $\circ$ ), and college graduates ( $+$ ). The three panels on the right plot the same series for model-simulated data using constructed skill groups. The three groups are drawn from the following overlapping segments the population distribution of  $x$ : workers below the thirty-third percentile ( $\diamond$ ), workers below the sixty-sixth percentile ( $\square$ ), and workers above the thirty-third percentile ( $\nabla$ ).

data since the model does not make any predictions about wages without additional assumptions. Bilal, Chang, and Kim (2012) calibrate a Mortensen-Pissarides model with two levels of market productivity and two levels of home production, producing four worker types with approximately equal size in the population. All matches begin at the maximum possible surplus and endogenous match separations

result from idiosyncratic and aggregate productivity shocks. They calibrate worker heterogeneity to match average wages above and below the median, and to match average market hours above and below the median, and calibrate match-specific shocks to match wage variation. They conclude that heterogeneity is not capable of producing substantial volatility of unemployment in their model. The issue is that high-type workers display employment fluctuation that are too small relative to the data. The fact that our model has two-sided heterogeneity allows for the possibility that even high-type workers can be initially mismatched and as a result subject to endogenous separation in downturns. Additionally, the tractability of our model allows us to work with flexible distributions; in practice we simulate with 21 types of workers and 21 types of firms (compared to 4 *ex ante* types of workers and 1 type of firm). This flexibility is important as it allows us to model *ex ante* heterogeneity that varies continuously. With only four points of support there would be essentially no *ex ante* worker heterogeneity within quartiles. The extent to which our model is able to also match wage data is an open question. That said, we do know that the stationary version matches cross-sectional wage data very well (see Postel-Vinay and Robin 2002). We are currently in the process of specifying and estimating a richer version of the model that incorporates more direct empirical measures of worker and firm heterogeneity, such as measures based on education, occupation, wages, value added, and other conditional measures available in matched employer-employee data.

#### IV. Concluding Remarks

In this paper we develop an equilibrium on-the-job search model of the labor market featuring aggregate uncertainty and *ex ante* heterogeneity in worker and firm types. We show that the model has a recursive structure that makes it very tractable. We fit the model to US time-series from 1951 to 2012 to illustrate the ability of the model to generate aggregate business-cycle dynamics. We then explore the interaction between heterogeneity and aggregate shocks in the fitted model in terms of the cyclical properties of the distributions of vacancies, unemployed workers, and worker-firm matches.

In ongoing work we develop the implications of the model for the business-cycle fluctuations in the joint distribution of wages over worker-firm matches. Using the wage-distribution implications (which requires more structure on wage setting than what is assumed in the current paper) will allow us to incorporate richer micro-level shocks at the worker and firm level, and permits estimation using linked employer-employee data.

#### APPENDIX A

##### A1. Model Extensions

In the body of the paper we have restricted our attention to a single source of aggregate volatility modeled as an aggregate productivity shock  $z$ . The one-shock model is sufficient to demonstrate the theoretical result and generates sufficiently rich aggregate dynamics to illustrate how the aggregate shock interacts with the

heterogeneity. While the one-shock model is useful for the purposes of the current paper, further empirical work based on microdata will certainly want to incorporate additional stochastic processes and sources of nonstationarity. We briefly discuss how the model can accommodate these extensions, while preserving the recursive equilibrium that makes the model tractable.

*Aggregate Growth.*—All allocations and decisions are identical in a model where market production, home production, and the cost of posting vacancies all grow at the same rate  $g$ .

*Idiosyncratic Shocks.*—Shocks to worker skill and/or firm TFP are easily incorporated by modifying the surplus function

$$S(x, y, z) = s(x, y, z) + \frac{1 - \delta}{1 + r} E_{x', y', z'}[S(x', y', z') | x, y, z],$$

and augmenting the flow equations (8) and (9) appropriately.

*Nonstationarity in the Meeting and Posting Technology.*—One implication of the recursive structure of the equilibrium is that the future is fully summarized by  $S(x, y, z_t)$ , making the vacancy creation problem effectively static (there is no fixed point to solve). This implies that the meeting technology and the cost of posting vacancies can vary with time. For example, we can allow for time-varying aggregate matching efficiency  $\alpha_t$ , or time-varying posting costs  $c_{0t}$ .

*Nonstationary Distribution of Worker Skills.*—For simplicity in Section I, we assumed that the distribution of worker skills in the population was given by the time-invariant distribution  $\ell(x)$ . This is unlikely to be a good approximation due to the fact that the workforce has become much more educated over time, as well as the fact that the flow of new immigrants into a country is likely to have a very different skill composition than the native population: for example, we may expect it to be overrepresentative of the very low and very high skilled. Such nonstationarity can easily be accommodated by introducing a birth-death process for workers. At the beginning of each period, all workers die with probability  $\mu$  and are replaced with newborn unemployed workers  $n_t(x)$ . The flow equations (4) and (5) become

$$\begin{aligned} u_{t+}(x) &= (1 - \mu)u_t(x) + n_t(x) \\ &+ \int [\mathbf{1}\{S_t(x, y) < 0\} + \delta \mathbf{1}\{S_t(x, y) \geq 0\}](1 - \mu)h_t(x, y) dy, \\ h_{t+}(x, y) &= (1 - \delta)(1 - \mu)\mathbf{1}\{S_t(x, y) \geq 0\}h_t(x, y), \end{aligned}$$

and the effective discount factor in the surplus function is adjusted to incorporate the mortality rate. It is possible to accommodate both population growth and an evolving skill distribution via an evolving flow of newborn workers  $n_t(x)$ .

The model can accommodate both substantial heterogeneity and nonstationary while remaining tractable. Of course the data requirements necessary to identify such a flexible model will be great and require further investigation.

## APPENDIX B: DATA AND ESTIMATION DETAILS

### B1. Data Moments

Wherever possible we use publicly available aggregate data. The unemployment data are from the US Bureau of Labor Statistics (BLS) and cover the period 1951:1 to 2012:12. We use the BLS series of seasonally adjusted monthly employment and unemployment levels for all persons aged 16 and over. In addition to the number of unemployed, we also use the number of unemployed with unemployment durations of more than 5, 15, and 27 weeks.<sup>12</sup> We divide the unemployment levels each month by the sum of unemployment and employment to obtain rates. This gives us monthly series for  $U_m$ ,  $U_m^5$ ,  $U_m^{15}$ , and  $U_m^{27}$  corresponding to the fraction of individuals unemployed and the fraction unemployed more than 5, 15, and 27 weeks, where the  $m$  subscript refers to a monthly frequency.

From these series we construct monthly transition rates between unemployment and employment and between employment and unemployment as follows:

$$UE_m = 1 - U_{m+1}^5/U_m, \quad EU_m = (U_{m+1} - U_{m+1}^5)/E_m.$$

The BLS does not provide a series for job-to-job transitions. We construct this series using the Current Population Survey (CPS) for the period 1994:1 to 2012:12. This series is constructed following Moscarini and Thomsson (2007).

A time series of monthly vacancies can be constructed by combining the BLS monthly Help Wanted Index (HWI) 1951:1 to 2006:12 and the Job Openings and Labor Turnover Survey (JOLTS) 2001:1–2012:12.<sup>13</sup> To obtain a consistent series we project the HWI series onto the JOLTS series for the overlapping months in the years. We then obtain a combined series using predicted HWI based on the JOLTS to put them into the same scale. We rescale the vacancy series so that the mean vacancy to unemployment rate matches Table 1 of Hagedorn and Manovskii (2008). This normalization fixes the scale for the vacancy posting costs.

Output data are provided on a quarterly frequency. We use the BLS quarterly series 1951:I–2012:IV of seasonally adjusted real value added in the nonfarm business sector.<sup>14</sup> Since the value-added series are only provided at the quarterly frequency, we aggregate all series up to this frequency by taking the quarterly average for the monthly series. We HP-filter the log-transformed quarterly series using smoothing parameter  $10^5$  (Shimer 2005).

Finally, we add moments for the mean and standard deviation of productivity dispersion, as well as the correlation with output growth at an annual frequency from Bloom et al. (2014, Table A1 and supplementary data).

<sup>12</sup> Series LNS12000000, LNS13000000, LNS13008396, LNS13008756, LNS13008516, and LNS13008636

<sup>13</sup> Series JTS00000000JOL

<sup>14</sup> Series PRS85006043

## B2. Model Simulated Moments

Using the subscript  $t$  to denote weekly time-series simulated from the model, we first construct the weekly series of aggregate value added, (un)employment and vacancies,

$$\begin{aligned} U_t &= \int u_t(x) dx, & V_t &= \int v_t(y) dy, \\ E_t(y) &= \int h_t(x, y) dx, & E_t &= \int E_t(y) dy, \\ p_t(y) &= \int p(x, y, z_t) h_t(x, y) dx, & p_t &= \int p_t(y) dy. \end{aligned}$$

We calculate the weekly series of the number of unemployed workers with durations of 5, 15, and 27 weeks or more as

$$U_t^s = \int u_{t-s}(x) \prod_{j=0}^{s-1} \left[ 1 - \int \lambda_{t-s+j} \frac{v_{t-s+j}(y)}{V_{t-s+j}} \mathbf{1}\{S(x, y, z_{t-s+j}) \geq 0\} dy \right] dx,$$

where  $U_t^s$  is the number of unemployed workers at period  $t$  who have been unemployed for  $s$  periods (weeks) or more. We then construct monthly transition rates exactly as we did from the BLS data,

$$\begin{aligned} UE_m &= 1 - U_{m+1}^5 / U_m, \\ EU_m &= (U_{m+1} - U_{m+1}^5) / E_m, \end{aligned}$$

where the subscript month corresponds to the same week which would be sampled by the BLS.<sup>15</sup> The monthly job-to-job transition rate is constructed to match the construction from the CPS and is calculated as the sum of weekly transition rates within a month,

$$EE_m = \sum_{t \in m} EE_t.$$

The resulting series are aggregated exactly as the data to obtain the corresponding quarterly moments. For example, quarterly output is obtained by summing over output from the 13 weeks in the quarter and quarterly unemployment rates are obtained by averaging over the monthly rates in the quarter.

To calculate the standard deviation of productivity dispersion, we define annual output-per-worker at jobs of type  $y$  as

$$\varpi_a(y) = \log \left( \sum_{t \in a} p_t(y) / \sum_{t \in a} E_t(y) \right),$$

<sup>15</sup>The BLS survey is done each month in the week containing the twelfth day of the month. We sample from our simulated data to replicate this strategy. For example, in the first year of the simulation we sample the monthly data from simulation weeks {2, 7, 11, 15, 19, 24, 28, 33, 37, 42, 46, 50}.

where  $a$  indexes year, and  $\overline{\varpi}_a = \int \varpi_a(y) dy$ . We calculate our annual series of the cross-sectional standard deviation of value added per worker across firms as

$$\text{sd labor prod}_a = \left( \int (\varpi_a(y) - \overline{\varpi}_a)^2 dy \right)^{\frac{1}{2}}.$$

We fit the model to moments of the US data from 1950:I to 2012:IV.<sup>16</sup> Given the specification above, we have 17 parameters to determine:  $\alpha$ ,  $\omega$ ,  $s$ ,  $c_0$ ,  $c_1$ ,  $\delta$ ,  $\sigma$ ,  $\rho$ ,  $\beta_1$ ,  $\beta_2$ ,  $p_1$ ,  $\dots$ ,  $p_6$ , and  $r$ . We fix the discount rate to 5 percent annually, and fix  $\omega$  at 0.5.<sup>17</sup> This leaves 15 parameters which are estimated to best fit the 28 moments.

To obtain simulated time-series we begin with a distribution of workers across employment states and jobs implied by the economy in the absence of any aggregate shock. We then simulate the economy for 700 years at a weekly frequency, discard the first 100 years to reduce the impact of initial conditions, aggregate to quarterly or annual data, and calculate the simulated moments. The GMM objective function is nonsmooth and displays many local minima. We use a variant of simulated annealing and many starting values to address these issues.

While none of the parameters has a one-to-one relationship to a moment, we can provide a heuristic description of identification. The main variation can be described as follows: the mobility parameters  $\alpha$ ,  $s$ , and  $\delta$  are identified by the average rates at which workers transit between unemployment and employment, between jobs, and from employment to unemployment. The parameters of the latent productivity shock,  $\sigma$  and  $\rho$ , are identified by the standard deviation and autocorrelation of output (corrected for selection via the model). The flow cost of creating new vacancies,  $c_0$  and  $c_1$ , governs the response of vacancies to changes in profitability and is identified by the standard deviation of vacancies and the correlation of vacancies with output.

The last set of parameters  $\beta_i$  and  $p_k$  govern the distribution of worker types in the population and value added. The distribution of worker types is identified by the pattern in the number of workers unemployed 5, 15, and 27 or more weeks (homogeneous workers would imply this distribution is exponential). The contribution of firm type to value added can be then identified by the cross-sectional variation in value added per job. Correlations over time between the various measurement variables finally determine the last remaining parameters governing complementarities in the production function.

In practice, the standard errors of parameter estimates convey information about local identification for our sample data. The standard errors reflect not only the sampling variability of the data moments, but also depend on the partial derivative of each simulated moment with respect to each estimated parameter.<sup>18</sup>

<sup>16</sup>The productivity moments are based on data from 1971 to 2011 and the job-to-job transitions moments on data from 1994 to 2011. See the Appendix for details on constructing the covariance matrix for the moments in the presence of missing data.

<sup>17</sup>Without using direct information on the costs of vacancy creation, there is little hope to separately identify the vacancy cost function from the meeting function.

<sup>18</sup>Think of the OLS formula for the variance of the estimator:  $\sigma^2(X^\top X)^{-1}$ . Parameters are imprecisely estimated if the error variance  $\sigma^2$  is large or if the model is weakly identified because of near-multicollinearity (i.e.  $X^\top X$  is near-singular). See Appendix BB3 for the precise expression of the asymptotic variance of the estimates in the GMM case of this paper.

### B3. The GMM Estimator

Let  $\hat{m} = (\hat{m}_1, \dots, \hat{m}_N)$  denote the  $N \times 1$  vector of data moments that we want to fit. Let  $m(\theta) = (m_1(\theta), \dots, m_N(\theta))$ , for  $\theta$  a vector of  $K$  parameters, be the corresponding theoretical moments. We estimate  $\theta$  by

$$\hat{\theta} = \arg \min_{\theta} \sum_{i=1}^N \omega_i \left( \frac{\hat{m}_i - m_i(\theta)}{\hat{m}_i} \right)^2,$$

where  $\omega_i$  are fixed weights and we standardize by the moments themselves instead of their standard errors, which are less precisely estimated, particularly when the variables are autocorrelated. Under standard regularity conditions,

$$\left[ \sum_{i=1}^N \frac{\omega_i}{\hat{m}_i^2} \frac{\partial m_i(\theta_0)}{\partial \theta} \frac{\partial m_i(\theta_0)}{\partial \theta^\top} \right] (\hat{\theta} - \theta_0) = \sum_{i=1}^N \frac{\omega_i}{\hat{m}_i^2} (\hat{m}_i - m_i(\theta_0)) \frac{\partial m_i(\theta_0)}{\partial \theta} + o_P(1).$$

Assuming that, for a large sample size  $T$ ,

$$\hat{m} \sim \mathcal{N}(m(\theta_0), \hat{\Sigma}),$$

with  $\hat{\Sigma} = o_P(1/T)$ , then

$$\hat{\theta} \sim \mathcal{N}(\theta_0, \hat{J}^{-1} \hat{I} \hat{J}^{-1}),$$

for

$$\begin{aligned} \hat{J} &= \sum_{i=1}^N \frac{\omega_i}{\hat{m}_i^2} \frac{\partial m_i(\hat{\theta})}{\partial \theta} \frac{\partial m_i(\hat{\theta})}{\partial \theta^\top} = \hat{M}^\top \hat{\Omega} \hat{M}, \quad \hat{M} = \frac{\partial m(\hat{\theta})}{\partial \theta^\top} \\ \hat{\Omega} &= \text{diag}\left(\frac{\omega_1}{\hat{m}_1^2}, \dots, \frac{\omega_L}{\hat{m}_L^2}\right), \quad \hat{I} = \hat{M}^\top \hat{\Omega} \hat{\Sigma} \hat{\Omega} \hat{M}. \end{aligned}$$

### B4. The Variance-Covariance Matrix of the Vector of Moments

The vector of moments consists in sample averages, standard deviations, and correlations of some vector  $y_t = (y_{1t}, \dots, y_{Lt})$  of variables. Let

$$f_{1i}(y_t, m) = y_{it} - \mu_i, \quad f_{2ij}(y_t, m) = (y_{it} - \mu_i)(y_{jt} - \mu_j) - \rho_{ij} \sigma_i \sigma_j,$$

where  $m = [\mu; \sigma; \rho]$  is the vector of parameters,<sup>19</sup> for  $\mu = (\mu_1, \dots, \mu_L)$  the vector of means,  $\sigma = (\sigma_1, \dots, \sigma_L)$  the vector of standard deviations, and  $\rho = (\rho_{ij})_{i>j}$  the vector of (nontrivial) correlations.

The vector of moments  $\hat{m}$  is obtained as the solution to

$$\hat{E} f_{1i}(y_t, \hat{m}) = \frac{1}{T} \sum_{t=1}^T f_{1i}(y_t, \hat{m}) = 0, \quad \hat{E} f_{2ij}(y_t, \hat{m}) = 0, \quad \forall i, j.$$

<sup>19</sup>  $x = [x_1; x_2]$  denotes the vertical stacking of vectors  $x_1, x_2$ .

Let  $\mathbf{f}_1 = (f_{11}, \dots, f_{1L})$  and  $\mathbf{f}_2 = (f_{2ij})_{i \geq j}$ . Let also  $\mathbf{f} = [\mathbf{f}_1; \mathbf{f}_2]$ .

First, we calculate the Jacobian of the transformation  $m \mapsto \hat{\mathbf{E}}\mathbf{f}(y_t, m)$ , that is

$$\hat{D} = \left. \frac{\partial \hat{\mathbf{E}}\mathbf{f}(y_t, m)}{\partial m^\top} \right|_{m=\hat{m}}.$$

We have

$$\left. \frac{\partial \hat{\mathbf{E}}f_{1i}(y_t, m)}{\partial \mu_k} \right|_{m=\hat{m}} = -\mathbf{1}_{i=k},$$

where  $\mathbf{1}_{i=k} = 1$  if  $i = k$  and 0 otherwise,

$$\left. \frac{\partial \hat{\mathbf{E}}f_{1i}(y_t, m)}{\partial \rho_{k\ell}} \right|_{m=\hat{m}} = \left. \frac{\partial \hat{\mathbf{E}}f_{2ij}(y_t, m)}{\partial \mu_k} \right|_{m=\hat{m}} = 0,$$

and

$$\left. \frac{\partial \hat{\mathbf{E}}f_{2ij}(y_t, m)}{\partial \sigma_k} \right|_{m=\hat{m}} = -\rho_{ij} [\mathbf{1}_{i=k} \sigma_j + \mathbf{1}_{j=k} \sigma_i], \quad \left. \frac{\partial \hat{\mathbf{E}}f_{2ij}(y_t, m)}{\partial \rho_{k\ell}} \right|_{m=\hat{m}} = -\mathbf{1}_{ij=k\ell} \sigma_i \sigma_j.$$

Second, we need to estimate the variance of  $\mathbf{f}(y_t, m)$ . Given the autocorrelated nature of  $y_t$ , we use the Newey-West estimator

$$\hat{S} = \sum_{p=-q}^q \frac{q-|p|}{q} \sum_{t=1+p}^{T-p} \mathbf{f}(y_t, \hat{m}) \mathbf{f}(y_{t-p}, \hat{m})^\top,$$

where  $q$  is of the order of  $T^{-1/3}$ . Start low and increase progressively until  $\hat{S}$  stabilizes.

Finally, we can estimate the asymptotic variance of  $\hat{m}$  as

$$\hat{\Sigma} = (\hat{D}^\top \hat{S}^{-1} \hat{D})^{-1}.$$

### B5. Implementation Details

In practice, we use a lag order of 8 in the Newey-West covariance estimator. The fixed weights used in estimation  $\omega$ , are equal to 100 for  $E[U]$  and  $\text{sd}[VA]$ ; 10 for  $E[J2J]$ ,  $E[\text{sd labor prod}]$ ,  $\text{sd}[U]$ ,  $\text{sd}[V/U]$ ,  $\text{sd}[\text{sd labor prod}]$ ,  $\text{sd}[V]$ ,  $\text{autocorr}[VA]$ ,  $\text{corr}[\text{sd labor prod}, VA]$ ; and 1 for all other moments. These weights were selected to ensure the model replicated the variability and persistence of output, the level and variability of the unemployment rate and variability of vacancy creation: moments that have been of primary interest in the related literature. The calculation of standard errors outlined above fully accounts for the use of these fixed weights.

The moments are not necessarily smooth functions of the parameters due to simulation noise. To estimate the derivative of each moment with respect to each parameter  $\partial m(\hat{\theta})/\partial \theta$ , we simulate the model for 101 equally spaced values for each parameter  $\theta_k \in [0.5\hat{\theta}_k, 1.5\hat{\theta}_k]$  holding all other parameters at their estimated values, and saving the vector of moments for each evaluation. We then fit a polynomial of degree 9 for each moment as a function of each parameter. The derivative of this polynomial evaluated at  $\hat{\theta}$  is our estimate of  $\partial m(\hat{\theta})/\partial \theta$ .

Finally, missing data for the job-to-job and productivity series means that the cells of the covariance matrix of the data are calculated using only the maximal available data for each cell. As a result the covariance matrix  $\hat{S}$  is not guaranteed to be positive semidefinite (although it will be asymptotically). To ensure invertability, we multiply the diagonal of  $\hat{S}$  by  $1 + \lambda$  where  $\lambda$  is chosen as small as possible such that all eigenvalues of  $\hat{S}$  are positive. In the current case  $\lambda = 0.6003$ . As a robustness check, we calculate the standard errors of the moments using only the data available for all series and find only minor differences.

## REFERENCES

- Barlevy, Gadi. 2002. "The Sullyng Effect of Recessions." *Review of Economic Studies* 69 (1): 65–96.
- Bils, Mark, Yongsung Chang, and Sun-Bin Kim. 2012. "Comparative Advantage and Unemployment." *Journal of Monetary Economics* 59 (2): 150–65.
- Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry. 2014. "Really Uncertain Business Cycles." Center for Economic Studies Working Paper 14-18.
- Burdett, Kenneth, and Dale T. Mortensen. 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review* 39 (2): 257–73.
- Coles, Melvyn G., and Dale T. Mortensen. 2016. "Equilibrium Labor Turnover, Firm Growth, and Unemployment." *Econometrica* 84 (1): 347–63.
- Hagedorn, Marcus, and Iourii Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review* 98 (4): 1692–1706.
- Hall, Robert. 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review* 95 (1): 50–65.
- Jolivet, Grégory, Fabien Postel-Vinay, and Jean-Marc Robin. 2006. "The Empirical Content of the Job Search Model: Labor Mobility and Wage Distributions in Europe and the US." *European Economic Review* 50 (4): 877–907.
- Kaas, Leo, and Philipp Kircher. 2015. "Efficient Firm Dynamics in a Frictional Labor Market." *American Economic Review* 105 (10): 3030–60.
- Lise, Jeremy, and Jean-Marc Robin. 2017. "The Macrodynamics of Sorting between Workers and Firms: Dataset." *American Economic Review*. <https://doi.org/10.1257/aer.20131118>.
- Menzio, Guido, and Shouyong Shi. 2010a. "Block Recursive Equilibria for Stochastic Models of Search on the Job." *Journal of Economic Theory* 145 (4): 1453–94.
- Menzio, Guido, and Shouyong Shi. 2010b. "Directed Search on the Job, Heterogeneity, and Aggregate Fluctuations." *American Economic Review* 100 (2): 327–32.
- Menzio, Guido, and Shouyong Shi. 2011. "Efficient Search on the Job and the Business Cycle." *Journal of Political Economy* 119 (3): 468–510.
- Moscarini, Giuseppe, and Fabien Postel-Vinay. 2010. "Unemployment and Small Cap Returns: The Nexus." *American Economic Review* 100 (2): 333–37.
- Moscarini, Giuseppe, and Fabien Postel-Vinay. 2013. "Stochastic Search Equilibrium." *Review of Economic Studies* 80 (4): 1545–81.
- Moscarini, Giuseppe, and Kaj Thomsson. 2007. "Occupational and Job Mobility in the US." *Scandinavian Journal of Economics* 109 (4): 807–36.
- Pissarides, Christopher A. 1985. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *American Economic Review* 75 (4): 676–90.
- Postel-Vinay, Fabien, and Jean-Marc Robin. 2002. "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity." *Econometrica* 70 (6): 2295–2350.
- Postel-Vinay, Fabien, and Hélène Turon. 2010. "On-The-Job Search, Productivity Shocks, and the Individual Earnings Process." *International Economic Review* 51 (3): 599–629.

- Robin, Jean-Marc.** 2011. "On The Dynamics of Unemployment and Wage Distributions." *Econometrica* 79 (5): 1327–55.
- Rudanko, Leena.** 2011. "Aggregate and Idiosyncratic Risk in a Frictional Labor Market." *American Economic Review* 101 (6): 2823–43.
- Sahin, Aysegül, Joseph Song, Giorgio Topa, and Giovanni L. Violante.** 2014. "Mismatch Unemployment." *American Economic Review* 104 (11): 3529–64.
- Schaal, Edouard.** 2015. "Uncertainty and Unemployment." [https://sites.google.com/site/edouardschaal/research/schaal\\_uncertainty\\_unemployment.pdf](https://sites.google.com/site/edouardschaal/research/schaal_uncertainty_unemployment.pdf) (accessed February 2017).
- Shi, Shouyong.** 2009. "Directed Search for Equilibrium Wage-Tenure Contracts." *Econometrica* 77 (2): 561–84.
- Shimer, Robert.** 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95 (1): 25–49.

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