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Marking to Market versus Taking to Market*

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Abstract

Building on the idea that accounting matters for corporate governance, this paper studies the equilibrium interaction between the measurement rules that firms find privately optimal (which interestingly involve gains trading), firms' governance, and the liquidity in the secondary market for their assets. This equilibrium approach reveals a socially excessive use of market-value accounting: Corporate performance measures rely excessively on the information generated by other firms' asset sales and insufficiently on the realization of a firm's own latent capital gains. This dries up market liquidity and reduces the informativeness of price signals.

Keywords: corporate governance, agency, accounting measures, gains trading.

JEL numbers: D82, M41, M52.

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1 Introduction

Accounting statements are the primary source of verified information that firms provide to their stakeholders, and therefore an important ingredient of corporate governance. Accounting measurements are in particular explicit inputs in executive compensation contracts, debt covenants, and regulations such as prudential rules for financial institutions. They also play a more implicit but pervasive role in the enforcement of stakeholders' rights during events that are defining for corporations, such as takeovers, proxy contests, bankruptcy procedures. In venture-capital (VC) funding, limited partners rely on marks provided by the general partners in order to track progress, review allocations to the asset class, determine compensation and fulfil their own reporting requirements. Measurements from the last funding round become stale and provide only very imperfect estimates of the value of the VC's portfolio investments. Due to their illiquidity, firms often prefer to stay with the same financiers rather than to sell (take to market) for the next round of financing. VC then use fair value accounting to measure the value of the portfolios.¹

Amidst a global debate that has been raging for years, accounting conventions have evolved from the use of historical costs towards "fair-value" measurements of assets and liabilities. The International Accounting Standard Board defines fair value as "the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date."² This contrasts with historical-cost accounting whereby, broadly, items remain recorded at their entry value (net of amortizations) instead of reflecting all relevant data accruing from markets for similar items.³

The goal of this paper is to offer a framework for the study of accounting measures that builds on the primitive ingredients of information economics. We introduce accounting measurements as important corporate governance tools, and we determine the extent to which they should reflect market data. We are particularly interested in the mutual feedback between the design of privately optimal measurements by firms, and the efficiency of the secondary market for the items in their balance sheets.

Our starting point is a standard agency model of corporate finance in which the outside stakeholders of a firm need to provide inside stakeholders with incentives to figure out a value-maximizing strategy, which we simply model as the selection of a good project/asset. Insiders' rewards (control rights, rents from office, pecuniary transfers,...) must be decided before the asset pays off, and must therefore be based on measurements of this payoff.

¹Sloan (2001) surveys these implicit and explicit uses of accounting information in corporate governance and the related empirical evidence.

²See International Financial Reporting Standard 13.

³Some impairment rules may take sufficiently negative market signals into account under historical-cost accounting. By contrast, mildly negative or positive market signals per se do not lead to a change in book value.

Two such measurements are available to outsiders. The latter can avail themselves of a costless but noisy public signal from a market for similar assets. They can also obtain a costly measure of asset value by reselling the asset to imperfectly competitive informed buyers. Insiders are in charge of such resales and privately observe the informed bids received by the firm for its project. Outsiders observe only whether the project is sold and the price at which it is sold.

In the full-fledged equilibrium model, we solve for informed buyers' equilibrium bidding strategies, and endogenize the public market signal as publicly observable transactions by other firms seeking to resell comparable assets. The initial partial equilibrium approach of Section 2 by contrast takes the precision of the market signal and the distribution of informed bids for the asset as exogenous, and solves for the (privately) optimal contract.

This optimal contract has the following simple structure. If the market signal is above a threshold, then insiders get rewarded through sustained control or monetary compensation. If the signal is below this threshold, insiders are allowed to resell the asset above a given reserve price, and get rewarded if the sale is actually executed above this price. This abstract contract admits a realistic implementation, whereby insiders are rewarded if and only if a carefully designed accounting measurement of the project is above a threshold. The important feature of the measurement is its degree of conservatism. Under a more conservative regime, the book value of the project recognizes the latent capital gains suggested by market data less aggressively. Insiders are therefore induced to realize these latent gains more often in order to get rewarded. In the limit of a most conservative regime, only realized gains are recognized, as is the case under pure historical-cost accounting.

Interestingly, this (privately) optimal contract trades off costs that closely mirror those mentioned by each side in the policy debate on fair-value accounting. Advocates of fair-value accounting have argued that historical-cost accounting induces distortions such as costly and unnecessary realizations of latent capital gains by firms ("gains trading"), whereas the opponents of fair value point at the irrelevant noise that market data may add to corporate performance measure. Accordingly, our optimal contract (and accounting measure) minimizes the sum of two types of costs, those incurred when validating insiders' claims with an ex-post inefficient asset resale and the costs induced by the inefficient reward of insiders based on noisy market data absent such a resale ("reward for luck").

Section 3 then endogenizes the bids that firms receive for their projects. Firms can sell their assets to the informed buyers in a decentralized market. Buyers are randomly matched to firms. Without observing how many fellow buyers are matched with a given firm, they submit bids. In equilibrium, the stakeholders of each firm adopt an optimal contract based on their beliefs about the liquidity in the market for their assets, and informed buyers submit bids rationally anticipating such contracts. There can be two equilibrium

degrees of liquidity in the market for assets, that correspond to two equilibrium degrees of conservatism used by firms when recognizing latent gains in their contracts. There exists a liquid, constrained-efficient equilibrium in which firms set high reserve prices when allowing resales, are conservative when recognizing latent gains in their books, and in which informed buyers bid aggressively. There may also exist an illiquid equilibrium in which firms rely too aggressively on market data, allow for asset resales too rarely at deeply discounted reserve prices, and in which informed buyers accordingly submit low bids. This illiquid equilibrium comes with a higher cost of capital for firms. It is unstable in the following sense. A regulation that forces firms to use (even a slightly) higher degree of conservatism than they find privately optimal when recognizing latent gains based on market data leads the constrained-efficient equilibrium to be the unique equilibrium.

Finally, Section 4 endogenizes the competitiveness of the asset market among informed buyers through a free-entry condition. It also endogenizes market signals as publicly observed asset resales by firms. When liquidity is endogenous, laissez-faire can no longer be constrained efficient. It leads to an excessive reliance on market data by firms in the form of an overly aggressive recognition of latent gains when measuring performance. The reason is that firms fail to internalize the effect of their accounting conventions on the liquidity of the items that they seek to measure, where liquidity is defined both in terms of ease of trading and of the informativeness of price signals. Under laissez-faire, firms contract too much on transactions by other firms. They sell their own assets too rarely, and at deep discounts when they do so. A regulation that forces them to adopt more conservative accounting measures reduces their cost of capital by spurring informed buyers' entry. Taking to market is more efficient because asset resales occur at higher prices. So is marking to market because resale prices are more informative.

Section 5 provides further discussion and discusses alleys for future research.

Related Literature

This paper relates to the burgeoning accounting literature on the real effects of accounting regimes. Marinovic (2016) in particular studies the effect of the measurement regime applied to an asset on the outcome of an auction for that asset.⁴ We extend this literature by developing a full-fledged economic theory of optimal accounting measures. In our framework, both corporate governance mechanisms, including measurement regimes, and liquidity in the markets for balance sheet items are the endogenous outcome of equilibrium optimizing behaviors by all agents.

The three sections of the paper each relate to different literatures. Section 2 that

⁴Other contributions include Allen-Carletti (2008), Bleck-Gao (2012), Bleck-Liu (2007), Ellul et al. (2015), Heaton et al. (2010), Laux-Leuz (2010), Otto-Volpin (2015), and Plantin et al. (2008).

derives firms' privately optimal contracts is most related to the agency literature on informativeness of performance measurement. Holmström (1979) proves that incentives should be based solely on a sufficient statistic of unobservable effort. Kim (1995) shows that information systems are ranked if the likelihood ratio distribution of an action choice under an information system is a mean-preserving spread of the likelihood ratio distribution under the other. Section 2 derives the optimal mix between using a free, but noisy external signal and using a costly, but more precise one obtained through resale. The paper shares with the literature on costly state verification initiated by Townsend (1979) and with the "variance-investigation" literature in accounting the idea that the optimal agency contract uses costly inspections so as to verify the agent's claim. Whereas negative reports lead to verification in this literature, positive reports do in our setup. Dye (1986) studies a principal-agent model in which the principal faces the related problem of optimally combining two sources of information, the agent's output and a direct but costly verification of her effort level. Our paper interprets the "verification cost" as a discount on an asset resale, and to the best of our knowledge, is the first to derive marking to market and gains trading as optimal features of an optimized information system.

Section 3 endogenizes resale costs by positing a matching process and a first-price auction among bidders. The reserve price is secret as in Elyakime et al (1994), in which the number of bidders is unlike here known.⁵ The key new feature relative to the auctions literature is that the seller does not have a set valuation. Rather, the reserve price is derived from a contracting problem, where the transaction cost of selling the asset is compared with the imprecision of the market signal and both jointly contribute to set the agent's incentives.

Like the literature on auctions with an endogenous number of bidders (e.g. Levin-Smith 1994, Jehiel-Lamy 2015b), Section 4 endogenizes entry through a zero expected profit condition. And the seller benefits from a liquid market.⁶ The novelty is that liquidity depends on the accounting choices made by the other firms. This externality is at the core of our welfare analysis.

One can also draw an interesting analogy with the literature on thick-market externalities (Admati-Pfleiderer 1988, Pagano 1989). In that literature, investors with liquidity needs who are able to select their trading date prefer to bunch with other liquidity traders as this limits the ability of informed buyers to exploit mispricing and further may induce more competition among informed buyers. A common feature with our paper is that sell-

⁵In their framework, Elyakime et al. prove that the seller would be strictly better off if he could commit to a reserve price. The same holds in our framework as well. Jehiel and Lamy (2015a) endogenize secret reserve prices through heterogeneous seller valuations.

⁶In Aghion et al (2004), the firm elicits buyer interest in the asset (which is also sold for performance measurement purposes) by forcing passive shareholders to tender their shares (drag-along institution). Here liquidity relies not on an internal design, but from accounting choices made by other firms.

ers’ decisions (when to trade, extent of fair-value accounting) affect the welfare of other sellers through the impact on informed trading. For instance, Admati and Pfleiderer endogenize information acquisition through a free-entry condition and show that the patterns of trading volume that exist in the model with a fixed number of informed traders become more pronounced if the number of informed traders is endogenous. Besides the obvious differences in focus (intraday trading volatility vs. accounting choices) and modelling (our model captures the decision of whether to bring the asset to the market rather than the choice of when or where to bring it to the market), our paper emphasizes the benefit (performance measurement), rather than the cost of informed trading.

2 Marking to market versus taking to market: A simple framework

2.1 Model

There are three dates 0, 1, 2. There are two parties, a principal and an agent, involved in a project—a “firm”—that is initiated at date 0 and pays off at date 2. The principal stands for outsiders—the constituencies that have a stake in the firm but do not operate its assets, such as diffuse shareholders, arm’s length creditors, or a prudential supervisor in the case of financial institutions. The agent stands for insiders—the stakeholders who run the firm or closely oversee its operations, such as controlling blockholders, directors, or top managers.

The principal does not discount time and is risk neutral over consumption at each date. The cashless agent derives utility at date 1 only. The principal can provide the agent with any utility level $u \in [0, 1]$ at date 1 at a monetary cost u . The important feature is that the agent’s utility cannot be costlessly backloaded to date 2, so that date-1 measurements of the final cash flow matter for performance evaluation.⁷

This date-1 utility transfer from the principal to the agent lends itself to three standard interpretations that we will use when discussing the practical implications of the optimal contract for accounting measures:

- *Interpretation: (1) Continuation/expansion versus liquidation/downsizing.* Under this interpretation, the principal may entrust the agent with a new project, or with the continuation of the current one (as opposed to liquidation) at date 1. The agent

⁷In particular, the agent can be risk neutral and care about the sum of his consumption at dates 0 and 1 without any change in the treatment. Section E.2 in the online appendix discusses the more general case in which the agent discounts date-2 utility at a higher rate than the principal (has payoff $u_0 + u_1 + \delta u_2$, where u_t is date- t utility and $\delta < 1$). Also, setting the cost of delivering utility u to the agent to u is only a normalization.

derives a private benefit from this new project or from continuation (normalized to 1), but the project value net of this private benefit is negative (normalized to -1).

- *Interpretation: (2) Transfer of corporate control.* A second interpretation is that the principal has the option to fire the agent at date 1 in order to hire a new one who can create more value for him out of the project. The initial agent derives a private benefit from remaining involved with the firm, but this comes at the opportunity cost for the principal of giving up this value.
- *Interpretation: (3) Managerial compensation.* Finally, the transfer may correspond to managerial compensation. That the agent's utility over consumption is locally linear and bounded above captures risk-aversion in the simplest fashion.⁸

Project selection. The agent must select a project at date 0. Projects may be of two types, 1 or 2. Both types require the same date-0 investment outlay. One type of project pays off h at date 2, whereas the other type delivers l , where

$$h > l.$$

The principal and the agent have the common prior that a payoff of h is associated with the type-1 project with probability $1/2$. The principal observes the type of the project selected by the agent.⁹

At date 0, the agent receives a private signal about the payoff associated with each project type. The signal's precision depends on an effort level secretly chosen by the agent. If the agent "behaves," the signal matches the true payoffs with probability p . If he "shirks," the signal is correct with probability $p - \Delta p$ only, where

$$\frac{1}{2} \leq p - \Delta p < p < 1.$$

By shirking, the agent derives a private benefit added to any utility he may receive from the principal and that we denote $b > 0$.

In other words, the principal-agent relationship is plagued by a moral-hazard problem that takes the form of a nonobservable forecasting effort exerted by the agent. This effort stands for any time and resources that insiders devote to figuring out the strategy that generates the highest firm value instead of devoting them to tasks that they find more rewarding. Depending on the context, such strategic decisions encompass asset allocation, market entry or exit, risk-management decisions, etc...This incentive problem creates a role for performance-based contracts.

⁸Setting the upper bound at 1 saves on notation and comes at no loss of generality.

⁹This is immaterial until Section 4.

Neither the principal nor the agent observe the project payoff before date 2.¹⁰ The principal has access to two measurements of the project's payoff at date 1: a public signal and resale opportunities.

Measurement: (1) Public signal. First, a public signal $s \in \mathbb{R}$ is available at date 1. The distribution of this signal conditional on a final payoff $y \in \{h; l\}$ admits a continuous density $f_y(s)$ such that f_h/f_l is strictly increasing. For brevity, we will rule out some corner solutions by assuming that the signal can be arbitrarily informative and so $f_h(s)/f_l(s)$ spans $(0, +\infty)$ as s spans \mathbb{R} . We denote by F_y the conditional c.d.f. of the signal.

We interpret this abstract date-1 signal as publicly observable transaction data for assets that are comparable to that chosen by the agent. This interpretation corresponds to the way we endogenize the signal in Section 4. For notational simplicity, we assume that this signal is freely available. In practice, external pricing services and audit, or assessment of fair values may create a cost of obtaining this signal. The qualitative results however would not be affected by the introduction of a cost of observation.

Measurement: (2) Resale opportunities. The second source of information available to the principal are resales of the project to informed buyers at date 1. We introduce imperfectly competitive bidding in the date-1 market for projects as follows. The agent is in charge of reselling the project and thereby solicits bids. The agent privately observes the bids received by the firm. The principal observes only whether the project is sold and the price at which it is sold. So, bids are unverifiable unless they are acted upon, i.e. generate a trade.

We make assumptions about the distribution of the bids received by the firm that later will be rationalized through optimal bidding by informed buyers. We assume that a project that pays off $y \in \{l; h\}$ never elicits bids above y , that a h -payoff project elicits no bid with some probability q_0 and, with the complementary probability, receives a finite number of bids that take values in $(l, h]$. Finally, we assume that bids and the public signal are independently drawn conditionally on the project type. We denote H the absolutely continuous c.d.f. of the highest bid received by a firm that has a h -payoff project, such that $H(x) = q_0$ for all x below the lowest bid that the firm can receive.¹¹ We denote L the c.d.f. of the highest bid received by an l -payoff project.

Summary of timing. Figure 1 summarizes the sequence of events:

¹⁰The case in which the agent privately observes the payoff before date 2 yields qualitatively similar results.

¹¹This amounts to formalizing the absence of bids as receiving a highest bid equal to $-\infty$.

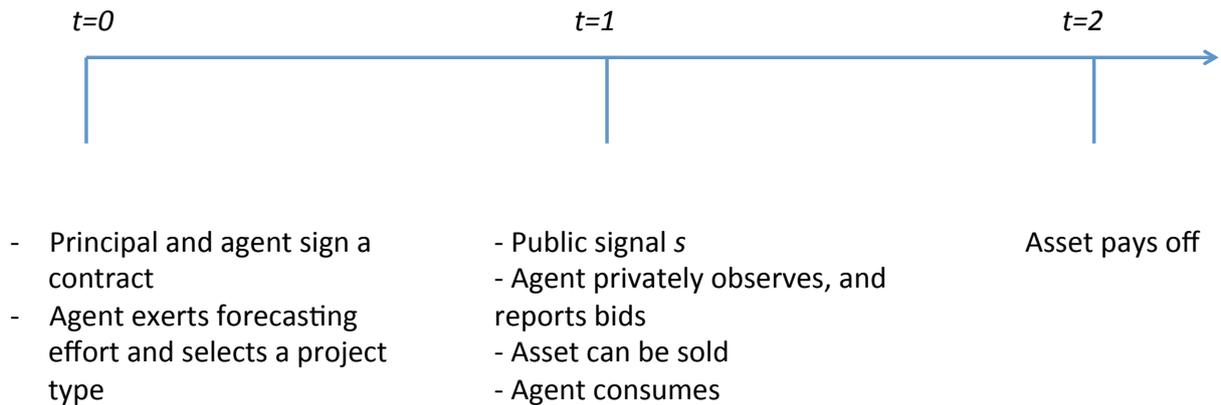


Figure 1. Timeline.

2.2 Optimal contract

We suppose that the parameters are such that the relationship is socially desirable only if the agent behaves. We solve for the contract that minimizes the cost to the principal from inducing the agent to behave.¹² We suppose that effort incentives can be provided:

$$\Delta p \geq b.$$

This means that if the principal observed the date-2 payoff at date 1, he could elicit effort by granting the agent utility $b/\Delta p$ whenever $y = h$ and 0 when $y = l$. The ratio $b/\Delta p$ of the private benefit divided by the impact of shirking captures the extent of moral hazard. We denote it

$$\beta \equiv \frac{b}{\Delta p}.$$

In this second-best case, in which the principal-agent relationship is plagued only by a moral-hazard problem but not by a measurement problem, the expected cost for the principal to induce the agent to behave would be $p\beta$. We are interested in assessing the additional costs induced by the measurement problem.

Because the principal at date 1 only has access to measures of the terminal payoff, he writes a contract that specifies the circumstances under which either measure (signal or resale) is used to determine the compensation of the agent. We suppose that the principal can fully commit to a contract and we solve for the optimal contract. A general

¹²For brevity, we suppose that the participation constraints of the principal and the agent are always satisfied under this contract, so there is no binding financing constraint.

mechanism is such that after the signal s is realized, the agent reports the received bids. The principal then makes resale and compensation decisions that depend on the signal, the report, and (for the compensation decision and if the asset is sold) on the resale price. We let:

$$\bar{\beta} \equiv F_l(\sigma^*) - q_0 F_h(\sigma^*), \quad (1)$$

where σ^* is defined by

$$\frac{f_h(\sigma^*)q_0}{f_l(\sigma^*)} = 1. \quad (2)$$

The following proposition shows that the optimal contract has a simple structure.

Proposition 1. (*Optimal contract*) *If $\beta > \bar{\beta}$, there exists no contract that elicits high effort. Otherwise, the optimal contract is characterized by a threshold σ and a reserve price r such that:*

- *if the signal is above σ , then the agent receives utility 1;*
- *if the signal is below σ , then the principal allows the agent to sell the asset above the reserve price r , and provides utility 1 if the sale is executed;*
- *the agent receives zero utility otherwise.*

Either the reserve price satisfies $r > l$, or all bids strictly above l are accepted, in which case we adopt the notation $r = l^+$. The cost of capital at the optimal contract is

$$p\beta + 1 - F_l(\sigma) + pF_h(\sigma) \int_r^h (h - t)dH(t). \quad (3)$$

Proof. See the appendix. ■

In words, the generic optimal contract rewards the agent if the signal is above a cut-off, or if it is below this cut-off and the agent is able to demonstrate a high value by reselling the asset above reservation price r . Expression (3) is the expected cost of the project for the principal above and beyond the initial investment outlay. It is comprised of expected payments to the agent and expected resale costs. Slightly abusing terminology, we deem it the “cost of capital” throughout the paper, as it represents the difference between the expected future cash flows of the firm and outsiders’ actual return. The optimal contract is the one that minimizes this cost of capital. Each of the three terms in (3) admits a simple interpretation. The first term $p\beta$ is the second-best cost that would prevail absent measurement frictions. The two other terms represent the cost of the measurement frictions that we added to this standard agency problem. The second term, $1 - F_l(\sigma)$, is the cost from rewarding the agent for luck when mistakenly using

the public signal, whereas the last term, $pF_h(\sigma) \int_r^h (h-t)dH(t)$, represents the expected transaction cost from sales. The optimal contract trades off these latter two costs.

It is impossible to elicit high effort by the agent when $\beta > \bar{\beta}$. An inspection of (1) shows that $\bar{\beta}$ is small when the project is illiquid (q_0 large), and the signal poorly informative ($F_l \simeq F_h$).

A noteworthy feature of the optimal contract is that asset resales consist only in realizing latent gains, never latent losses. This is so regardless of the cost of selling a low cash flow project (that is, even if an l -payoff sells at l almost surely). The reason is that addressing the ex-ante moral-hazard problem requires that the agent be rewarded for having selected a good project and punished for having selected a bad one. Such a compensation scheme implies that ex post, the agent has incentives to report bids that are consistent with the project being good. Only reports of resale options that are consistent with the project being good therefore need to be verified with an ex-post inefficient sale.

The determination of the parameters (σ, r) that characterize the optimal contract is instructive. It amounts to finding a cut-off signal value σ above which the agent is rewarded and a reserve resale price r when the signal realization is below this cut-off value such that the cost of capital is minimized subject to incentive-compatibility:

$$\min_{\{\sigma, r\}} \left\{ p\beta + 1 - F_l(\sigma) + pF_h(\sigma) \int_r^h (h-t)dH(t) \right\} \quad (4)$$

s.t.

$$F_l(\sigma) - F_h(\sigma) + [1 - H(r)]F_h(\sigma) = \beta, \quad (5)$$

$$q_0 \leq H(r) \leq 1. \quad (6)$$

The incentive-compatibility constraint (5) is derived as follows. Effort raises the probability of a payoff h by Δp at the cost of the private benefit b . An h payoff raises in turn the agent's expected utility due to favorable signal realizations by $(1 - F_h(\sigma)) - (1 - F_l(\sigma)) = F_l(\sigma) - F_h(\sigma)$ and due to asset resales by $[1 - H(r)]F_h(\sigma)$. Thus the term $F_l(\sigma) - F_h(\sigma)$ in (5) reflects the incentives that are provided to the agent through the market signal. The term $[1 - H(r)]F_h(\sigma)$ corresponds to those provided through asset resales. The feasibility constraint (6) states that the probability of successful resale of an h -payoff project is bounded above by the probability $1 - q_0$ that the firm receives at least one bid.

Ignoring this feasibility constraint (6), the first-order condition for this program reads:

$$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h-r) + 1}{p \int_r^h H(t)dt}. \quad (7)$$

We show in the proof of Proposition 1 that $\{(5), (7)\}$ admits at most one feasible solution (σ, r) . If there is one solution, it characterizes the optimal contract.¹³ Otherwise, there is a corner solution (σ, r) such that $f_h(\sigma)/f_l(\sigma) > [p(h-r) + 1]/(p \int_r^h H(t)dt)$ and $H(r) \in \{q_0; 1\}$. Denoting $[r_-, r_+]$ the support of H , this corner solution can be of two types:

- Either there are no resales: $H(r) = 1$, and then there must exist σ such that

$$F_l(\sigma) - F_h(\sigma) = \beta, \quad (8)$$

and the optimal contract has a signal cut-off equal to the largest solution to (8).

- Or $H(r) = q_0$ and the optimal contract is (σ, r_-) , where σ is the smallest solution to $F_l(\sigma) - q_0 F_h(\sigma) = \beta$.

Condition (7), that characterizes the interior solution of this contracting problem together with the incentive-compatibility constraint (5), admits the following interpretation. It states that at (σ, r) , the marginal cost of rewarding the agent based on good signals (“marking to market”) is equal to that of rewards based on successful resales (“taking to market”). The corner solution such that $H(r) = 1$ corresponds to the case in which the signal is sufficiently informative or/and the resale costs sufficiently large that the optimal contract relies only on market data. At the corner solution $H(r) = q_0$, ex-ante incentives are too small for all values of (σ, r) that satisfy the indifference condition (7). Incentive-compatibility then requires to rely more on resales, and thus to use a cut-off σ at which the marginal cost of taking to market exceeds that of marking to market.

It is interesting to define the *degree of marking to market* of a contract as the fraction of the total incentives in (5) that stem from the market signal:

$$\frac{F_l(\sigma) - F_h(\sigma)}{\beta}.$$

Holding signal distributions constant, the degree of marking to market decreases in σ in the relevant range where $f_h/f_l > 1$.¹⁴ The degree of marking to market is 100% in the corner solution in which the optimal contract involves no resale. If f_h/f_l were bounded above (a case we assumed away for brevity), it could be that the degree of marking to market be zero. This is akin to pure historical-cost accounting, whereby only realized capital gains, as opposed to latent ones based on market signals, are recognized.

¹³Section E.1 in the online appendix shows that the second-order condition for a maximum is globally satisfied.

¹⁴The first-order condition (7) implies that at the optimal contract, $f_h(\sigma)/f_l(\sigma) \geq 1 + 1/[p(h-r)] > 1$.

2.3 Implementation with an accounting measure

The abstract optimal contract (σ, r) admits a simple and realistic implementation that builds on an appropriate accounting measure of the project. We generally define an accounting measure as a date-1 valuation of the project equal to:

- the resale proceeds if the project was resold at date 1;
- a value $m(s)$ otherwise, where $m(\cdot)$ is increasing in the public signal s .

Such a generally defined accounting measure has the realistic properties that cash-on-hand at date 1 is booked at face value, and that the only input used to value the future cash flow absent a resale is public information s .

Suppose that such an accounting measure satisfies in addition $m(\sigma) = r$. A mechanism that transfers utility to the agent from the principal if and only if the book value of the firm is larger than r at date 1 implements the optimal contract. This is because the book value is above r if market news is such that $s \geq \sigma$, or if $s < \sigma$ and the agent successfully sells the project at a price above r . Under this implementation, there is no explicit contracting about resales. The accounting measure induces insiders to realize latent gains optimally.

Before explicitly constructing such an appropriate accounting measure, it is worthwhile noting that this elementary mechanism resembles arrangements that prevail in practice in the three interpretations of the model mentioned earlier:

- *Continuation/expansion versus liquidation/downsizing.* The mechanism can be interpreted in this case as a transfer of control rights from shareholders (the agent) to creditors (the principal) following the breach of a debt covenant stipulating that the book value of the firm be above the threshold r . Shareholders continue/expand the firm if they remain in control whereas creditors liquidate or downsize it otherwise. As in Dewatripont-Tirole (1994), one can endogenize the respective payoffs of creditors and shareholders from each course of action as resulting from the respective curvatures of their claims.¹⁵ This mechanism can also be interpreted as a stylized representation of prudential regulation in the particular case in which the creditors are the depositors of a bank or the policyholders of an insurance company.
- *Transfer of corporate control.* Under this interpretation, the mechanism best describes conflicts of interests between diffuse and large shareholders, whereby the former are more likely to tender their shares in a hostile takeover if the incumbent insiders fail to meet some objectives in terms of firm book value. Note that this role

¹⁵Creditors who have a concave payoff favor the risk reduction associated with liquidation/downsizing whereas shareholders with their convex claims (and managers) favor continuation/expansion.

of accounting measures in the market for corporate control is implicit in practice, rather than resulting from explicit contracts. Sloan (2001) surveys the various channels through which accounting measures play an important implicit role in corporate governance.

- *Managerial compensation.* In this case the optimal mechanism admits a straightforward interpretation as a performance-based bonus.

Construction of an optimal accounting measure. We now explicitly construct an accounting measure that implements the optimal contract as follows. We start from the verbatim official definition of a fair-value measurement as “the price that would be received to sell an asset at the measurement date.” In line with this definition, we build a measure based on a market-consistent estimate of the resale value of the asset at date 1.

Formally, after observing a market signal s at date 1, an econometrician would infer that the distribution of the price at which the project would be sold at this date were it taken to the market admits a c.d.f. $D_s(x)$ that satisfies

$$D_s(x) = \frac{pf_h(s)H(x) + (1-p)f_l(s)L(x)}{pf_h(s) + (1-p)f_l(s)}. \quad (9)$$

Recall that H and L are the respective distributions of the highest bid for projects with h and l payoffs. For every $\alpha \in (0, 1)$, we define the measure $m_\alpha(s)$ as the value that the realized resale price would exceed with probability α were the project taken to market at date 1. Formally, for every signal realization s , $m_\alpha(s)$ is the unique solution in m to:

$$D_s(m) = 1 - \alpha. \quad (10)$$

The value $m_\alpha(s)$ is increasing in s and decreasing in α . One can therefore interpret α as the *degree of conservatism* of the accounting measure $m_\alpha(s)$ because it quantifies the extent to which the measure $m_\alpha(s)$ recognizes the possible latent gains on the project suggested by signal realization s . As α tends to 1, $m_\alpha(s)$ tends to the lower bound of the support of L for a fixed s . This corresponds to a very conservative measure that recognizes latent gains only when they are very likely (s very large). Conversely, as α becomes arbitrarily small, the measure is very aggressive in that it books the project at a value close to the upper bound of the support of H unless the realization of s is very small.

Proposition 2. (*Optimal degree of conservatism*) *The optimal contract (σ, r) can be implemented using an accounting measure $m_\alpha(s)$ with degree of conservatism α such that*

$$\alpha = \frac{pf_h(\sigma)[1 - H(r)]}{pf_h(\sigma) + (1-p)f_l(\sigma)}. \quad (11)$$

Proof. The optimal contract (σ, r) is implemented with a degree of conservatism α such that $m_\alpha(\sigma) = r$. Injecting this condition in (10) and noting that $r \geq l^+$ implies $L(r) = 1$ yields (11). ■

Note that a sufficiently large degree of conservatism is akin to historical-cost accounting: If $\alpha > 1 - q_0$, then it must be that $m_\alpha(s) \leq l$ for all s , and only realized gains affect corporate governance.

Relation to fair value hierarchy. The International Financial Reporting Standard 13 that defines fair value uses a “fair value hierarchy” that classifies balance-sheet items according to the nature of the inputs required to determine their fair value. Level 1 items are those valued using only “quoted prices in active markets for identical assets or liabilities that the entity can access at the measurement date.” Conversely, level 2 and 3 items cannot be directly marked to market based on listed prices this way, and are rather “marked to model.” Their fair value is based on a valuation model, and as such it involves assumptions (e.g., absence of arbitrage opportunities or complete markets) and the use of proprietary information.

This hierarchy is a coarse characterization of the ease with which one can identify the price at which an item would trade at the measurement date. In our setup, this concept of liquidity evolves continuously as the public signal becomes more accurate and the informed bids less dispersed. Ideally, a level-1 signal would be generated by publicly observed transactions for assets identical to that of the firm provided these assets face a perfectly elastic demand. This is of course an abstraction. Even liquid assets usually exhibit a bid-ask spread. Furthermore, the lag between measurement and disclosure by itself introduces noise as assets fundamentals and prices fluctuate over time.

The remainder of the paper endogenizes the environment (market signals and resale costs) facing each firm. We are interested in particular in studying whether the degree of conservatism that each firm finds privately optimal is also socially optimal.

3 Short-term equilibrium: endogenous resale prices

This section endogenizes the bids received at date 1 as resulting from the interaction of many firms and informed buyers in a decentralized market. Suppose there are a continuum of firms with unit mass. Each faces the same situation as that described in the previous section. In particular, the payoff from each project type is the same for all firms. Date-0 private signals are independent across agents. The project type chosen by a firm is not observed by other firms. Each firm receives a signal about its project payoff at date 1 whose conditional distributions F_h, F_l are identical across firms and have the same properties as in the previous section. The joint distribution of the signals is immaterial

for our analysis.

The economy is also populated by a continuum of informed buyers with mass λ who are risk-neutral over consumption.¹⁶ Firms can sell their assets to the informed buyers in a decentralized market at date 1. Firms and buyers interact as follows. Each buyer is randomly matched to a firm and privately observes the payoff from its project.¹⁷ Without observing how many fellow buyers are matched with this firm, he submits a bid. Only the agent of the firm observes the bids submitted by the matched buyers (if any). The principal observes only if a sale goes through and at which price if it does. The matching technology is such that each firm is matched with k buyers with probability q_k .

We solve for equilibria with incentive-compatible contracts. We show in the appendix that all bids for h -projects must lie within $(l, h]$ in equilibrium.¹⁸ From Proposition 1, in this case, each firm designs a contract (σ, r) such that the agent is rewarded if the signal is above σ , or if it is below σ and he manages to sell the asset above a reserve price $r \geq l^+$. Anticipating such contracts, but without observing them, informed buyers place bids for the good project type according to a distribution with c.d.f. S . Put differently, equilibrium bidding involves a symmetric mixed strategy over bids with c.d.f. S .

An equilibrium with incentive-compatible contracts is then a triplet (σ, r, S) such that:

- Each firm finds the contract (σ, r) optimal given S .
- Each bidder is indifferent between each bid for a good project in the support of S .

The bidding strategies of buyers matched with l -projects are irrelevant as these projects are never resold. In order to characterize these interior equilibria, we first study buyers' strategies and then firms' contracting decisions.

Bidding strategies. We show in the appendix that the distribution according to which buyers mix their bids for a good project, S , is either a Dirac delta at h , or is atomless.¹⁹ The lower bound of its support, r_- , must in equilibrium be equal to the bidders' (correct) expectation of r .²⁰

This latter property has the key implication that the equilibrium probability that the sale of a good project fails to go through depends only on the absence of an informed buyer due to random matching. Thus it must be that in equilibrium, the probability of resale of an asset taken to market is $1 - q_0$, or $H(r) = q_0$. This implies that the incentive-compatibility constraint (5) reads in equilibrium:

$$F_l(\sigma) - q_0 F_h(\sigma) = \beta. \tag{12}$$

¹⁶Section E.3 in the online appendix tackles the case in which λ is uncertain at the contracting date.

¹⁷That each buyer meets with one firm is only a normalization.

¹⁸See proof of Lemma 3.

¹⁹See proof of Lemma 3.

²⁰Id.

Optimal contracts. We suppose that the bidders' mixing strategy S is not degenerate. Each firm then faces a highest-bid distribution H such that

$$H = \sum_{k \geq 0} q_k S^k,$$

with the convention that an absence of bid is a bid equal to $-\infty$. Optimal bidding requires that the optimal contract (σ, r) must be such that $r = r_-$ and that σ is a solution to (12). From Section 2, it must also satisfy

$$\frac{f_h(\sigma)}{f_l(\sigma)} \geq \frac{p(h-r) + 1}{p \int_r^h H(t) dt}. \quad (13)$$

The case in which (13) holds as an equality corresponds to an interior solution to the contracting problem, whereas the inequality may be strict for a corner solution. In the remainder of the paper, we assume away equilibria with corner solutions such that $r = r_- > l$, as the reservation price is then a weakly dominated strategy. One can eliminate such equilibria by assuming, for example, that buyers submit bids with arbitrarily small trembles, so that the project can be sold at a price below r_- with an arbitrarily small probability. This rules out a continuum of uninteresting equilibria with arbitrarily aggressive bids. This implies the following restrictions on the contracts that arise in equilibrium:

Lemma 3. *An equilibrium contract (σ, r) is either such that $\sigma = +\infty$ and $r = h$ or such that σ is finite and solves*

$$F_l(\sigma) - F_h(\sigma)q_0 = \beta. \quad (14)$$

In this latter case, the contract is either such that

$$r > l \text{ and } \frac{f_h(\sigma)q_0}{f_l(\sigma)} = \frac{1 + \frac{1}{p(h-r)}}{1 + \frac{\lambda q_1}{q_0(1-q_0)}}, \quad (15)$$

or such that

$$r = l^+ \text{ and } \frac{f_h(\sigma)q_0}{f_l(\sigma)} \geq \frac{1 + \frac{1}{p(h-l)}}{1 + \frac{\lambda q_1}{q_0(1-q_0)}}.$$

This latter corner contract must be such that σ is the smallest solution to (14).

Proof. See the appendix. ■

The case $\sigma = +\infty$ and $r = h$ corresponds to the second-best in which the asset can be resold at a fair price. Equilibria with a finite signal cut-off σ are of two types. If the optimal contracting problem admits an interior solution, then it satisfies the respective equilibrium versions of the incentive-compatibility constraint and the first-order condition, (14) and (15) respectively, together with the condition $r > l$. The only equilibrium in

which the optimal contract is a corner solution to the optimal contracting problem is that in which $r = l^+$.

Since (15) and (14) pin down $h - r$ independently of the value of l , $r > l$ holds as long as l is sufficiently small or, equivalently, $h - l$ sufficiently large. On the other hand, if $h - l$ is sufficiently small that all the (σ, r) that solve (15) and (14) are such that $r < l$, then the only possible equilibrium contract with a finite σ is the corner solution such that $r = l^+$. For brevity only, the remainder of the paper focuses on the case in which l is sufficiently small that there exists at least one equilibrium with an interior contract (provided there exists at least an equilibrium). The existence and number of equilibria can then be fully characterized as follows.

Proposition 4. (Equilibria with endogenous resale prices) *If $\beta > \bar{\beta}$, there is no equilibrium with incentive-compatible contracts. If $\beta \leq \bar{\beta}$,*

- *there exists a unique equilibrium in which the contract $(\bar{\sigma}, \bar{r})$ is such that $f_h(\bar{\sigma})q_0/f_l(\bar{\sigma}) \geq 1$. If $\beta > 1 - q_0$, then $\bar{\sigma}$ is finite and $\bar{r} < h$. If $\beta \leq 1 - q_0$, the equilibrium contract consists in selling the good asset at price h with probability $\beta/(1 - q_0)$ (second-best).*
- *There also exists a unique equilibrium in which the contract $(\underline{\sigma}, \underline{r})$ is such that $f_h(\underline{\sigma})q_0/f_l(\underline{\sigma}) \leq 1$. It is such that $(\underline{\sigma}, \underline{r}) \leq (\bar{\sigma}, \bar{r})$.*

Proof. The results in Proposition 4 are easy to see from Figure 2, which depicts the left-hand side of the incentive-compatibility constraint (14).

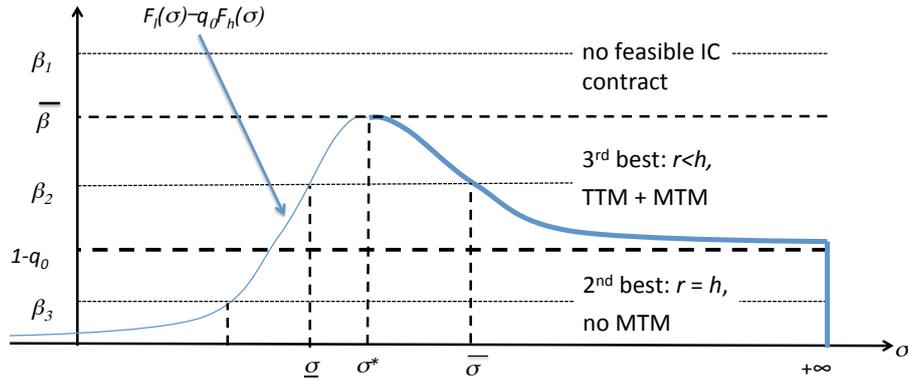


Figure 2. Equilibria for three values of β .

The situation in which incentive-compatible contracts cannot be supported in equilibrium corresponds to $\beta = \beta_1$ in Figure 2.

Otherwise, if $\beta \in (1 - q_0, \bar{\beta}]$ (case $\beta = \beta_2$ in Figure 2), then the incentive-compatibility constraint has two solutions, $\underline{\sigma} \leq \sigma^*$ where the function $F_l - q_0 F_h$ is increasing ($f_h(\underline{\sigma})q_0 < f_l(\underline{\sigma})$), and $\bar{\sigma} \geq \sigma^*$ where the function $F_l - q_0 F_h$ is decreasing ($f_h(\bar{\sigma})q_0 > f_l(\bar{\sigma})$). We assumed l sufficiently small that the latter solution always corresponds to an interior equilibrium (\bar{r} is then given by (15)), whereas the former may either correspond to an interior equilibrium or to the corner solution in which $\underline{r} = l^+$ below a threshold value of β .

If $\beta \leq 1 - q_0$ (case $\beta = \beta_3$ in Figure 2), then the second-best in which firms provide incentives only by selling their good projects at price h with probability $\beta/(1 - q_0)$ is the unique feasible equilibrium such that $f_h q_0 / f_l > 1$ because contracts with a finite σ cannot be incentive compatible unless $f_h q_0 / f_l < 1$. Finally, the unique cut-off $\underline{\sigma}$ that solves (14) when $\beta \leq 1 - q_0$ may again either correspond to an interior equilibrium (for β sufficiently large) or to the corner one. ■

The equilibrium probability of success of a resale is $1 - q_0$, imposed by the matching technology, because buyers always bid above their (correct) expectation of the reserve price set by firms in equilibrium. But then this reserve price must adjust so that firms are indifferent between taking to market and marking to market at the signal cut-off of the optimal contract. There can be two pairs $(\underline{\sigma}, \underline{r}) \leq (\bar{\sigma}, \bar{r})$ for which this is the case. If informed buyers expect a high reservation price \bar{r} , they bid aggressively. In this liquid equilibrium, resale costs are low, and thus firms rely on market data only when it is very informative—hence a low degree of marking to market $\bar{\sigma}$ which justifies in turn aggressive bids. When $\beta \leq 1 - q_0$, this liquid equilibrium takes the extreme form of the second best in which firms sell their assets at the maximum price and do not use market data at all. There may also exist an illiquid equilibrium in which sales are costly, which in turn justifies relying less on sales, and relying instead heavily on mediocre signal realizations that vindicate in turn low bids. In this illiquid equilibrium, the degree of marking to market is higher than in the liquid one. Asset sales are rare and occur at distressed prices upon the realization of very negative signals. This illiquid equilibrium is inefficient in the sense that it comes at a higher cost of capital for firms.²¹

The possibility that the equilibrium outcome yields excessive marking to market jointly with distressed sales is interesting. Yet, equilibria such that $f_h q_0 / f_l < 1$ are unstable whereas those such that $f_h q_0 / f_l > 1$ are stable and constrained-efficient in the following sense. Suppose that an abstract regulation prevents firms from rewarding their agents based solely on the signal for signal realizations $s < \sigma'$. We deem such regulated contracts “ σ' -contracts”.

Proposition 5. (*Regulating marking to market*) *Suppose there are two equilibria $(\underline{\sigma}, \underline{r}) \leq (\bar{\sigma}, \bar{r})$.*

²¹See proof of Lemma 3.

- *The illiquid equilibrium $(\underline{\sigma}, \underline{r})$ is unstable: For every $\sigma' \in (\underline{\sigma}, \bar{\sigma}]$, the only equilibrium with σ' -contracts is the one with the contract $(\bar{\sigma}, \bar{r})$.*
- *The liquid equilibrium $(\bar{\sigma}, \bar{r})$ is constrained efficient: If $\sigma' > \bar{\sigma}$, there is no equilibrium with incentive-compatible σ' -contracts.*

Proof. See the appendix. ■

In other words, it is possible to move the economy towards the equilibrium with the lowest cost of capital with a simple upper bound on the degree of marking to market— a lower bound σ' on the informativeness of the signal that firms can use to mark to market. The broad intuition why the inefficient equilibrium is unstable is as follows. Because $f_h q_0 / f_l < 1$ at this equilibrium, imposing an increase in the cut-off σ generates higher incentive provisions. Firms would like to respond with higher reserve prices because they can then afford a lower probability of successful resale. But since buyers always at least match this reserve price, this probability cannot decrease, and the reserve price and signal cut-off must keep increasing until the efficient equilibrium is reached. On the other hand, when this efficient equilibrium features a finite $\bar{\sigma}$, such a regulation of marking to market, if too tight ($\sigma' > \bar{\sigma}$), destroys firms' ability to write incentive-compatible contracts at all.

Implementation: Imposing a higher degree of accounting conservatism

The abstract regulation of the cutoff σ described in Proposition 5 admits a concrete interpretation under the implementation with the accounting measure introduced in Section 2.3. It corresponds to a regulation of the degree of conservatism α used when measuring firm value. Since $H(r) = q_0$ in equilibrium, an equilibrium contract (σ, r) is implemented with an accounting measure that uses an equilibrium degree of conservatism

$$\alpha(\sigma) = \frac{p f_h(\sigma)[1 - q_0]}{p f_h(\sigma) + (1 - p) f_l(\sigma)} \quad (16)$$

that depends only on the equilibrium signal cut-off σ .²²

Thus, in the presence of multiple equilibria, one can ensure that the economy reaches the efficient equilibrium simply by imposing a degree of conservatism within $(\alpha(\underline{\sigma}), \alpha(\bar{\sigma}))$. In other words, forcing firms to recognize latent gains in a less aggressive fashion than they find privately optimal in the inefficient equilibrium ensures that the economy reaches the efficient equilibrium.

²²Expression (16) simply stems from injecting $H(r) = q_0$ in (11).

4 Long-term equilibrium: endogenous liquidity

Finally, we allow the mass λ of informed buyers to be endogenously determined through a free-entry condition. We also endogenize the market signal received by each firm as the imperfect observation of transactions by other firms. Endogenous liquidity λ affects firms' environment by impacting the matching process between firms and informed buyers, and in turn the ease of asset resale and the quality of market data resulting from their interactions. We first assume a general form for the dependence of the date-1 signal on the liquidity parameter λ in Section 4.1, and then offer examples of microfoundations in Section 4.2.

4.1 A general model

We suppose that there are a continuum of initially uninformed potential buyers with arbitrarily large mass. By incurring a cost $\kappa > 0$, each of them can acquire knowledge so as to be able to privately observe the payoff of the project once matched to a firm. The mass of informed bidders is now an equilibrium outcome λ .

Section 3 corresponds to the situation in which the marginal information acquisition cost is a step function of λ from zero to a large value so that λ is completely inelastic with respect to firms' accounting strategies. By contrast, with such a fixed cost κ , liquidity λ now responds elastically to firms' decisions and impacts them in turn as follows. First, we assume that each firm is matched with k informed buyers with probability $q_k(\lambda)$, where the functions $\{q_k(\lambda)\}_{k \in \mathbb{N}}$ are continuously differentiable and such that:

Assumption 1. (*Informed trading generates liquidity.*) *The ratio $q_{k+1}(\lambda)/q_k(\lambda)$ increases in λ for all k , and $q_1(\lambda)/[q_0(\lambda)(1 - q_0(\lambda))]$ is increasing.*

The condition that q_{k+1}/q_k increases implies that the number of buyers per firm increases with λ in the sense of first-order stochastic dominance. The condition on $q_1/[q_0(1 - q_0)]$ means that the probability q_0 that a firm faces no informed buyer decreases at a faster rate than the conditional probability of facing only one buyer $q_1/(1 - q_0)$ when λ increases. These conditions are satisfied, for example, with an urn-ball process ($q_k(\lambda) = \lambda^k e^{-\lambda}/k!$).

Second, we also suppose that the c.d.f. of the date-1 signal conditional on a payoff $y \in \{h; l\}$ is of the form $F_y(s, \lambda)$, continuously differentiable with respect to λ , and satisfies:

Assumption 2. (*Informed trading generates better market data.*) *For all (s, λ) ,*

$$f_l(s, \lambda) \frac{\partial F_h(s, \lambda)}{\partial \lambda} \leq \min \left\{ 0; f_h(s, \lambda) \frac{\partial F_l(s, \lambda)}{\partial \lambda} \right\}. \quad (17)$$

This means that F_h must increase with λ in the sense of first-order stochastic dominance, and sufficiently more so than F_l . This property will arise naturally in the next section that endogenizes the signal.²³

We keep assuming that l is sufficiently small other things being equal. As in the previous section, it amounts to assuming that the condition $r > l$ for interior equilibria is always satisfied. It also ensures that uninformed bidders cannot find it optimal to bid above r because the expected cost of receiving an l -project against such a bid is too high.²⁴

An equilibrium is then a triplet (σ, r, λ) such that (σ, r) defines an equilibrium in the sense of the previous section given λ , and potential buyers are indifferent between becoming informed or not given (σ, r, λ) . Note that the second-best equilibrium whereby firms use only sales at $r = h$ can no longer be sustained as the absence of sales precludes information acquisition by buyers. As a result, an interior equilibrium with information acquisition ($\lambda > 0$) must satisfy the first-order condition (15), the incentive-compatibility constraint (14), and a free-entry condition:

$$\frac{f_h(\sigma, \lambda)q_0(\lambda)}{f_l(\sigma, \lambda)} = \frac{1 + \frac{1}{p(h-r)}}{1 + \frac{\lambda q_1(\lambda)}{q_0(\lambda)(1-q_0(\lambda))}}, \quad (18)$$

$$F_l(\sigma, \lambda) - F_h(\sigma, \lambda)q_0(\lambda) = \beta, \quad (19)$$

$$pF_h(\sigma, \lambda)\frac{q_1(\lambda)(h-r)}{1-q_0(\lambda)} = \kappa. \quad (20)$$

Condition (18) is the first-order condition, (19) is the incentive-compatibility constraint, and (20) is the free-entry condition stating that the expected bidding profit of an informed buyer is equal to the information acquisition cost κ .

In this section we focus on equilibria such that $f_h q_0 / f_l > 1$ that are stable in the sense defined in Section 3. We have:

Proposition 6. (*Existence of a stable equilibrium*) *If there exists λ such that $q_0(\lambda) > 1 - \beta$,²⁵ there exists $\bar{\kappa}$ such that for all $\kappa \leq \bar{\kappa}$, there exists a stable equilibrium.*

Proof. See the appendix. ■

Section 3 showed that stable equilibria are constrained-efficient when λ is inelastic. We now show that, conversely, laissez-faire is generically inefficient when λ responds elastically to firms' behavior. To see this, suppose that there exists a stable equilibrium (σ, r, λ) , but

²³Condition (17) obviously holds if F_h increases whereas F_l decreases in λ in the sense of first-order stochastic dominance.

²⁴One can alternatively ensure that this latter property holds by assuming that q_0 is sufficiently small in the relevant range so that uninformed bidders face a sufficiently severe adverse selection problem.

²⁵This condition ensures that (19) admits a solution such that $f_h q_0 / f_l > 1$. It is satisfied for every $\beta \in (0, 1)$ for an urn-ball matching process.

that firms are forced to use a regulated contract such that they must use a cut-off $\sigma' > \sigma$ for signal-based rewards, whereas they are free to set whichever reserve price they see fit for resale-based rewards.

Proposition 7. (*Excessive marking to market under laissez-faire*) *Let (σ, r, λ) be a stable equilibrium. If $\sigma' > \sigma$ is sufficiently close to σ , then*

1. *There exists an incentive-compatible equilibrium with regulated contracts with cut-off σ' .*
2. *This equilibrium features a lower cost of capital than under laissez-faire. There are more informed buyers, firms set more aggressive reserve prices, bidders bid more aggressively, and market signals are more informative.*

Proof. See the appendix. ■

Bidders bid more aggressively in the regulated equilibrium in the sense that the distribution of their bids dominates the laissez-faire one according to first-order stochastic dominance. Thus, laissez-faire is associated with excessive reliance on market signals and resales at excessively large discounts.

The intuition for this result is simple. Firms fail to internalize the positive liquidity externalities that they create for each other by taking to market instead of relying on the public signal. Forcing them to adopt a higher confidence level when relying on market-based evidence induces more trades and yields an increase in λ . This increase in λ reduces firms' cost of capital through two channels. First, even if signal distributions were not affected by λ , curbing marking to market would still reduce firms' cost of capital as they would benefit from a larger pool of informed buyers and could afford more aggressive resale pricing. Second, in addition, they also benefit from a higher quality of market data through the assumed impact of λ on F_h and F_l . This latter effect exists only if inequality (17) holds strictly at the equilibrium.

Implementation: Imposing a higher degree of accounting conservatism

Again, to an equilibrium (σ, r, λ) corresponds an implementation of the equilibrium contract with an accounting measure using an equilibrium degree of conservatism that is increasing with respect to σ and λ :

$$\alpha = \frac{pf_h(\sigma)[1 - q_0(\lambda)]}{pf_h(\sigma) + (1 - p)f_l(\sigma)} \quad (21)$$

and that is too small under laissez-faire. The abstract small increase in σ imposed on firms in Proposition 7 admits an interpretation as imposing an increase in the degree of conservatism that firms must use when booking latent gains.

4.2 Public signal as observed transaction prices

Let us now interpret the signal s received by a firm as the price fetched by comparable assets sold by other firms. If, as implied by the optimal contract, only h -payoff assets are sold and the principal in a firm is able to relate the project selected by the agent to those sold on the market (there is no misclassification error), actual transactions are perfectly informative and reveal that the agent has selected a high-payoff project. So two routes to noisy market measurement can be taken. The first involves misclassification. The second posits that assets may trade for other reasons than the provision of incentives. We formalize each microfoundation in turn.

We endogenize firms' date-1 signal using rational expectations equilibrium as our equilibrium concept (see, e.g., Grossman 1981 or Grossman-Stiglitz 1980). Namely, we suppose that all asset resales take place at date 1 but that each firm can condition its own resale decision on the observation of transactions by other firms.²⁶

Misclassification

A firm perfectly observes the transaction prices of resold assets (which are from the equilibrium contract only h -payoff assets). It however cannot ascertain perfectly how similar the resold assets are to its own asset. The accuracy of its classification is denoted ρ and has a continuously increasing density $g(\rho)$ over $[0, 1]$ such that $g(0) = 0$.²⁷ When endowed with an asset of type $k \in \{1; 2\}$, a firm assigns a fraction ρ of any sample of assets in category k to category k , and misleadingly, a fraction $1 - \rho$ to the other category. The realizations of ρ are independent across firms, and firms do not observe their own ρ . A firm's signal s is then the fraction of resold assets to which it assigns the same type as its own asset and has conditional densities:

$$f_h(s) = f_l(1 - s) = g(s), \quad (22)$$

and

$$\frac{f_h(s)}{f_l(s)} = \frac{g(s)}{g(1 - s)} \quad (23)$$

is increasing in s because so is g . The distribution of the signal does not depend on λ , and thus condition (17) is satisfied.

Proposition 8. (*Misclassification risk*) *Under misclassification risk, Proposition 7 applies.*

²⁶This is similar to REE in Walrasian environments where agents condition their demand schedules on contemporaneous prices.

²⁷For example, $g(\rho) = (1 + \chi)\rho^\chi$ for $\chi > 0$.

Proof. Discussion above. ■

With such misclassification risk, the signal does not depend on λ and so condition (17) is binding. This implies that imposing a higher degree of conservatism on firms reduces their cost of capital only through the channel of a lower cost of resales due to more aggressive bids. The informativeness of market signals is unaffected by an increase in λ . We now develop an alternative microfoundation in which inequality (17) will be strict: An increase in λ will affect both the costs of taking to market and that of marking to market in this case.

Idiosyncratic risk

We now suppose that firms perfectly identify asset types when observing transactions by other firms, but that payoffs across assets of the same type differ along an idiosyncratic component. This corresponds to assets that are heterogenous in nature (over-the-counter derivatives, real estate,...). As mentioned in Section 2.3, this can also stand for changes in an asset fundamental between measurement and disclosure dates. Such idiosyncratic noise per se does not generate noisy inference if only h -payoff assets are resold in equilibrium. So we also add a reason why l -payoff assets are occasionally resold. We describe in turn each ingredient and explain how it affects the equilibrium characterized in Section 4.1.

First, we suppose that the date-2 payoff of each project is equal to $y + z$, where $y \in \{l; h\}$ is identical across projects of the same type. The new terms z are independently and identically distributed across firms with a c.d.f. Ψ that admits a log-concave density with full support over the real line.²⁸ Each firm (principal and agent) and all the buyers who are matched with it (informed or not) observe the realization of z for that firm at date 1, whereas other agents do not.

Second, we suppose that in addition to receiving a signal and bids, a firm (principal and agent) privately observes the value of its project's payoff at date 1 with probability $\gamma < \beta$ (so far we had $\gamma = 0$). This affects the provision of incentives to agents as follows. In the event of such an early payoff discovery, the agent receives utility 1 if the payoff is h and 0 if it is l . In the absence of early discovery, the agent is as before rewarded if the signal is above a cut-off σ , or if he successfully resells the asset at a price above $r + z$, where z is the firm's idiosyncratic shock. The pair $\{\sigma, r\}$ solves (4) subject to constraints (5) and (6), with the only change that β is replaced by $\beta' = (\beta - \gamma)/(1 - \gamma) < \beta$ in these equations.

Third, we also assume that a principal, when indifferent between reselling the firm's asset or not, always chooses to do so. This is for expositional simplicity: In the proof of Proposition 9, we show that such a preference for trading arises endogenously from

²⁸We could impose positive payoffs at the cost of some additional complexity.

small gains from trades between principals and potential buyers. This affects equilibrium transactions as follows. We keep assuming an arbitrarily large mass of uninformed buyers, so that each firm always faces competitive uninformed buyers. Firms that discover a l -payoff at date 1 sell their asset to uninformed buyers at the price $l + z$, and do not reward the agent upon such a sale. Whereas l -payoff assets are sold if discovered early by a firm, h -payoff assets are sold only for measurement purposes in the absence of early discovery and when the firm receives a signal below σ and at least one bid above $r + z$. Indeed, indifference between bids implies that informed bids for h -assets are bounded away from h , the principal's valuation of the asset.

Finally, for tractability, we preserve the information structure assumed thus far with signals that are univariate and identically distributed across firms by assuming that each firm observes the price fetched by one asset of the same type as its own one before making its resale decision.²⁹ This observed transaction price therefore plays the role of the exogenous date-1 signal assumed thus far. We have:

Proposition 9. (*Idiosyncratic risk*) *For κ and l sufficiently small, there exists a stable equilibrium with endogenous signals such that Proposition 7 applies and inequality (17) is strict. This implies that imposing a higher degree of conservatism reduces firms' cost of capital by making both marking to market and taking to market strictly more efficient.*

Proof. See the appendix. ■

The equilibrium behaviors described above imply that l -payoff assets trade only at the price $l + z$ set by competitive uninformed buyers. Conversely, as in Section 4.1, h -payoff assets are resold to imperfectly competitive informed buyers who mix bids over an interval $[r, r^+]$, where $r^+ < h$. These transaction prices then generate the following signal distributions. Let $H_1 \equiv (H - q_0)/(1 - q_0)$ denote the c.d.f. of the highest bid for a h -payoff asset net of z conditionally on receiving at least one informed bid. We have:

$$F_l(s) = \Psi(s - l), \tag{24}$$

$$F_h(s) = \int H_1(s - z)d\Psi(z). \tag{25}$$

As λ increases, informed buyers are more aggressive and H_1 increases in the sense of first-order stochastic dominance. On the other hand the resale prices of l -projects are unaffected by λ because they are snapped up by (an arbitrarily large mass of) competitive uninformed buyers. Inequality (17) therefore holds strictly.

This implies that Proposition 7 applies. Interestingly, this microfoundation is also suggestive of contagion phenomena. If many firms observe the resale price generated by the same transaction by a firm with an h -project but a negative idiosyncratic shock, then

²⁹Each firm could more generally observe any statistic from a finite sample of transactions.

this firm's resale sends low market signals, thereby inducing a large number of ex-post inefficient resales by firms with h -assets.

5 Discussion and concluding remarks

This paper augments a standard agency-based model of corporate finance with a measurement friction. It offers a theory of optimal accounting in an environment in which both contractual relations between firms' stakeholders and liquidity in the market for firms' assets are the endogenous outcome of optimizing behaviors.

Gains trading arises naturally in privately optimal contracts as a substitute for relying on market data (marking to market). These contracts admit a natural implementation with an accounting measure that recognizes latent capital gains with an appropriate degree of conservatism.

Our main equilibrium result is that *laissez-faire* generically leads to a socially insufficient degree of accounting conservatism. When the liquidity of firms' assets is exogenously given, inefficient equilibria may arise whereby firms rely too much on market data of poor quality and sell their assets at excessively deep discounts following negative market signals. With endogenous asset liquidity, the equilibrium exhibits an excessive form of bootstrapping. It is inefficient because firms fail to internalize the negative liquidity externalities that they create for each other by contracting too much on transactions by other firms and too little on their own transactions. A higher degree of conservatism would enable firms to trade their assets at a lower cost, and would enhance the quality of market data.

Our model admits broader interpretations. The transaction cost of learning the value of one's asset was traced to buyer market power in the resale or securitization market. Relatedly, the rents of informed buyers could stem from the presence of noise trading³⁰; the firm can then control the extent of market monitoring by affecting the liquidity of the market for its asset (through ownership concentration or the easiness with which shares can be traded), rather than by setting a reserve price like in our model. Again there would be externalities, since more active trading would benefit other firms through the presence of a larger community of informed traders and more informative transaction prices. In another reinterpretation of the model, the informed buyers would be capital constrained at date 1 when purchasing resold assets (as in the classic fire sales literature) and would accordingly accumulate costly reserves at date 0. The prospect of advantageous acquisitions would make them hoard more reserves, making assets markets more liquid at date 1. Again there would be an externality and too much reliance on marking to market from the industry's point of view.

³⁰As in for instance Holmström-Tirole (1993) on the monitoring role of the stock market.

An interesting feature of the model for the political economy of accounting is that the total cost of capital for firms is split into expected payments to insiders and resale costs. Assuming as we have done that informed buyers and sellers are different entities³¹, the less conservative the recognition of latent gains, the larger the fraction of these payments that accrue to insiders: Rewards for luck are more likely when the signal cut-off is lower. Managers benefit from a higher degree of marking to market. More generally, the study of the political economy and not only the normative features of accounting choices would be worth expanding upon.

There are many interesting routes for future research. For instance, firms' balance sheets are in practice comprised of very heterogeneous items. Consider for example the case of an insurance company with marketable assets backing complex and illiquid liabilities. We could formalize this in an extension in which firms run several "projects" for which market signals and resale options vary. Another important question in the design of accounting norms is that of the optimal degree of standardization across heterogeneous industries or/and countries. We believe that our framework is a useful starting point for an economic analysis of the trade-offs at stake.

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³¹This need not be the case; a firm might be on the selling side in some states of nature and on the buying side in others, depending on the liquidity shocks it itself faces; see e.g. chapters 7 and 8 in Holmström-Tirole (2011), which however do not consider the measurement issues that are central to this paper.

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Appendix

Proof of Proposition 1

Let us denote by $u_y(s)$ the expected utility that the optimal contract grants to the agent conditionally on a signal realization s and on a project payoff $y \in \{l; h\}$, and by $x_y(s)$ the probability that the project is sold at date 1, also conditional on s and y . The expected cost of selling a project with payoff y with a probability x is a convex continuously differentiable function $c_y(x)$: The principal can achieve the probability x at the lowest cost by allowing the agent to sell the asset at the highest bid received if this bid is above some reservation price.³²

For a given signal s , any report of bids that does not lead to a resale (including for example reports that no bid has been received) must lead to the same compensation, which we denote $u_0(s)$. Ignoring constraints on $x_y(s)$, the optimal contract then solves:

$$\min \left\{ \int [p f_h(s) [u_h(s) + c_h(x_h(s))] + (1-p) f_l(s) [u_l(s) + c_l(x_l(s))]] ds \right\} \quad (26)$$

s.t.

$$\int [f_h(s) u_h(s) - f_l(s) u_l(s)] ds \geq \beta, \quad (27)$$

for all s, y ,

$$u_0(s) \leq u_y(s) \leq x_y(s) + [1 - x_y(s)] u_0(s). \quad (28)$$

Condition (27) is the ex-ante incentive-compatibility constraint ensuring that the agent exerts maximum forecasting effort. Condition (28) reflects the fact that compensation is bounded by 1 if there is resale and equal to $u_0(s)$ if there is none. A simple inspection of the program shows that at the optimal contract, (27) must be binding (otherwise all expected utilities could be scaled down), and it must be that

$$x_l(s) = 0, u_l(s) = u_0(s), u_h(s) = x_h(s) + [1 - x_h(s)] u_0(s),$$

otherwise it is possible to both generate higher-powered incentives and reduce (26). Substituting these into (27), and $\int [p f_h(s) u_h(s) - p f_l(s) u_0(s)] ds$ with $p\beta$ in (26) yields a Lagrangian:

$$\mathfrak{L}(u_0(\cdot), x_h(\cdot), \mu) = \int \left[\begin{array}{l} [\mu [f_h(s) - f_l(s)] - f_l(s)] u_0(s) \\ + [\mu [1 - u_0(s)] x_h(s) - p c_h(x_h(s))] f_h(s) \end{array} \right] ds - \mu \beta - p \beta, \quad (29)$$

³²An arbitrary small increase in the compensation of the agent as a function of the resale price ensures that he has incentives to sell at the highest possible price.

where μ is the shadow price of (27). Pointwise optimization yields that $u_0(s) = 1_{\{s \geq \sigma\}}$, where $\mu[f_h(\sigma)/f_l(\sigma) - 1] = 1$, and $x_h(s) = x1_{\{s < \sigma\}}$ for some fixed x .

Introducing the reserve price r such that $x = 1 - H(r)$, we have $c_h(x) = \int_r^h (h-t)dH(t)$. This implies that the optimal contract is characterized by a pair (σ, r) that solves

$$\min_{\{\sigma, r\}} \left\{ p\beta + 1 - F_l(\sigma) + pF_h(\sigma) \int_r^h (h-t)dH(t) \right\} \quad (30)$$

s.t.

$$F_l(\sigma) - H(r)F_h(\sigma) = \beta, \quad (31)$$

$$q_0 \leq H(r) \leq 1. \quad (32)$$

If there exists no σ such that

$$F_l(\sigma) - q_0F_l(\sigma) = \beta, \quad (33)$$

then there exists no contract that elicits high effort.

Looking for an interior solution (that is, ignoring (32)), the first-order constraint reads:

$$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h-r) + 1}{p \int_r^h H(t)dt} \equiv T(r). \quad (34)$$

The system of equations $\{(31);(34)\}$ has at most one solution (σ, r) . It is best seen graphically on Figure 3:

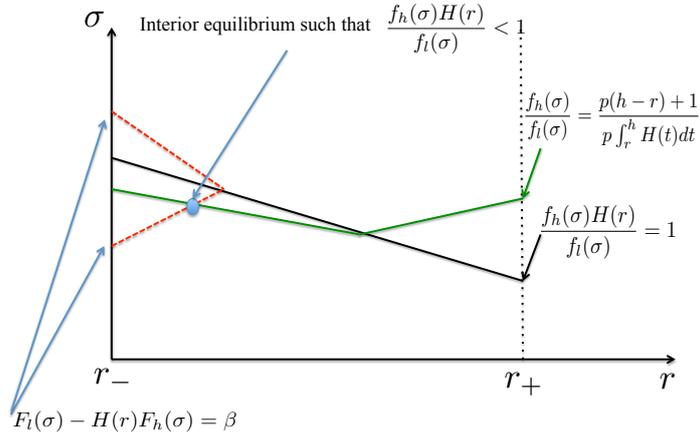
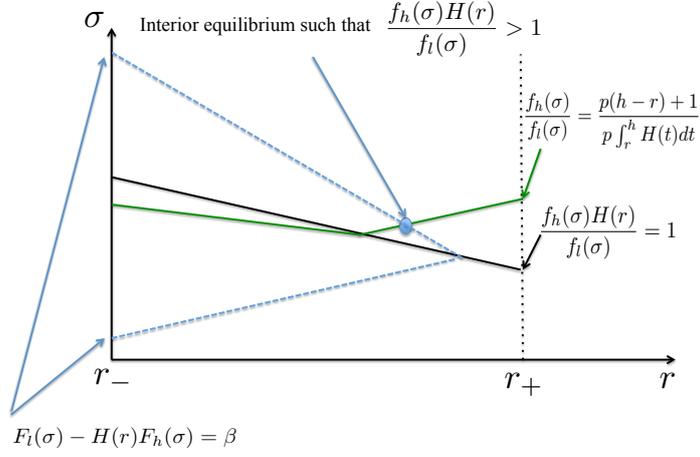


Figure 3. First-order condition and incentive-compatibility constraint in the (σ, r) plane. Each graph shows a possible configuration for the IC constraints.

The equation $[f_h(\sigma)/f_l(\sigma)]H(r) = 1$ defines a decreasing frontier in the plane (r, σ) over $r \in [r_-, r_+]$, the support of H . The first-order condition (34) implicitly defines σ as first decreasing in r below this frontier and then increasing above it over $[r_-, r_+]$. The incentive-compatibility condition defines implicitly two thresholds σ as functions of $r \in [r_-, r_+]$, one that is increasing and lies below the frontier and one that is decreasing and lies above it. These two graphs intersect at the frontier at the maximum value of r for which there exists at least one σ such that the incentive-compatibility constraint is satisfied.

From Figure 3, it is easy to see that there is at most one (σ, r) that solves $\{(31);(34)\}$. Otherwise, there is a corner solution (σ, r) such that

$$\frac{f_h(\sigma)}{f_l(\sigma)} \geq T(r) \quad (35)$$

and $H(r) \in \{q_0; 1\}$.³³ At such a corner solution, it would be optimal to lower $f_h(\sigma)/f_l(\sigma)$ by reducing σ and to raise the value of $T(r)$, but a binding feasibility constraint prevents it:

- If the corner solution is such that $f_h H/f_l > 1$, $T(r)$ is locally increasing in r and the constraint $H(r) = 1$ is binding. There are no resales and the optimal contract has a signal cut-off equal to the largest solution to:

$$F_l(\sigma) - F_h(\sigma) = \beta. \quad (36)$$

- If the corner solution is such that $f_h H/f_l < 1$, $T(r)$ is locally decreasing in r and the constraint $H(r) = q_0$ is binding. The optimal contract is (σ, r_-) , where σ is the smallest solution to $F_l(\sigma) - q_0 F_h(\sigma) = \beta$.

Finally, the expression of the cost of capital (3) stems directly from computing the objective (26) under the optimal contract.

Proof of Lemma 3

Proof that bids for h -projects must be strictly above l

Suppose otherwise. Since buyers cannot find it optimal to place bids to which they assign a zero-probability of execution, this means that equilibrium contracts must be such that firms with h -projects allow for resale at prices lower than l . This cannot be optimal as these resales are costly and do not provide (actually impair) ex-ante incentives: The probabilities of such resales of h and l projects are identical in equilibrium, equal to q_0 .

Equilibrium bidding strategies

We first show that bidding strategies are either degenerate or must have a continuous c.d.f. If a bidding strategy has h in its support then it must be a Dirac delta at h since bidders must be indifferent between all bids. Suppose a bidding strategy is nondegenerate. We show that it cannot have an atom at any point of its support. Suppose otherwise that

³³The direction of inequality (35) is the same for both corner solutions because the first-order condition in σ defines σ as either increasing or decreasing in the shadow cost of (31) depending on the sign of $[f_h H/f_l] - 1$.

it is not left-continuous at some point of the support (necessarily strictly smaller than h). Then bids in the right-neighborhood of this point strictly dominate a bid at this point, which cannot be.

The fact that bids must have a continuous c.d.f. when nondegenerate implies that the contracts set by firms are given by Proposition 1 in this case.

We then show that the lower bound of the support of S , r_- , is equal to the reservation price r anticipated by bidders. It must be that $r_- \geq r$. Suppose $r_- > r$. We know from the above that S is atomless at r_- . This implies that bidding r (or any value within (l, r_-) when $r = l^+$) strictly dominates bidding r_- , which cannot be. Thus $r = r_-$.

We then compute π , the expected profit of an informed buyer. We have just seen that the distribution of bids S is atomless with support of the form $[r, r_+] \subset [r, h]$. Thus the ex-ante (before matching) expected profit π of a bidder satisfies:

$$\text{For all } t \in [r, r_+], pF_h(\sigma) \sum_{k \geq 1} \frac{q_k}{1 - q_0} (h - t) S^{k-1}(t) = \pi \quad (37)$$

which for $t = r$ yields

$$\pi = \frac{pF_h(\sigma)q_1(h - r)}{1 - q_0}. \quad (38)$$

Equilibrium incentive-compatibility constraint and first-order condition

There are λ buyers per firm on average so the expected resale cost for a firm is

$$pF_h(\sigma) \int_r^h (h - t) dH(t) = \lambda\pi. \quad (39)$$

The incentive-compatibility constraint (14) is derived in the body of the paper, and the first-order condition (15) stems from

$$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h - r) + 1}{pH(r)(h - r) + p \int_r^h (h - t) dH(t)}, \quad (40)$$

$$H(r) = q_0, \quad (41)$$

$$p \int_r^h (h - t) dH(t) = \frac{\lambda p q_1 (h - r)}{1 - q_0}. \quad (42)$$

Finally, note that, using the incentive-compatibility constraint, the cost of capital can be re-written as

$$1 - \beta + p\beta + F_h(\sigma) \left[\frac{\lambda p q_1 (h - r)}{1 - q_0} - q_0 \right]. \quad (43)$$

The term in square brackets is positive (negative) if $f_h q_0 / f_l \leq 1$ ($f_h q_0 / f_l \geq 1$).

Proof of Proposition 5

It is easy to see that an optimal σ' -contract consists in rewarding the agent based on a signal larger than $\sigma^R \geq \sigma'$ and allowing him to sell the asset above some reserve price otherwise, where σ^R may be infinite. That the incentive-compatibility constraint must be binding, and that the equilibrium probability of resale is q_0 whenever resale is allowed, imply that for every $\sigma' \in (\underline{\sigma}, \bar{\sigma}]$, the only equilibrium with σ' -contracts is the one with the contract $(\bar{\sigma}, \bar{r})$. If $\sigma' > \bar{\sigma}$, there is no equilibrium with incentive-compatible σ' -contracts.

Proof of Proposition 6

For ϵ sufficiently small, for each λ such that $(1 - q_0(\lambda)) \in [\beta - \epsilon, \beta)$, there exists a unique σ such that (19) holds and $f_h q_0 / f_l > 1$ for (λ, σ) . Continuity implies that the set of σ defined this way is an interval $[\sigma_1, +\infty)$. Further, differentiating the incentive-compatibility (19) constraint yields:

$$\frac{\partial \sigma}{\partial \lambda} = \frac{\frac{\partial q_0}{\partial \lambda} F_h + q_0 \frac{\partial F_h}{\partial \lambda} - \frac{\partial F_l}{\partial \lambda}}{f_l - q_0 f_h} > 0 \quad (44)$$

because

$$q_0 \frac{\partial F_h}{\partial \lambda} - \frac{\partial F_l}{\partial \lambda} \leq \frac{\partial F_h}{\partial \lambda} \left(q_0 - \frac{f_l}{f_h} \right) \leq 0. \quad (45)$$

from condition (17).

For any $\sigma \geq \sigma_1$, one can then uniquely define $\lambda(\sigma)$ and $r(\sigma)$ such that (19) and (20) are satisfied for $(\sigma, r(\sigma), \lambda(\sigma))$. Define then $\Sigma(\sigma)$ as the solution to (18) for such $(r(\sigma), \lambda(\sigma))$. For κ sufficiently small, $r(\sigma)$ takes values in a compact set that is sufficiently close to h and thus $\Sigma(\sigma) \geq \sigma_1$ for all $\sigma \geq \sigma_1$. Also, Σ is bounded from above because $\lambda(\sigma)$ and $r(\sigma)$ take values in compact sets. Denoting σ_2 this upper bound, Σ is an (obviously continuous) mapping over $[\sigma_1, \sigma_2]$ and thus admits a fixed point which is a stable equilibrium by construction.

Proof of Proposition 7

Note first that the equilibrium distribution of the highest bid $S(\cdot, \lambda, r)$ is given by:

$$\sum_{k \geq 1} \frac{q_k(\lambda)}{q_1(\lambda)} (h - t) S^{k-1}(t, \lambda, r) = h - r, \quad (46)$$

That q_k/q_1 increases w.r.t. λ for all $k \geq 1$ implies that $S(t, \lambda, r)$ must decrease w.r.t. λ for all r, t . It is also transparent from (46) that $S(t, \lambda, r)$ decreases in r for all λ, t . Also,

$$H = \sum_{k \geq 0} q_k S^k \quad (47)$$

implies that H increases in the sense of first-order stochastic dominance when S and $\{q_k\}$ do.

Second, taking λ , σ and r as given, an individual firm chooses a contract (σ', r') that solves:

$$\max_{(\sigma', r')} V(\sigma', r') = F_l(\sigma', \lambda) - pF_h(\sigma', \lambda) \int_{r'}^h (h-t) \frac{\partial H(t, \lambda, r)}{\partial t} dt \quad (48)$$

s.t.

$$F_l(\sigma', \lambda) - F_h(\sigma', \lambda)H(r', \lambda, r) = \beta. \quad (49)$$

The Lagrangian of this program is

$$\begin{aligned} \mathcal{L} &= V(\sigma', r') + p(h-r')[F_l(\sigma', \lambda) - F_h(\sigma', \lambda)H(r', \lambda, r) - \beta] \\ &= [1 + p(h-r')]F_l(\sigma', \lambda) - F_h(\sigma', \lambda) \left[p \int_{r'}^h H(t, \lambda, r) dt \right] - \beta p(h-r'). \end{aligned} \quad (50)$$

The envelope theorem then yields that at the equilibrium values $(\sigma', r') = (\sigma, r)$,

$$\begin{aligned} \frac{\partial V}{\partial \lambda} &= \left[[1 + p(h-r)] \frac{\partial F_l(\sigma, \lambda)}{\partial \lambda} - \frac{\partial F_h(\sigma, \lambda)}{\partial \lambda} \left[p \int_r^h H(t, \lambda, r) dt \right] \right] \\ &\quad - pF_h(\sigma, \lambda) \int_r^h \frac{\partial H(t, \lambda, r)}{\partial \lambda} dt, \\ \frac{\partial V}{\partial r} &= -pF_h(\sigma, \lambda) \int_r^h \frac{\partial H(t, \lambda, r)}{\partial r} dt. \end{aligned}$$

Conditions (17) and (18) imply that

$$[1 + p(h-r)] \frac{\partial F_l(\sigma, \lambda)}{\partial \lambda} - \frac{\partial F_h(\sigma, \lambda)}{\partial \lambda} \left[p \int_r^h H(t, \lambda, r) dt \right]$$

is positive in equilibrium. Thus an increase in λ yields more efficient equilibrium contracting through a lower cost of marking to market, strictly so if (17) is strict. Even when (17) binds, an increase in λ improves contracting because it reduces the cost of taking to market by inducing more aggressive bidding (term $-pF_h(\sigma, \lambda) \int_r^h \partial H(t, \lambda, r)/\partial \lambda dt$), and so does an increase in the reserve price r (term $-pF_h(\sigma, \lambda) \int_r^h \partial H(t, \lambda, r)/\partial r dt$).

Suppose now that the planner forces a contract with cut-off $\sigma' > \sigma$. An incentive-compatible equilibrium must be such that firms reward their agents if the signal is above σ' and use otherwise a reserve price r' that leads to a speculative activity λ' such that (19) and (20) hold given σ' :

$$\begin{aligned} F_l(\sigma', \lambda') - F_h(\sigma', \lambda')q_0(\lambda') &= \beta, \\ pF_h(\sigma', \lambda') \frac{q_1(\lambda')(h-r')}{1-q_0(\lambda')} &= \kappa. \end{aligned}$$

By continuity there exists such a triplet (σ', r', λ') provided σ' is sufficiently close to σ . It only remains to show that $r' > r$ and $\lambda' > \lambda$. The latter result we already established in the proof of Proposition 6 where we showed that (19) implicitly defines a strictly increasing $\lambda(\sigma)$. Rewriting (20) as:

$$p[F_l(\sigma, \lambda) - \beta] \frac{q_1(\lambda)(h - r)}{q_0(\lambda)(1 - q_0(\lambda))} = \kappa \quad (51)$$

shows that an increase in σ also yields an increase in r since $q_1/q_0(1 - q_0)$ increases in λ , and (19) and condition (17) imply:

$$f_l + \frac{\partial F_l}{\partial \lambda} \frac{\partial \lambda}{\partial \sigma} \geq \frac{\partial \lambda}{\partial \sigma} \frac{\frac{\partial q_0}{\partial \lambda} F_h}{\left(1 - \frac{q_0 f_h}{f_l}\right)} > 0. \quad (52)$$

Proof of Proposition 9

Step 1. Optimal contracting in the presence of early payoff discovery

We first revisit the contracting problem of Section 2 in the case in which, with a probability $\gamma < \beta$, the firm discovers the project value at date 1. It is clearly optimal that, in the event of such an early discovery, the agent receives utility 1 if the payoff is h and 0 if it is l . It is easy to see that the optimal course of action in the absence of early discovery solves the same problem as that in the case $\gamma = 0$ up to a replacement of the parameter β with $\beta' = (\beta - \gamma)/(1 - \gamma) < \beta$.

Step 2. Optimal contracting and bidding in the presence of early payoff discovery

This implies that in the setting of Section 4.1 with exogenous signals and endogenous number of informed bidders, Lemma 6 applies in the presence of such early discovery: For κ, l sufficiently small, a stable equilibrium exists and solves (18), (19), and (20) where the parameters β and κ are replaced by $\beta' = (\beta - \gamma)/(1 - \gamma)$ and $\kappa' = \kappa/(1 - \gamma)$ respectively.

Step 3. Optimal contracting and bidding in the presence of early payoff discovery and gains from trade

We now study how the presence of gains from trade between firms' principals and potential buyers affects such a stable equilibrium with possible early discovery described in Steps 1 and 2. We suppose that potential buyers value a payoff $y + z$ at $y + z + \epsilon$, where $\epsilon > 0$. We show that in the limiting case in which $\epsilon \rightarrow 0$ (infinitesimal gains from trade), these gains from trade induce the preference for trading that is directly assumed in the body of the paper. For ϵ sufficiently small, firms that discover an h -payoff and firms that receive

a bid larger than r and a signal larger than σ never sell as their valuation of the project, h , exceeds that of the highest possible bid. This is because the condition that informed buyers be indifferent between bids for h -projects implies that their bids are bounded away from h . Sales that take place above r are therefore only for incentive purposes. Firms that discover an l -payoff always sell as uninformed bids are competitive and thus weakly larger than $l + z + \epsilon$.

Denote l_u the value of uninformed bids (net of z). Firms that do not discover the payoff nor receive a bid (net of z) larger than r sell at this price provided their signal is smaller than σ_1 such that

$$\frac{pq_0 f_h(\sigma_1)h + (1-p)f_l(\sigma_1)l}{pq_0 f_h(\sigma_1) + (1-p)f_l(\sigma_1)} = l_u. \quad (53)$$

Therefore,

$$l_u = \int_{-\infty}^{\sigma_1} \frac{pq_0 f_h(s)h + (1-p)f_l(s)l}{pq_0 f_h(s) + (1-p)f_l(s)} ds + \epsilon, \quad (54)$$

and

$$\lim_{\epsilon \rightarrow 0} l_u = l; \lim_{\epsilon \rightarrow 0} \sigma_1 = -\infty. \quad (55)$$

Thus, in the limiting case $\epsilon \rightarrow 0$, gains from trade induce only sales of l -payoff projects discovered by firms.

Note that the resales meant to reap gains from trade do not affect informed bidding strategies nor informed bidders' expected profits. Thus the equilibrium values $\{\sigma; r; \lambda\}$ are as stated in Step 2.

Step 4. Endogenous signals

Step 3 shows that there exists a stable equilibrium in the extension of Section 4.1 to early payoff discovery and arbitrarily small gains from trade. It remains to show that this applies to the case in which the signals are given by:

$$F_l(s) = \Psi(s - l), \quad (56)$$

$$F_h(s, \lambda, r) = \Psi * H_1(s, \lambda, r), \quad (57)$$

where Ψ is the c.d.f. of z and

$$H_1(s, \lambda, r) = \sum_{k \geq 1} \frac{q_k(\lambda)}{1 - q_0(\lambda)} S^k(s, \lambda, r), \quad (58)$$

where S is implicitly defined by (46). Such F_h , F_l satisfy (17) because $\partial F_l / \partial \lambda = 0$, $\partial F_h / \partial \lambda \leq 0$.

These signals depart from the assumptions of section 4.1 to the extent that F_h also depends on r with $\partial F_h / \partial r \leq 0$. We leave it to the reader to check that the proofs of Lemma 6 and Proposition 7 can be simply adapted to this case.

Online appendix: Extensions

This appendix states and proves various results that extend the results discussed throughout the paper.

E.1 Second-order condition for program $\{(4); (5)\}$

The Lagrangian of the program is

$$L(\sigma, r, \nu) = f(\sigma, r) + \nu g(\sigma, r) \quad (\text{E.1})$$

where

$$f(\sigma, r) = F_l(\sigma) - pF_h(\sigma) \int_r^h (h-t)dH(t), \quad (\text{E.2})$$

$$g(\sigma, r) = F_l(\sigma) - H(r)F_h(\sigma). \quad (\text{E.3})$$

We need to compute the determinant of the bordered Hessian matrix

$$\begin{bmatrix} 0 & g_\sigma & g_r \\ g_\sigma & L_{\sigma\sigma} & L_{\sigma r} \\ g_r & L_{r\sigma} & L_{rr} \end{bmatrix}. \quad (\text{E.4})$$

We have

$$L_\sigma = f_l - pf_h \int_r^h (h-t)dH(t) + \nu(f_l - H(r)f_h) = 0, \quad (\text{E.5})$$

$$L_r = p(h-r)F_h \frac{dH(r)}{dr} - \nu F_h \frac{dH(r)}{dr} = 0 \rightarrow \nu = p(h-r), \quad (\text{E.6})$$

$$L_{\sigma r} = p(h-r)f_h \frac{dH(r)}{dr} - \nu f_h \frac{dH(r)}{dr} = 0, \quad (\text{E.7})$$

$$L_{r\sigma} = p(h-r)f_h \frac{dH(r)}{dr} - \nu f_h \frac{dH(r)}{dr} = 0, \quad (\text{E.8})$$

$$L_{rr} = -pF_h \frac{dH(r)}{dr}, \quad (\text{E.9})$$

$$L_{\sigma\sigma} = f'_l - pf'_h \int_r^h (h-t)dH(t) + \nu(f'_l - H(r)f'_h) = f_l(1 + p(h-r))(f'_l/f_l - f'_h/f_h), \quad (\text{E.10})$$

$$g_\sigma = f_l - H(r)f_h, \quad (\text{E.11})$$

$$g_r = -\frac{dH(r)}{dr}F_h. \quad (\text{E.12})$$

This yields a determinant

$$(H(r)f_h - f_l)^2 pF_h \frac{dH(r)}{dr} + F_h^2 f_l(1 + p(h-r)) \frac{dH(r)}{dr} (f'_h/f_h - f'_l/f_l) > 0. \quad (\text{E.13})$$

E.2 Business model and measurement regime

Throughout the paper, we take the agent's reward date as fixed ($t = 1$) for expositional simplicity. In this section, we more generally follow the literature on incentives provision for agents with liquidity needs³⁴ and assume that delaying compensation to date 2 involves a social cost. Suppose that the agent derives utility at both dates 1 and 2 and has preferences

$$u_1 + \delta u_2, \tag{E.14}$$

where $\delta, u_1, u_2 \in [0, 1]$. The principal can still provide the agent with utility u_t at cost u_t for $t \in \{1; 2\}$. The baseline model corresponds to the case in which $\delta = 0$. The second-best case without measurement frictions corresponds to $\delta = 1$. It is straightforward to extend the analysis to the case in which $\delta \in (0, 1)$.

In case of a date-1 resale, the price reveals the project's payoff, and thus the principal knows it at date 2. If the firm holds on to the asset until date 2, the payoff is revealed at this date. This implies that either way, the principal knows the payoff at date 2. The information conveyed by the date-1 signal and resale (if any) is thus immaterial at this date. Also, the cost of providing utility at date 2 is independent of any utility already provided at date 1. Thus, one can without loss of generality consider only contracts whereby the date-2 compensation does not depend on the contracting history, only on the realized payoff. Denote (σ, r, y) such a contract. It is such that the agent is rewarded at date 1 if the signal is above σ , or if it is below σ and he manages to sell the project at a price above r . He may also be rewarded at date 2 with probability y if the project pays off h . Ignoring feasibility constraints, an optimal contract solves the counterpart of $\{(4), (5)\}$

$$\min_{\{\sigma, r, y\}} \left\{ 1 - F_l(\sigma) + pF_h(\sigma) \int_r^h (h-t)dH(t) + p(1-\delta)y \right\} \tag{E.15}$$

s.t.

$$F_l(\sigma) - H(r)F_h(\sigma) + \delta y = \beta. \tag{E.16}$$

For brevity, we discuss only the case in which the optimal contract corresponds to an interior solution of this program. The interior solution is then characterized by (E.16) and two first-order conditions:

$$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h-r) + 1}{p \int_r^h H(t)dt}, \tag{E.17}$$

$$h - r = \frac{1 - \delta}{\delta}. \tag{E.18}$$

³⁴See e.g. Aghion et al (2004), or Faure-Grimaud-Gromb (2004).

It is then obvious that an increase in δ , starting from a contract such that $f_h H(r)/f_l > 1$, yields an increase in σ , r , and y .

This extension of the model bears interesting relationship to the accounting standard IFRS 9 issued in 2014. In this standard, the business model used by an entity for managing an asset affects the measurement of this asset. The “hold and collect” business model, whereby firms acquire assets to collect their cash flows until maturity, is the one that corresponds to the lowest degree of marking to market. In line with this, this simple extension predicts that firms with more patient agents (a higher δ other things being equal) rely more on the “hold to collect” model and, at the same time, use a lower degree of marking to market because (σ, r, y) increases in δ . Thus we rationalize this connection between “business model” and measurement regime.

E.3 Uncertain liquidity and the suspension of fair-value accounting

Suppose that firms must write contracts before observing the mass of informed buyers λ . They share the prior that λ is distributed according to the c.d.f. Λ . After agents have exerted their forecasting effort, the value of λ is publicly observed by firms and buyers. The optimal equilibrium contract is now $(r(\lambda), \sigma(\lambda))$, contingent on the realization of λ . Suppose that the matching probabilities $\{q_k(\lambda)\}_{k \in \mathbb{N}}$ are such that $q_{k+1}(\lambda)/q_k(\lambda)$ increases in λ for all k . For brevity we focus on the stable equilibrium with a finite σ . We have:

Proposition E.1. (*Uncertain liquidity*) *The equilibrium contract is such that the reserve price is a constant r and the signal threshold $\sigma(\lambda)$ is an increasing function of λ .*

Proof. We skip the proof that an equilibrium contract must be of the form $(r(\lambda), \sigma(\lambda))$. Such a contract solves

$$\max_{(r(\cdot), \sigma(\cdot))} \left\{ \int \left[1 - F_l(\sigma(\lambda)) + p F_h(\sigma(\lambda)) \int_{r(\lambda)}^h (h-t) \frac{\partial H(t, \lambda)}{\partial t} dt \right] d\Lambda(\lambda) \right\} \quad (\text{E.19})$$

s.t.

$$\int [F_l(\sigma(\lambda)) - H(r(\lambda), \lambda) F_h(\sigma(\lambda))] d\Lambda(\lambda) = \beta \quad (\text{E.20})$$

The first-order condition yields

$$r(\lambda) = r, \quad (\text{E.21})$$

$$\frac{f_h(\sigma(\lambda))}{f_l(\sigma(\lambda))} = \frac{p(h-r) + 1}{p \int_r^h H(t, \lambda) dt}. \quad (\text{E.22})$$

It only remains to show that $H(t, \lambda)$ increases in λ in the sense of first-order stochastic dominance ($\partial H/\partial \lambda \leq 0$). Note first that the distribution of bids $S(\cdot, \lambda)$ satisfies

$$\sum_{k \geq 1} \frac{q_k(\lambda)}{q_1(\lambda)} (h - t) S^{k-1}(t, \lambda) = h - r. \quad (\text{E.23})$$

That q_k/q_1 increases in λ implies that $S(t, \lambda)$ must decrease w.r.t. λ for all t . Second,

$$H = \sum_{k \geq 0} q_k S^k \quad (\text{E.24})$$

implies that so does H from first-order stochastic dominance. ■

In the presence of uncertain liquidity, firms equate the marginal cost of a resale across states (r constant). Because the average resale cost decreases in λ , firms raise the cut-off above which they reward agents based on the signal as the market becomes more liquid, and thus rely more on resales in this case. Otherwise stated, the recognition threshold α that implements the optimal contract is larger in illiquid times.

Following the 2008 financial crisis, regulators across the globe have temporarily suspended marking to market (or marking to model) for several banks' asset classes. The motivation was that, due to an abnormal lack of buyers, transaction data was irrelevant and would have triggered inefficient fire sales. The ex-ante optimal rule established in Proposition E.1 relates to such decisions. In illiquid times, firms optimally ignore low transaction signals: They do not punish agents following such signals and avoid very costly resales.