On the Possibility of Price Decreasing Bubbles

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ON THE POSSIBILITY OF PRICE DECREASING BUBBLES

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A RATIONAL SPECULATIVE BUBBLE on a stock which can be freely disposed of—a form of limited liability—is nonnegative by definition:[2] it represents what an investor might be willing to pay to buy a stock *forever* stripped of its dividends. The fundamental, on its part, represents the present discounted value of the dividend stream. In legal terms, the fundamental is the value of the *usufruct*, while the bubble measures the value of the stock minus its usufruct. For instance, a claim to an apple tree could be split into two assets: a claim to the apple crop (whose value is the fundamental), and a claim to the useless[3] wood itself (the bubble). Because of the free disposal assumption, the price of the latter claim cannot be negative; a rational speculative bubble on an asset which can be costlessly destroyed is, therefore, nonnegative.

Much has been made of this fact in some recent discussions.[4] It has been in particular argued that the definition given *supra* of a rational bubble poorly captures the essence of bubbles in actual economies. Two reasons are advanced. Firstly, it is proposed, "a bubble cannot start; to exist, it must be present since the origin of time." This is true, but in the somewhat trivial sense that, with perfect markets without transaction costs, the equilibrium price today of any one of the innumerable "assets" which have an even infinitesimal probability of being valued in the future cannot be zero; by backward recursion, every bubble which may conceivably arise tomorrow must exist today. Secondly, and more importantly, the above definition of a rational bubble is judged to be unsatisfactory because it cannot account for the existence of "undervalued" assets. "Because bubbles are positive," it is claimed, "the presence of a bubble increases the equilibrium price of an asset."

The latter argument is of course correct if the fundamental is independent of the presence or absence of a bubble:[5] the sum of a positive bubble and a *fixed* fundamental indeed exceeds the fundamental, and the "bubbly" price—in Tirole's terminology—is, therefore, larger than the nonbubbly price. As soon as either dividends or discount rates depend on the presence or absence of a bubble, however, the fundamental is affected by the presence of a bubble. For instance—and this is the scenario explored in this paper,[6] the existence of a bubble may lead to an increase in interest rates which so depresses the fundamental that the sum of the positive bubble and of the bubbly fundamental falls short of the nonbubbly fundamental. Hence, *a positive rational bubble may in fact decrease the overall price of a stock*, contrary to what is commonly believed.

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[2]In the following, I adopt Tirole's (1982, 1985) definition of a rational bubble.

[3]I assume, for the sake of discussion, that apple wood can neither be burned nor serve any other useful purpose, and is costless to destroy.

[4]See, for instance, Diba and Grossman (1987) and West (1988).

[5]This will in particular occur whenever the dividend is always zero, or for any asset in zero net supply which can be, as a consequence, priced at the margin.

[6]The other possibility—namely that managers may pay attention, when setting dividends or issuing new shares, to the existence of a bubble on their company's stock—is not analyzed here. A result akin to Wallace's (1980) theorem on the maximum rate of growth of the money supply is likely to hold in that setting: the bubble on a stock must be driven to zero if new bubbly shares are issued "too fast" by the firm.

The purpose of this paper is to formalize the foregoing argument, and to determine conditions under which price decreasing bubbles on a stock may indeed exist, in general equilibrium, when the discount rate is not exogenous. In Section 1, I construct a simple general equilibrium model in which a stock need not be priced at its fundamental. In Section 2, I derive existence conditions for price decreasing rational bubbles. The conclusion summarizes the results.

1. THE MODEL

To make sense of the question asked in this paper—can price decreasing bubbles exist in general equilibrium?—it is of course necessary to first construct a model in which asset bubbles are at all possible. This is achieved, most simply, by considering a simple two-period overlapping generation model without intergenerational altruism which provides, through the perpetual arrival of new cohorts, the *new* entrants into the market required, as shown by Tirole (1982), for the existence of rational speculative bubbles. What is therefore crucial, in this overlapping generation setting, for the existence of bubbles is *not* the finiteness of the agents' horizon but, instead, the fact that successive generations are economically distinct.[7]

1.1. *Consumers*

Consumers live for two periods. Population is constant, with reproduction assumed to take place by parthenogenesis.[8] All consumers are identical, both within a generation and across cohorts. There is only one nonstorable good, call it a fruit. A (selfish) representative consumer born at time t wishes to maximize the time separable utility

$$U_t \equiv u(c_{1t}) + v(c_{2t}) \tag{1}$$

he derives from his consumption bundle (c_{1t}, c_{2t}) when young and old. The functions $u(.)$ and $v(.)$ are defined on R^+, are continuous, increasing, and concave, and satisfy the following assumptions:

ASSUMPTION 1: $\lim_{c \to 0} u'(c) = \lim_{c \to 0} v'(c) = +\infty$.

ASSUMPTION 2: $\lim_{c \to +\infty} u'(c) = \lim_{c \to +\infty} v'(c) = 0$.

Each agent receives an endowment (e_1, e_2), $e_1 > 0$, $e_2 \geqslant 0$ of the consumption good. It is in addition assumed that the following inequality holds:

ASSUMPTION 3: $u'(e_1)/v'(e_2) < 1$.

Assumption 3 implies the dynamic inefficiency, in this zero population growth economy, of any competitive equilibrium yielding (e_1, e_2) as a consumption allocation. This crucial assumption will be used *infra*.

[7]See my (1989) paper for an example of an economy in which bubbles are possible, despite the infinite horizon of every agent alive, because of the continuous arrival into the economy of *new* infinitely-lived families.

[8]See Bernheim and Bagwell (1988) for the complications introduced by marriage.

To smooth out, if desired, his consumption profile, our consumer can buy x_t fruit producing trees, at a price of p_t fruits each. His budget constraints are thus:[9]

$$c_{1t} + p_t x_t = e_1, \tag{2}$$

$$c_{2t} = e_2 + (p_{t+1} + y_{t+1})x_t, \tag{3}$$

$$c_{1t}, c_{2t} \geqslant 0, \tag{4}$$

where $y_{t+1} > 0$ denotes the fruit output of a tree at time $t+1$. The first-order condition for an interior[10] utility maximum is simply

$$p_t u'(c_{1t}) = (p_{t+1} + y_{t+1})v'(c_{2t}). \tag{5}$$

Equation (5) can be solved, using (2) and (3), for the optimum tree holding:

$$p_t x_t = S\left(\frac{p_{t+1} + y_{t+1}}{p_t}\right), \tag{6}$$

where $S(.)$ denotes the savings function.

1.2. *Technology*

Trees have the following features. There is a constant number of identical trees, normalized without loss of generality to be equal to the size of the population; the number of trees per capita is therefore equal to 1. It is known from the origin of time that

$$y_t = \begin{cases} y > 0 & \text{for } 0 < t \leqslant T-1, \\ 0 & \text{for } t \geqslant T, \end{cases} \tag{7}$$

where T is a possibly very large but finite integer. In other terms, a tree produces fruits for only a finite number of periods. In the long run ($t \geqslant T$), trees become sterile. It is further assumed, in the spirit of the discussion above, that the wood of the tree is intrinsically useless and costless to destroy; the only reason a consumer might therefore choose to buy a tree in or after period $T-1$ is for speculative purposes: "buying in order to resell." Prior to period $T-1$, however, trees have a positive fundamental—the present discounted value of future dividends.

The rationale for specifying the dividend process in that fashion is best understood by remembering Tirole's (1985) results: if (i) dividends are capitalized *ex ante* (i.e., all the trees ever present in the economy are traded today), and (ii) dividends per head do not become zero in the long run, there cannot be a bubble on a asset with a fundamental. Our setup violates the second condition—the dividend per head becomes zero after time T—which leaves open the possibility that a bubble may exist on trees.[11]

1.3. *Competitive Equilibria*

In a competitive equilibrium, a representative consumer must hold—because of the choice of units—precisely one tree:

$$x_t = 1, \qquad \forall\, t \geqslant 0. \tag{8}$$

[9] I do not explicitly introduce a consumption-loans market, as it will be inactive in equilibrium.
[10] Corner solutions are not characterized, as equilibrium considerations impose interiority.
[11] See Tirole (1985, Proposition 1).

Therefore, substituting (8) into (5), one finds that the equilibrium fruit price of trees must satisfy the following first-order difference equation:

$$(9) \qquad p_t u'(e_1 - p_t) = (p_{t+1} + y_{t+1}) v'(e_2 + p_{t+1} + y_{t+1}),$$

which is forced by the dividend process given in (7). Equivalently, the equilibrium fruit price of a tree must follow

$$(10) \qquad p_t = S\left(\frac{p_{t+1} + y_{t+1}}{p_t}\right).$$

Given the structure of the dividend process, it is simplest to divide the solution procedure into two parts: $t \geqslant T-1$ and $t \leqslant T-2$.

1.3.1. *Long run ($t \geqslant T-1$)*

Trees become intrinsically useless as of period T. Their fundamental value is thus zero as of period $T-1$ whether or not a bubble is present. Determining the equilibrium price of trees thus amounts to studying whether or not a bubble on an intrinsically useless asset may exist in this economy starting in period $T-1$. Now we know, from Gale (1973), from Wallace's (1980) results on fiat money equilibria and Tirole's (1985) related work on bubbles, that, in many cases, the dynamic inefficiency of the *bubbleless* economy is necessary and sufficient for the existence of bubbly equilibria. As the following shows, this holds true in the present model.

If trees are not valued in and after period $T-1$ ($p_t = 0$ for $t \geqslant T-1$), the competitive equilibrium consumption allocation is, trivially, the autarkic one (e_1, e_2)—which, by construction (see Assumption 3), is dynamically inefficient. This equilibrium is one in which there is no bubble: the price of a tree is equal to its zero fundamental.

But then, because Assumption 3 holds, it is easy to check that there also exists a stationary *golden rule* bubbly equilibrium characterized by

$$(11) \qquad p_t = \hat{p}, \qquad \forall\, t \geqslant T-1,$$

where $\hat{p}$ is given, from (10) and the fact that the dividend is zero after $T-1$, by

$$(12) \qquad \hat{p} = S(1) > 0,$$

with the inequality implied by the dynamic inefficiency of the autarkic equilibrium imposed by Assumption 3.[12] *Because the long run fundamental is constructed to be zero independently of equilibrium interest rates*, $\hat{p}$ represents a pure bubble. Bubbles are thus trivially price increasing *in the long run* in this economy—in accordance with the intuition given in the introduction.

Before proceeding further, it is necessary to note that many nonstationary equilibria exist in addition to the stationary equilibria described above. Following standard[13] arguments, it is easy to show that if the interest elasticity of savings is not too negative, these nonstationary paths converge to the inefficient nonbubbly steady state. If income effects are strong ($S'(.) \ll 0$), however, cyclical or chaotic equilibria may occur. For the sake of simplicity, I henceforth concentrate on equilibria which are stationary as of period $T-1$—i.e., equilibria which are such that either $p_t = 0$ or $p_t = \hat{p}$ for all $t \geqslant T-1$.

[12] Equivalently, $\hat{p}$ is the stationary solution to (9).

[13] See, for instance, Grandmont (1985), Tirole (1985), and Wallace (1980).

1.3.2. *Short-run ($t < T-1$)*

In the short run, trees have a positive fundamental. Let

$$R_{t+1} \equiv \frac{u'(e_1 - p_t)}{v'(e_2 + p_{t+1} + y)} \tag{13}$$

denote the (implicit) gross interest rate on consumption loans between periods t and $t+1$. The fundamental value f_τ of a tree at some date $\tau < T-1$ is simply the present discounted value of dividends distributed by trees until $T-1$, i.e.,

$$f_\tau \equiv \sum_{i=\tau+1}^{T-1} \frac{y}{\Pi_{s=\tau+1}^{i} R_s}. \tag{14}$$

Note that the discount factors depend on the asset price itself, so that the fundamental will differ in general across bubbly and nonbubbly equilibria.

The equilibrium behavior of tree prices in the short run is characterized, from (7) and (10), by *backward*[14] iterations of the difference equation

$$p_t u'(e_1 - p_t) = (p_{t+1} + y)v'(e_2 + p_{t+1} + y), \tag{15}$$

subject to either of the following "terminal" conditions:

$$p_{T-1} = 0 \quad \text{or} \quad p_{T-1} = \hat{p}. \tag{16}$$

Because the function $pu'(e_1 - p)$ is increasing and maps, using Assumptions 1 and 2, the interval $[0, e_1)$ onto $[0, +\infty)$, it has an inverse, so that the backward perfect foresight dynamics are uniquely defined, and can be represented, from (15), as

$$p_t = \varphi(p_{t+1}), \tag{17}$$

where φ, continuous and differentiable, maps $[0, \infty]$ onto $[0, e_1)$.

Using (17) and the terminal conditions (16), two competitive (long-run stationary) equilibrium paths emerge:

- a non-bubbly equilibrium path $\bar{p} = (\bar{p}_0, \bar{p}_1, \ldots, \bar{p}_{T-2}, 0, 0, \ldots)$;
- a bubbly equilibrium path $\hat{p} = (\hat{p}_0, \hat{p}_1, \ldots, \hat{p}_{T-2}, \hat{p}, \hat{p}, \ldots)$;

where $\bar{}$'s and $\hat{}$'s henceforth denote, respectively, the nonbubbly and bubbly price sequences. Notice that, for each type of equilibrium, the associated fundamental (which need not be explicitly computed) is unique.[15] It of course differs *across* equilibria.

Although in the long run a bubble is necessarily price increasing since $\hat{p} > 0$, it is possible, as the next section will demonstrate, that it is price decreasing in the short run.

2. PRICE DECREASING BUBBLES

I now turn to the comparison of the nonbubbly and bubbly price sequences, $\bar{p}$ and $\hat{p}$, and ask whether it is possible to have, for some $\tau < T-1$, $\hat{p}_\tau < \bar{p}_\tau$. From basic economic reasoning, the answer to this question must crucially depend on the preferences of consumers, and more precisely on the sign and magnitude of their interest elasticity of savings.

To understand why this is so, it is necessary to consider in detail the channels through which the presence of a bubble might lead to a decrease of the overall price of the asset.

[14]Backward perfect foresight dynamics are the appropriate solution concept in this model, and not just an analytical device as in Grandmont (1985).

[15]This is true in this model because the asset stops paying dividends in finite time. In more general frameworks, there might be, for instance, several nonbubbly fundamentals (e.g., Obstfeld and Rogoff (1983)).

As mentioned *supra*, the main requirement is that the presence of a bubble decrease the asset's fundamental. In this world in which the dividends distributed by the tree are exogenous, such a decrease can only occur if the interest rates at which future dividends are discounted are higher in the bubbly path than in the nonbubbly path. The existence of a price decreasing bubble therefore must require that *higher* equilibrium interest rates be associated with a *lower* overall real value of the stock of trees—which cannot occur if the interest elasticity of savings is positive. The critical role of income effects is formalized in the following proposition:

PROPOSITION 1: *If the interest elasticity of savings, $S'(.)$, is positive, bubbles are price increasing in the short run as well as in the long run.*

PROOF: The long run part of the proposition was proved in Section 1.3.1. The short run part is proved by backward induction. We know that, as of $T-1$, the bubble is price increasing, since $\hat{p} > 0$. Can it be non-price increasing at $T-2$? Suppose it is, so that $\hat{p}_{T-2} \leqslant \bar{p}_{T-2}$. Then $(\hat{p}+y)/\hat{p}_{T-2} > y/\bar{p}_{T-2}$. But if $S'(.) > 0$, this implies in equilibrium that $\hat{p}_{T-2} = S[(\hat{p}+y)/\hat{p}_{T-2}] > S[y/\bar{p}_{T-2}] = \bar{p}_{T-2}$—a contradiction! Hence $\hat{p}_{T-2} > \bar{p}_{T-2}$: the bubble is price increasing at $T-2$. By a similar argument, one shows that if the bubble is price increasing at $\tau \leqslant T-2$, it is price increasing at $\tau - 1$ if $S'(.) > 0$. Therefore the bubble is price increasing at every date, in the short run and in the long run.

The intuition underlying the proof of this proposition is straightforward. For interest rates to be higher in the bubbly equilibrium than in the nonbubbly one (a necessary condition for a bubble to be price decreasing), it must be the case, if the interest elasticity of savings is positive, that trees crowd out consumption loans in the bubbly equilibrium relative to the nonbubbly case.[16] But if the bubble is price decreasing, the real value of the stock of trees is lower in the bubbly than in the nonbubbly equilibrium —i.e., trees crowd consumption loans *in*, *not out*! Whence the contradiction on which the proof is based, and which can only be resolved by allowing for a negative interest elasticity of savings.[17]

What happens when the interest elasticity of savings is negative critically depends, as in endogenous business cycle theory,[18] on the particular specification of the utility function and of the endowment vector. The striking pattern of a bubble which is price increasing in, say, even periods and price decreasing in odd periods emerges in the following case:

PROPOSITION 2: *Suppose the interest elasticity of savings is negative, and that the second period endowment is zero. Then, in the short run, the bubble on the tree is price decreasing in periods $T-2k$, and price increasing in periods $T-2k+1$, where k is a positive integer.*

PROOF: Since $e_2 = 0$ (a specification which satisfies, because of Assumption 1, Assumption 3), it is straightforward to show from (5) that the interest elasticity of savings is negative if the function $g(x) \equiv xv'(x)$ is decreasing over the interval $[0, e_1)$. But then, from (15) and (17), the map φ is decreasing. Therefore, $\hat{p}_{T-2} = \varphi(\hat{p}) < \varphi(0) = \bar{p}_{T-2}$: the

[16]This crowding is of course only incipient, since the interest rate adjusts to make the consumption loans market inactive in equilibrium.

[17]The borderline case in which intertemporal substitution and income effects cancel each other out to yield $S'(.) = 0$ is not considered for the sake of brevity. It can be agglomerated into the positive interest elasticity of savings case with a slight change in the proofs.

[18]See Grandmont (1985).

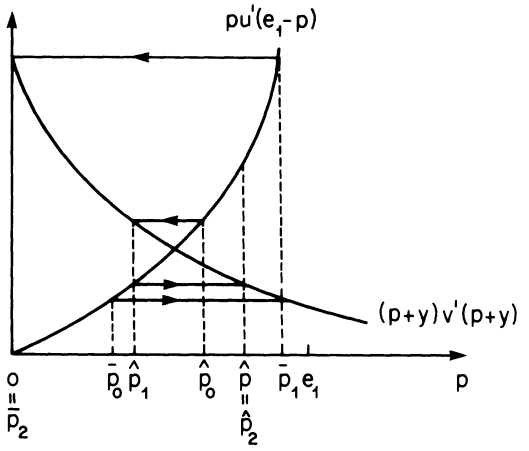

FIGURE 1.—Equilibrium dynamics ($S' < 0$; $e_2 = 0$; $T = 3$).

bubble is price decreasing at $T-2$. This implies that $\hat{p}_{T-3} = \varphi(\hat{p}_{T-2}) > \varphi(\bar{p}_{T-2}) = \bar{p}_{T-3}$: the bubble is price increasing at $T-3$. Similarly, it is price decreasing at $T-4$, price increasing at $T-5$, etc., until the origin of time is reached.

The results of Proposition 2—which Figure 1 illustrates for the case $T = 3$[19]—cannot be dismissed as irrelevant on the ground that economies with negative interest elasticities of savings are pathological, since recent estimates of the elasticity of intertemporal substitution point out the existence of strong income effects.[20]

Extending this Proposition to allow for a nonzero second period endowment or an interest elasticity of savings nonuniformly negative presents little interest (except that of a theoretical *curiosum*) since Proposition 2 decisively establishes that there is indeed no reason to presume, as is often done, that the presence of a bubble always raises the price of an asset.

Deriving general necessary and sufficient existence conditions for the existence of price-decreasing bubbles, while a worthwhile endeavor, is outside the scope of this note. These existence conditions are likely to be intimately linked, but not identical, to those under which cyclic or sunspot equilibria on useless assets may exist—for the model of perpetually useless assets studied so far by the sunspot literature is but a special case of the one presented here. The specific way in which the dividend vanishes—deterministically in T periods (as here), or probabilistically with the stopping time following a Poisson process (a variant of my (1987) model of stochastic bubbles)[21]—is, of course, a crucial determinant of the properties of bubbly and nonbubbly equilibria; in other terms, one cannot provide technology-independent conditions for the existence of price-decreasing bubbles.

[19]I thank an anonymous referee for suggesting this diagrammatic representation.

[20]See, for instance, Hall (1988).

[21]Suppose the dividend crashes to zero with a constant probability π, and never comes back once it has collapsed (this scenario was suggested by one of the referees). Maintain the dynamic inefficiency condition of Assumption 3. In that model, unlike in the one in Proposition 2, pseudo-stationary, price-increasing bubbles (which are constant before and after the crash) may exist even if $S' < 0$. Thus, in general, a negative interest elasticity of savings, while necessary for the existence of price-decreasing bubbles, does not rule out price-increasing bubbles—as, of course, is clear from the theory of bubbles on intrinsically useless assets.

CONCLUSION

This paper has established that, contrary to what is often claimed, rational speculative bubbles, which are positive by definition, need not increase the price of the asset to which they are attached—because the fundamental is not in general independent, in equilibrium, of the bubble. It is therefore possible to observe situations in which bubbles are price decreasing: the price of an asset subject to a bubble might be lower than the price of that asset in the absence of a bubble.

Whether that situation should be construed as one in which the asset is "undervalued" is mainly a question of semantics. If by undervaluation it is meant that the price falls short of the fundamental in the particular equilibrium under consideration, an asset is indeed, by this narrow definition, never undervalued: the price of an asset which can be freely disposed of always is never lower than the value of its usufruct. If undervaluation is taken in the broader acceptation of "existence of a bubbly equilibrium with lower asset prices than the nonbubbly equilibrium," then an asset can indeed be undervalued.

Psychological factors, "animal spirits" and extraneous waves of pessimism can, in any case, depress as well as raise an asset price relative to its "fundamental" nonbubbly value.

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