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## HAL Authorization

# Public Sector Wage Policy and Labor Market Equilibrium: A Structural Model\*

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## Abstract

We develop and estimate a structural model that incorporates a sizable public sector in a labor market with search frictions. The wage distribution and the employment rate in the public sector are taken as exogenous policy parameters. Overall wage distribution and employment rate are determined within the model, taking into account the private sector's endogenous response to public sector employment policies. Job turnover is sector specific and transitions between sectors depend on the worker's decision to accept alternative employment in the same or different sector by comparing the value of employment in the current and prospective jobs. The model is estimated on British data by a method of moments. We use the model to simulate the impact of various counterfactual public sector wage and employment policies.

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# 1 Introduction

We formulate a search-theoretic model that incorporates interactions between the public and private sectors. The wage offer distribution and job offer rate of the public sector are treated as exogenous policy parameters, and conditional on these and exogenous transitional parameters, the private sector wage distribution is then derived endogenously. Exploiting data from the British Household Panel Survey (BHPS) the model is then estimated by minimum-distance matching of some key moments from the data. These estimates allow us to make counterfactual policy analysis of different public sector wage policies.

There has been very little done in modeling the public sector explicitly within an equilibrium model of the labor market and nothing to our knowledge that estimates such a model. This is a major oversight when one thinks that in our data 24% of employed individuals were employed by the public sector.<sup>1</sup> It is, of course, naive to believe that with an employment share this large the public sector will not influence wage determination and by extension overall employment.

Instead of modeling the behavior of private sector firms explicitly, the literature thus far has been dominated by reduced form comparisons of the two sectors.<sup>2</sup> The general consensus of stylized facts emerging from the empirical literature is that the public sector wage distribution is more compressed than that in the private sector and workers receive a small public sector wage premium which is more prevalent in low-skilled workers.

With these stylized facts being known for some time, it is fairly surprising that so little has been done in explicitly modeling the interaction between the two sectors. The existing literature that does this has largely focused on assessing the impact of the public sector on the level or volatility of aggregate wages and employment. Papers in that vein include Algan et al. (2002), Quadrini and Trigari (2007), Hörner et al. (2007), and Gomes (2014). All model search is directed to a particular sector and none has direct job-to-job reallocation (within or between sectors). All model wages are determined by bargaining over the surplus from a match. Algan et al. (2002) find that

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<sup>1</sup>Algan et al. (2002) report that, based on a slightly narrower definition of public sector employment, the OECD finds an average public sector share of total employment of 18.8% in 2000 over a sample of 17 major OECD countries.

<sup>2</sup>For a survey of the literature, see Bender (1998).

the creation of public sector jobs has a massive crowding out effect on private-sector job creation, such that the marginal public sector job may destroy as many as 1.5 private sector jobs in some OECD countries. This crowding out effect is especially strong when public sector wages are high and/or when public and private-sector output are close substitutes. Focusing on the cyclicalities of employment, Quadrini and Trigari (2007) examine a public sector wage policy that is acyclical (a single wage) and procyclical (government wage is an increasing function of private sector wages). Calibrating the model for the US economy, they find that public sector employment and wage policy increased employment volatility by four and two times, respectively, over the periods 1945-70 and 1970-2003. They attribute this downward trend in employment volatility to an increasingly procyclical compensation policy adopted by the state. Hörner et al. (2007) model two economies: one where a benevolent social planner aims to maximize individual's welfare with public sector wages and employment (amongst other matters); the other in the absence of a public sector. The equilibrium of the model allows the authors to draw two conclusions. Firstly, that the public sector has an ambiguous effect on overall employment and secondly, that in more turbulent times there will be higher unemployment in the economy with the public sector. The latter result comes from the individuals being risk averse and therefore crowding into the safer sector (the public sector) in more uncertain times. Finally, Gomes (2014) builds a dynamic stochastic general equilibrium model with search and matching frictions calibrated to U.S. data and shows that high public sector wages induce too many unemployed to apply for public sector jobs and raise unemployment. He further argues that the cyclicalities of public sector wage policy has a strong impact on unemployment volatility.

To our knowledge, other than this paper, the extent of the literature that explicitly models private sector firm behavior for a given public wage setting policy is Albrecht et al. (2011) and Burdett (2011). Albrecht et al. (2011) extend the canonical Diamond-Mortensen-Pissarides model (Pissarides, 2000) to incorporate the public sector posting an exogenous number of job vacancies. They also introduce match-specific productivity and heterogeneity in worker's human capital, which generate wage dispersion in the private sector. Burdett (2011) is closer to ours in the sense that firms

post wages rather than bargain over the surplus and, crucially, that workers are allowed to search on the job. On-the-job search — and the possibility for workers to switch jobs between sectors — is an essential ingredient of the model as it captures rich and complex aspects of the competition between sectors over the labor services of workers. Moreover, as is understood since at least Burdett and Mortensen (1998), on-the-job search begets endogenous wage dispersion. However, in Burdett’s model the public sector sets a single wage, leading to the counterfactual prediction that the private sector’s response to competition from the public sector is to post a wage distribution with a hole in its support. In our model the public sector’s policy is to post wages from a distribution. This allows us to have wage differences in the public sector and a continuous private sector wage distribution with connected support (we discuss the intuition for this result below). Unlike the two models discussed above we allow for differences in cross-sector job destruction and job offer arrival rates. Crucially, this paper is unique in the literature insofar as the parameters of the model are structurally estimated.

Methodologically, a similar paper to ours is Meghir et al. (2014) who develop an equilibrium wage posting model to determine the interaction between a formal and informal sector in a developing country. Here the two sectors vary in the degree of regulatory tightness, the formal sector firms incurring additional costs to wages in the form of corporation tax, income tax, social security contributions, severance pay and unemployment insurance. While firms in the informal sector are not exposed to these labor market regulations they do face the chance (with given probability) of incurring a non-compliance cost. Private sector firms endogenously select into either the formal or informal sector and the equilibrium wage offer distributions of both are determined endogenously. Similarly to this paper, Meghir et al. estimate their model using indirect inference.

Our estimated model fits the distribution of wages and transition rates that are observed empirically very well. The estimates suggest that inference based on just cross-sections of wages will result in an overestimate of the public sector premium. The estimated differences in lifetime earnings, taking into account job mobility and job loss probabilities, are far smaller than differences in wages between sectors. Public sector workers on average command higher wages. But the overwhelming

majority of private sector workers would require a large pay increase to motivate them to take up a job in the public sector. The increased value of a private sector job is primarily driven by a higher probability of receiving other private sector job offers, which implies faster upward wage mobility (or equivalently higher returns to experience) in the private sector. This dominates the increased value from more job security offered by the public sector.

In order to demonstrate how the modeling approach can be used to assess public sector wage and employment policies we assess policies that attempt to mimic some of the austerity measures implemented across Europe after the 2008 recession. The specific policies we consider are: a reduction in public sector hiring; an increase in the rate the public sector lays workers off; and progressive and proportional decreases to the distribution of wages offered by the public sector. The optimal policy depends on the government's objective function. We find the best performing policy for the welfare of workers is a progressive wage cut, and for maximizing GDP, a reduction in new public sector hiring performs best. Further, the rise in unemployment, associated with austerity will be minimal from cutting public sector jobs as the private sector responds by hiring many more workers. Similarly, there is no or a very small depression in the wages of private sector workers associated with the policies. In fact, if one includes the savings in public sector spending as a lump sum transfer to all workers, the policies actually increase the aggregate discounted lifetime value of workers. The results also suggest that output as a whole will rise with more labor inputs going towards the more productive, private sector.

The paper is organized as follows. In the next section we derive the equilibrium structural model. Section 3 gives an overview of the properties of the data used for estimation. Sections 4 and 5 outline the estimation protocol and present the estimation results. In Section 6 we use the results obtained to run counterfactual policy analysis and in Section 7 we conclude.

## 2 The Model

### 2.1 Basic Environment

We consider a model of wage-posting akin to Burdett and Mortensen (1998). Time is continuous and the economy is in steady state. A  $[0, N]$  continuum of infinitely lived, risk neutral, ex-ante homogeneous workers face a fixed continuum of employers in a frictional labor market. A key aspect of our approach is that the set of employers comprises a continuum of infinitesimally small heterogeneous, profit-maximizing firms which we interpret as Private Sector employers, that coexist with a single, non-infinitesimal, non-profit maximizing employer which we interpret as the Public Sector. Private-sector firms behave in the same way as employers in the standard Burdett and Mortensen (1998) model, while public sector wages and level of labor demand are taken as exogenous. Because of the public sector's non-infinitesimal size as an employer, changes in public sector employment policies will have a non-trivial impact on labor market equilibrium, both directly and through the private sector's response to said changes in policy. The main objective of this paper is to quantify that impact for various policy changes.

### 2.2 Workers and Jobs

A worker can be in one of three states, either unemployed or employed in the public or private sector. Throughout the paper we indicate a worker's labor market state using a subscript  $s \in \{u, p, g\}$  for unemployment, employment in the private sector and employment in the public sector, respectively. The steady-state numbers of workers in each employment state are denoted as  $N_u$ ,  $N_g$  and  $N_p$ .

A job is fully characterized by a constant wage  $w$  and the sector it is attached to. Workers receive job offers at a Poisson rate that depends on the worker's state. Jobs are not indefinite and also face a Poisson destruction shock. The notation used to describe all those shocks is largely consistent with the previous literature,  $\delta$  being used to denote job destruction shocks and  $\lambda$  for job offer arrival rates. To explain the two states between which the particular worker transits a two letter index is used. The first letter designates the sector of origin and the second the sector of destination. So for example,  $\lambda_{pg}$  is the arrival rate of public-sector offers to private sector

employees,  $\lambda_{ug}$  is the arrival rate of public-sector offers to unemployed workers, and so on. As job destruction always results in the worker becoming unemployed, a single index is used to specify the job destruction shock,  $\delta_p$  or  $\delta_g$ .

To summarize, a worker employed in sector  $s \in \{p, g\}$  faces three random shocks: a job destruction shock  $\delta_s$ , after which the worker becomes unemployed and gets flow utility  $b$ , a within sector job offer  $\lambda_{ss}$  and a cross-sector job offer  $\lambda_{ss'}$ . While the arrival rates from the public sector are exogenous, arrival rates from the private sector and the associated acceptance decisions are determined endogenously.

As in Burdett and Mortensen (1998), a job offer from the private sector consists of a draw from a wage offer distribution  $F_p(\cdot)$  which results from uncoordinated wage posting by the set of infinitesimally small private employers, each maximizing its profit taking as given the strategies of other firms and that of the public sector.  $F_p(\cdot)$  will be determined endogenously in equilibrium. By contrast, a job offer from the public sector consists of a draw from a (continuous) wage offer distribution  $F_g(\cdot)$  which is taken as an exogenous policy tool. We thus assume from the outset that the public sector offers jobs at different wage levels to observationally similar workers. This assumption is both realistic — residual wage dispersion among similar public-sector employees is observed in the data — and needed to avoid the counterfactual prediction of Burdett (2011) that equilibrium features a wage distribution with disconnected support. In Burdett’s model, the overall (public + private) wage offer distribution has an atom at the unique public sector wage, say  $w_g^*$ . Moreover, private firms are homogeneous, hence achieve equal profits in equilibrium. Now, if a private firm posted a wage epsilon below  $w_g^*$ , it would make the same profit as a private firm posting a wage epsilon above  $w_g^*$ , but lose a mass of workers to the public sector compared to the private firm posting  $w_g^* + \varepsilon$ , resulting in discretely lower profits going to the firm posting  $w_g^* - \varepsilon$ . Therefore, there can be no equilibrium with private firms posting wages arbitrarily close to  $w_g^*$  on both sides of  $w_g^*$ , hence the hole in the offer distribution.

We finally recognize that public and private sector jobs may differ along other dimensions than

just the wage and the transition parameters. There may be, for example, systematic differences in working conditions. Also workers may enjoy a utility surplus ('public service glow') or suffer a utility loss ('public service stigma') from working in the public sector. In order to capture those unobserved differences in a parsimonious way, we assume that the flow utility that workers derive from working in sector  $s \in \{p, g\}$  for a wage of  $w$  is equal to  $w + a_s$ , where  $a_s$  is a sector-specific 'amenity'. Finally, and without further loss of generality, we normalize  $a_p$  to zero, so that  $a_g$  reflects the relative utility surplus (or loss) from working in the public sector. This utility surplus is assumed to be the same for all workers.

### 2.2.1 Worker Values and Reservation Wages

An individual's utility is given by the present discounted future stream of wages. For a given worker, the transitional parameters will be unchanged if he moves job within a sector. The acceptance decision for an offer within the worker's current sector is therefore entirely determined by the worker's current wage and the new wage being offered. If the new offer,  $x$ , is higher than the worker's current wage,  $w$ , he will accept and otherwise reject. However, since a change in sector is not only associated with a different wage but also with a change in transitional parameters, the acceptance decision is not so trivial when the job offer is from another sector. Thus depending on the two sets of transition parameters, an individual may accept a job offer from a different sector with a wage cut, or conversely, require a higher wage in order to accept. These acceptance decisions can be characterized by a set of reservation wages, accepting job offers greater or equal to these wages and rejecting those below. With the three states we have defined, there will be four corresponding reservation wages, which we define using the same double-index system as for transition parameters:  $R_{up}$ ,  $R_{ug}$ ,  $R_{pg}(w)$  and  $R_{gp}(w)$ .

When employed, a worker's reservation wage will be a function of their current wage. The reservation wage applying to private (public) sector offers made to a public (private) sector worker earning  $w$  makes said worker indifferent between his current present value and the present value of private (public) sector employment at his reservation wage. Formally, that is  $W_p(R_{gp}(w)) = W_g(w)$  and  $W_g(R_{pg}(w)) = W_p(w)$ , where  $W_p(w)$  and  $W_g(w)$  are the values of working in the private and

public sectors at wage  $w$ . It follows from those definitions that the two reservation wages described are reciprocal of each other:

$$R_{pg}(R_{gp}(w)) = w. \quad (1)$$

The reservation wage of an unemployed worker receiving an offer from the public (private) sector is the wage at which they are indifferent between unemployment and the public (private) sector. Formally, the two reservation wages solve the equality,  $U = W_p(R_{up}) = W_g(R_{ug})$ , where  $U$  is the present value of a worker in unemployment. Hence applying (1) to this equality one can derive a second property of the reservation wages:

$$R_{pg}(R_{up}) = R_{ug}. \quad (2)$$

Note that the analogous property for  $R_{gp}(\cdot)$  also holds.

### 2.2.2 Bellman Equations

The value function for an unemployed worker is defined by the following Bellman equation, where  $r$  is the rate of time preference, constant across workers:

$$rU = b + \lambda_{up} \int_{R_{up}}^{+\infty} [W_p(x) - U] dF_p(x) + \lambda_{ug} \int_{R_{ug}}^{+\infty} [W_g(x) - U] dF_g(x), \quad (3)$$

The first term,  $b$  is the flow utility an individual gets from being in unemployment. Offers arrive from the public (private) sector at a rate of  $\lambda_{ug}$  ( $\lambda_{up}$ ). Wage offers,  $x$  are drawn from the private sector from an endogenous distribution,  $F_p(w)$ , which will be derived from the firm side later. An unemployed worker will accept the job offer if the wage is higher than the worker's reservation wage for that sector, the lower bound of the integral. Inside the integral is the gain the worker makes from switching from unemployment to private sector employment at wage  $w$ . The final term is the public sector analogue to the second. The theoretical difference between the two is that the distribution from which public-sector job offers are drawn is an exogenous policy parameter of the model.

Similar value functions define a worker employed in the private and public sectors. Below is the

example for a private sector employee:

$$rW_p(w) = w + \delta_p [U - W_p(w)] + \lambda_{pp} \int_w^{+\infty} [W_p(x) - W_p(w)] dF_p(x) + \lambda_{pg} \int_{R_{pg}(w)}^{+\infty} [W_g(x) - W_p(w)] dF_g(x) \quad (4)$$

A worker employed in the private sector and earning a wage  $w$  has a discounted value from employment given by the right hand side of (4). The first term  $w$  is the instantaneous wage paid in the current private sector firm. The next term, is the loss of value an individual would get if he were to transit into unemployment  $[U - W_p(w)]$  multiplied by the flow probability of such an event occurring, the private sector job destruction rate,  $\delta_p$ . At rate  $\lambda_{pp}$  the worker receives an offer from another private sector firm, where the offer is drawn from the distribution  $F_p(x)$ . If this offer is greater than his current wage  $w$  he will accept, the lower bound of the integral. Given the offer is received and it meets his acceptance criterion, the individual will make an unambiguous gain in value given by  $[W_p(x) - W_p(w)]$ . The next term represents the equivalent, except for offers from the public sector. Thus the wage is drawn from a different distribution and the acceptance criteria, the lower bound of the integral is instead  $R_{pg}(w)$ . An analogous Bellman equation defines the value function for a worker in the public sector:

$$rW_g(w) = w + a_g + \delta_g [U - W_g(w)] + \lambda_{gp} \int_{R_{gp}(w)}^{+\infty} [W_p(x) - W_g(w)] dF_p(x) + \lambda_{gg} \int_w^{+\infty} [W_g(x) - W_g(w)] dF_g(x). \quad (5)$$

Note the presence of the additional flow utility term  $a_g$ , the ‘public-sector amenity’ discussed above.

The value functions given by (3), (4) and (5) allow us to obtain the reservation wage required to leave the private for the public sector and vice-versa as a function of the transition parameters. This is done using the identity  $W_p(R_{pg}(w)) = W_g(w)$  and  $W_g(R_{gp}(w)) = W_p(w)$  and assuming differentiability of the value functions. This manipulation is performed in Appendix A and the solution for a private sector worker’s reservation wage from the public sector solves the following

non-linear ODE:<sup>3</sup>

$$R'_{pg}(w) = \frac{r + \delta_g + \lambda_{gp}\bar{F}_p(w) + \lambda_{gg}\bar{F}_g(R_{pg}(w))}{r + \delta_p + \lambda_{pp}\bar{F}_p(w) + \lambda_{pg}\bar{F}_g(R_{pg}(w))}, \quad (6)$$

with initial condition  $R_{pg}(R_{up}) = R_{ug}$ . It should be noted that  $R_{up}$  and  $R_{ug}$  themselves depend on the functions  $R_{pg}(\cdot)$  and  $R_{gp}(\cdot)$  as they are obtained by solving  $W_s(R_{us}) = U$  for  $s = p$  or  $g$ . However, they also depend on unemployment income flows (the  $bs$ ), which are free parameters, so those reservation wages can themselves be estimated as free parameters.

### 2.2.3 Flow-Balance Equations and Worker Stocks

The economy being in steady-state, the flows in and out of any given sector, for each class of workers, are equal. Applying this to unemployment, one obtains:

$$(\lambda_{up} + \lambda_{ug}) N_u = \delta_p N_p + \delta_g N_g \quad (7)$$

The left hand side of (7) is the rate at which workers leave unemployment toward the two sectors of employment. This occurs when a worker receives a job offer from a given employment sector and the associated wage offer is higher than his appropriate reservation wage. Assuming homogeneous workers, there is no reason why a firm would offer wages below a worker's reservation wage (and if it did, it would employ no worker and therefore become irrelevant to market equilibrium). Therefore we assume without (further) loss of generality that  $F_p(R_{up})$  and  $F_g(R_{ug})$  are equal to zero. The right hand side is the unemployment inflow, which consists of workers being hit by job destruction shocks in their sector of employment. A worker can only be in one of three states,  $u, p$  or  $g$  so:  $N_u + N_p + N_g = N$ , where  $N$  is the total population of workers, a given number.

Equation (8) is the flow-balance equation for private sector workers, equating the flow into the private sector below a wage  $w$  to the flow out, thus imposing that not only is the share of private sector workers constant in the steady state, but so is the distribution of wages amongst them. The left hand side is the flow out of private employment.  $N_p G_p(w)$  is the number of private sector workers earning less than a wage  $w$ . They can exit to unemployment through job destruction shocks  $\delta_p$ . The second and third terms are the exit rates into the public sector and higher paid

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<sup>3</sup>Here and throughout the rest of the paper, a bar over a c.d.f. denotes the survivor function, so for example  $\bar{F}_p(\cdot) := 1 - F_p(\cdot)$ .

private sector jobs, respectively, upon receiving a job offer ( $\lambda_{pg}$  and  $\lambda_{pp}$ ). The right hand side is the flow into private sector employment, the first term being the flow from unemployment and the second, from the public sector.

$$\begin{aligned} N_p \delta_p G_p(w) + N_p \lambda_{pg} \int_{R_{up}}^w \bar{F}_g(R_{pg}(x)) dG_p(x) + N_p \lambda_{pp} \bar{F}_p(w) G_p(w) \\ = N_u \lambda_{up} F_p(w) + N_g \lambda_{gp} \int_{R_{ug}}^{R_{pg}(w)} [F_p(w) - F_p(R_{gp}(x))] dG_g(x) \end{aligned} \quad (8)$$

Rearranging equation (8) and differentiating with respect to the wage rate,  $w$ , one obtains:

$$\begin{aligned} \frac{d}{dw} \{ [\delta_p + \lambda_{pp} \bar{F}_p(w)] N_p G_p(w) \} + N_p g_p(w) \lambda_{pg} \bar{F}_g(R_{pg}(w)) \\ - N_g \lambda_{gp} G_g(R_{pg}(w)) f_p(w) = N_u \lambda_{up} f_p(w). \end{aligned} \quad (9)$$

This would be a fairly straightforward ODE in  $g_p(w)$  (the density of observed wages in the private sector), if it was not for the term featuring  $G_g(R_{pg}(w))$ . This term can be derived by manipulation of the flow balance equation for public sector workers earning less than  $R_{pg}(w)$  (instead of  $w$ ). This manipulation is performed in Appendix A. Plugging this solution into (9), we obtain an ODE that defines  $G_p(w)$ .

An additional hurdle at this point is the determination of  $N_p$  and  $N_g$  (with  $N_u = N - N_p - N_g$ ). Those numbers are needed to solve for  $G_p(\cdot)$  in the ODE resulting from the combination of (9) and the isolation of  $N_g G_p(R_{pg}(w))$ , given in the appendix. Now,  $N_p$  and  $N_g$  are jointly defined by the balance of flows in and out of employment (7), and the flow balance in and out of, say, the private sector, which is given by evaluating the flow-balance equation of private sector workers, equation (8) at  $w \rightarrow +\infty$ :

$$\begin{aligned} N_p \delta_p + N_p \lambda_{pg} \int_{R_{up}}^{+\infty} \bar{F}_g(R_{pg}(x)) dG_p(x) \\ = N_g \lambda_{gp} \int_{R_{ug}}^{+\infty} \bar{F}_p(R_{gp}(x)) dG_g(x) + N_u \lambda_{up} \bar{F}_p(R_{up}) \end{aligned} \quad (10)$$

The distribution,  $G_g(\cdot)$ , can be derived using the identity  $R_{pg}(R_{gp}(w)) = w$  applied to the derivation of  $G_p(R_{pg}(w))$  in the appendix. The latter equation involves  $G_p(\cdot)$ , which in turns depends

on  $N_p$  and  $N_g$ , so that those three objects have to be solved for simultaneously. This will be done using an iterative procedure.

### 2.3 Private Sector Firms

There exists a  $[0,1]$  continuum of private sector firms who are profit maximizers and heterogeneous in their level of productivity,  $y$ , where  $y \sim \Gamma(\cdot)$  over the support  $[y_{\min}, y_{\max}]$  in the population of firms. Firms set their wage  $w$  and their search effort (number of vacancies, advertising, recruitment agencies) in order to make a number of hires  $h$ . The pair  $(w, h)$  is chosen so as to maximize steady-state profit flow. A private sector firm choosing to pay  $w$  will experience a quit rate of  $\Delta(w)$  of its employees, where:

$$\Delta(w) = \delta_p + \lambda_{pp}\bar{F}_p(w) + \lambda_{pg}\bar{F}_g(R_{pg}(w))$$

As a consequence, the steady-state size of this firm will be  $\ell(w, m)$ :

$$\ell(w, h) = \frac{h}{\Delta(w)} \quad (11)$$

and its steady-state profit flow:

$$\Pi(w, h) = (y - w)\ell(w, h) - c(h) = (y - w)\frac{h}{\Delta(w)} - c(h) \quad (12)$$

where  $c(h)$  is the cost incurred by the firm to make  $h$  hires <sup>4</sup>. Optimal wage and search policies  $w^*(y)$  and  $h^*(y)$  can thus be characterized using the following first-order conditions:

$$y = w^* - \frac{\Delta(w^*)}{\Delta'(w^*)} \quad (13)$$

$$c'(h^*) = \frac{-1}{\Delta'(w^*)} \quad (14)$$

The proportion of all offers that are accepted from a private sector firm offering a wage  $w$  is  $\alpha(w)$ .

$$\alpha(w) = \frac{\lambda_{up}N_u + \lambda_{pp}N_p G_p(w) + \lambda_{gp}N_g G_g(R_{pg}(w))}{\lambda_{up}N_u + \lambda_{pp}N_p + \lambda_{gp}N_g}$$

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<sup>4</sup>We think of  $c(h)$  as a training cost. This differs slightly from the standard specification in the literature, which conventionally models recruitment costs as a vacancy posting cost (although there are increasingly many exceptions to this tradition). We adopt this training cost specification for the algebraic simplicity it affords, and conjecture that not much would change if we specified the recruitment cost as a cost per vacancy.

It follows that the total number of contacts in the economy is the number of hires divided by the acceptance rate:

$$M = \int_{y_{\min}}^{y_{\max}} \frac{h^*(y)}{\alpha(w^*(y))} d\Gamma(y), \quad (15)$$

and that the fraction of these contacts that is attached to wage lower than a given  $w$ , in other words the probability that a wage offer is less than  $w$  can be written in the two following manners (the left and right hand sides of equation (16)):

$$F_p(w) = \frac{1}{M} \int_{y_{\min}}^{\tilde{y}} \frac{h^*(y)}{\alpha(w^*(y))} d\Gamma(y), \quad (16)$$

where  $\tilde{y}$  is such that  $w^*(\tilde{y}) = w$ .

Similarly, the fraction of employees earning a wage less than  $w^*(y)$  do so because they are employed by firms with a productivity lower than  $y$ . Thus:

$$H[\ell(w^*(y), h^*(y))] = G_p[w^*(y)], \quad (17)$$

where  $H(\cdot)$  is the distribution of firm sizes among employed workers.

We are now in a position to close the model given public sector policy choices and endogenize private sector job offer arrival rates. To this end, we need to make one final assumption –that the relative search intensities of workers in the three labor market states, i.e. unemployment, employment in the private sector and employment in the public sector are constant. These will be denoted  $s_{up}$  (normalized to 1 without loss of generality),  $s_{pp}$  and  $s_{gp}$  respectively. The arrival rates of private sector offers hence have the following expressions:

$$\begin{aligned} \lambda_{up} &= \lambda_p \\ \lambda_{pp} &= s_{pp} \cdot \lambda_p \\ \lambda_{gp} &= s_{gp} \cdot \lambda_p. \end{aligned}$$

The private sector job offer arrival rate  $\lambda_p$  (per search efficiency unit) can then be recovered from the following and equation (15), for a given distribution of productivities in the population of firms  $\Gamma(y)$ :

$$M = \lambda_p (N_u + s_{pp}N_p + s_{gp}N_g). \quad (18)$$

This and equation (16) illustrate the private sector firms' response to changes in public sector policy in terms of search effort (or number of offers) and wage offer distribution. Those equations determine  $\lambda_p$  and  $F_p(\cdot)$  respectively, given public sector hiring policy, embodied in the  $\{\lambda_{ug}, \lambda_{pg}, \lambda_{gg}\}$  rates, wage offer policy, embodied in the distribution  $F_g(\cdot)$  and “job security” policy, embodied in the public sector layoff rate  $\delta_g$ . Note however that we do not consider any response of the private sector in terms of its layoff rate  $\delta_p$  as this is considered exogenous and unresponsive to labor market changes.

### 3 Data and Estimation

We now outline our estimation protocol, which is based on minimum-distance matching of certain descriptive moments of the data.<sup>5</sup> We set the discount rate  $r$  ex-ante at 0.004 (where one unit of time is a month), implying an annual rate of approximately 5%.  $\theta$ , given below is the exogenous parameter vector which we intend to estimate.

$$\theta = (b, a_g, \delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg}, F_p, F_g, \Gamma, c(\cdot))'$$

Note that the two offer distributions, the distribution of firm types ( $\Gamma$ ) and the cost function of making  $h$  hires  $c(h)$  feature in the list of parameters. As we will argue below, those distributions are non-parametrically identified. However, for numerical tractability, we will make parametric assumptions on  $F_p$  and  $F_g$  as outlined later.

The rest of this section focuses on obtaining estimates for the vector  $\theta$ . We begin by describing the moments we match and how we obtain them from the data, and we then describe in detail the estimation procedure. Results are presented in the next section.

#### 3.1 The Sample

The data used in the analysis are taken from the BHPS, a longitudinal data set of British households. Data were first collected in 1991 and the households selected were determined by an equal

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<sup>5</sup>For a comprehensive overview of related simulation-based methods, see Gouriéroux et al. (1993).

probability sampling mechanism.<sup>6</sup> Since then, there have been 18 further waves, collected annually. The model outlined is derived under a steady state assumption. Therefore it is necessary that the time period used is short and has approximately constant shares in each of the three states across time. We choose data from 2004 to 2008 to satisfy this assumption, allowing long enough time after the Conservatives’ drive toward privatization in the 80s and 90s but before the Great Recession of 2008.

Using retrospective accounts of employment history we construct a panel dataset of respondents at a monthly frequency. We include in our data those who across our panel reach at least 21 years of age and don’t exceed 60. Income is adjusted for inflation according to the consumer price index and we trim the income distributions in each sector, treating data as missing if it is below the 2nd or above the 98th percentile in the distribution of wage in either employment sector. We also exclude individuals with holes in their employment history and once someone becomes inactive they are from then on excluded. Thus, consistent with our model, an agent can be in one of three states, unemployment or employment in the private or public sectors. We define private sector employment as anyone who declares themselves as employed in a private sector firm, non-profit organization or in self-employment and public sector employment as in the civil service, central or local government, the NHS, higher education, a nationalized industry, the armed forces or a government training scheme. Finally, we restrict our sample to men, to avoid issues of labor supply that are more prevalent amongst women, particularly part-time work and inactivity.

The two sectors vary in their composition of workers; particularly in gender and human capital (see Table 1). We therefore divide our sample into two strata, those who have acquired A-levels and those who haven’t. A-levels are taken by 18 year-old British students at the end of further education. We provide estimates separately for the two subsamples. Our sample contains 1,307 high-skill and 1,083 low skill males who we follow for a maximum of 5 years. There is some attrition which we assume to be exogenous. Table 1 shows some basic descriptive statistics. Consistent with the literature on the public-private sector relationship, we find the British public sector is better

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<sup>6</sup>From wave 9 the BHPS was extended to include Scotland and Wales and from wave 11, Northern Ireland. All three regions are over represented in the sample and therefore we weight the data accordingly.

Table 1: Descriptive Statistics

	Private Sector	Public Sector	Unemployment
size of each sector:			
all	79.64 %	14.73%	5.63%
A-levels and above	75.98%	19.00%	5.02%
less than A-levels	83.67%	10.01%	6.32%
mean hourly earnings			
all	12.42	14.15	-
A-levels and above	14.99	16.04	-
less than A-levels	10.41	11.04	-
standard deviation of hourly wages:			
all	10.53	9.32	-
A-levels and above	11.64	9.99	-
less than A-levels	8.37	5.61	-

educated, on average receive higher wages for which there is less dispersion within the sector. Note the proportion of public sector workers appears low as women are over represented in the public sector.

Table 2 conveys information about the extent of job mobility, both within and between sectors. Our data is an unbalanced monthly panel of workers, starting from the first interview date and ending with the last. Counting in each month the number of people making each type of transition and the number in each state, we construct monthly cross-sector transition matrices. Averaging these across our time period, we obtain the transition matrix shown in Table 2.

Private sector workers are, on the whole, more mobile than public sector workers. Both high skill and low skill private sector workers have approximately a 0.1% better chance of changing jobs in a given month than their public sector counterparts. A closer look reveals that private sector workers experience much more frequent *within-sector* job changes than their public-sector counterparts. Mobility *between* employment sectors, however, is dominated by public sector employees moving to the private sector, cross-sector mobility in the other direction being a comparatively rare event. It is also striking to see that the separation rate into unemployment is only marginally smaller in the public sector than in the private sector. Finally, perhaps the most important conclusion to be drawn from Table 2 is that direct, job-to-job reallocation *between employment sectors* is substantial:

Table 2: Job mobility within and between sectors

A-levels and above:	Private Sector	Public Sector	Unemployment
Private Sector	0.0153	0.0020	0.0044
Public Sector	0.0078	0.0077	0.0038
Unemployment	0.0865	0.0175	—
Less than A-levels:	Private Sector	Public Sector	Unemployment
Private Sector	0.0152	0.0010	0.0050
Public Sector	0.0101	0.0044	0.0041
Unemployment	0.0834	0.0080	—

**Note:** Transition rates are monthly. Rows do not add up to one. The two entries on the main diagonal are the fractions of workers *changing jobs* within the private and the public sector, respectively.

given the transition rates in Table 2 and the various sectors' relative sizes given in Table 1, one can infer that about 20 percent of the employment inflow into the private sector comes from the public sector, and that about 30 percent of the private sector employment outflow goes into the public sector. High skill workers seem to have the best of both worlds, with higher rates of job movement and lower job destruction.

In addition, we have data on the distribution of firm sizes in the population of employed workers in the private sector. These data are taken from the Inter Departmental Business Register (IDBR) which contains information on VAT traders and PAYE employers in a statistical register representing nearly 99% of economic activity.

A caveat of this data is that it refers to employers' sizes in terms of all employees' skills combined, whereas our estimations are carried out on subsets of the data stratified by skills. Given our stylized modeling of the firm as a single-input constant-returns-to-scale production unit, we shall ignore this issue by assuming that either the distribution of firms' sizes among employees is the same across skill groups, or that the optimal number of hires  $h$  derived by a firm is shared in constant proportions between the different skill groups.

### 3.2 Estimation

Identification of the model's parameters ( $b, a_g, \delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg}, F_p, F_g, \Gamma, c(\cdot)$ ) comes from two data sources: observed transitions between labor market states and observed wage distri-

butions. Data on the distribution of firms' sizes allow us to retrieve estimates of  $(\Gamma, c(\cdot))$ .

Observed sector-specific wage distributions are direct empirical counterparts to  $G_p(\cdot)$  and  $G_g(\cdot)$  in the model. While neither has a closed-form solution in the model, both can be simulated given parameter value. In order to map the wage distribution well, we take as moments to be matched 50 quantiles of the distributions, giving 100 moments in total:  $\{w_{s,j}\}_{s=1,2,j=1..50}$ .

Turning to transition moments, we match the eight transition rates reported in Table 2. Denoting these as  $\pi_{ss'}$  where  $s$  is the state of origin and  $s'$  the state of destination, we thus add eight moments to match:  $(\pi_{up}, \pi_{ug}, \pi_{pu}, \pi_{pp}, \pi_{pg}, \pi_{gu}, \pi_{gp}, \pi_{gg})$ . The theoretical counterparts of those monthly transition rates are given by the probabilities of a certain type of transition occurring within a one-month period. The theoretical counterparts of  $\pi_{pu}$ ,  $\pi_{pp}$ ,  $\pi_{pg}$ ,  $\pi_{gu}$ ,  $\pi_{gp}$  and  $\pi_{gg}$  all have similar expressions:  $\pi_{ss'}^{\text{model}}$  is constructed by taking the probability that an exit from state  $s = p$  or  $g$ , given wage  $w$ , occurs before one month has elapsed, multiplying it by the conditional probability of exiting toward  $s'$ , given that an exit occurs and given initial wage  $w$ , then finally integrating out  $w$  using the relevant initial wage distribution,  $dG_s(w)$ . For example:

$$\pi_{pp}^{\text{model}} = \int_{R_{up}}^{+\infty} \frac{\lambda_{pp} \bar{F}_p(w) \left(1 - e^{-(\delta_p + \lambda_{pp} \bar{F}_p(w) + \lambda_{pg} \bar{F}_g(R_{pg}(w))) \times 1}\right)}{\delta_p + \lambda_{pp} \bar{F}_p(w) + \lambda_{pg} \bar{F}_g(R_{pg}(w))} dG_p(w),$$

where the “ $\times 1$ ” term in the exponential is there as a reminder that  $\pi_{pp}$  is a monthly transition probability and that all the flow parameters ( $\delta_p$ ,  $\lambda_{pp}$ , etc.) are monthly. The theoretical counterparts of the transition rates from unemployment are simpler (as there is no wage to integrate out):

$$\pi_{up}^{\text{model}} = \frac{\lambda_{up} (1 - e^{-(\lambda_{up} + \lambda_{ug}) \times 1})}{\lambda_{up} + \lambda_{ug}},$$

and symmetrically for  $\pi_{ug}^{\text{model}}$ .

As for the estimation of the function  $c(h)$ , i.e. the cost of making  $h$  hires for a private sector firm, we will be using the 12 cutoffs of the distribution of firms' sizes within private sector employment, denoted  $\{H(\ell_i^c)\}_{i=1..12}$ , where  $H(\cdot)$  is the cumulative distribution function of firm sizes among private sector employees and the  $\ell_i^c$ 's are the 12 size cutoffs for which employment sizes are grouped into in the IDBR. As will be discussed in the next section, the distribution of firm productivities

in the population of firms,  $\Gamma(y)$ , will then be estimated by matching 50 points of the private sector wage *offer* distribution corresponding to the observed wage quantiles seen above,  $\{F_p(w_{p,j})\}_{j=1..50}$ .

### 3.3 Estimation Procedure

We first estimate the first twelve components of  $\theta$  by matching the 108 moments described above, leaving  $\Gamma(\cdot)$  out. The latter is backed out in a final step as the underlying private firm productivity distribution that rationalizes the estimates of  $F_p(\cdot)$  and  $F_g(\cdot)$  obtained in previous steps. We also make parametric assumptions about  $F_p(\cdot)$  and  $F_g(\cdot)$ . First we assume that:

$$F_p(w) = \begin{cases} \frac{1-(w/R_{up})^{\alpha_p}}{1-(\bar{w}_p/R_{up})^{\alpha_p}} & \text{if } w \in [R_{up}, \bar{w}_p] \\ 0 & \text{if } w < R_{up} \\ 1 & \text{if } w > \bar{w}_p, \end{cases}$$

where  $\bar{w}_p$  is set equal to the top percentile in the observed wage distribution. It then proves convenient to parameterize  $F_g(\cdot)$  as:

$$F_g(R_{pg}[F_p^{-1}(x)]) = x^{\alpha_g}, \quad \text{for } x \in [0, 1]. \quad (19)$$

Note that the latter parameterization carries the implicit assumption that the lower support of  $F_g(\cdot)$  is precisely  $R_{ug}$ . This assumption, although not implausible, has no real theoretical justification as the public sector is not assumed to be profit maximizing and as such may offer wages that are all strictly greater than the workers' common reservation wage. Experimenting with richer specifications, allowing for the lower support of  $F_g(\cdot)$  to be strictly above  $R_{ug}$ , led to the conclusion that those two numbers are indeed very close and that (19) is a valid approximation.

In order to match the moments described we implement a two-step algorithm. In a first step, we use the eight flow parameters  $(\delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg})$  to fit the eight transition rates derived from our model to those observed in the data, conditional on initial guesses about the offer distributions  $F_p(\cdot)$  and  $F_g(\cdot)$ . This first, just-identified step produces a perfect fit to observed transition rates. Then, in a second step, conditional on the transition rates obtained from the first step, we derive the offer distributions such that the distance between the vector of quantiles of the empirical and theoretical wage distributions  $G_p(\cdot)$  and  $G_g(\cdot)$  is minimized. The process

is repeated until convergence. We find that this iterative two-step protocol performs better than one-step procedures.

Now turning to the estimation of the last two components of  $\theta$ , namely  $c(h)$  and  $\Gamma(y)$ , we use the fact the larger firms pay higher wages, i.e.  $\ell$  is increasing in  $w$ , which is consistent with our model (see appendix). This and data on  $H(\ell_i^c)$  allows us to infer the wage rates  $w_i^c$  paid at each cutoff size  $\ell_i^c$ :

$$G_p(w_i^c) = H(\ell_i^c) \quad \text{for } i = 1..10, \quad (20)$$

where  $\ell_i^c = \ell(w_i^c, h_i^c) = \ell(w^*(y_i^c), h^*(y_i^c))$ . Now, with the pair  $(\ell_i^c, w_i^c)$  at the 12 cutoff sizes, we are able to infer both  $h_i^c$  and  $c'(h_i^c)$  at these 12 points thanks to the following (see equations (11) and (14)):

$$\begin{aligned} h_i^c &= \ell_i^c \Delta(w_i^c) \\ c'(h_i^c) &= \frac{-1}{\Delta'(w_i^c)} \end{aligned}$$

We thereby obtain a non-parametric estimate of the shape of  $c'(\cdot)$  over a set of 12 points, from which we extrapolate the derivative of the cost function over the whole range of wages<sup>7</sup>.

All that is left to estimate now is the distribution of productivities in the population of firms,  $\Gamma(y)$ . Productivity levels are derived from the first-order condition (13), which gives us a relationship  $\tilde{y}(w)$ , where  $\tilde{y}$  is such that  $w^*(\tilde{y}) = w$ . The number of hires  $h^*(y)$  is estimated by inverting the cost of hire function, equation (14), thus giving us an  $h^*(y)$ . Manipulation of the expression for the wage offer distribution, equation (16) gives us an expression for  $\Gamma(y(w))$ , this performed in the appendices.

$$\Gamma(y(w)) = M \int_0^{F_p(w)} \frac{\tilde{\alpha}(x)}{\tilde{h}(x)} dx \quad (21)$$

Thus we obtain the  $\Gamma(\cdot)$  distribution that matches 50 points of the  $F_p$  distribution previously estimated.

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<sup>7</sup>Note that we can retrieve the cost function itself by assuming that the cost of making zero hires is zero and by integrating  $c'$ .

## 4 Results

### 4.1 Labor Market Transitions

Parameter estimates of the model are given in Table 3, of which the top panel contains all transition parameters. The unit of time associated with the transition/offer arrival rates is a month. Again, given our estimates of the rest of the parameter vector, those transition rate values produce a perfect fit to the observed monthly transition rates reported in Table 2.

A striking feature of our parameter estimates is the large on-the-job offer arrival rates for workers in both sectors. Comparison of our estimates of  $\lambda_{pp}$ ,  $\lambda_{pg}$ ,  $\lambda_{gp}$ , and  $\lambda_{gg}$  with the corresponding monthly transition rates  $\pi_{pp}$ ,  $\pi_{pg}$ ,  $\pi_{gp}$ , and  $\pi_{gg}$  (see Table 2) suggests that employed workers only accept between 2.5 and 3.5 percent of the offers they receive. Moreover, employed workers, regardless of sector, receive far more frequent offers from both sector than unemployed workers. Those results contrast sharply with standard findings from simple, one-sector wage-posting model. However, the same pattern arises in estimates obtained by Meghir et al. (2014) in a different two-sector wage posting model, with a formal and an informal sector estimated on Brazilian data.

Given in the parenthesis are the 95% confidence intervals for the parameter estimates. These are obtained by resampling the data, allowing for repetition and running the estimation protocol outlined previously on repeated redraws of the data. Transition rates will be non-negative by construction across all redraws, thus so will the transitional parameters and their distributions will be non-symmetric, as can be seen in Table 3. We therefore find displaying confidence intervals more informative than standard errors.

### 4.2 Wages and Worker Values

Table 3 also reports estimates of the wage offer distribution parameters,  $\alpha_p$  and  $\alpha_g$ , and an estimate of the flow value of unemployment,  $b$ , at about £11/hr for high skill workers and £7.50/hr for low skill. The latter implies values of the unemployed workers' reservation wages of  $R_{up} = £2.29/\text{hr}$  ( $R_{up} = £2.74/\text{hr}$ ) and  $R_{ug} = £4.75/\text{hr}$  ( $R_{ug} = £4.85/\text{hr}$ ) for high skilled workers (low skilled workers), respectively. The fact that  $R_{up}$  and  $R_{ug}$  are both lower than  $b$ , which is unusual in empirical

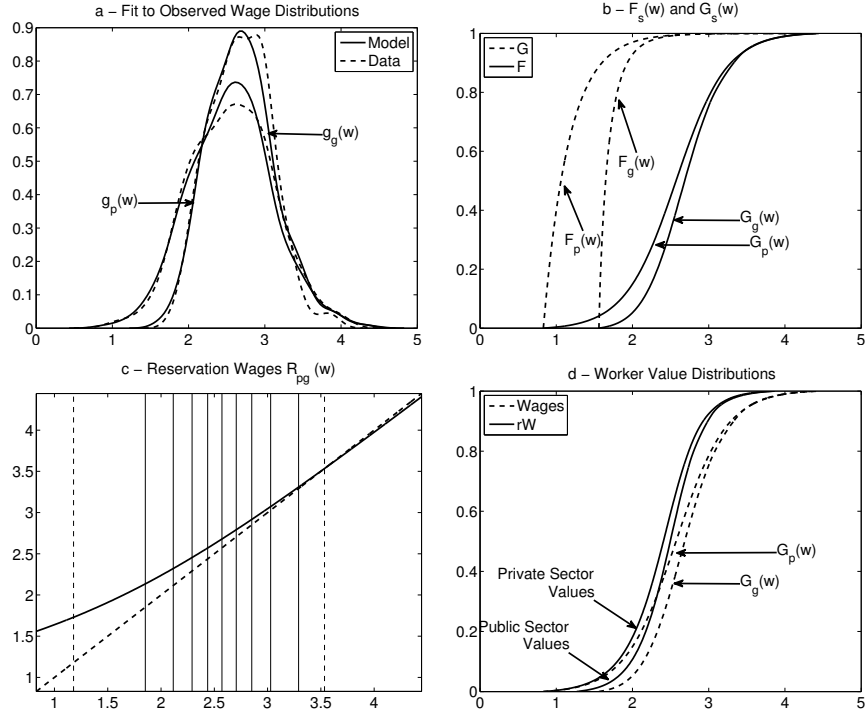
Table 3: Parameter Estimates

A-Levels and above			
$\delta_p$	$\lambda_{up}$	$\lambda_{pp}$	$\lambda_{pg}$
0.0044 (0.0039,0.0054)	0.0913 (0.0768,0.1063)	0.7160 (0.3327,1.5937)	0.1157 (0.0331,0.2996)
$\delta_g$	$\lambda_{ug}$	$\lambda_{gp}$	$\lambda_{gg}$
0.0039 (0.0031,0.0060)	0.0185 (0.0126,0.0250)	0.3661 (0.1427,0.7870)	0.4369 (0.1129,1.0956)
$\alpha_p$	$\alpha_g$	$b$	$a$
2.9958 (2.7586,3.2298)	0.8208 (0.2815,3.4827)	11.16 (9.7215,11.8487)	-1.42 (-2.4755,0.1276)
Less than A-Levels			
$\delta_p$	$\lambda_{up}$	$\lambda_{pp}$	$\lambda_{pg}$
0.0050 (0.0044,0.0063)	0.0875 (0.0726,0.1026)	0.3523 (0.2057,0.6379)	0.0294 (0.0077,0.0524)
$\delta_g$	$\lambda_{ug}$	$\lambda_{gp}$	$\lambda_{gg}$
0.0042 (0.0026,0.0078)	0.0084 (0.0049,0.0123)	0.2210 (0.0898,0.4832)	0.1156 (0.0356,0.2735)
$\alpha_p$	$\alpha_g$	$b$	$a$
3.5016 (3.2544,4.1731)	0.7972 (0.6487,3.3715)	7.45 (6.4323,8.2692)	-1.09 (-2.4745,-0.2823)

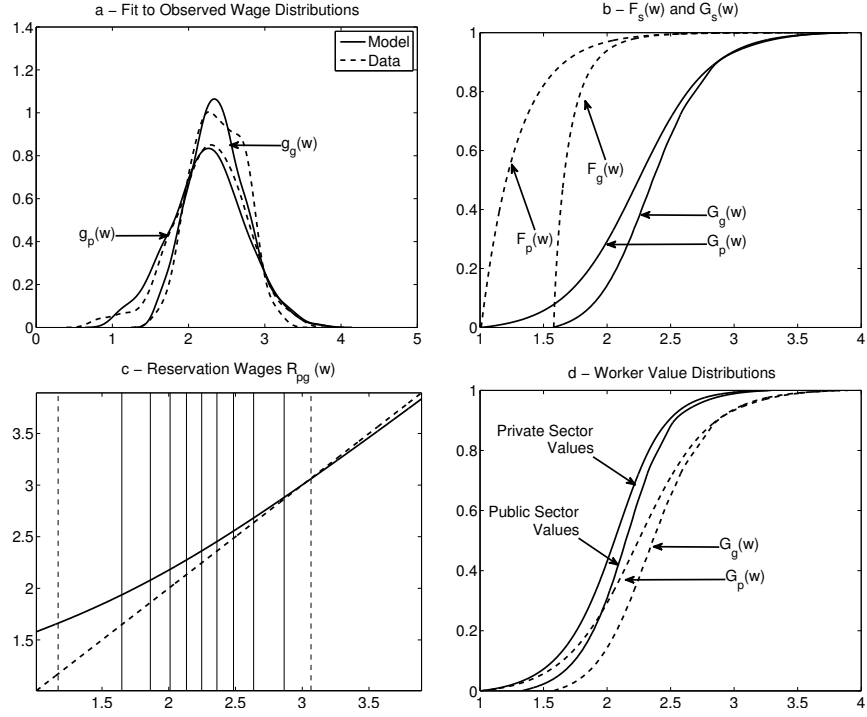
wage posting models, is a consequence of the relative values of offer arrival rates on- and off-the job: unemployed workers are prepared to give up a substantial amount in terms of income flow to benefit from the more efficient on-the-job search technology. It is also the higher job arrival rate coupled with lower job destruction that means high skilled workers have lower reservation wages from unemployment. There is a small public sector *stigma*, but it is small relative to wages and is not statistically significantly different from zero for the higher skilled workers.

We now turn to an analysis of wage distributions. Panels (a) in Figure 1 shows the model's fit to observed cross-section (log-) wage distributions in both sectors of employment. The fit is reasonably good in both sectors, although the model has some difficulty fitting the high quantiles of the public-sector distribution. In passing, we note that, as is well documented elsewhere in the literature, the public-sector wage distribution dominates the private-sector one except in the top two deciles. There is also markedly less wage dispersion in the public sector.

Panels (b) in Figure 1 next shows estimated log-wage offer distributions in both sectors,  $F_p(\cdot)$  and  $F_g(\cdot)$ , together with the distributions of accepted log-wage offers,  $G_p(\cdot)$  and  $G_g(\cdot)$ . Both



A-Levels and above



Less than A-levels

Figure 1: Wage distributions and reservation wages

offer distributions are fairly concentrated, much more so than the corresponding accepted offer distributions. Indeed the large estimated offer arrival rates imply a large extent of stochastic dominance of  $G_s(\cdot)$  over  $F_s(\cdot)$  for  $s = p$  or  $g$ . We also see that the public-sector offer distribution strongly dominates its private-sector equivalent. However part of that dominance is “undone” by later reallocation within and between sectors: the dominance of  $G_g(\cdot)$  over  $G_p(\cdot)$  is less marked than the one of  $F_g(\cdot)$  over  $F_p(\cdot)$ . The main driver here is the rate  $\lambda_{pp}$  at which private sector workers receive offers of alternative jobs in the private sector, which is very high compared to its public sector counterpart  $\lambda_{gg}$ , implying much quicker upward wage mobility in the private than in the public sector. As a consequence, the distribution of private sector wages  $G_p(\cdot)$  dominates the private sector offer distribution  $F_p(\cdot)$  by much more than  $G_g(\cdot)$  dominates  $F_g(\cdot)$ .

Panels (c) in Figure 1 then plots  $R_{pg}(w)$ , the reservation wage of private-sector employees presented with offers from the public sector. The dashed line on that graph is the main diagonal and the vertical lines materialize the deciles of the private-sector wage distribution,  $G_p(\cdot)$ . It appears on this plot that, over most of the support of  $G_p(\cdot)$  (up to about the 95th percentile),  $R_{pg}(w) > w$ , i.e. private-sector employees will only accept to go into the public sector with a wage increase. The likely reason is again that upward wage mobility is quicker in the private sector, mainly because of the high value of the private-sector offer arrival rate  $\lambda_{pp}$ .

Finally, our model allows us to examine the public-private sector pay gap. While this pay gap is conventionally assessed in terms of wages, in our model workers care not only about their wages but also about future wages, which depend on transition rates and expected future wage progression patterns that differ between sectors. Following Postel-Vinay and Turon (2007), we thus assess the public-private pay gap in terms of lifetime values of employment, as well as raw wages. Specifically, panels (d) in Figure 1 takes up a plot of the private- and public-sector wage distributions  $G_p(\cdot)$  and  $G_g(\cdot)$ , together with corresponding plots of the distributions of worker “permanent income” in both sectors, where we define permanent income as the annuitized worker value  $rW_p(w)$  and  $rW_g(w)$  as defined by equation (4). Again, the  $x$ -axis on panels (d) is on a logarithmic scale. We draw two conclusions from this graph. First, assessing the public pay gap based on wages only may lead one

to overstate the public sector premium: the public- and private-sector distributions of worker values are much closer together than the corresponding wage distributions. In other words, differences in patterns of wage mobility (due to differences in offer arrival rates) between sectors rub off a large part of the public-sector wage premium. Second, even when comparing lifetime values, a public premium persists in the lower quantiles of the value distribution in the sense that the distribution of public-sector values dominates its private-sector equivalent in the bottom quantiles, consistent with the documented fact that low-wage workers tend to queue for public sector jobs much more than high wage workers. This dominance is particularly pronounced for low skilled workers. Both of those conclusions are consistent with the findings of Postel-Vinay and Turon (2007), who estimate a descriptive model of wages and job mobility across the public and private sectors, also on BHPS data, but with a much richer representation of worker heterogeneity and wage dynamics.

## 5 Counterfactual Policy Analysis

Using the estimated parameters of the structural model we simulate the effects of various changes in public sector wage and employment policy. While the model allows the simulation of many possible policy changes, from a topical perspective, an assessment, by simulations, of the various public sector austerity measures being enacted across Europe seems to be a sensible subject to pursue.

Before beginning, it is important to make clear that this paper has nothing to say on whether or not austerity is good economic policy. Rather, conditional on wanting to implement it, this model can inform policy makers on the best way to go about it. There are two further considerations to make when reviewing the results presented in this section. The results only concern public spending on the wages of public sector employees; no changes in government revenue through taxation are taken into account. Wages for public sector workers cost the UK government £174 billion in 2008<sup>8</sup>. This accounts for around 30% of total expenditure and 50% of non-investment expenditures.

Policies we consider are categorized into employment and wage policies. Employment policies are reducing hiring in the public sector and increasing firing. They are modeled as changes to  $\delta_g$

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<sup>8</sup>ONS, United Kingdom National Accounts: Blue Book, 2009, <http://www.statistics.gov.uk/statbase/tsdtables1.asp?vlnk=bb>.

and  $\lambda_{sg}$ , respectively, where,  $s \in \{u, p, g\}$ . Wage policies are treated as changes to the distribution  $F_g(\cdot)$ .

## 5.1 Counterfactual Policies

Formally, the specific policies we simulate are represented by equation (22), where new policies are denoted by a  $\star$ . The parameter  $\pi$  is the degree by which the parameters change (see Appendix B for the protocol for simulation).

$$\begin{aligned}
&\text{Increase in Fires: } \delta_g^\star = (1 + \pi) \times \delta_g \\
&\text{Decrease in Hires: } \lambda_{sg}^\star = (1 - \pi) \times \lambda_{sg} \text{ where } s \in \{u, p, g\} \\
&\text{Proportional reduction in wages: } w_g^\star = (1 - \pi) \times w_g \\
&\text{Progressive reduction in wages: } w_g^\star = (1 - \pi) \times w_g + \pi \times \underline{w}_g
\end{aligned} \tag{22}$$

These policies are aimed at approximating the types of policies implemented across Europe during the “*age of austerity*”<sup>9</sup>. Specifically, the policies documented in equation (22) are intended to replicate policies implemented in Italy, UK, Spain and Portugal, respectively. While within each country a variety of policies have been undertaken, these four countries all adopted different principal tactics in reducing the wage bill of public sector employees. Italy froze all new recruitment; the UK actively cut public sector jobs; Spain froze public sector pay across the board; and Portugal implemented a 5% pay cut on the higher earners in the public sector.

It is difficult to compare policies without a clearly defined metric for assessment. For each policy we compute the savings the public sector makes as a proportion of its initial expenditure. This savings rate is given by equation (23), where the superscript  $\star$  denotes a simulated policy<sup>10</sup>. Conveniently, for each of the policies, this savings rate is monotonically increasing in  $\pi$ . This means

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<sup>9</sup>The phrase was coined by Prime Minister David Cameron at the Conservative party forum in Cheltenham, 26th April 2009.

<sup>10</sup>This formula is only valid, strictly speaking, for the cases of an increase in public job destruction and of a cut in public wages. Indeed, a reduction in public hiring is likely associated with extra savings on (unmodeled) hiring costs - the public-sector counterparts to our private-sector  $c(h)$ . We should bear this caveat in mind when interpreting the simulation results.

it is possible to plot a variety of labor market outcomes on the degree of savings made, rather than an arbitrary parameter  $\pi$ . This is exactly what is done in all the remaining figures.

$$\text{savings} = \frac{N_e \int_{R_{ug}}^{\infty} wdG_g(w) - N_e^* \int_{R_{ug}^*}^{\infty} wdG_g^*(w)}{N_e \int_{R_{ug}}^{\infty} wdG_g(w)} \quad (23)$$

Figures 2 and 3 show the implications of a given level of saving on employment rates and the first two moments of the distributions of wages in both sectors. As one may expect, all four austerity measures are associated with a fall in public sector employment, panels (c) and (d), and a fall in the mean wage of a public sector employee, panels (e) and (f). An important question is then: how does the private sector respond? A common feature of all four policies across both strata is that the primary response of the private sector is to increase employment. The private sector responds by increasing its employment by over 80% of the fall in public sector employment. The progressive wage policy has the smallest impact on unemployment for both levels of skills, with the private sector increasing employment by 87% and 90% of the fall in public sector employment. By comparison, the change in the composition of wages within the private sector is minimal. The mean wage in the private sector never exhibits even as much as a 0.25% change and the policies' effect on the variance of wages is similarly meagre.

The large employment and small wage effect have two notable policy implications. Firstly, a government implementing the policies assessed need not worry about large increases in unemployment. The majority of the fall in public sector employment will be soaked up by the private sector. Secondly, for a policy to make a significant impact on private sector wages, it would need to be so draconian it would probably be politically infeasible. This suggests a Eurozone country with no autonomy over monetary policy cannot increase their international competitiveness with fiscal policy (in the form of changes to public sector wages and employment). Further, from a theoretical perspective, it highlights the importance of endogenous private labor demand in the model. As a response to public sector policy, the private sector's primary response is to post more vacancies, rather than change their wage policy.

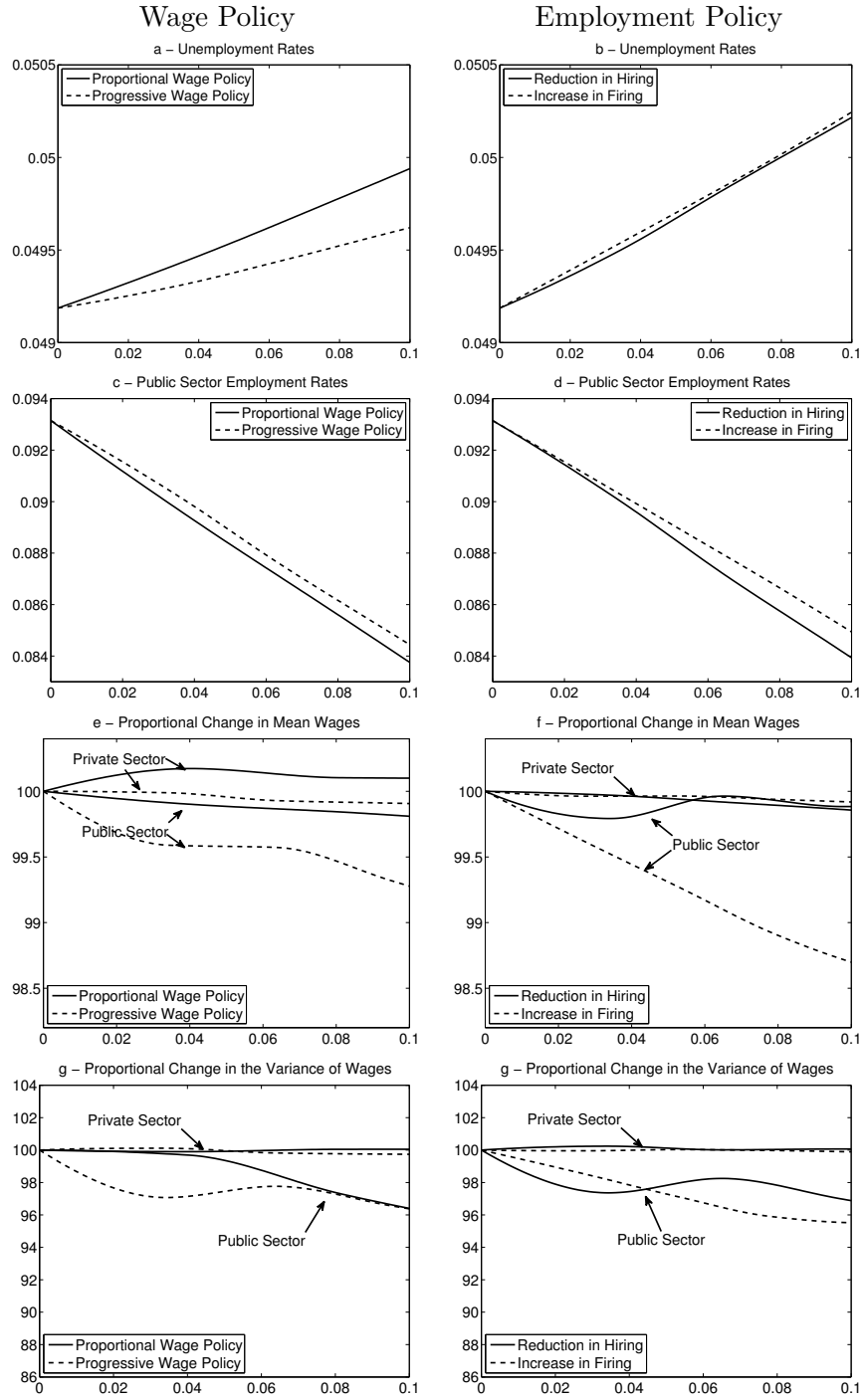


Figure 2: Labor Market Outcomes for Low Skill Workers

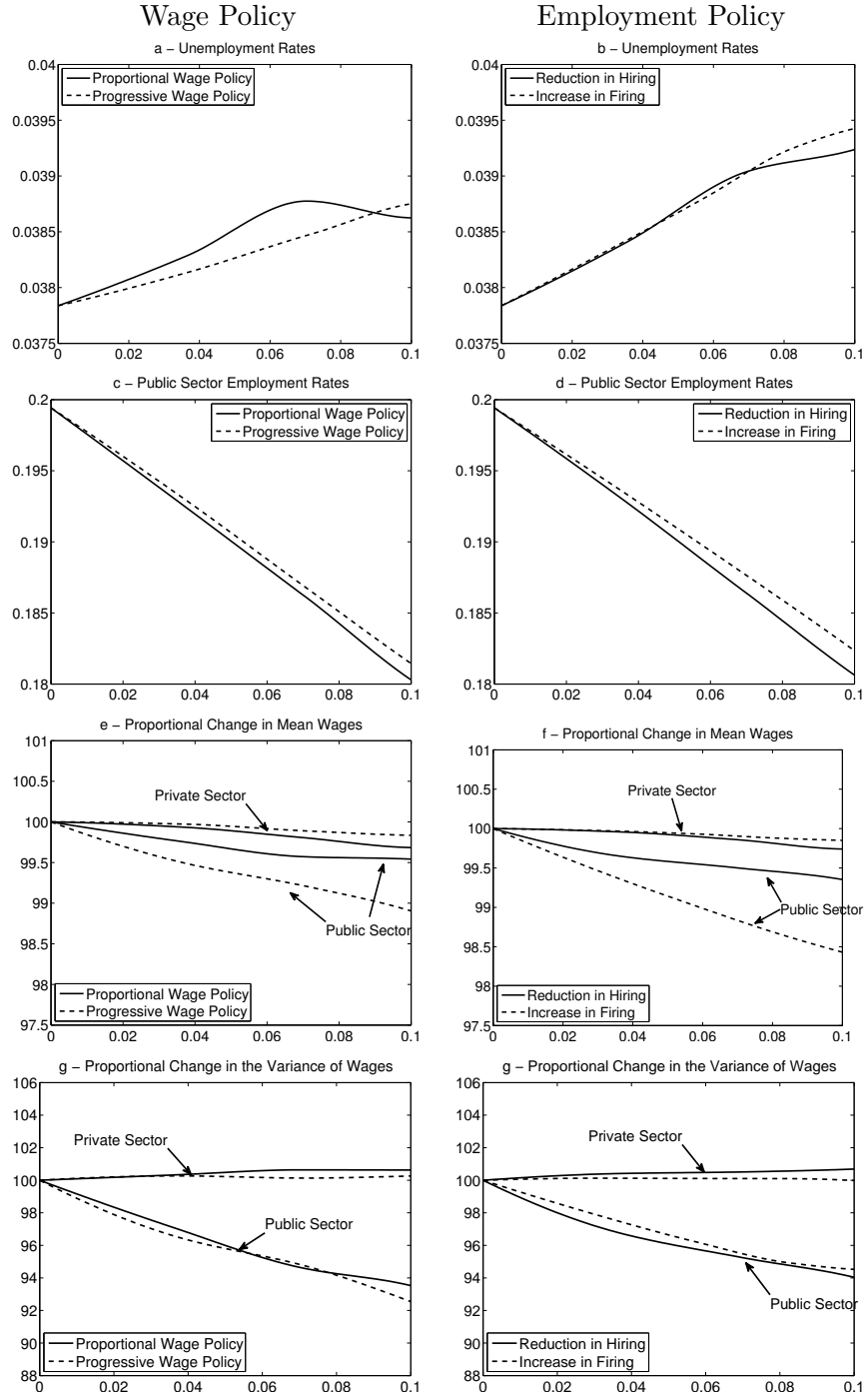


Figure 3: Labor Market Outcomes for High Skill Workers

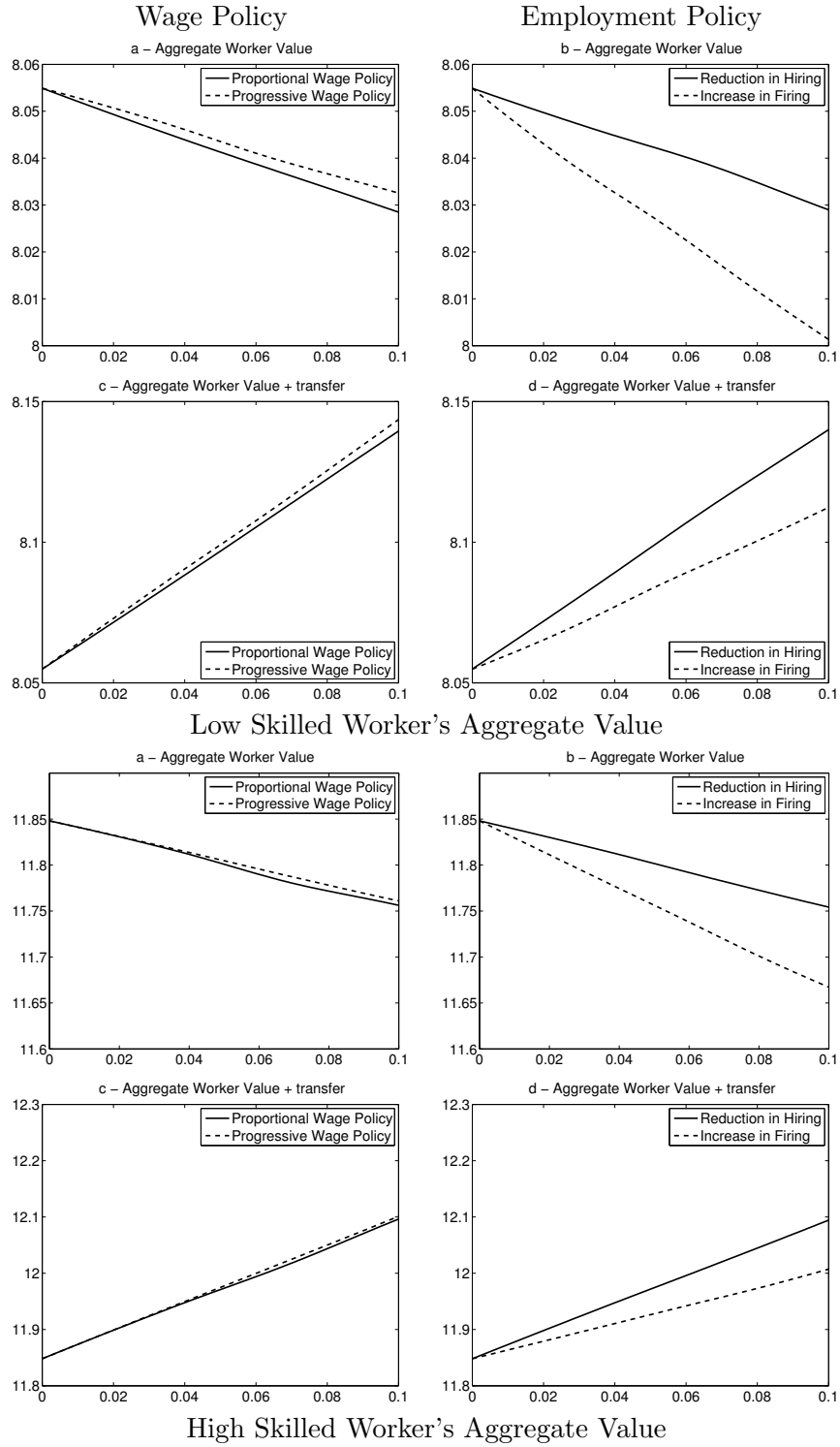


Figure 4: Aggregate Worker Value

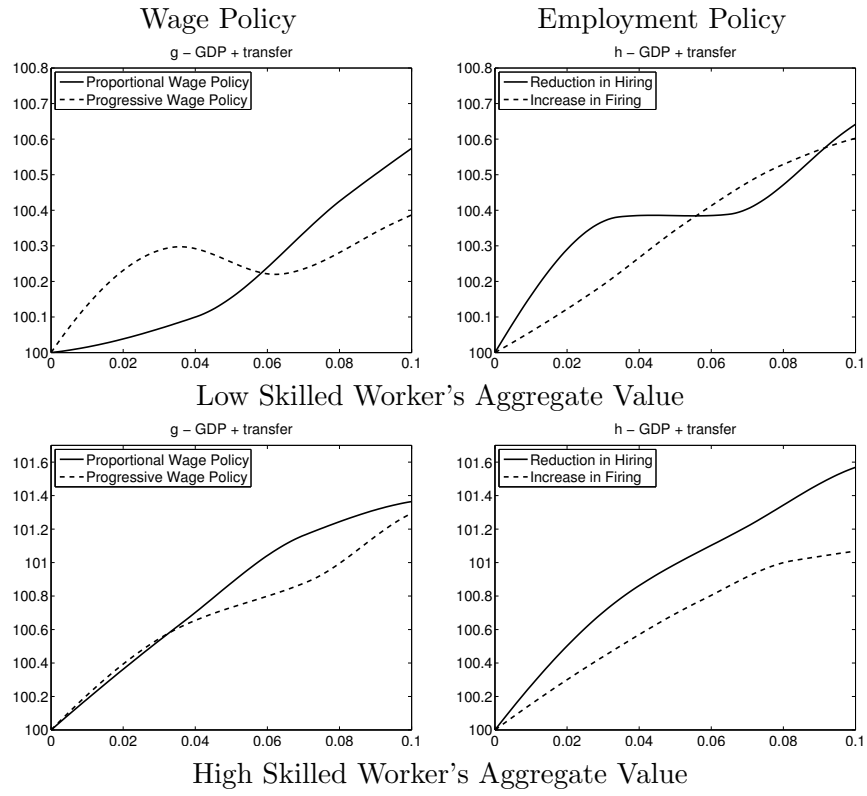


Figure 5: Output of Economy

## 5.2 Welfare

A desirable feature of the model is that it allows us to look at the long-term welfare of the worker. We assess how these policies affect aggregate worker value, where, aggregate worker value is defined as the mean present discounted value of a worker in each of the three states of employment, weighted by the size of that state. The present discounted value of a worker in a given state is calculated by computing equations (3), (4) and (5). In equation (24) the aggregate worker value is denoted as  $rO$ .

$$rO := N_u rU + N_p r \int_{R_{up}}^{\infty} W_p(x) dG_p(x) + N_g r \int_{R_{ug}}^{\infty} W_g(x) dG_g(x) \quad (24)$$

Figure 4, panels (a) and (b), shows how the aggregate worker value, as defined by equation (24) varies with increasingly severe austerity policies. Unsurprisingly, austerity has a negative impact on worker's value. Increasing the layoff rate of public sector workers has the largest negative impact on aggregate worker value across both strata. The other three policies perform similarly, but for both the high and low skilled, a progressive wage policy has the least detrimental effect on the average worker's lifetime value. Interestingly, if we include in the lifetime value a transfer, equal to the savings of austerity, panels (c) and (d), even an increase in firing has a positive impact on worker value. If one were to ignore the provision of public goods and use this to assess optimal policy, it would be in the interest of a social planner to cut public sector employment spending by more than 10% and probably do so using a progressive cut to public sector wages.

## 5.3 GDP

A further important implication of austerity is the effect it will have on a nation's GDP. The output of the private sector is a primitive of the model and changes in private sector output resulting from policy changes can easily be quantified. More problematic however, is quantifying changes in output in the public sector, as thus far, we have no concept of public sector productivity.

Using national accounts data<sup>11</sup> we can calculate the total output of the public sector as a

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<sup>11</sup>ONS, United Kingdom National Accounts: Blue Book, 2013, <http://www.ons.gov.uk/ons/rel/naa1-rd/united-kingdom-national-accounts/the-blue-book-2013-edition/tsd—blue-book-2013.html>

proportion of total GDP. Over our time period, this remained relatively stable at approximately 13%. However, with no data regarding the relative production of output by strata of education we need to make further assumptions regarding public sector output. Firstly, it is assumed that within strata all public sector workers produce the same amount of public output. Secondly, the output of a public sector worker ( $y_g$ ) is assumed to be equal to the mean output of a private sector worker in the same strata multiplied by a scalar, ( $y_g = a_g \bar{y}_p$ ), where  $a_g$  is independent of strata. Given these two assumptions, the output of the public sector is calculated from the value of  $a_g$  that rationalizes the proportion of output that the public sector is responsible for. The public sector is found to be about three quarters as productive as the private sector. Aggregate output is defined by equation (25).

$$output := N_g y_g + N_p \int y dG_p(w(y)) \quad (25)$$

This output, plus the additional savings from austerity are shown in Figure 5. In all instances, there is the potential to increase output by implementing austerity policy. This is true even without the addition of the savings. These results, no doubt, hinge on the assumptions made about public sector productivity. Firstly, as the private sector is on average one third more productive than the public sector, policies that contract the public sector and expand the private sector are likely to increase aggregate output. Secondly, within the private sector, with on the job search, workers allocate more efficiently than if randomly assigned to firms. All policies reduce the number of private sector workers poached from high productive firms and hence increase the level of allocative efficiency.

The increases in output are fairly substantial, especially for high skilled workers, where there is a greater proportion of public sector employees compared with the low skilled (see Table 1). A 10% saving in the public sector wage bill through a reduction in hiring, increases the aggregate output of high skilled workers by 1.6%. Abstracting from crude measures of output by the public sector and the provision of public goods as inputs into the private sector's production function, austerity policies, as modeled here will not lead to a contraction in output.

## 6 Concluding Remarks

This paper explores the interaction between the public and private sectors by developing and estimating an equilibrium model of the labor market. The rates at which the public sector hire and layoff workers (employment policy) and the distribution of wages offered (wage policy) are exogenous. The rate at which workers receive offers from the private sector is endogenous as is the distribution from which wages are drawn. The paper allows for mobility between and within three labor market states and the model is estimated using British data.

The parameter estimates allow us to draw two conclusions regarding the public and private sectors. Firstly, comparisons of the welfare of workers drawn only from information on wages will be misleading and lead to a gross overestimate of the premium associated with work in the public sector. Once one accounts for different wage mobility rates between sectors, the size of this premium diminishes substantially. Secondly, although a premium still exists, it really only exists at the lower end of the wage distribution and is far more prevalent amongst the lower skilled workers.

While these empirical facts are interesting, the key contribution of this paper is that with this modeling approach, policies relating to wages and employment in the public sector can be assessed prior to implementation. To demonstrate this *user friendly* approach we apply the model to see what the effects on the British economy would have been under a variety of policies implemented across Europe aimed at cutting the wage bill of the public sector. We find surprisingly little impact on private sector wages and employment as a whole. If one cuts public sector wages/employment, the private sector soaks up the majority of the fall in employment and wages remain relatively stable. The policy that seems to be most in the interest of the workers is a progressive wage cut of the public sector, akin to the primary policy of Portugal post 2008. However, if the objective of the government were to maximise its GDP, arguably a better policy would be to simply reduce new hires as implemented by Mario Monti's government in Italy.

This model can inform policy makers and sits in a small subset of equilibrium search models of the labor market that are designed with policy primarily in mind. It is the hope of the authors that similar models are developed to critically assess future policy proposals.

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## APPENDIX

### A Theory: Intermediate Derivations

#### A.1 Derivation of the Reservation Wage, Equation (6)

The value function for a private sector worker earning a wage  $w$ , is given in equation (4). Assuming differentiability:

$$W'_p(w) = [r + \delta_p + \lambda_{pp}\bar{F}_p(w) + \lambda_{pg}\bar{F}_g(R_{pg}(w))]^{-1} \quad (26)$$

This also gives  $W'_p(w)$  by analogy. Integrating by parts in (4) yields:

$$(r + \delta_p) W_p(w) = w + \delta_p U + \lambda_{pp} \int_w^{+\infty} W'_p(x) \bar{F}_p(x) dx + \lambda_{pg} \int_{R_{pg}(w)}^{+\infty} W'_g(x) \bar{F}_g(x) dx \quad (27)$$

Plugging the various value functions into the definition of  $R_{pg}(w)$  given in the paper, one obtains the following, fairly complicated expression:

$$\begin{aligned} R_{pg}(w) = -a_g + \frac{r + \delta_g}{r + \delta_p} w + \left\{ \frac{r + \delta_g}{r + \delta_p} \delta_p - \delta_g \right\} U + \left\{ \frac{r + \delta_g}{r + \delta_p} \lambda_{pp} - \lambda_{gp} \right\} \int_w^{+\infty} W'_p(x) \bar{F}_p(x) dx \\ + \left\{ \frac{r + \delta_g}{r + \delta_p} \lambda_{pg} - \lambda_{gg} \right\} \int_{R_{pg}(w)}^{+\infty} W'_g(x) \bar{F}_g(x) dx \end{aligned}$$

Differentiating yields (6).

#### A.2 Derivation of the Private-Sector Wage Distribution, Equation (9)

Equation (9) would be a simple ODE if it was not for the term featuring  $G_g(R_{pg}(w))$ . We now show how to express that term as a function of  $w$  and  $G_p(w)$ . Writing the flow-balance equation for the public sector yields:

$$\begin{aligned} \{\delta_g + \lambda_{gg}\bar{F}_g(w)\} N_g G_g(w) + N_g \lambda_{gp} \int_{R_{ug}}^w \bar{F}_p(R_{gp}(x)) dG_g(x) \\ - N_p \lambda_{pg} \int_{R_{up}}^{R_{gp}(w)} [F_g(w) - F_g(R_{pg}(x))] dG_p(x) = N_u \lambda_{ug} [F_g(w) - F_g(R_{ug})]. \end{aligned}$$

Now applying the latter equation at  $R_{pg}(w)$  (instead of  $w$ ), we get:

$$\begin{aligned} \{\delta_g + \lambda_{gg}\bar{F}_g(w)\} N_g G_g(R_{pg}(w)) + N_g \lambda_{gp} \int_{R_{ug}}^{R_{pg}(w)} \bar{F}_p(R_{gp}(x)) dG_g(x) \\ - N_p \lambda_{pg} \int_{R_{up}}^w [F_g(R_{pg}(w)) - F_g(R_{pg}(x))] dG_p(x) = N_u \lambda_{ug} [F_g(R_{pg}(w)) - F_g(R_{ug})]. \quad (28) \end{aligned}$$

Adding (28) to (8):

$$\begin{aligned}
& N_p G_p(w) \{ \delta_p + \lambda_{pp} \bar{F}_p(w) + \lambda_{pg} \bar{F}_g(R_{pg}(w)) \} \\
& + N_g G_g(R_{pg}(w)) \{ \delta_g + \lambda_{gp} \bar{F}_p(w) + \lambda_{gg} \bar{F}_g(R_{pg}(w)) \} \\
& = N_u \lambda_{ug} [F_g(R_{pg}(w)) - F_g(R_{ug})] + N_u \lambda_{up} [F_p(w) - F_p(R_{up})], \quad (29)
\end{aligned}$$

which can be solved for  $N_g G_g(R_{pg}(w))$ . Plugging the solution into (9), we obtain an ODE defining  $G_p(w)$ . Note that by considering  $w \rightarrow +\infty$  in the latter equation, one obtains (7).

### A.3 Productivity Distribution

Using a change of variable and assuming the that  $w^*(y_{min}) = R_{up}$ , equation (16) is equivalent to:

$$\begin{aligned}
F_p(w) &= \frac{1}{M} \int_{R_{up}}^w \frac{h(z)}{\alpha(z)} \gamma(y(z)) y'(z) dz \quad (30) \\
f_p(w) &= \frac{1}{M} \frac{h(w)}{\alpha(w)} \gamma(y(w)) y'(w) \\
\frac{d}{dw} \{ \Gamma(y(w)) \} &= \frac{M f_p(w) \alpha(w)}{h(w)} \\
\Gamma(y(w)) &= M \int_{R_{up}}^w \frac{\alpha(x)}{h(x)} dF_p(x) \\
\Gamma(y(w)) &= M \int_0^{F_p(w)} \frac{\tilde{\alpha}(x)}{\tilde{h}(x)} dx
\end{aligned}$$

Where,  $\alpha(x) = \tilde{\alpha}(F_p(x))$ ,  $h(x) = \tilde{h}(F_p(x))$  and  $h(w) = c'^{-1} \left( \frac{y-w}{\Delta(w)} \right)$

## B Simulation Protocol

Previously the wage offer distribution of the public sector ( $F_g(w)$ ) was parameterized as a function of the wage offer distribution of the private sector ( $F_p(w)$ ), equation (19). As  $F_p(w)$  is an endogenous object it will change with changes to public policy, and we therefore need to fix  $F_g(w)$  ex-ante. Taking the point estimates reported in Section 4 we fit the distribution derived from the transform of equation (19) and the distribution in equation (31). We include the shifting parameter  $\tilde{\alpha}$  so as to match the distribution more closely, as the lower bound  $R_{ug}$  also affects the curvature of the distribution.

$$F_g(w) = \begin{cases} \frac{1-(w+\tilde{\alpha}/R_{ug})^{\tilde{\alpha}_g}}{1-(\bar{w}_g/R_{ug})^{\tilde{\alpha}_g}} & \text{if } w \in [R_{ug}, \bar{w}_p] \\ 0 & \text{if } w < R_{ug} \\ 1 & \text{if } w > \bar{w}_g, \end{cases} \quad (31)$$

We implement policy changes as changes to the job offer arrival rates of public sector jobs, public sector job destruction and the wage offer distribution. Simulating the new equilibrium is performed using an iterative procedure:

1. We begin by taking initial values for two endogenous parameters, the values of the Pareto index ( $\alpha_p$ ) of the  $F_p(\cdot)$  distribution and the private sector job offer arrival rate ( $\lambda_p$ ). The initial values  $\alpha_p^0$  and  $\lambda_p^0$  are the estimated values of these before the implementation of any policy.
2. The worker side is re-evaluated as before; with new parameters, equations (6) through (9) are solved as before, giving us new values of  $N_u$ ,  $N_p$ ,  $N_g$ ,  $G_p(w)$ ,  $G_g(w)$ ,  $R_{up}$ ,  $R_{ug}$  and  $R_{pg}(w)$ .
3. Turning to the firm side: the cost of hires and firm productivity distributions are treated as primitives of the model. Firms maximize profit according to their first order conditions, equations (13) and (14). Conditional on  $\alpha_p^0$ ,  $\lambda_p^0$  and the parameters calculated in step 2, the new optimal search policy  $h^*(y)$  is derived. Then our updated  $F_p(\cdot)$  is obtained from equation (16) and  $\lambda_p$  from equations (15) and (18). Then, fitting a Pareto distribution through the updated  $F_p(\cdot)$ , one obtains  $\alpha_p$ .
4. The values of  $\alpha_p$  and  $\lambda_p$  are continuously updated until they have converged.