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OPTIMAL FISCAL POLICY WITH HETEROGENEOUS AGENTS AND AGGREGATE SHOCKS

François Le Grand and Xavier Ragot

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Optimal fiscal policy with heterogeneous agents and aggregate shocks*

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Abstract

We provide a theory of truncation for incomplete insurance-market economies with aggregate shocks, which is shown to be a consistent representation of standard incomplete-market economies. This representation allows deriving optimal policies with capital and aggregate shock. We apply this framework to an economy where the government can use capital and labor taxes, positive transfers and public debt to smooth aggregate shocks. The average capital tax is shown to be positive if and only if credit constraints are binding for some households. In a quantitative exercise, the capital tax appears to be more volatile than the labor tax and public debt is countercyclical and mean-reverting.

Keywords: Incomplete markets, optimal policy, public debt.

JEL codes: E21, E44, D91, D31.

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1 Introduction

Incomplete insurance-market economies provide a useful framework to think about many relevant aspects of inequalities and individual risk in general equilibrium. In these models, infinite-lived agents face incomplete insurance markets and borrowing limits that prevent them from perfectly hedging their idiosyncratic risk, in the tradition of the Bewley-Huggett-Aiyagari literature (Bewley 1983, Imrohoroğlu 1989, Huggett 1993, Aiyagari 1994). These frameworks are becoming increasingly popular and are now widely used, since they fill a gap between micro- and macroeconomics and enable the inclusion of aggregate shocks and a number of additional frictions on both the goods and labor markets. However, considering normative analysis, little is known about optimal policies in these environments, due to the difficulties generated by the large and time-varying heterogeneity across agents. This is unfortunate, since a vast literature, reviewed below, suggests that the interaction between wealth inequalities and capital accumulation has first-order implications for the optimal design of time-varying fiscal policies.

This paper presents a methodological contribution that offers a general and tractable representation of incomplete insurance-market economies. This representation allows us to easily solve for the Ramsey program in incomplete insurance-market economies with both capital and aggregate shocks. We apply our framework to provide a theoretical and quantitative analysis of optimal fiscal policy. We derive new results about the optimal dynamics of public debt, distorting capital and labor taxes and transfers, considering rich trade-offs involving redistribution, insurance, and incentives.

When insurance markets are incomplete, heterogeneity increases as time goes by, because agents differ according to the whole history of their idiosyncratic risk realizations. Huggett (1993), using the results of Hopenhayn and Prescott (1992), and Aiyagari (1994) have shown that these economies without aggregate risk have a recursive structure when the distribution of wealth is introduced as a state variable. Unfortunately, the distribution of wealth has an infinite support, which is the root of many analytical difficulties. Our methodological contribution is to show that incomplete-market economies can be represented as the limit of economies with finite support. More precisely, we construct an environment where the heterogeneity across agents depends only on a finite but arbitrarily

large number, denoted N, of consecutive past realizations of the idiosyncratic risk. As a theoretical outcome, agents having the same idiosyncratic risk history for the previous N periods choose the same consumption and wealth levels. The interest of this *truncated* representation of incomplete insurance-market economies lies in four properties. First, the allocation can be represented as the solution of the program of a constrained planner, which ensures the existence of the equilibrium with aggregate shocks. Second, the policy rules of our truncated representation converge, for large N, toward those of a standard incomplete-market model under general conditions. Third and more importantly, as our representation has a finite state-space, we can use the tools derived in Marcet and Marimon (2011) to derive Ramsey programs. These tools rely on the extensive use of Lagrange coefficients. Finally, the finite state-space simplifies to a large extent the simulation of the model, as standard perturbation methods can be used.

Regarding the Ramsey program in an incomplete-market economy with technological shock, and following the literature, we consider that the planner has four instruments: taxes on capital, taxes on labor, public debt, and positive transfers. We derive three sets of results.

First, for any truncation N, the average optimal long-run capital tax is directly related to the severity of credit constraints. More formally, the tax on capital is proportional to the sum of the Lagrange multipliers of agents' credit constraints. The capital tax is thus always non-negative (as already found in Aiyagari 1995) and is positive if and only if credit constraints bind for some agents in equilibrium. This result contributes to clarify the deviations from the Chamley (1986) and Judd (1985a) no-capital tax result found in the literature. In addition, the pre-tax marginal return on capital is uniquely pinned down by the planner discount factor, as originally found by Aiyagari (1995). Finally, labor taxes are shown to directly depend on the elasticity of labor supply.

Second, the dynamics of the fiscal policy is mainly driven by the gap in two valuations of liquidity. The first one is the planner valuation of government liquidity, measured by the Lagrange multiplier of the government budget constraint. The second one is the social valuation of the liquidity of private agents. Since the planner internalizes agents' saving incentives, this valuation differs from private agents' valuation. The difference in these liquidity valuations, the so-called *liquidity valuation gap*, is key to understand optimal

fiscal policy dynamics.

Third, we simulate the model to determine the optimal fiscal policy after a technology in a model where households face employment risk. We find that the steady-state level of public debt is negative, and its level is countercyclical. The capital tax is procyclical and much more volatile than the labor tax.

Related literature. This paper first contributes to the literature on the theory of incomplete insurance-market economies with aggregate shocks. Some environments already provide a tractable framework. This is the case of no-trade equilibria with permanent idiosyncratic shocks (Constantinides and Duffie 1996), used for instance in Heathcote, Storesletten, and Violante (2016). More recently, Krusell, Mukoyama, and Smith (2011) study a class of no-trade equilibria in an economy without capital and with a tight-enough credit constraint, as in Ravn and Sterk (2017). Departing from no-trade, a class of "small trade" equilibria, featuring "reduced heterogeneity" with a finite number of wealth levels, has been studied (Challe and Ragot 2016, LeGrand and Ragot 2016, Challe, Matheron, Ragot, and Rubio-Ramirez 2015, Ragot 2016, Bilbiie and Ragot 2017). The current paper extends these previous works and provides a general theory of truncated representations of incomplete-market economies, which is a consistent representation of Bewley economies. In addition, it derives new tools to study optimal policies, based on the dynamic structure of Lagrange coefficients.

Second, our paper contributes to the literature on distortions and optimal policies in incomplete insurance-market models. Many contributions identify a number of relevant trade-offs, but the general case with capital accumulation and aggregate shocks has not been studied yet, to the best of our knowledge.[1] In economies without aggregate shocks, Aiyagari (1995) shows that the capital tax is non-negative. Aiyagari and McGrattan (1998) compute the optimal steady-state level of debt. Dávila, Hong, Krusell, and Ríos-Rull (2012) show that the capital stock can be too low. Açikgöz (2015) solves for the Ramsey program to obtain the steady-state optimal level of public debt. Dyrda and Pedroni (2016) solve numerically for the optimal policies along the transition between

[1] Many papers have considered incomplete insurance markets to analyze the positive effect of fiscal policies. The positive effects are studied in Heathcote (2005), who considers aggregate shocks. A recent contribution is Kaplan and Violante (2014), who introduce transaction costs for some assets.

two steady-states. Gottardi, Kajii, and Nakajima (2014) compute the Ramsey allocation in a model with human capital accumulation. Nuño and Moll (2017) use a continuous-time approach and mean-field games to characterize differences in inequalities for economies without aggregate shocks. Shin (2006) studies a two-agent economy to derive additional results. Recently, Bhandari, Evans, Golosov, and Sargent (2013, 2016b) derive results about optimal policies in environments with incomplete insurance markets and aggregate shocks. They study an economy without capital, with lump-sum taxes, and where credit constraints are loose enough such that they never bind in equilibrium. They show that public debt is irrelevant, what simplifies the state space and allows introducing additional features. Instead, we study an economy with capital (and capital tax) and we allow for binding credit constraints. Besides being closely connected to the Chamley-Judd literature on capital taxation, our analysis also relates to the literature on the optimal quantity of safe assets, as the steady-state level of public debt is well-defined in our economy (see Gorton, Lewellen, and Metrick 2012 and Golec and Perotti 2017, for a survey and references).[2]

Third, this paper is also related to the vast literature on optimal fiscal policy with aggregate shocks. Seminal contributions consider a complete-market economy with a representative agent (Barro 1979, Lucas and Stokey 1983, surveyed in Chari and Kehoe 1999). More recent contributions consider incomplete markets for the aggregate risk, introducing non state-contingent public debt (Aiyagari, Marcet, Sargent, and Seppälä 2002, Farhi 2010, Bhandari, Evans, Golosov, and Sargent 2016a). Several papers have additionally introduced ex-ante heterogeneity among agents (see Bassetto 2014, Azzimonti, de Francisco, and Krusell 2008a and 2008b, Azzimonti and Yared 2017, Correia 2010, Greulichy, Laczo, and Marcet 2016). The New Dynamic Public Finance literature focuses on optimal fiscal policy in environments with heterogeneous and private information (Mirrlees 1971, Golosov and Tsyvinski 2007, Werning 2007 among others). Here, we focus on a Ramsey approach, limiting the number of instruments (see Farhi and Werning 2013, and Golosov,

[2]Some papers have introduced incomplete insurance markets in overlapping generation models to quantitatively investigate optimal fiscal policies (Imrohoroğlu 1998, and Conesa, Kitao, and Krueger 2009). There is a large literature on the effect of public debt, for instance explained by generational accounting: Auerbach, Gokhale, and Kotlikoff (1991, 1994) for seminal contributions and Bassetto and Kocherlakota (2004) for an extension to distortionary taxes. We do not explicitly consider intergenerational transfers in our analysis.

Tsyvinski, and Werquin 2016 for a discussion).

This paper is also, but more indirectly, related to the computational literature studying incomplete insurance markets with perturbation methods. Reiter (2009) uses perturbation methods to solve for aggregate dynamics, by discretizing the wealth distribution to obtain a finite-dimensional state space.[3] Instead, we construct economies which deliver a finite-dimensional state-space in the space of histories, as a theoretical outcome. This last property is key to be able to derive optimal policies with a number of instruments.

The rest of the paper is organized as follows. In Section 2, we present the environment. We describe the family head problem and derive the associated allocation in Section 3. We then show in Section 4 the decentralization mechanism. We solve the Ramsey program in Section 5 and discuss in Section 6 the Ramsey program fiscal policy. In Section 7, we provide a numerical application illustrating our findings. Finally, conclusions are given in Section 8.

2 The environment

Time is discrete, indexed by $t \geq 0$. The economy is populated by a continuum of agents of size 1, distributed on a segment J following a non-atomic measure ℓ: $\ell(J) = 1$.[4]

2.1 Risk

Aggregate risk. The aggregate risk is represented by a probability space $(\mathcal{S}^\infty, \mathcal{F}, \mathbb{P})$. In any period t, the aggregate state, denoted s_t, takes values in the state space $\mathcal{S} \subset \mathbb{R}^+$ and follows a first-order Markov process. The history of aggregate shocks up to time t is denoted $s^t = \{s_0, \ldots, s_t\} \in \mathcal{S}^{t+1}$. Finally, the period-0 probability density function of any history s^t is denoted $m^t(s^t)$.

[3] Other numerical methods using perturbation methods are developed in Mertens and Judd (2012), Preston and Roca (2007), Kim, Kollmann, and Kim (2010) or Winberry (2016) who approximates the wealth distribution by a finite number of parameters.

[4] We assume that the law of large numbers holds. See Green (1994) for a proper construction of J and ℓ. See also Feldman and Gilles (1985), Judd (1985b), and Uhlig (1996) for other solutions.

Idiosyncratic risk. At the beginning of each period, agents face an uninsurable idiosyncratic labor productivity shock e_t that can take $E+1$ values in the set $\mathcal{E} = \{0, \ldots, E\} \in \mathbb{R}_+^{E+1}$. Agents in state $e \in \mathcal{E}$, $e \neq 0$, have a labor productivity $\theta_e > 0$, which is assumed to be increasing in e, without loss of generality. Agents in state $e = 0$ have a zero market productivity but devote a fixed amount of $\delta > 0$ labor units to earn a home production of δ units of final goods. The former agents can be considered as employed workers with various productivities, while the latter can be considered as unemployed workers. This modeling choice enables us to cover the various cases that can be found in the literature.

The productivity shock e_t follows a discrete first-order Markov process with transition matrix $M(s_t) \in [0,1]^{(E+1)\times(E+1)}$. The probability $M_{e,e'}(s_t)$ is the probability for an agent to switch from state e at date t to state e' at date $t+1$, when the aggregate state is s_t in period t. The history of idiosyncratic shocks up to date t is denoted $e^t = \{e_0, \ldots, e_t\} \in \mathcal{E}^{t+1}$.

Remark 1 (Notations) *For the sake of clarity, for any random variable $X_t : \mathcal{S}^t \to \mathbb{R}$, we will denote X_t, instead of $X_t(s^t)$, its realization in state s^t, and for any random variable $Y_t : \mathcal{S}^t \times \mathcal{E}^t \to \mathbb{R}$, we will denote Y_{t,e^t} its realization in state (s^t, e^t).*

2.2 Preferences

In each period, the economy has two goods: a consumption-capital good and labor. Agents rank consumption c and labor l according to a smooth period utility function $U(c,l)$, satisfying standard regularity properties. As is standard in this class of models, we consider a Greenwood-Hercowitz-Huffman (GHH) utility function, exhibiting no wealth effect for the labor supply:[5]

$$U(c,l) = u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right), \tag{1}$$

[5] All our results can be derived with a general utility function $U(c,l)$. A GHH utility function slightly simplifies the algebra, especially when deriving the Ramsey program in Section 5. Admittedly, and as shown by Marcet, Obiols-Homs, and Weil (2007), considering an alternative utility function would affect the optimal tax schedule, as aggregate labor supply would depend on the wealth distribution.

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ scales labor disutility, and $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously derivable, increasing, and concave, with $u'(0) = \infty$. Each agent ranks consumption and labor streams, denoted respectively as $(c_t)_{t\geq 0}$ and $(l_t)_{t\geq 0}$, according to the intertemporal criterion $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$, where $\beta \in (0, 1)$ is the discount factor.

2.3 Production and assets

In any period t, a production technology with constant returns to scale (CRS) transforms capital K_{t-1} and labor L_t into $F(K_{t-1}, L_t, s_t)$ units of output. The production function is smooth in K and L and satisfies the standard Inada conditions. Capital must be installed one period before production, and the state of the world possibly affects productivity through a technology shock. This formulation allows for capital depreciation, which is subsumed by the production function F, as in Farhi (2010) for instance. Labor L_t is measured in efficient units, and is equal to the sum of the individual labor efforts expressed in efficient units: $L_t = \int_{i\in J} \theta_{e_t^i} l_{i,t} \ell(di)$. The good is produced by a unique profit-maximizing representative firm. We denote as $\tilde{w}_t$ the real before-tax wage rate in period t and as $\tilde{r}_t$ the real before-tax rental rate of capital in period t. Profit maximization yields in each period $t \geq 1$:

$$\tilde{r}_t = F_K(K_{t-1}, L_t, s_t) \quad \text{and} \quad \tilde{w}_t = F_L(K_{t-1}, L_t, s_t). \tag{2}$$

Finally, agents save using two assets, which are claims on the capital stock and public debt. In addition, agents cannot borrow more than an exogenous borrowing limit $-\bar{a} \leq 0$.

2.4 Government, fiscal tools, and resource constraints

In each period t, the government has to finance an exogenous public good expenditure G and it can choose a positive lump-sum transfer $T_t \geq 0$ paid to all agents. The government can levy distorting taxes on capital income τ_t^K or on labor income τ_t^L or issue an amount B_t of a riskless one-period public bond.[6] As in Heathcote (2005), we assume that the

[6]The question of the optimal mix of these financing tools will be the focus of the second part of the paper and in particular of the Ramsey program studied in Section 5.

public debt pays the economy-wide interest rate $\tilde{r}_t$ for any aggregate history $s^t \in \mathcal{S}^t$. The same tax rate τ_t^K applies to public bonds and capital shares. In consequence, both assets are perfect substitutes for agents. Positive lump-sum transfers $T_t > 0$ are allowed because Heathcote, Storesletten, and Violante (2016) show that they are needed to properly approximate the current US fiscal system. Following the tradition of Lucas and Stokey (1983), lump-sum taxes (or negative T_t) are not available.[7]

As is standard, we also assume that the date-0 capital tax rate, bearing on initial capital, is exogenously set. Indeed, taxing capital in the first period is non-distorting, and the government would heavily tax the initial capital stock (see Farhi 2010, or Sargent and Ljungqvist 2012, Section 16.7 for a discussion). The period-t budget constraint of the government is:

$$G + (1 + \tilde{r}_t)B_{t-1} + T_t \leq \tau_t^L \tilde{w}_t L_t + \tau_t^K \tilde{r}_t A_{t-1} + B_t. \tag{3}$$

We denote the after-tax real interest and wage rates respectively as:

$$r_t = (1 - \tau_t^K)\tilde{r}_t \quad \text{and} \quad w_t = (1 - \tau_t^L)\tilde{w}_t. \tag{4}$$

Using the CRS property of the production function, the budget constraint (3) becomes:

$$G + r_t K_{t-1} + w_t L_t + (1 + r_t)B_{t-1} + T_t \leq F(K_{t-1}, L_t, s_t) + B_t. \tag{5}$$

Finally, if C_t^{tot} denotes the total consumption in period t, the economy-wide resource constraint is $G + C_t^{tot} + K_t \leq F(K_{t-1}, L_t, s_t) + K_{t-1} + S_{t,0}(s^t)\delta$, where $S_{t,0}(s^t)$ denotes the size of the population in state $e = 0$ at date t and thus producing δ.

3 The island economy

In general, the previous economy features a growing heterogeneity in wealth levels over time, because agents with different idiosyncratic histories will choose different savings. This heterogeneity can be represented by a time-varying distribution of wealth levels with

[7] We assume the absence of non-distorting taxes ($T_t < 0$) to follow the literature (see also Aiyagari, Marcet, Sargent, and Seppälä 2002), but the case where these taxes are available can be easily studied, as shown below.

infinite support, which raises considerable theoretical and computational challenges. We now present an environment in which the savings of each agent depends on the realizations of the idiosyncratic risk for only a given number of consecutive past periods, and not on the whole history. As an endogenous outcome, the heterogeneity among the population is summarized by a finite (but possibly large) number of agent types.

To simplify the exposition, we present this economy using the family and island metaphor (see Lucas 1975 and 1990, or Heathcote, Storesletten, and Violante 2016 for a recent reference). The gain of this presentation strategy is that equilibrium existence can be proved using standard techniques. In Section 4 below, we show that the island allocation can be decentralized.

We denote by $N \geq 0$ the length of the truncation for idiosyncratic histories.

Island description. There are $(E+1)^N$ different islands, where we recall that the cardinal of the set $\mathcal{E}$ of idiosyncratic risk realizations is $E+1$. Agents with the same idiosyncratic history for the last N periods are located on the same island. Any island is represented by a vector $e^N = (e^N_{-N+1}, \ldots, e^N_0) \in \mathcal{E}^N$ summarizing the N-period idiosyncratic history of all island inhabitants. At the beginning of each period, agents face a new idiosyncratic shock. Agents with history $\hat{e}^N \in \mathcal{E}^N$ in the previous period are endowed in the current period with history e^N, and we denote $e^N \succeq \hat{e}^N$ when e^N is a possible continuation of $\hat{e}^N$. The specification $N = 0$ corresponds to the full insurance case (only one island), and thus to the standard representative-agent assumption. Symmetrically, the case $N = +\infty$ corresponds to a standard incomplete-market economy with aggregate shocks, à la Krusell and Smith (1998).

The family head. The family head maximizes the welfare of the whole family, attributing an identical weight to all agents and being price-taker.[8] The family head can freely transfer resources among agents within the same island, but cannot do so across islands. All agents belonging to the same island are treated identically and will therefore receive the same allocation, as is consistent with welfare maximization. For agents in any island

[8] As the family head does not internalize the effect of its choice on prices, the allocation is not constrained-efficient, and the distortions identified by Davila et al. (2012) are present in the equilibrium allocation. The planner will reduce them with its instruments, defined in Section 5.

$e^N \in \mathcal{E}^N$, the family head will choose the per-capita consumption level c_{t,e^N}, the labor supply l_{t,e^N}, and the end-of-period savings a_{t,e^N} (remember that capital and public debt are substitutes).

Agents face borrowing constraints, and their asset holdings must be higher than $-\bar{a}$.[9] Some proofs below require that agents cannot save more than a^{max}. This maximal amount can be chosen to be arbitrarily large, in particular such that it is never a binding constraint.[10] Finally, we assume that all agents enter the economy with an initial wealth $(a_{-1,e^N})_{e^N \in \mathcal{E}^N}$.

Island sizes. The probability $\Pi_{t,\hat{e}^N,e^N}$ that an agent with history $\hat{e}^N = (\hat{e}^N_{-N+1}, \ldots, \hat{e}^N_0)$ in period t experiences history $e^N = (e^N_{-N+1}, \ldots, e^N_0)$ in period $t+1$ is the probability to switch from state $\hat{e}^N_0$ at t to state e^N_0 at $t+1$, provided that histories $\hat{e}^N$ and e^N are compatible. Formally, we have $\Pi_{t,\hat{e}^N,e^N} = 1_{e^N \succeq \hat{e}^N} M_{\hat{e}^N_0, e^N_0}(s_t)$, where $1_{e^N \succeq \hat{e}^N} = 1$ if e^N is a possible continuation of history $\hat{e}^N$ and 0 otherwise. We can thus deduce the law of motion of island sizes $(S_{t,e^N})_{t \geq 0, e^N \in \mathcal{E}^N}$:

$$S_{t+1,e^N} = \sum_{\hat{e}^N \in \mathcal{E}^N} S_{t,\hat{e}^N} \Pi_{t,\hat{e}^N,e^N}, \tag{6}$$

where the initial size of each island $(S_{-1,e^N})_{e^N \in \mathcal{E}^N}$, with $\sum_{e^N \in \mathcal{E}^N} S_{-1,e^N} = 1$, is given. The law of motion (6) is thus valid from period 0 onwards.

Timing. At the beginning of each period t, agents learn their current idiosyncratic shock and have to move from an island $\hat{e}^N$ to the relevant island e^N. The family head cannot change the allocation in the period, before agents leave the island. As a consequence, agents move by taking with them their wealth, equal to the per-capita saving $a_{t-1,\hat{e}^N}$. On island e^N, the wealth of all agents coming from island $\hat{e}^N$ (equal to $S_{t-1,\hat{e}^N} \Pi_{t-1,\hat{e}^N,e^N} a_{t-1,\hat{e}^N}$) – and for all islands $\hat{e}^N$– are pooled together and then equally divided among the S_{t,e^N}

[9] See Aiyagari (1994) for a discussion of the relevant values for of $\bar{a}$, called the natural borrowing limit in an economy without aggregate shocks. See Shin (2006) for a discussion with aggregate shocks. A standard value in the literature is $\bar{a} = 0$, which ensures that consumption is positive in all states of the world.

[10] As for instance in Szeidl (2013), the assumption on the maximal bound a^{max} enables us to consider a general utility function. An alternative option is to assume a bounded periodic utility function u, as in Miao (2006).

agents of island e^N. Therefore, each of these agents holds, in the beginning of period t, the wealth $\tilde{a}_{t,e^N}$ equal to:

$$\tilde{a}_{t,e^N} = \sum_{\hat{e}^N \in \mathcal{E}^N} \frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}} \Pi_{t-1,\hat{e}^N,e^N} a_{t-1,\hat{e}^N}. \tag{7}$$

The program of the family head can now be expressed as follows:[11]

$$\max_{\left(a_{t,e^N}, c_{t,e^N}, l_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U\left(c_{t,e^N}, l_{t,e^N}\right) \right], \tag{8}$$

$$a_{t,e^N} + c_{t,e^N} = w_t \theta_{e_0^N} l_{t,e^N} 1_{e_0^N \neq 0} + \delta 1_{e_0^N = 0} + (1 + r_t)\tilde{a}_{t,e^N} + T_t, \text{ for all } e^N \in \mathcal{E}^N, \tag{9}$$

$$c_{t,e^N},\ l_{t,e^N} \geq 0, a_{t,e^N} \geq -\bar{a}, \text{ for all } e^N \in \mathcal{E}^N, \tag{10}$$

$$(S_{-1,e^N})_{e^N \in \mathcal{E}^N} \text{ and } (a_{-1,e^N})_{e^N \in \mathcal{E}^N} \text{ are given,} \tag{11}$$

and subject to $l_{t,e^N} = \delta$ if $e_0^N = 0$, the law of motion (6) for $(S_{t,e^N})_{t\geq 0}^{e^N \in \mathcal{E}^N}$, and to the definition (7) of $(\tilde{a}_{t,e^N})_{t\geq 0}^{e^N \in \mathcal{E}^N}$.[12]

The family head maximizes the aggregate welfare (8) subject to the budget constraints (9) on all islands, to positivity and borrowing constraints (10), and to initial conditions (11). As the objective function is concave, constraints are linear (i.e., the admissible set is convex), and allocations are bounded (a^{max} guarantees a compact admissible set), the existence of the equilibrium can be proved using standard techniques – see Stokey and Lucas (1989), and we omit it to save some space.[13] If $\beta^t \nu_{t,e^N} m(s^t)$ denotes the Lagrange multiplier of the credit constraint of island e^N, first-order conditions are:

$$U_c(c_{t,e^N}, l_{t,e^N}) = \beta \mathbb{E}_t \left[\sum_{\tilde{e}^N \succeq e^N} \Pi_{t,e^N,\tilde{e}^N} U_c(c_{t+1,\tilde{e}^N}, l_{t+1,\tilde{e}^N})(1 + r_{t+1}) \right] + \nu_{t,e^N}, \tag{12}$$

$$l_{t,e^N} = \left(w_t \theta_{e_0^N}\right)^{\varphi} 1_{e_0^N > 0} + \delta 1_{e_0^N = 0}, \tag{13}$$

$$\nu_{t,e^N}(a_{t,e^N} + \bar{a}) = 0 \text{ and } \nu_{t,e^N} \geq 0. \tag{14}$$

To anticipate Section 4 below, these first-order conditions (12)–(14) have the same form

[11] We denote e_0^N the current idiosyncratic state in island e^N, and $1_{e_0^N=0}$ equals 1 if $e_0^N = 0$ and 0 otherwise.

[12] Note that $\mathbb{E}_t[\cdot]$ in (8) is the expectation operator at date $t \geq 0$ over all future aggregate histories.

[13] Due to the finite heterogeneity representation, we could also prove the existence of a recursive equilibrium. To save some space, we do not present this recursive formulation, as it is not necessary to derive first-order conditions.

as the ones derived in standard incomplete insurance-market models. Indeed, although the family head cares about agents moving across islands, the result is similar to the one of individuals self-insuring against income risk, due to the law of large numbers.

Labor market. On any island e^N, the market labor supply in efficient units at date t amounts to $\theta_{e_0^N} S_{t,e^N} l_{t,e^N}$ (recall that $\theta_0 = 0$). Summing across all islands yields the total labor supply:

$$L_t = \sum_{e^N \in \mathcal{E}^N} \theta_{e_0^N} S_{t,e^N} l_{t,e^N}. \tag{15}$$

Financial market. The total end-of-period savings of all agents, denoted A_t at date t is:

$$A_t = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} a_{t,e^N} = \sum_{e^N \in \mathcal{E}^N} S_{t+1,e^N} \tilde{a}_{t+1,e^N}, \tag{16}$$

where the last equality stems from the pooling equation (7). The clearing of the financial market at date t implies that at any date t, the following equality holds:

$$A_t = B_t + K_t. \tag{17}$$

We can state our sequential equilibrium definition, similarly to Aiyagari, Marcet, Sargent, and Seppälä (2002) and Farhi (2010).

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual allocations* $\left(c_{t,e^N}, l_{t,e^N}, \tilde{a}_{t,e^N}, a_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$*, of island population sizes* $\left(S_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$*, of aggregate quantities* $(L_t, A_t, B_t, K_t)_{t\geq 0}$*, of price processes* $(w_t, r_t, \tilde{r}_t, \tilde{w}_t)_{t\geq 0}$ *and of a fiscal policy* $(T_t, \tau_{t+1}^K, \tau_t^L, B_t)_{t\geq 0}$*, such that, for an initial distribution of island population and wealth* $\left(S_{-1,e^N}, a_{-1,e^N}\right)_{e^N \in \mathcal{E}^N}$*, and for initial values of the capital stock* $K_{-1} = \sum_{e^N \in \mathcal{E}^N} S_{-1,e^N} a_{-1,e^N}$*, of the public debt* B_{-1}*, of the capital tax* τ_0*, and of the initial aggregate shock* s_{-1}*, we have:*

1. *given prices, individual strategies* $\left(a_{t,e^N}, c_{t,e^N}, l_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$ *solve the agents' optimization program in equations (8)–(11);*

2. *island sizes and beginning of period individual wealth* $\left(S_{t,e^N}, \tilde{a}_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$ *are consistent with law of motions (6) and (7);*

3. *labor and financial markets clear at all dates: for any $t \geq 0$, equations (15)–(17) hold;*

4. *the government budget constraint (5) holds at any date;*

5. *factor prices $(w_t, r_t, \tilde{r}_t, \tilde{w}_t)_{t\geq 0}$ are consistent with (2) and (4).*

The equilibrium has a simple structure defined at each date by $6(E+1)^N + 8$ variables and $6(E+1)^N + 8$ equations for a given fiscal policy $(T_t, \tau^K_{t+1}, \tau^L_t, B_t)_{t\geq 0}$, which is endogenized below.

4 Decentralization and convergence properties

We now show that the previous program can be decentralized, and we prove that the policy rules converge, for large N, toward the ones of a Bewley economy, under general conditions. We start with given factor prices and without aggregate shocks. Two main reasons motivate these restrictions. First, dropping aggregate shocks implies that we have existence proof of a recursive representation in this case (see Huggett 1993). Second, fixing factor prices avoids issues related to equilibrium multiplicity that may otherwise emerge, as shown in Açikgöz (2016) for instance.[14]

The economy is now similar to the one of Section 2, except for the following differences. First, we consider as given a constant after-tax interest rate r – with $\beta(1+r) < 1$ – an after-tax wage w, and a constant transfer T. We discuss below the case with aggregate shocks. Second, no family head imposes allocations, and agents are expected-utility maximizers taking fiscal policy as given. Finally, each agent receives at each date a lump-sum transfer $\Gamma_{N+1}(e^{N+1})$, which is contingent on her individual history e^{N+1} over the previous $N+1$ periods. This is the actual difference with a standard incomplete-market framework.

[14]This section can be skipped if the reader is convinced by the relevance of the island economy and wants to directly consider Ramsey policy in this environment.

Using standard techniques, the agents' program can be written recursively as:[15]

$$V_{N+1}(a, e^{N+1}) = \max_{a',c,l} U(c,l) + \beta \mathbb{E}\left[\sum_{(e^{N+1})' \succeq e^{N+1}} \Pi_{e^{N+1},(e^{N+1})'} V_{N+1}(a', \left(e^{N+1}\right)')\right], \quad (18)$$

$$a' + c = w\theta_{e_0^N} l 1_{e_0^N \neq 0} + \delta 1_{e_0^N = 0} + (1+r)a + T + \Gamma_{N+1}(e^{N+1}), \quad (19)$$

$$c,\, l \geq 0, a' \geq -\bar{a}, \quad (20)$$

with $l = \delta$ if $e_0^N = 0$, and where $V_{N+1} : [-\overline{a}, a^{max}] \times \mathcal{E}^{N+1} \to \mathbb{R}$ is the value function, and e_0^N the current idiosyncratic shock realization. Compared to the economies studied by Huggett (1993) and Aiyagari (1994), the individual history e^{N+1} is a state variable, as it determines the transfer $\Gamma_{N+1}(e^{N+1})$. The Lagrange coefficient of the credit constraint $a' \geq -\bar{a}$ is denoted ν, and the solution of this program consists of the policy rules g_c^{N+1}, $g_{a'}^{N+1}$, g_l^{N+1} and g_ν^{N+1} – defined over $[-\overline{a}, a^{max}] \times \mathcal{E}^{N+1}$ determining respectively consumption, savings, labor supply, and the Lagrange multiplier of the individual budget constraint. We now state our characterization result.

Proposition 1 (Finite state space) *There exists a balanced transfer Γ^*_{N+1}, such that any optimal allocation of the family head program (8)–(11) is also a solution to the decentralized program (18)–(20).*

The previous proposition states that the family head program presented in Section 3 can be decentralized by the balanced lump-sum transfer Γ^*_{N+1}. This transfer consists in pooling the resources of all agents having the same idiosyncratic history for $N+1$ periods, and redistributes the same amount to agents having the same idiosyncratic history for N periods, such that there are only $(E+1)^N$ possible wealth levels. Thus, the transfer Γ^*_{N+1} mimics the wealth pooling of the island economy, formalized in equation (7), that occurs when agents transfer from one island to another one.

Note that the standard Bewley economy is simply defined as the previous program where we further impose $\Gamma_{N+1}(e^{N+1}) = 0$ for all periods. In this case, we need only the current idiosyncratic state as a state variable (instead of the whole history e^{N+1}). The value function is then denoted $V^{Bewley} : [-\overline{a}, a^{max}] \times \mathcal{E} \to \mathbb{R}$, while we denote V^*_{N+1} the

[15] Following the literature, we denote as a' the savings choice in the current period. The value a is thus the beginning-of-period wealth.

value function of the program (18)–(20) for the transfer Γ^*_{N+1}. We can now state our convergence result.

Proposition 2 (Convergence) *For given factor prices and for the transfer Γ^*_{N+1}, if there exists $\kappa \in (0,1)$ and $\overline{N} \geq 1$, such that for all $N \geq \overline{N}$, such that for all $(e_{\bar{N}-1}, \ldots, e_0) \in \mathcal{E}^N$, and $(f_N, \ldots, f_{\bar{N}}), (g_N, \ldots, g_{\bar{N}}) \in \mathcal{E}^{N-\bar{N}+1}$ and $a_1, a_2 \in [-\overline{a}, a^{max}]$:*

$$\left| g^{N+1}_{a'}(a_1, (f_N, \ldots, f_{\bar{N}}, e_{\bar{N}-1}, \ldots, e_0)) - g^{N+1}_{a'}(a_2, (g_N, \ldots, g_{\bar{N}}, e_{\bar{N}-1}, \ldots, e_0)) \right| < \kappa \left| a_1 - a_2 \right|, \tag{21}$$

then $\lim_{N\to\infty} |\Gamma^*_{N+1}| = 0$, and for all $a \in [-\overline{a}, a^{max}]$ and $(e^N, e) \in \mathcal{E}^{N+1}$:

$$\lim_{N\to\infty} V^*_{N+1}\left(a, (e^N, e)\right) = V^{Bewley}(a, e).$$

Though involved, condition (21) has a simple meaning. It states that the marginal propensity to save is always smaller than 1 for all agents, as soon as N is high enough. When this condition is fulfilled, the transfer tends toward 0 as the length of idiosyncratic history N increases. Indeed, if the saving propensity is strictly lower than one, initial differences in wealth vanish and agents experiencing the same history of idiosyncratic shocks end up having the same wealth as time goes by. As a consequence, the wealth pooling generated by the transfer Γ^*_{N+1} concerns wealth levels which tend to be closer to each other, and the transfer Γ^*_{N+1} tends toward 0. In this case, we can show that the value function of the truncated economy converges toward the value function of the Bewley economy, which depends only on the current idiosyncratic shock.

Although condition (21) involves the savings policy function rather than model exogenous parameters, it is useful to understand and to check the convergence properties of the truncated representation. By contradiction, it is easy to show that a necessary condition for inequality (21) to be fulfilled is that the propensity to save is always strictly smaller than one for all agents in the corresponding Bewley economy (i.e., with $\Gamma_{N+1}(e^{N+1}) = 0$). To our knowledge, all calibrated Bewley models in the literature share this property, such that one can be confident about the general relevance of this truncated representation of incomplete-market economies.[16]

[16] It is possible to implement numerical methods to fasten the convergence of the truncated economy to the Bewley allocation for small N. We don't discuss here these computational considerations and focus instead on the theoretical properties of optimal policies in this environment.

In the economy with aggregate shocks, the limit of the truncated economy can be proven to exist, following the same steps, if a condition similar to (21) is fulfilled for any realization of the aggregate shock. However, it is difficult to compare this limit with other models as, to our knowledge, there is no proof in the general case of the existence of a recursive representation for incomplete-market economies with aggregate shocks when the distribution of wealth is the only state variable (see Kubler and Schmedders 2002 and 2003, Miao 2006, Cao 2016, or Cheridito and Sagredo 2016b and 2016a, for example). The current construction of a truncated economy could be the foundation of such a proof, but we leave this possibility for future research.[17]

5 Optimal fiscal policy: The Ramsey problem

5.1 The Ramsey problem

We now solve the Ramsey program in our incomplete-market island economy with aggregate shocks. The Ramsey program consists for the government to choose a fiscal policy that maximizes the aggregate welfare. This aggregate welfare, computed using a utilitarian criterion, is simply the objective of the family head in equation (8).[18] The following definition formalizes this program, using the notations of Section 3.

Definition 2 (Ramsey program for a truncated economy) *Let $N > 0$. Given initial conditions about the wealth distribution $\left(S_{-1,e^N}, a_{-1,e^N}\right)_{e^N \in \mathcal{E}^N}$, the initial public debt B_{-1}, the initial capital tax τ_0^K, and the initial aggregate state s_{-1}, the Ramsey program consists in choosing, at date 0, a fiscal policy made of lump-sum, capital, and labor tax paths $(T_t, \tau_{t+1}^K, \tau_t^L)_{t\geq 0}$, and of public debt paths $(B_t)_{t\geq 0}$, that maximizes the aggregate welfare defined in (8) among the set of competitive equilibria characterized in Definition 1.*

[17] In this section, we achieved decentralization through a fiscal transfer Γ_{N+1}^*, but this is not the only option. Indeed, following the constructions of Alvarez et al. (2009) and Khan and Thomas (2015), it is possible to provide a sequential decentralization of the island economy. Indeed, islands are devices to pool income at each date t (and as such to provide insurance) for idiosyncratic risks occurring before date $t-(N+1)$. As a consequence, if all agents are ex-ante identical, it is possible to achieve decentralization using insurance contracts, which hedge at any date t the risks occurring before date $t-(N+1)$ among agents with the same N-period history.

[18] Alternative social welfare functions can be used, but we focus on the most standard case.

Since the period-0 capital tax rate is given, the capital tax path starts at date 1. Equation (4) implies that the government can equivalently decide the post-tax interest rate $(r_t)_{t\geq 1}$ and the post-tax wage rate $(w_t)_{t\geq 0}$ instead of the distorting taxes $(\tau_t^K)_{t\geq 1}$ and $(\tau_t^L)_{t\geq 0}$, as in Chamley (1986). As a consequence, we can formalize the Ramsey program as follows:

$$\max_{\left(T_t, r_{t+1}, w_t, B_t, (a_{t,e^N}, c_{t,e^N}, l_{t,e^N})_{e^N\in\mathcal{E}^N}\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \sum_{e^N\in\mathcal{E}^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N})\right], \tag{22}$$

$$B_t + F(K_{t-1}, L_t, s_t) \geq G + (1+r_t)B_{t-1} + r_t K_{t-1} + w_t L_t + T_t, \tag{23}$$

for all $e^N \in \mathcal{E}^N$:

$$a_{t,e^N} + c_{t,e^N} = w_t\theta_{e_t^N} l_{t,e^N} 1_{e_0^N\neq 0} + \delta 1_{e_0^N=0} + (1+r_t)\tilde{a}_{t,e^N} + T_t, \tag{24}$$

$$U_c\left(c_{t,e^N}, l_{t,e^N}\right) - \nu_{t,e^N} = \beta\mathbb{E}_t\left[\sum_{\tilde{e}^N\in\mathcal{E}^N}\Pi_{t+1,e^N,\tilde{e}^N} U_c\left(c_{t+1,\tilde{e}^N}, l_{t+1,\tilde{e}^N}\right)(1+r_{t+1})\right], \tag{25}$$

$$l_{t,e^N} = \left(w_t\theta_{e_t^N}\right)^{\varphi} 1_{e_0^N>0} + \delta 1_{e_0^N=0}, \tag{26}$$

$$\nu_{t,e^N}(a_{t,e^N} + \overline{a}) = 0, \tag{27}$$

$$A_t = \sum_{e^N\in\mathcal{E}^N} S_{t,e^N} a_{t,e^N},\ L_t = \sum_{e^N\in\mathcal{E}^N} S_{t,e^N}\theta_{e_t^N} l_{t,e^N},\ K_t = A_t - B_t, \tag{28}$$

$$c_{t,e^N}, l_{t,e^N}, (a_{t,e^N} + \overline{a}) \geq 0, \tag{29}$$

with the law of motion (6) of $(S_{t,e^N})_{t\geq 0, e^N\in\mathcal{E}^N}$, and the definition (7) of $(\tilde{a}_{t,e^N})_{t\geq 0, e^N\in\mathcal{E}^N}$. All constraints (23)–(29) should be understood, unless specified, for all $s^t \in \mathcal{S}^t$ and all $e^N \in \mathcal{E}^N$.[19]

Maximization devices in the Ramsey program are on the one hand individual quantities – consumption level, labor effort, and asset holdings – and on the other hand fiscal instruments: public debt, lump-sum taxes, and post-tax interest and wage rates. Equation (23) is the government budget constraint, while the individual budget constraint is given in equation (24). The individual Euler equations for consumption and labor are provided in equations (25) and (26), respectively. The complementary slackness condition is stated in equation (27). Equation (28) gathers the aggregation for individual wealth and the labor supply, as well as the financial market clearing. Finally, positivity and borrowing

[19] Again, $\mathbb{E}_t[\cdot]$ is the conditional expectation at date t with respect to aggregate shocks.

constraints appear in equation (29).

5.2 Simplification of the Ramsey program

We simplify the formulation of the Ramsey program exposed in equations (22)–(29), following Marcet and Marimon (2011). We first denote $\beta^t m^t(s^t) S_{t,e^N} \lambda_{t,e^N}$ the (discounted) Lagrange multiplier of the Euler equation of agent e^N in state s^t. We also define for all $e^N \in \mathcal{E}^N$:

$$\Lambda_{t,e^N} = \frac{\sum_{\hat{e}^N \in \mathcal{E}^N} S_{t-1,\hat{e}^N} \lambda_{t-1,\hat{e}^N} \Pi_{t,\hat{e}^N,e^N}}{S_{t,e^N}}, \tag{30}$$

which can be interpreted as the average for agents of island e^N of their previous period Lagrange multipliers for the Euler equation. Finally, we can notice that $\lambda_{t,e^N} = 0$ if $a_{t,e^N} = -\overline{a}$: λ_{t,e^N} is zero when the credit constraint is binding. The product $\lambda_{t,e^N} \nu_{t,e^N}$ (for any t and any e^N) is thus always equal to 0. The following lemma summarizes our simplification of the Ramsey program.

Lemma 1 (Simplified Ramsey program) *The Ramsey program in equations (22)–(29) can be simplified into:*

$$\max_{\left(r_{t+1}, w_t, B_t, T_t, (a_{t,e^N}, c_{t,e^N}, l_{t,e^N})_{e^N \in \mathcal{E}^N}\right)_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) \tag{31}$$

$$+ U_c(c_{t,e^N}, l_{t,e^N}) \left(\Lambda_{t,e^N}(1 + r_t) - \lambda_{t,e^N} \right) \Big),$$

$$s.t.\ \lambda_{t,e^N} = 0 \ \textit{if}\ a_{t,e^N} = -\overline{a}, \tag{32}$$

and subject to equations (6), (7), (23)–(26), (28)–(29), and (30).

The proof is relegated to Appendix C. The simplification of the Ramsey program, which eases the computation of the maximization problem, is based on a re-writing of the Lagrangian to introduce Lagrange coefficients into the objective, as done by Marcet and Marimon (2011). It could also provide a recursive formulation of the Ramsey program that we do not need, as the sequential representation allows us to derive first-order conditions, expressed in a way which eases the interpretation.

6 Understanding fiscal policy when markets are incomplete

6.1 First-order conditions

An understanding of optimal fiscal policy can be obtained from the first-order conditions of the program (31), which are necessary conditions. A central concept for interpreting all these conditions is a new valuation of liquidity, which we call the *social valuation of liquidity for agents* e^N and denote ψ_{t,e^N}. It is formally defined as:

$$\psi_{t,e^N} \equiv U_c(c_{t,e^N}, l_{t,e^N}) - U_{cc}(c_{t,e^N}, l_{t,e^N}) \left(\lambda_{t,e^N} - \Lambda_{t,e^N}(1 + r_t)\right). \tag{33}$$

The valuation ψ_{t,e^N} differs from the marginal utility of consumption $U_c(c_{t,e^N}, l_{t,e^N})$ – which can be seen as the private valuation of liquidity for agents e^N– since ψ_{t,e^N} takes into consideration the Euler equations from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t to $t+1$ and thus makes her Euler equation more "binding". This more "binding" constraint decreases the utility by the algebraic quantity $U_{cc}(c_{t,e^N}, l_{t,e^N})\lambda_{t,e^N}$, where λ_{t,e^N} is the Lagrange multiplier of the agent's Euler equation at date t. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ to t and therefore "relaxes" the constraint of date $t-1$. This is reflected in Λ_{t,e^N}.

It is easy to show that if the government could implement island-specific lump-sum transfers (such as unconstrained T_{t,e^N}), it would implement $\mu_t = \psi_{t,e^N}$ for all $e^N \in \mathcal{E}^N$. The difference $\mu_t - \psi_{t,e^N}$ is thus a measure of the cost for island e^N of imperfect and distorting policy tools. Furthermore, since μ_t *is* the (normalized) Lagrange multiplier of the government budget constraint *(23)*, it measures the social value of liquidity for the government. We will therefore call the difference $\mu_t - \psi_{t,e^N}$ the *liquidity valuation gap for agents* e^N, as it is equal to the marginal gain of transferring resources from the budget of island e^N to the budget of the government. The liquidity valuation gap $\mu_t - \psi_{t,e^N}$ can be either positive or negative depending on the island, but, as shown below, the sum of social values over all islands is non-negative.

We now present and discuss the first-order conditions of the planner using these concepts. We derive them formally in Appendix D.

Social valuation of government liquidity, μ_t. The dynamics of μ_t is

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1}\right) \right]. \tag{34}$$

Equation (34) sets equal the marginal benefit of one additional unit of debt at date t to the marginal extra cost at date $t+1$, using the before-tax return $\tilde{r}_t$ to value the next period. On the one hand, the extra debt unit relaxes the government budget constraint at date t by one unit and thus implies a benefit that amounts to the Lagrange multiplier of the government budget constraint, μ_t. On the other hand, the extra debt unit implies debt reimbursement and interest payment in the next payment, i.e., a total payment of $1+\tilde{r}_{t+1} = 1+F_K(A_t - B_t, L_{t+1})$ that makes the next-period government budget constraint stricter.

Liquidity valuation gaps, $\mu_t - \psi_{t,e^N}$. We begin with defining $\mathcal{C}_t$ as the set of islands on which agents are credit-constrained at date t. Formally:

$$\mathcal{C}_t = \{e^N \in \mathcal{E}^N,\ \nu_{t,e^N} > 0\}. \tag{35}$$

Then, for non credit-constrained islands, the dynamics of the liquidity valuation gap is:

$$\forall e^N \in \mathcal{E}^N \setminus \mathcal{C}_t,\ \mu_t - \psi_{t,e^N} = \beta \mathbb{E}_t \left[\sum_{\tilde{e}^N \in \mathcal{E}^N} (1 + r_{t+1}) \Pi_{t+1,e^N,\tilde{e}^N} \left(\mu_{t+1} - \psi_{t+1,\tilde{e}^N} \right) \right]. \tag{36}$$

Equation (34) can be interpreted as a *modified* Euler equation for non credit-constrained agents. It equalizes the current *liquidity valuation gap* $\mu_t - \psi_{t,e^N}$ to its discounted value tomorrow. The Euler equation for the *liquidity valuation gap* is similar to the Euler equation for the *private valuation of liquidity* for the same agents (equation 25). Both the agents and the planner perceive that the marginal gain to transfer resources to the next period is r_t.

Labor taxes. The first-order condition for the post-tax real wage w_t is:

$$\sum_{e^N \in \mathcal{E}^N} \frac{S_{t,e^N} l_{t,e^N} \theta_{e_t^N}}{L_t} \left(\mu_t - \psi_{t,e^N}\right) = \mu_t \varphi \frac{\tau_t^L}{1-\tau_t^L}. \tag{37}$$

Equation (37) sets equal the social gain of financing the government budget using labor tax τ_t^L (the left-hand side) to its cost (the right-hand side). More precisely, the left-hand side is the marginal gain of transferring resources for all islands $e^N \in \mathcal{E}^N$ to the budget of the government using an increase in labor tax τ_t^L. This implies a liquidity valuation gap $\mu_t - \psi_{t,e^N}$, for every island e^N, which is weighted by its share in the total labor effort $\frac{S_{t,e^N} l_{t,e^N} \theta_{e_t^N}}{L_t}$, expressed in efficient units. This weight is thus proportional to the labor-tax base. The right-hand side is the cost of labor tax distortion, which reflects the reduction in the base of the labor tax. The magnitude of the distortion depends positively on the fiscal wedge generated by labor tax τ_t^L, the Frisch elasticity of labor supply φ, which determines how agents adapt their labor effort to the tax distortion, and the government liquidity valuation μ_t.

Capital taxes. The first-order condition for the post-tax interest rate r_t can be written as:

$$\sum_{e^N \in \mathcal{E}^N} \left[S_{t,e^N} \tilde{a}_{t,e^N} \left(\mu_t - \psi_{t,e^N}\right)\right] = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N}, \tag{38}$$

where $\tilde{a}_{t,e^N}$ is given by (7). Equation (38) sets equal the social gain of financing the government budget using the distorting capital tax τ_t^K (the left-hand side) to its cost (the right-hand side). More precisely, the left-hand side is the average liquidity valuation gap weighted by the before-tax wealth on each island, which is the tax base. The right-hand side is the sum of individual distortions of a higher capital tax that affects individual Euler equations, and more precisely, consumption smoothing between the previous and the current periods. Therefore, individual distortions are measured by Λ_{t,e^N}, which assesses the tightness of Euler equations between $t-1$ and t, and thus the willingness to smooth out consumption between both periods.

Lump-sum transfer T_t. The first-order condition for the transfer T_t is:

$$\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \left(\mu_t - \psi_{t,e^N}\right) \geq 0, \text{ with equality when } T_t > 0. \tag{39}$$

This equation states that, when the positivity constraint on the lump-sum transfer is not binding, the government sets the population-weighted sum of liquidity valuation gaps to 0. In particular, this would also be the case in the absence of any positivity constraint. However, when the constraint $T_t \geq 0$ is binding and when the government would actually like to tax some island using the lump-sum instrument, the constraint $T_t \geq 0$ binds, and the social benefit of liquidity for the government is higher than its average cost over all islands: $\mu_t > \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \psi_{t,e^N}$.[20]

6.2 Steady-state fiscal policy

Using first-order conditions, we derive theoretical implications about the steady-state optimal fiscal policy. We assume here that the steady-state solution is interior – and we will numerically check that this is the case in our quantitative exercise.[21] Indeed, Straub and Werning (2014) have provided examples of economies where this is not the case. This avoids discussing economies with zero long-run wealth. The main results, which are independent of N, are summarized in the next proposition. To denote steady-state variables, we simply drop the subscript t.

Proposition 3 (Steady-state) *In the interior steady-state of the Ramsey equilibrium:*

1. *the marginal productivity of capital is pinned down by the discount factor β:*

$$1 + F_K(K, L) = \frac{1}{\beta}, \tag{40}$$

2. *the capital tax is non-negative; it is positive if and only if credit-constraints bind for*

[20] Allowing for negative transfers does not imply that (39) holds with equality. Indeed, the ability of the lowest income agents to pay lump-sum taxes may provide a binding bound on negative taxes.

[21] More precisely, we show that such a steady state exists, and we use perturbation methods (for the aggregate shock) to show that it is locally stable considering both the agents and the government optimal policy. See Chen, Chien, and Yang (2017) for a discussion and a formal proof of the existence of well-defined steady state in a related environment.

some agents, or more formally:

$$\tau^K = \frac{\sum_{e^N \in \mathcal{C}} S_{e^N} \nu_{e^N}}{(1-\beta) \sum_{e^N \in \mathcal{E}^N} S_{e^N} U_c \left(c_{e^N}, l_{e^N}\right)}, \tag{41}$$

where we recall that $\mathcal{C}$ is the set of islands where credit constraints bind at the steady-state, and ν_{e^N} is the Lagrange multiplier of credit constraint for island e^N;

3. *the labor tax is determined by the net average liquidity valuation gap:*

$$\mu\varphi \frac{\tau^L}{1-\tau^L} = \sum_{e^N \in \mathcal{E}^N} \frac{S_{e^N} l_{e^N} \theta_{e^N}}{L} \left(\mu - \psi_{e^N}\right). \tag{42}$$

The first item in equation (40) of Proposition 3 is a direct implication of the government Euler equation (34). As a consequence, the marginal productivity of capital is determined by the discount factor β only, as originally explained by Aiyagari (1995). This important restriction is the so-called "modified golden rule".

To the best of our knowledge, the second item in Proposition 3 is new in the literature. We prove that the capital tax is always non-negative and that its value is determined by the severity of credit constraints. If credit constraints do not bind for any agent – for instance, if they are chosen to be below the natural borrowing limit, as defined by Aiyagari (1994) – then the equilibrium capital tax will be 0.[22] Conversely, the steady-state capital tax will be positive if and only if some agents are credit-constrained. Indeed, when credit constraints are binding for some agents, credit-constrained agents cannot borrow as much as they would like to, while non-constrained agents save too much to self-insure. Both effects contribute to create an oversupply of liquidity. To correct this oversupply, the government raises the capital tax, which decreases the post-tax interest rate and thus the incentives to save.[23] Aiyagari (1995) proves that the capital tax is positive in a similar environment without aggregate shocks. Our contribution is to connect the capital tax rate to the severity of credit constraints. The steady-state level of public debt is determined as a residual of the government budget constraint, and it can be either positive or negative.

[22]The denominator in equation (42) is indeed bounded away from zero because the economy is finite, i.e., not all marginal utilities can be simultaneously zero.

[23]This logic is already present in Woodford (1990), and discussed in Davila et al. (2012). The last paper differs from ours, because the authors analyze the constrained optimality of the capital stock in a situation where the planner can change agents' saving decisions without distortion, while we focus on distorting fiscal instruments.

When credit constraints do not bind, we can check that the level of public debt is indeterminate, which relates our results to those recently obtained by Bhandari, Evans, Golosov, and Sargent (2016b). Indeed, as no agent is credit constrained, the capital tax is null, $\tau^K = 0$, and the post-tax gross interest rate amounts to $1/\beta$. When public spending is not too large, a continuum of values for the steady-state public debt and transfers both satisfy the first-order conditions of the government and the government budget constraint.[24]

7 Quantitative properties of the optimal tax system

We now investigate quantitatively the properties of our finite-state model. After describing the calibration – which is standard – and the steady-state, we carefully analyze the convergence of the economy as a function of N. We will explain that a small N is sufficient for capturing the steady-state of the limit economy. We finally explore the response of the limited-heterogeneity economy to a technology shock in terms of impulse-response functions and second-order moments.

7.1 Calibration.

The utility function is $u\left(c - \chi\frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) = \log\left(c - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)$, with a Frisch elasticity of labor supply set to $\varphi = 0.5$, as is consistent with empirical estimates (Chetty, Guren, Manoli, and Weber 2011).

The aggregate state is assumed to follow an AR(1) process: $s_t = \rho_s s_{t-1} + \varepsilon_t^s$, where $(\varepsilon_t^s)_{t\geq 0}$ is a white-noise process with a distribution $\mathcal{N}(0, \sigma_s^2)$. The production function is a Cobb-Douglas with a constant capital depreciation μ: $F(K, L, s) = \xi(s)K^\alpha L^{1-\alpha} - \mu K$, where $\xi(s) = \exp(s)$ is the technology level. The labor share is $\alpha = 0.33$.

We calibrate the idiosyncratic risk to the unemployment risk, as in Imrohoroğlu (1992), Krusell and Smith (1998) or Challe and Ragot (2016) among others. Households can be either employed, in which case they choose their labor supply, or unemployed, in which

[24] It is now well understood that the optimal public debt is well-defined only if there are some constraints on fiscal instruments. Otherwise, a form of Ricardian equivalence holds in incomplete-market economies. In our economy, the absence of lump-sum taxes is a sufficient condition.

case they obtain δ consumption units from home production. This corresponds to $E = 1$. The period is a quarter. For the employment risk, we use the calibration for the US of Challe and Ragot (2016), which is based on the strategy of Shimer (2003). The parameter δ is set such that home production amounts to 50% of market income. The quarterly job finding rate is 80% and the quarterly job separation rate is 5%. The transition matrix is thus $M = \begin{bmatrix} 0.2 & 0.8 \\ 0.05 & 0.95 \end{bmatrix}$. The quarterly depreciation rate equals $\mu = 2.5\%$. The persistence of the standard values technology shock is set to $\rho_s = 0.95$ and the standard deviation is $\sigma_s = 1\%$. Finally, public spending is set to $G = 0.71$, which implies a steady-state public spending-to-GDP ratio equal to 19%, the postwar average value in the US. Table 1 summarizes parameter values.

β	μ	$\delta/(wl)$	G	ρ_s	σ_s
0.99	2.5%	50%	0.71	0.95	1%

Table 1: Parameter values

The remaining parameter is N, which parametrizes the truncation of idiosyncratic histories. In the baseline model, we consider $N = 8$, meaning an economy populated by $2^8 = 256$ different households. Increasing N further does not significantly affect the convergence, as we discuss in Section 7.3.

7.2 Solution method and steady-state results

Our truncated equilibrium enables us to solve the model using a standard perturbation method. First, we determine the steady-state allocation without aggregate shocks, using an iteration over the post-tax interest rate, as described in Appendix F. Second, we linearize all equations around the steady-state (including first-order conditions of the Ramsey problem) to obtain the dynamics of the fiscal system and constrained-optimal allocations after a small technology shock. The technology shock is small enough such that, for all simulations, the set of credit-constrained islands remains unchanged and does not vary along the business cycle, which can be shown by simulating the model for a high number of periods.

For $N = 8$, the dynamic system is composed of a total of 1554 variables (including all Lagrange coefficients), which are simulated using Dynare. The gains of the perturbation method are twofold. First, we can choose a continuous state-space for the technology shock. Second, we can use the same mature methodology as the one used in the DSGE literature to compute IRFs and second-order moments. This means that we can rely on tools (e.g., Dynare) that have already been developed and make our results directly comparable with a large branch of the literature. The additional gain is that the model is solved in three seconds.[25]

Steady-state results. We simulate our incomplete-market economy with $N = 8$. Results are gathered in Table 2. First, the marginal productivity of capital is pinned down by the discount factor of households (see Proposition 3). In consequence, the annualized real interest rate is $\tilde{r} = 4.17\%$, the wage rate is $\tilde{w} = 1.0161$, and the aggregate capital stock is $K/Y = 2.35$.

The public debt amounts to -8.64 or -231% of GDP. The government doesn't hold all the capital stock and households hold a small amount of assets. A negative public debt allows the government to obtain resources in a non-distorting way. One can show that if markets for the idiosyncratic risk were complete (case $N = 0$ in our setup), the government would hold all the capital stock to finance its expenditures out of capital income. The small amount of assets held by households is thus held for self-insurance purpose. The labor tax is equal to 14%, the capital tax amounts to 27%, and 5% of the population is credit constrained. Note that the capital tax base is very small.

$\tilde{r}(\%)$	$\tilde{w}$	$\tau^K(\%)$	$\tau^L(\%)$	$B/Y(\%)$	$K/Y(\%)$	L	Y
4.17	1.0161	27	14	-231	2.35	1.24	3.74

Table 2: Steady-state outcome

How do our results compare with the literature studying the optimal steady-state debt level? Aiyagari and McGrattan (1998) find an optimal debt over GDP of 60%, maximizing steady-state welfare. Açikgöz (2015) shows that the results are quantitatively different

[25] Pröhl (2017) uses polynomial chaos expansions to discretize cross-sectional distributions, which offer global solutions and avoid using perturbation methods.

if one solves for the Ramsey program instead of maximizing steady-state welfare. He finds an optimal quantity of debt to be above 300% of GDP. Dyrda and Pedroni (2016) find a negative optimal debt-to-GDP ratio, below −300% of GDP. These substantial quantitative differences can for a large be explained by parameter choices, such as the elasticity of labor supply or the specification of the income process, for which there is no strong consensus. In what follows, we focus on the dynamics of the fiscal system, what is the contribution of our paper.

7.3 Convergence results

We now simulate the model for different values of N and justify our choice of $N = 8$

Amount of insurance. We simulate the economy for different values of N, and compute the amount of risk-sharing induced by the truncation of histories at the steady-state. The natural metric is the pooling transfer (expressed as a percentage of the household's labor income for normalization purpose) that households with the same N-period histories should receive for having the same wealth. Formally, with the notations of Section 4, this measure is $\Gamma^*_{N+1}(e^{N+1})/(w\theta_{e^N} l_{e^N})$ and the average value across histories of this transfer is 0 by construction. In Table 3, we report the average standard deviation across all histories e^{N+1} of the transfer Γ^*_{N+1}, normalized by the labor income, $sd_{e^{N+1}}(\Gamma^*_{N+1}(e^{N+1})/(w\theta_{e^N} l_{e^N}))$. This is our proxy for measuring the distance to a standard Bewley economy, for which this quantity should be zero.

N	1	2	3	4	5	6	7	8	9	10
$Sd(\%)$	18.54	6.94	5.99	4.98	4.05	3.20	2.60	2.10	1.70	1.30

Table 3: Transfer as a function of N

One can observe that the standard deviation of the transfer is decreasing as N increases and converges to 0. For $N = 8$, the standard deviation is only 2.1% of income. We checked that the policy rules (for each history) are very close to the one implied by the true Bewley model.

Tax system. As a second investigation of the convergence properties of the model, we determine the evolution of the optimal tax system as a function of N. Note that for all economies, the before-tax prices are $\tilde{r} = 4.17\%$, and $\tilde{w} = 1.0161$, while the aggregate capital stock is $K/Y = 2.35$. Table 4 presents, for different values of N, steady-state values of the equilibrium tax system (i.e., capital tax τ^K, labor tax τ^L, and debt-to GDP ratio B/Y).

N	1	2	3	4	5	6	7	8	9	10
τ^K	0.23	0.23	0.24	0.24	0.25	0.26	0.26	0.26	0.27	0.27
τ^L	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
B/Y	−2.29	−2.30	−2.31	−2.31	−2.31	−2.31	−2.31	−2.31	−2.31	−2.31

Table 4: Fiscal system as a function of N

We can observe that the tax system is converging rapidly to the steady-state tax system. Furthermore, the total labor supply hardly changes when we increase N, as it increases from $L = 1.2408$ for $N = 1$ to $L = 1.2414$ for $N = 10$. We have also checked that the dynamic properties presented below are unchanged when we increase N. As a consequence, we are confident that $N = 8$ is a good approximation of the limit economy.

7.4 Effects of a technology shock.

We now simulate the economy for 200 periods after a transitory technology shock of 1% on impact. Figure 1 plots the dynamics of 12 key variables. All variables are presented in percentage deviation from steady-state, except taxes (taul and tauk) and the real interest rate (r), which are in level deviation.

The first panel in Figure 1 plots the TFP shock. GDP increases by 1.2%. Aggregate consumption (Ctot) reaches its maximum after periods and then decreases smoothly. The capital stock (K) increases progressively, with a maximal increase of 0.8%, whereas the aggregate savings of private agents (A) increases rapidly, by 2% at the maximum. The public debt (B) decreases – in other words, public savings increase what contributes to an increase in the capital stock to benefit from the high TFP. Public debt slowly reverts to its steady-state level.

The third line plots the labor and capital tax rates. The labor tax (taul) increase a little bit after the technology shock, while the capital tax (tauk) increases a lot before converging after one period back to its steady-state value. As a consequence, one finds that the capital tax is very volatile and increases a lot after the technology shock to increase the resources of the government and to decrease public debt. These qualitative properties are also found with a representative agent (Farhi 2010). With incomplete markets, public debt is (slowly) mean-reverting toward its steady-state value. The Lagrange multiplier of the government budget (mu) decreases, because the budget constraint of the government is relaxed as the economy is wealthier. The last line plots after tax real wage (w), real interest rate (r) and total labor (L).

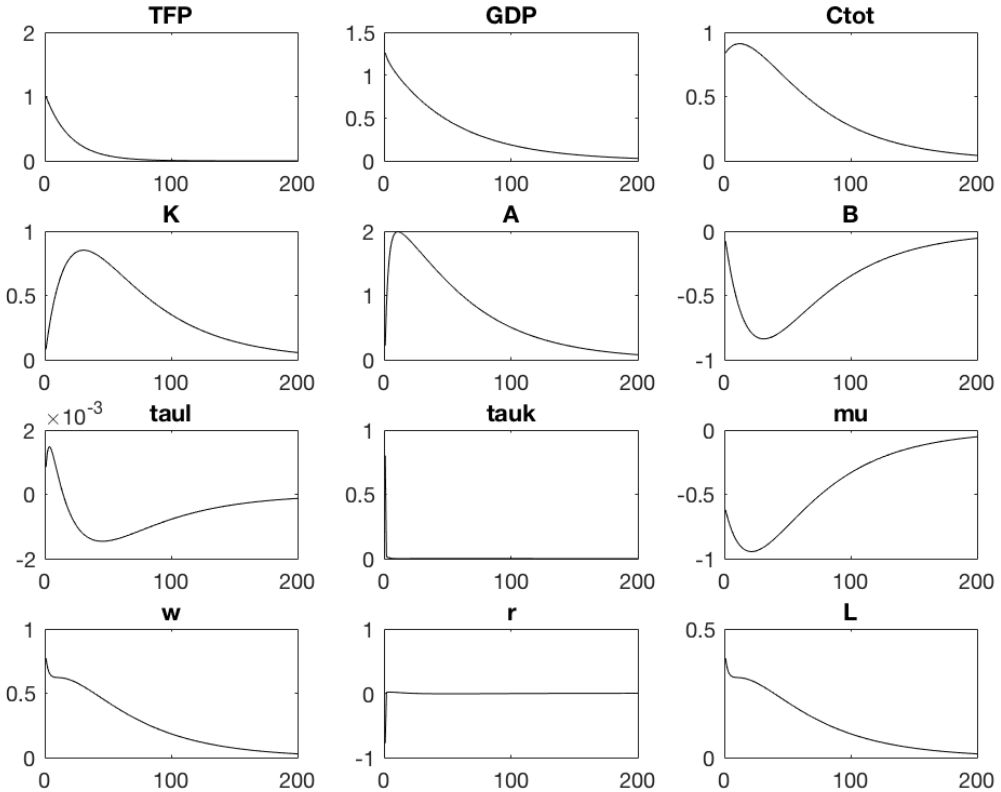

Figure 1: Aggregate IRFs after a 1% increase in TFP.

Finally, Table 5 provides unconditional second-order moments generated by the simu-

lated model. As consistent with our analysis of IRFs, the standard deviation of the labor tax is low (0.012) and the one of the capital tax (0.80) and of the public debt (2.28) are high compared to output (0.23). Public debt is volatile because it reverts slowly to its mean, with an autocorrelation of 0.99. The persistence of capital tax is very low (autocorrelation of 0.02) compared to labor taxes (0.99) and public debt (0.99), as observed discussing IRFs.

$sd(Y)$	$sd(\tau^K)$	$sd(\tau^L)$	$sd(B)$	$corr(Y, Y_{-1})$
0.23	0.80	0.012	2.28	0.97
$corr(B, Y)$	$corr(\tau^K, Y)$	$corr(\tau^L, Y)$	$corr(\tau^K, K)$	$corr(\tau^L, K)$
-0.857	-0.026	-0.435	0.027	-0.839

Table 5: Second-order moments

The previous results provide some insights for other approximation methods, which are used in similar setups with aggregate shocks. In particular, the aggregate law of motion of the capital stock used in Krusell and Smith (1998) is not a sufficient statistics for the evolution of aggregate variables. Indeed, the capital tax is much more volatile than the capital stock and both variables are almost uncorrelated (correlation below 3%). For this reason, it is not easy to use the procedure of Krusell and Smith (1998) to compare our results to the one implied by the application of an approximate aggregation procedure. It may be possible to introduce an additional aggregate law of motion for the capital tax for households to form correct expectations with simple rules. Such an investigation is obviously outside the scope of the current paper.[26]

8 Concluding remarks

The market equilibrium in an incomplete insurance-market economy with aggregate shocks can be represented as the allocation of a family-head program. This representation allows to generate a finite-dimensional state-space equilibrium, in which the Ramsey outcome

[26] More generally, we compare our truncated procedure with an approximate-aggregation procedure in a companion paper (in an economy without optimal fiscal policy), using additional computational tricks to improve the fit of the truncated economy.

can be studied with aggregate shocks and various fiscal tools. We apply this framework to study optimal fiscal policy, when positive transfers, distorting taxes on capital and labor, and public debt are available. A first interest of this framework is to allow for the analytical derivation of properties of the tax system, such as the steady-state level of capital and labor taxes. A second interest is the ability to simply simulate these economies. We apply this quantitative investigation to a standard economy, where the employment risk is uninsurable. We carefully investigate the convergence of this economy as a function of N and show that a reasonably small value of N is sufficient for capturing the main characteristics of the limit economy.

The methodology presented in this paper could be applied to different settings to understand distortions in more general incomplete insurance-market economies. First, we could increase the number of instruments available to the government, such as non-linear tax schedules. Second, additional heterogeneity, such as the structure of qualification, could be introduced to make more extensive use of empirical estimates of key parameters. Considering, health and family shocks and a labor market with different education groups is an obvious route to follow in future works. Third, and just as importantly, it is also possible to include other distortions, such as nominal frictions or frictional labor markets, to derive optimal policies in these richer environments.

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Appendix

A Proof of Proposition 1

Consider an agent endowed with the $N+1$-period history $e^{N+1} = (\hat{e}^N, e) \in \mathcal{E}^{N+1}$. The history e^{N+1} can also be written as $e^{N+1} = (e_N, e^N)$. In the former notation, e^{N+1} is seen

as the history $\tilde{e}^N \in \mathcal{E}^N$ with the successor state $e \in \mathcal{E}$, while in the latter notation, e^{N+1} is seen as the state $e_N \in \mathcal{E}$ followed by history $e^N \in \mathcal{E}^N$. The solutions to the maximization program (18)–(20) are the policy rules denoted $c = g_c^{N+1}(a, e^{N+1})$, $a' = g_{a'}^{N+1}(a, e^{N+1})$, $l = g_l^{N+1}(a, e^{N+1})$ and the multiplier $\nu = g_\nu^{N+1}(a, e^{N+1})$ satisfying the following first-order conditions:

$$U_c(c, l) = \beta \mathbb{E}\left[\sum_{e' \in \mathcal{E}} M_{e,e'} U_c(c', l')(1+r)\right] + \nu, \tag{43}$$

$$l = (\chi w \theta_e)^\varphi 1_{e>0} + \delta 1_{e=0}, \tag{44}$$

$$\nu(a' + \bar{a}) = 0 \text{ and } \nu \geq 0. \tag{45}$$

We use a guess-and-verify strategy. The transfer is constructed such that all agents with the same N-period history will have the same after-transfer wealth. The measure of agents with history e^N follows the same law of motion as (6) in the island economy and this measure is also equal to S_{e^N}. If all agents with the same history $(\hat{e}^N, e), e \in \mathcal{E}$ have the same beginning-of-period wealth $a_{\hat{e}^N}$, the after-transfer wealth, denoted $\hat{a}_{e^N}$, of agents with history $e^N \succeq \hat{e}^N$ will be:

$$\hat{a}'_{e^N} = \sum_{\tilde{e}^N \in \mathcal{E}^N} \frac{S_{\tilde{e}^N}}{S_{e^N}} \Pi_{\tilde{e}^N, e^N} a'_{\tilde{e}^N}, \tag{46}$$

for agents with the same history to hold the same wealth. By construction, $\hat{a}_{e^N}$ follows a dynamics similar to equation (7) of the "after-pooling" wealth $\tilde{a}_{t,e^N}$ in the island economy. The transfer denoted Γ^*_{N+1} that enables all agents with the same history to have the same wealth is:

$$\Gamma^*_{N+1}(e^{N+1}) = (1+r)(\hat{a}_{e^N} - a_{\hat{e}^N}), \tag{47}$$

where we use $e^{N+1} = (\hat{e}^N, e) = (e_N, e^N)$. The transfer Γ^*_{N+1} defined in (47) swaps the beginning-of-period wealth $(1+r)a_{\hat{e}^N}$ by the *average* wealth $(1+r)\hat{a}_{e^N}$. By construction, all agents with current history e^N will have an identical after-transfer wealth, independently of the previous-period history $\hat{e}^N$. Since there is a continuum with mass $S_{\tilde{e}^N}$ of agents with history $\tilde{e}^N$, in which each individual agent is atomistic, all agents take the transfer Γ^*_{N+1} as given.

Finally, it is easy to check that the transfer scheme is balanced in each period. Us-

ing the definition (46) of $\hat{a}_{e^N}$, we obtain for $e^N = (e^N_{N-1}, \ldots, e^N_1, e^N_0) \in \mathcal{E}^N$, $S_{e^N}\hat{a}_{e^N} = \sum_{\hat{e}^N \in \mathcal{E}^N} S_{\hat{e}^N}\Pi_{\hat{e}^N, e^N} a_{\hat{e}^N} = \sum_{\hat{e}\in\mathcal{E}} S_{(\hat{e}, e^N_{N-1}, \ldots, e^N_1)} M_{e^N_1, e^N_0} a_{(\hat{e}, e^N_{N-1}, \ldots, e^N_1)}$. Therefore, we obtain: $\sum_{\tilde{e}\in\mathcal{E}} S_{(\tilde{e}, e^N)}\Gamma^*_{N+1}(\tilde{e}, e^N) = (1+r)\left[\sum_{\tilde{e}\in\mathcal{E}} S_{(\tilde{e}, e^N)}\left(\hat{a}_{e^N} - a_{(\tilde{e}, e^N_{N-1}, \ldots, e^N_1)}\right)\right] = 0$, where the last equality comes from the definition of $\hat{a}_{e^N}$ in equation (46).

B Proof of Proposition

The proof runs in three steps. In the remainder, we use the following notation. For $N > k > 0$, $e^k = (e_{k-1}, \ldots, e_0) \in \mathcal{E}^k$, $e^{N,k} = (e_N, \ldots, e_k) \in \mathcal{E}^{N+1-k}$, and $(e^{N,k}, e^k) = (e_N, \ldots, e_k, e_{k-1}, \ldots, e_0)$.

B.1 A contraction lemma

We denote by $Conv(A)$ the convex hull of the set $A \subset \mathbb{R}$, and $\mu_{\mathcal{L}}$ the Lebesgue measure on $\mathbb{R}$.

Lemma 2 (Contraction lemma) *Assume that $A \subset [-\bar{a}, a^{max}]$ and that the conditions of Proposition 2 are fulfilled. Let $B = \left\{g^{N+1}_{a'}(a, (\hat{e}^{N,\bar{N}}, e^{\bar{N}}))|\hat{e}^{N,\bar{N}} \in \mathcal{E}^{N+1-\bar{N}}, a \in A\right\}$ for any $e^{\bar{N}} \in \mathcal{E}^{\bar{N}}$,. We have then $\mu_{\mathcal{L}}(Conv(B)) \leq \kappa \times \mu_{\mathcal{L}}(Conv(A))$.*

Proof. Since $B \subset \mathbb{R}$, we have by definition of the convex hull, $Conv(A) = [\min(A), \max(A)]$ and $Conv(B) = [\min(B), \max(B)]$. Let $a' = \max(A)$ and $a = \min(A)$, then $\mu_{\mathcal{L}}(Conv(A)) = a' - a$ and $B \subset [g^{N+1}_{a'}(a, (\hat{e}^{N,\bar{N}}, e^{\bar{N}})), g(a', (\tilde{e}^{N,\bar{N}}, e^{\bar{N}}))]$ for some $\hat{e}^{N,\bar{N}}, \tilde{e}^{N,\bar{N}} \in \mathcal{E}^{N+1-\bar{N}}$. Therefore, we obtain $\mu_{\mathcal{L}}(Conv(B)) \leq g^{N+1}_{a'}(a', (\tilde{e}^{N+1-\bar{N}}, e^{\bar{N}})) - g^{N+1}_{a'}(a, (\hat{e}^{N+1-\bar{N}}, e^{\bar{N}}))$. Applying the Lipschitz property (21) directly yields $\mu_{\mathcal{L}}(Conv(B)) \leq \kappa \times \mu_{\mathcal{L}}(Conv(A))$. ■

B.2 Proof of the convergence of Γ^*_{N+1}

Let $N > 0$. Proposition 1 shows that when the transfer is Γ^*_{N+1}, there are $(E+1)^N$ possible end-of-period asset holdings denoted $(a'_{\hat{e}^N})_{\hat{e}^N \in \mathcal{E}^N}$. Let $A_{N,-1}$ be the set of all possible end-of-period asset holdings, in the previous period. We define:

$$A^{(N)}_N(e^N) = \{a'_{\hat{e}^N} \in A_{N,-1} | e^N \succeq \hat{e}^N\}, \text{ for } e^N \in \mathcal{E}^N, \tag{48}$$

as the set of all possible beginning-of-period and before-transfer asset holdings of agents with current history e^N. In other words, it is the set of all possible previous-period wealth levels of agents with current history e^N. Since the after-transfer wealth level $\hat{a}_{e^N}$ of (46) is by construction an average of before-transfer wealth levels $a'_{\hat{e}^N}$, we have $\hat{a}_{e^N} \in Conv\left(A_N^{(N)}(e^N)\right)$.

We define $\pi(e^N) = \{\hat{e}^N | e^N \succeq \hat{e}^N\}$ the set of possible predecessors of e^N. We rewrite (48) as: $A_N^{(N)}(e^N) = \{a'_{\hat{e}^N} \in A_{N,-1} | \hat{e}^N \in \pi(e^N)\}$. For any $a'_{\hat{e}^N} \in A_N^{(N)}(e^N)$, there exists $\tilde{e}^N \in \mathcal{E}^N$ such that $\hat{e}^N \succeq \tilde{e}^N$ and $a'_{\hat{e}^N} = g_{a'}^{N+1}(a_{\tilde{e}^N}, (\tilde{e}^N, \hat{e}_0^N)) = g_{a'}^{N+1}(a_{\tilde{e}^N}, (\tilde{e}^N, e_1^N))$ –since $e_1^N = \hat{e}_0^N$. In other words, $a'_{\hat{e}^N}$ is the optimal choice of an agent who had in the previous period the N-history $\tilde{e}^N$, which is thus a possible past of e^N. Using the notation $\pi^2 = \pi \circ \pi$, $\tilde{e}^N \in \pi^2(e^N)$, we can define:

$$A_{N-1}^{(N)}(e^N) = \{a'_{\tilde{e}^N} \in A_{N,-2} | \tilde{e}^N \in \pi^2(e^N) \text{ and } g_{a'}^{N+1}(a'_{\tilde{e}^N}, (\tilde{e}^N, e_1^N)) \in A_N^{(N)}(e^N)\},$$

which is the set of all possible end-of-period asset holdings two periods ago of agents with current history e^N. Similarly, we define for any $0 < k < N$:

$$A_{N-k}^{(N)}(e^N) = \{a'_{\tilde{e}^N} \in A_{N,-k-1} | \tilde{e}^N \in \pi^{k+1}(e^N) \text{ and } g_{a'}^{N+1}(a'_{\tilde{e}^N}, (\tilde{e}^N, e_k^N)) \in A_{N-k+1}^{(N)}(e^N)\},$$

which allows us to construct a sequence of sets $(A_{N-k}^{(N)})_{k=0,\ldots,N}$ (e_k^N is the element number k in e^N, i.e. the state k periods ago for an agent with current history e^N). In words, $A_{N-k}^{(N)}(e^N)$ is the set of all possible end-of-period asset holdings k periods ago of agents with current history e^N. Iterating backward to construct those sets, we thus go back in time to construct sets of possible wealth levels (instead of histories).

In the previous notation π^{k+1} denotes $\pi \circ \ldots \circ \pi$ ($k+1$ times). Note that we could equivalently define $A_{N-k+1}^{(N)}(e^N)$ as:

$$A_{N-k+1}^{(N)}(e^N) = \{g_{a'}^{N+1}(a'_{\tilde{e}^N}, (\tilde{e}^N, e_k^N)) | \tilde{e}^N \in \pi^{k+1}(e^N) \text{ and } a'_{\tilde{e}^N} \in A_{N-k}^{(N)}(e^N)\}.$$

In words, $A_{N-k+1}^{(N)}(e^N)$ is the set of successors (for relevant histories) of agents with wealth levels in $A_{N-k}^{(N)}(e^N)$. If $1 \leq k \leq N - \bar{N}$, we deduce, applying Lemma (2), $\mu_{\mathcal{L}}\left(Conv\left(A_{N-k+1}^{(N)}(e^N)\right)\right) \leq \kappa \mu_{\mathcal{L}}\left(Conv(A_{N-k}^{(N)}(e^N))\right)$, and iterating forward from $k = 1$

to $k = N - \bar{N} - 1$ to $k = N - 1$, one finds:

$$\mu_{\mathcal{L}}\left(Conv(A_N^{(N)}(e^N))\right) \leq \kappa^{N-\bar{N}}\mu_{\mathcal{L}}\left(Conv(A_{\bar{N}}^{(N)}(e^N))\right). \tag{49}$$

Since a^{max} $(-\bar{a})$ is the largest (lowest) wealth levels by definition, $A_N \subset [-\bar{a}, a^{max}]$ and for all k, $A_{N-k}^{(N)}(e^N) \subset A_N \subset [-\bar{a}, a^{max}]$. This implies that we have $\mu_{\mathcal{L}}\left(Conv(A_{\bar{N}}^{(N)}(e^N))\right) \leq a^{max} + \bar{a}$. Second we have showed that $\hat{a}_{e^N}, a_{\hat{e}^N} \in Conv\left(A_N^{(N)}(e^N)\right)$ for any $\hat{e}^N \in \pi(e^N)$. This implies that $|\hat{a}_{e^N} - a_{\hat{e}^N}| \leq \mu_{\mathcal{L}}\left(Conv\left(A_N^{(N)}(e^N)\right)\right)$. In consequence, we have from equation (49):$|\hat{a}_{e^N} - a_{\hat{e}^N}| \leq \kappa^{N-\bar{N}}(a^{max}+\bar{a})$, which can be made arbitrarily small ($0 < \kappa < 1$), when N increases. We deduce from (47) that $\lim_{N\to\infty} \sup_{e^{N+1}\in\mathcal{E}^{N+1}} |\Gamma_{N+1}^*(e^{N+1})| = 0$.

B.3 Convergence of the value function

The last steps is a standard convergence proof. Let $\varepsilon > 0$. There exists $\bar{N}$ such that for all $N \geq \bar{N}$: $|\Gamma_{N+1}^*| < \varepsilon$. Define:

$$V^{(+\varepsilon)}(a, e) = \max_{a'\geq -\bar{a}, c\geq 0, l\geq 0} U(c, l) + \beta\mathbb{E}\left[\sum_{\tilde{e}^N \succeq e^N} M_{e,e'} V_{N+1}(a', e')\right], \tag{50}$$

$$a' + c = w\theta_e l 1_{e\neq 0} + \delta 1_{e=0} + (1 + r)a + T + \varepsilon, \tag{51}$$

with $l = \delta$ if $e = 0$. Similarly, $V^{(-\varepsilon)}(a, e)$ is the value function where the budget constraint is diminished by $-\varepsilon$ (ε is low enough for all agents' resources to remain positive).

Using standard dynamic programming arguments with bounded returns (see Stokey, Lucas, and Prescott 1989 Section 9.2), one has for all $a \in [-\bar{a}, a^{max}]$, $e \in \mathcal{E}$, then $V^{(+\varepsilon)}(a, e) \leq V^{Bewley}(a, e) \leq V^{(-\varepsilon)}(a, e)$ and $V^{(+\varepsilon)}(a, e) \leq V_{N+1}(a, e^{N+1}) \leq V^{(-\varepsilon)}(a, e)$. The last inequality follows from the bounds on the transfers Γ_{N+1}^*. As a consequence, for all $a \in [-\bar{a}, a^{max}]$, $e \in \mathcal{E}$:

$$\left|V^{Bewley}(a, e) - V_{N+1}(a, e^{N+1})\right| \leq \left|V^{(+\varepsilon)}(a, e) - V^{(-\varepsilon)}(a, e)\right|,$$

which can be made arbitrarily small, as $[a^{max}, a] \times \mathcal{E}$ is compact and $V^{(\pm\varepsilon)}$ is continuous in ε.

C Proof of Lemma 1

We use the methodology of Marcet and Marimon (2011) to simplify the Ramsey program. Denoting $\beta^t m^t(s^t) S_{t,e^N} \lambda_{t,e^N}$ the Lagrange multiplier of the Euler equation for island e^N at date t, the objective of the Ramsey program (22)–(29) can be rewritten as:

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N}) - \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \lambda_{t,e^N} \tag{52}$$
$$\times \left(U_c(c_{t,e^N}, l_{t,e^N}) - \nu_{t,e^N} - \beta \mathbb{E}_t \left[\sum_{\hat{e}^N \in \mathcal{E}^N} \Pi_{t+1,e^N,\hat{e}^N} U_c(c_{t+1,\hat{e}^N}, l_{t+1,\hat{e}^N})(1 + r_{t+1}) \right] \right)$$

With $\lambda_{t,e^N} \nu_{t,e^N} = 0$ and the definition (30) of Λ_{t,e^N}, (52) yields after some manipulations the objective in (31), which is maximized subject to constraints (23)–(29), except (25).

D Derivation of first-order conditions for the Ramsey program

We compute the first-order conditions of the simplified Ramsey program (31)–(32). Let $\beta^t m^t(s^t) \mu_t$ be the Lagrange multiplier of the government budget constraint (23). The Lagrangian is:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \left(U(c_{t,e^N}, l_{t,e^N}) + U_c(c_{t,e^N}, l_{t,e^N}) \left(\Lambda_{t,e^N}(1 + r_t) - \lambda_{t,e^N} \right) \right) \tag{53}$$
$$- \mathbb{E}_0 \sum_{t=0}^{\infty} \mu_t \beta^t \left(G_t + B_{t-1} + r_t A_{t-1} + w_t L_t + T_t - B_t - F(A_{t-1} - B_{t-1}, L_t, s_{t-1}) \right),$$

where $c_{t,e^N} = w_t \theta_{e_t^N} l_{t,e^N} 1_{e_0^N \neq 0} + \delta 1_{e_0^N = 0} + (1 + r_t)\tilde{a}_{t,e^N} - a_{t,e^N} + T_t$, for all $e^N \in \mathcal{E}^N$, and $l_{t,e^N} = (\chi w_t \theta_{e^N})^{\varphi} 1_{e_0^N > 0} + \delta 1_{e_0^N = 0}$ (using (7), (24) and (26)).

First-order conditions with respect to B_t, r_t, and T_t are straightforward.

Derivative with respect to a_{t,e^N}. It yields for all $e^N \in \mathcal{E}^N \setminus \mathcal{C}_t$:

$$\psi_{t,e^N} = \beta \mathbb{E}_t \sum_{\tilde{e}^N \in \mathcal{E}^N} (1 + r_{t+1}) \Pi_{t+1,e^N,\tilde{e}^N} \psi_{t+1,e^N} - \beta \mathbb{E}_t \mu_{t+1} (r_{t+1} - \tilde{r}_{t+1}). \tag{54}$$

Combining (34) with (54) yields equation (36).

Derivative with respect to w_t. We obtain:

$$\mu_t L_t \left(1 + \frac{\varphi}{w_t}\left(w_t - F_L(K_{t-1}, L_t, s_{t-1})\right)\right) = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \theta_{e^N} l_{t,e^N} \psi_{t,e^N}. \tag{55}$$

Using $w_t - F_L(K_{t-1}, L_t, s_{t-1}) = w_t - \tilde{w}_t = -\frac{\tau_t^L}{1-\tau_t^L}\tilde{w}_t$, equation (55) becomes after some algebra $\mu_t L_t \left(1 - \varphi \frac{\tau_t^L}{1-\tau_t^L}\right) = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \theta_{e^N} l_{t,e^N} \psi_{t,e^N}$. Since $L_t = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \theta_{e^N} l_{t,e^N}$, we deduce equation (37).

E Proof of Proposition 3

First-order conditions (34) and (37) immediately imply (40) and (42) at the steady-state. Now, we sum individual consumption Euler equations (25) for all $e^N \in \mathcal{E}^N \setminus \mathcal{C}$ (i.e., when $\nu_{e^N} = 0$):

$$\sum_{e^N \in \mathcal{E}^N \setminus \mathcal{C}} S_{e^N} U_c(c_{e^N}, l_{e^N}) = \beta(1 + (1-\tau^k)\tilde{r}) \left[\sum_{\tilde{e}^N \in \mathcal{E}^N} \sum_{e^N \in \mathcal{E}^N \setminus \mathcal{C}} S_{e^N} \Pi_{e^N, \tilde{e}^N} U_c(c_{\tilde{e}^N}, l_{\tilde{e}^N})\right].$$

We now split the sum as $\sum_{e^N \in \mathcal{E}^N \setminus \mathcal{C}} = \sum_{e^N \in \mathcal{E}^N} - \sum_{e^N \in \mathcal{C}}$. After some manipulation, we get

$$\begin{aligned}\beta \tau^k \tilde{r} \sum_{e^N \in \mathcal{E}^N} S_{e^N} U_c(c_{e^N}, l_{e^N}) = \sum_{e^N \in \mathcal{C}} S_{e^N} \\ \times \left[U_c(c_{e^N}, l_{e^N}) - \beta(1 + (1-\tau^k)\tilde{r}) \left[\sum_{\tilde{e}^N \in \mathcal{E}^N} \Pi_{e^N, \tilde{e}^N} U_c(c_{\tilde{e}^N}, l_{\tilde{e}^N})\right]\right],\end{aligned}$$

where in the right-hand side we recognize the "Euler inequality" for constrained agents. Using equation (25) and $\beta\tilde{r} = 1 - \beta$, we obtain equation (41).

F Algorithm to solve the model

General method. Our algorithm, which relies on a guess-and-verify strategy, is as follows.

1. We determine the steady-state of the Ramsey program (see below).

2. We write a code that writes the set of dynamic equations in Dynare. We use the

Dynare solver to double-check our steady-state computations and to simulate the model.

3. We finally verify that the set of credit-constrained islands does not change in the presence of aggregate shocks.

Finding the steady-state. We now describe in more detail the algorithm to find the steady-state (step 2 of the general method).

1. We guess the set $\mathcal{C}$ of islands that are credit-constrained. We choose a post-tax interest rate r and a transfer value T (typically $T = 0$) and a post-tax wage rate w. Then:

   (a) We compute the labor supply on each island e^N, using agents' first-order conditions (26). We then deduce the aggregate labor L. We compute the aggregate capital K with $F_K(K, L) = \beta^{-1} - 1$.

   (b) We determine individual consumption levels and asset holdings using equations (24), (25), and (7). We deduce a corresponding value for public spending, given by $G = F(K, L) - rA - wL - T$.

   (c) We set a value of μ. We set values ψ_{e^N}, $e^N \in \mathcal{C}$ (for credit-constrained islands). Using (36), we then solve for ψ_{e^N}, $e^N \in \mathcal{E}^N \setminus \mathcal{C}$ (unconstrained islands). We obtain then the λ_{e^N}, $e^N \in \mathcal{E}^N$ using (33) defining ψ_{e^N}. We finally iterate on ψ_{e^N}, $e^N \in \mathcal{C}$, until we have $\lambda_{e^N} = 0$ for $e^N \in \mathcal{C}$ (constrained islands). We iterate on μ until equation (38) holds at the steady-state.

   (d) We iterate on w until (37) holds at the steady-state.

2. We iterate on r until G/Y matches its target. Check that the value $T = 0$ is indeed the equilibrium value, otherwise iterate. We finally verify that Euler inequalities are strict for islands $e^N \in \mathcal{C}$ to check that the set $\mathcal{C}$ of constrained islands is correct. Otherwise, we iterate on $\mathcal{C}$.