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# Capital Accumulation and the Dynamics of Secular Stagnation

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## Abstract

The economic and financial crisis of 2008 caused a deep crisis characterized by a weak potential growth and a persistent output gap. In this paper, we propose a model to explain the crisis and its persistence by a regime switching from a full employment equilibrium to an underemployment equilibrium. In the latter, the real equilibrium interest rate is negative and the economy is in deflation. Monetary policy targeting inflation becomes inactive due to a non-negativity constraint in the policy rate (zero lower bound). This secular stagnation equilibrium is achieved by introducing, in a standard overlapping generations model, two market imperfections: a credit rationing and a downward nominal wage rigidity. To exit from the secular stagnation trap, we study the impact of various economic policies. An increase in the inflation target (more accommodating monetary policy) is effective if the central bank is credible enough. A fiscal policy, by supporting aggregate demand, can help the economy to exit from the secular stagnation trap. But such a policy reduces the full employment potential regime due to less incentive to accumulate capital.

**Keywords:** Secular Stagnation, Capital Accumulation, Zero Lower Bound.

**JEL codes:** D91, E31, E52

# 1 Introduction

The economic state of slow growth and underemployment, coupled with low inflation or even deflation, has recently been widely discussed, in particular by Larry Summers (2014), under the label of “secular stagnation”. The hypothesis of secular stagnation was expressed for the first time in 1938 in a speech by A. Hansen, which was finally published in 1939. Hansen was worried about insufficient investment and a declining population in the United States, following a long period of strong economic and demographic growth.

A state of secular stagnation results when an abundance of savings relative to demand for credit pushes the “natural” real interest rate (what is compatible with full employment) below zero. But if the real interest rate permanently remains above the natural rate, then the result is a chronic shortage of aggregate demand and investment, with a weakened growth potential.

To counter secular stagnation, the monetary authorities first reduced their policy rates, and then, having reached the zero lower bound (ZLB), they implemented non-conventional policies called quantitative easing. The central banks cannot really force interest rates to be very negative, otherwise private agents would have an interest in keeping their savings in the form of banknotes. Beyond quantitative easing, what other policies might potentially help pull the economy out of secular stagnation?

To answer this crucial question, the model developed by Eggertsson and Mehrotra in 2014 has the great merit of clarifying the mechanisms behind a fall into long-term stagnation, and it is helping macroeconomic analysis to update its understanding of the multiplicity of equilibria and the persistence of the crisis. Their model is based on the consumption and savings behaviour of agents with a finite lifespan in a context of a rationed credit market and nominal wage rigidity. As for the monetary policy conducted by the central bank, this is set at a nominal rate using a Taylor rule.

According to this approach, secular stagnation was initiated by the 2008 economic and financial crisis. This crisis was linked to high household debt, which ultimately led to credit rationing. In this context, credit rationing leads to a fall in demand and excess savings. Consequently, the real interest rate falls. In a situation of full employment, if credit tightens sharply, the equilibrium interest rate becomes negative, which leaves conventional monetary policy toothless. In this case, the economy plunges into a lasting state of underemployment of labour, characterised by output that is below potential and

by deflation.

In the model proposed by Eggertsson and Mehrotra, there is no capital accumulation. As a result, the underlying dynamic is characterized by adjustments without transition from one steady state to another (from full employment to secular stagnation if there's a credit crisis, and vice versa if credit doesn't tighten much).

To extend the analysis, we considered the accumulation of physical capital as a prerequisite to any productive activity. This highlights an asymmetry in the dynamics of secular stagnation. If the credit constraint is loosened, then capital converges on its pre-crisis level. However, exiting the crisis takes longer than entering it. This property suggests that economic policies used to fight against secular stagnation must be undertaken as soon as possible.

There are several lessons offered by this approach.

To avoid the ZLB, there is an urgent need to create inflation while avoiding speculative asset "bubbles", which could require special regulation. The existence of a deflationary equilibrium thus raises the question of the appropriateness of monetary policy rules that are overly focused on inflation.

One should be wary of the deflationary effects of policies to boost potential output. The right policy mix is to support structural policies with a sufficiently accommodative monetary policy.

Cutting savings to raise the real interest rate (e.g. by facilitating debt) is an interesting possibility, but the negative impact on potential GDP should not be overlooked. There is a clear trade-off between exiting secular stagnation and depressing potential GDP.

The article consists of four parts. In the second section, we develop an overlapping generations model. This model makes the heart of the process of economic growth of productive capital accumulation. We introduce imperfections on the credit market (rationing) and the labor market (wage rigidity). In addition, we assume that the central bank conducts monetary policy to control inflation thanks to the setting of a nominal policy interest rates via a Taylor rule, knowing that the nominal rate can not be negative. In section 3, we characterize the economic dynamics (dynamic time paths and steady states) and the secular stagnation equilibrium. There are three configurations. If the long-run equilibrium is unique, there are two cases: full employment with inflation target or secular stagnation with underemployment and deflation. In both cases, the equilibria

are globally determined with dynamics characterized by a unique saddle path. Finally, in a third configuration, the two preceding equilibria coexist with a third equilibrium of full employment and missed inflation target. This equilibrium is undetermined and the other two are determined. These determined steady states are not unique. Therefore, they are only locally determined, not globally. Section 4 discusses the economic policy issues (monetary, fiscal and structural) to exit the secular stagnation trap. Without going into details, we show that an increase in the inflation target can be helpful but only if the central bank is credible enough, that increasing productivity or reducing rigidities can be counterproductive contrary to suggests the standard approach developped in the first part and finally that any reduction of savings induces inflationary pressure and can help the economy out of secular stagnation. The last section concludes.

## 2 The model

The economy is composed of four types of agents: individuals, firms, a government and a central bank.

Following Samuelson (1958) and Diamond (1965), we assume that individuals live for three periods: they are successively young, middle-aged-workers then retired. the number of young individuals,  $N_t$  at date  $t$ , is growing at the constant rate  $n$  so that:

$$N_t = (1 + n) N_{t-1} \tag{1}$$

Competitive firms produce one good, which is both a consumption good and an investment good, by using two factors: labor and capital. The government finances public expenditures by taxing workers (with balanced budget), and the central bank determines the nominal rate of interest to control inflation.

Accordingly, there are four markets in the economy: good, labor, capital and credit.

### 2.1 Individuals and credit rationing

During their first period of live, individuals borrow to invest in capital. We denote  $I_{t-1}$  this investment. One can think for example of human capital but for simplicity, we assume that physical and human capital are perfect substitutes. A period later, the investment is sold to firms with a return equal to  $R_t^k$ . When they are active they offer inelastically an amount

of work  $\bar{l}$  normalized to unity,  $\bar{l} = 1$ , and work for a real wage rate  $w_t$ . They consume  $c_t$  and save such that  $a_{t-1}^m$  is their real net asset, they also pay back their loans plus interest. They also pay a lump-sum tax  $T$ . In the final period of life, they consume  $d_{t+1}$ . Assuming that each individual works effectively an identical duration  $l_t \leq \bar{l}$ , budgetary constraints are as follows:

$$\begin{cases} a_{t-1}^y = -I_{t-1} \\ c_t + a_t^m = w_t l_t - T + R_t^k I_{t-1} + R_t a_{t-1}^y \\ d_{t+1} = R_{t+1} a_t^m \end{cases} \quad (2)$$

where  $a_t^y$  denotes the net real asset at date  $t$  of a young individual and  $R_t$  the real interest factor.

Note that for a pure question of analytical simplicity, we assume as usual in this type of literature that individuals do not consume in the first period of life (see for example Boldrin and Montes, 2005, and Docquier et al., 2007). Furthermore, it is assumed that there is no altruism, and so individuals start in life with zero assets. The preferences of an individual born in period  $t - 1$  are therefore characterized by the following utility function:

$$U_{t-1} = \log c_t + \beta \log d_{t+1} \quad (3)$$

where  $\beta$  denotes the psychological discount factor. It is easily shown that the optimal behavior of the consumer, obtained by utility maximization of eq. (3) under budgetary constraints (2), yields the following optimal asset:

$$a_t^m = s (w_t l_t - T + R_t^k I_{t-1} + R_t a_{t-1}^y) \quad (4)$$

where  $s = \frac{\beta}{1+\beta}$  denotes the saving rate.

An often preferred hypothesis to explain the economic and financial crisis of 2008 is that of an imperfect financial markets, and among them a household indebtedness resulting ultimately in the crisis by rationing credit these same households. To translate such a situation and study its ability to explain the crisis as part of a growth model, we assume following Aiyagari (1994), Krugman and Eggertsson (2012) or Eggertsson and Mehrotra (2014) that the credit market is rationed as<sup>1</sup>:

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<sup>1</sup>Microeconomic theories of credit rationing are based mostly on the non-observability either of the

$$-a_{t-1}^y \leq \frac{D}{R_t} \quad (5)$$

Such a constraint does not focus on the loanable proportion, but on the ability households will have the following period to repay their loans, i.e., to repay the capital borrowed plus interest. If this constraint bites (of course it is assumed when  $R_t^k > R_t$ ), we then have  $a_{t-1}^y = -\frac{D}{R_t}$ .

## 2.2 Firms

From the production side, we assume that the good is produced in a competitive sector characterized by a Cobb-Douglas technology with constant return to scale such that  $F(K_t, N_{t-1}l_t) = AK_t^\alpha (N_{t-1}l_t)^{1-\alpha}$ , where  $\alpha < 1$  and  $A$  denotes the total factor productivity (TFP). The profit maximization then yields:

$$w_t = A(1-\alpha)k_t^\alpha l_t^{-\alpha}, l_t \leq 1 \quad (6)$$

and

$$R_t^k = A\alpha k_t^{\alpha-1} l_t^{1-\alpha} + (1-\delta) \quad (7)$$

where  $k_t = \frac{K_t}{N_{t-1}}$  is the level of capital per worker and  $\delta$  the depreciation rate of capital,  $\delta \in (0, 1]$ .

## 2.3 Wage bargaining and nominal rigidity

Each generation of workers is negotiating a contract. We assume that at the beginning of each period a wage negotiation defines the profile of nominal wages throughout the period of activity. For simplicity, we define  $W_t(0)$  and  $W_t(1)$  as the levels of nominal wages at the beginning and at the end of period  $t$ . Assuming an aversion to the decline in nominal wages during the period, the wage at the end of the period is determined according to:

$$W_t(1) = \max\left(\tilde{W}_t, W_t^*\right) \quad (8)$$

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individual effort (moral hazard) or of skills (adverse selection) (see Stiglitz and Weiss, 1981; Aghion and Bolton, 1997; Piketty, 1997). They all have in common to explain that a higher collateral allows to borrow more.

where  $W_t^* = A(1 - \alpha)P_t k_t^\alpha$  is the full employment wage rate and  $\tilde{W}_t = \gamma W_t(0) + (1 - \gamma)W_t^*$ ,  $\gamma \in (0, 1)$  characterizes the aversion to the decline in nominal wages or the degree of downward rigidity of wages. Assuming that wage bargaining leads to setting a constant level of real wage over the period,  $w_t = \frac{W_t(0)}{P_{t-1}} = \frac{W_t(1)}{P_t}$ , we then have:

$$w_t = \max \left( \frac{(1 - \gamma) A (1 - \alpha) k_t^\alpha}{1 - \frac{\gamma}{\Pi_t}}, w_t^* \right) \quad (9)$$

where  $w_t^*$  is the full employment real wage rate. We observe straightforwardly that in this configuration, if the economy is in deflation, then the negotiated real wage level is above its full employment level:  $w_t = \frac{(1-\gamma)(1-\alpha)k_t^\alpha}{1-\frac{\gamma}{\Pi_t}} \geq (1 - \alpha) k_t^\alpha$  if  $\Pi_t \leq 1$ . Indeed, in case of deflation, maintaining both purchasing power and full employment means lower nominal wage. If the required drop is reduced, and especially as aversion to nominal wage decline is strong, then the real wage becomes stronger than the one that would allow full employment.

## 2.4 Central bank: Taylor rule and inflation target

We assume that the monetary authorities want to control inflation. Following Eggertsson and Mehrotra (2014), we express the Taylor rule as:

$$1 + i_t = \max \left( 1, (1 + i^*) \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right) \quad (10)$$

where  $\Pi^* \geq 1$  denotes the official inflation target,  $\phi_\pi > 1$ . When  $1 + i_t = (1 + i^*) \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi}$ , the Taylor rule will operate and, in this sense, we can say that monetary policy is active. By contrast, when  $i_t = 0$ , the central bank is constrained by the nominal zero lower bound. In this case, we say that monetary policy is inactive. According to the equation (10), we can highlight a level of inflation

$$\Pi_{kink} = (1 + i^*)^{\frac{-1}{\phi_\pi}} \Pi^* \quad (11)$$

such that  $i_t \geq 0 \Leftrightarrow \Pi_t \geq \Pi_{kink}$ . In addition, for the inflation target to be reached in the unconstrained regime we set  $1 + i^* = R_{eq} \Pi^*$ . Such Taylor rule means that the central bank pursues one goal: the inflation target  $\Pi^*$ .



## 2.5 Equilibrium with market imperfections

By assuming that the credit constraint is binding, i.e.  $R_t^k > R_t$ , an equilibrium is defined by clearings in the capital market

$$N_{t-1}k_t = N_{t-1}I_{t-1} \quad (12)$$

and in the credit market

$$N_t a_t^y + N_{t-1} a_t^m = 0 \quad (13)$$

By contrast, knowing that wages are downwardly rigid, the labor market does not necessarily clear. From eqs. (6) and (9), it then yields:

$$l_t = \min(1, \mathcal{L}(\Pi_t)) \quad (14)$$

where  $\mathcal{L}(\Pi_t) = \left(\frac{1-\frac{\gamma}{\Pi_t}}{1-\gamma}\right)^{\frac{1}{\alpha}}$  with  $\mathcal{L}' = \frac{\gamma}{\alpha\Pi^2} \left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{1}{\alpha}-1} > 0$ . We then observe that the under-employment of labor is particularly important that deflation is strong.

## 3 Characteristics of the secular stagnation equilibrium

### 3.1 The supply-demand equilibrium of good and the dynamics of capital

The supply of good per worker is obviously determined by:

$$y_t^s = Ak_t^\alpha \min(1, \mathcal{L}(\Pi_t))^{1-\alpha} + (1-\delta)k_t \quad (15)$$

From the constancy of scale returns, we immediately check that the demand per worker is equal to  $y_t^d = \frac{Y_t}{N_{t-1}} = w_t l_t + R_t^k k_t$ . Knowing that at equilibrium  $k_t = I_{t-1}$  (eq. 12), we deduce from the equation (4) that  $w_t l_t + R_t^k k_t = \frac{1}{s} a_t^m - R_t a_{t-1}^y + T$ . Using equilibrium relations (13) and (14), the dynamics of the population (1), the credit rationing (5) and the public balanced budget  $G = T$ , where  $G$  denotes the public spendings by number of workers at any date  $t$ , the aggregate demand per worker can be expressed as:

$$y_t^d = \frac{1+n}{s} I_t + D + G \quad (16)$$

Typically, it increases with its two components that are the private investment and the public demand. It also decreases with the saving rate  $s$  and increases with the population growth rate  $n$ . The aggregate demand is also impacted through the credit constraint (5). In particular, if  $D$  is lowered, consumption of the elderly is reduced so that  $d_t = R_t a_{t-1}^m = -R_t (1+n) a_{t-1}^y = (1+n) D$ .

In the good market, the supply-demand equilibrium  $y_t^s = y_t^d$  (with eq. 12) then determines the following dynamics of the capital stock:

$$k_{t+1} = \frac{1}{1+n} \mathcal{S}(k_t, \Pi_t) \quad (17)$$

where  $\mathcal{S}(k_t, \Pi_t) = sA k_t^\alpha \min(1, l(\Pi_t)^{1-\alpha}) - D - G + (1-\delta) k_t$  with  $\mathcal{S}'_k = sR^k > 0$ ,  $\mathcal{S}'_\Pi = \frac{sw}{(1+n)} \mathcal{L}' > 0$  if  $\Pi < 1$  and  $\mathcal{S}'_\Pi = 0$  if  $\Pi \geq 1$ .

If  $\Pi_t \geq 1$ , then the dynamics is  $k_{t+1} = \frac{1}{1+n} \mathcal{S}(k_t, 1)$ . In this case, it is easy to show that if  $D + G$  is small enough, there exists a unique stable steady state with full employment  $k_{FE}$  for any  $k_0 > k_{inst} > 0$ , where the second steady state  $k_{inst}$  is unstable. The dynamic properties of the capital accumulation process can be characterized by studying  $\Delta k_{t+1} = k_{t+1} - k_t$ . By definition at the steady state  $k_{FE}$ ,  $\Delta k_{t+1} = 0$ . Therefore, with decreasing productivity, if  $k_t > k_{FE}$  then  $\Delta k_{t+1} < 0$  and if  $k_{FE} > k_t > k_{inst}$  then  $\Delta k_{t+1} < 0$ . Note that this process is independent of inflation and thus the curve  $\Delta k = 0$  is vertical in the plane  $(k, \Pi)$ . If  $\Pi_t < 1$ , then the dynamics is expressed as  $k_{t+1} = \frac{1}{1+n} \mathcal{S}(k_t, \Pi_t)$ . When the economy falls in deflation, nominal rigidity of wages eliminates the full employment equilibrium. Thus, a decline in the amount of work will reduce the marginal product of capital. In this case, the equilibrium level of capital will be reduced. The curve describing the locus  $\Delta k_{t+1} = 0$  is growing in the plane  $(k, \Pi)$  when  $\Pi < 1$  and  $sR_t^k < 1$ .

### 3.2 Taylor rule and stabilization of inflation

According to the Fisher equation, we have:

$$R_{t+1} = (1+i_t) \frac{P_t}{P_{t+1}} = \frac{1+i_t}{\Pi_{t+1}} \quad (18)$$

Combining equations (10) and (18), we obtain:

$$\Pi_{t+1} = \frac{\max\left(1, (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right)}{R_{t+1}} \quad (19)$$

Knowing that  $k_{t+1} = I_t = \frac{D}{R_{t+1}}$ , it yields from eq. (17) that:

$$\frac{1}{R_{t+1}} = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)D}. \quad (20)$$

Including this result in equation (19), the dynamics of inflation becomes:

$$\Pi_{t+1} = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)D} \max\left(1, (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right) \quad (21)$$

For clarity, suppose  $\Pi_{kink} \geq 1$ . In this case, if  $\Pi_t \geq \Pi_{kink}$ , then this equation becomes  $\Pi_{t+1} = \frac{s}{(1+n)D} (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \mathcal{S}(k_t, 1)$  and we obtain:

$$\Delta\Pi_{t+1} = 0 \iff \Pi_t = \Pi^* \left(\frac{(1+n)k_{FE}}{\mathcal{S}(k_t, 1)}\right)^{\frac{1}{\phi_\pi-1}} \text{ if } \Pi_t \geq \Pi_{kink} \quad (22)$$

We observe that when monetary policy is active ( $\Pi_t \geq \Pi_{kink}$ ), then the stabilization of inflation requires that the level of inflation decreases when the level of capital is high (or the interest rate is low). To understand this configuration, rewrite the Fisher equation as  $\Pi_{t+1} = \frac{1+i_t}{R_{t+1}}$ . Stabilizing inflation  $\Delta\Pi_{t+1} = 0$  is then equivalent to  $\Pi_t = \frac{1+i_t}{R_{t+1}}$ , or after log-linearization to  $d\tilde{\Pi}_t = d\widetilde{1+i_t} - d\tilde{R}_{t+1}$ . According to equation (17), increasing the level of capital in  $t$  yields more capital in  $t+1$  and then a lower interest rate in  $t+1$ . The stabilization of inflation everything else being equal, i.e. with an unchanged nominal interest rate  $d\widetilde{1+i_t} = 0$ , requires then that the level of inflation increases in  $t$  such that  $d\tilde{\Pi}_t = d\tilde{\Pi}_{t+1} = -d\tilde{R}_{t+1} > 0$  when  $d\tilde{k}_t > 0$ . However, an active monetary policy yields an overreaction of the nominal interest rate such that  $d\widetilde{1+i_t} = \phi_\pi d\tilde{\Pi}_t$  where  $\phi_\pi > 1$ . Hence, the initial effect of an increase in the level of capital through a decrease in the interest rate is reversed such that  $d\tilde{\Pi}_t = d\tilde{\Pi}_{t+1} = \frac{d\tilde{R}_{t+1}}{\phi_\pi-1} < 0$  when  $d\tilde{k}_t > 0$ . Accordingly, we can verify that when monetary policy is inactive ( $\Pi_t < \Pi_{kink}$ ) and the inflation is positive ( $\Pi_t \geq 1$ ), this relationship is in the opposite direction. Indeed, in this case,  $\Pi_{t+1} = \frac{\mathcal{S}(k_t, 1)}{(1+n)D}$  which allows us to define:

$$\Delta\Pi_{t+1} = 0 \iff \Pi_t = \frac{\mathcal{S}(k_t, 1)}{(1+n)D} \text{ if } 1 \leq \Pi_t \leq \Pi_{kink} \quad (23)$$

Finally, when monetary policy is active in the deflationary area ( $\Pi_t < 1 \leq \Pi_{kink}$ ), we have  $\Pi_{t+1} = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)D}$  and we obtain:

$$\Delta\Pi_{t+1} = 0 \iff \Pi_t = \frac{\mathcal{S}(k_t, \Pi_t)}{(1+n)D} \text{ if } \Pi_t < 1 \leq \Pi_{kink} \quad (24)$$

Differentiating this equation in the neighborhood of  $\Pi = 1$  yields  $\left(1 - \frac{\mathcal{S}'_{\Pi}}{(1+n)D}\right) d\Pi_t = \frac{\mathcal{S}'_k}{(1+n)D} dk_t$ . As is obvious,  $\gamma = 0$  yields  $\frac{d\Pi_t}{dk_t} > 0$ . Indeed, in this case there is no wage rigidity and the curve is strictly similar to the previous one. By contrast, if the wage rigidity is strong enough such that  $\gamma > \frac{\alpha(1+n)D}{\alpha(1+n)D + (1-\alpha)sAk_{\Pi=1}^{\alpha}}$ , the relation that links the level of capital to inflation in order to guarantee a stable level of inflation is again decreasing. In that case, the increase in the capital in  $t$  yields a sufficiently strong decrease in employment (due to the gap between the effective and the full employment real wage) such that the product in  $t$  decreases. Therefore, savings in  $t$  decreases as well as the capital in  $t + 1$  such that  $R_{t+1}$  increases. In all of these configurations, it is easy to show that for any given level of  $\Pi_t$ ,  $\frac{\partial\Delta\Pi_{t+1}}{\partial k_t} > 0$ .

Representing the curves  $\Delta k$  and  $\Delta\Pi$  in the same phase plane  $(k, \Pi)$  can illustrate three configurations. If the equilibrium is unique, there are two cases: full employment with inflation target (Fig. 1a) with  $(k_{FE}, \Pi^*)$  the stable steady state or secular stagnation with underemployment and deflation (Fig. 1b) with  $(k_{SS}, \Pi_{SS})$  the stable steady state. In both cases, the equilibria are globally determined with dynamics characterized by a unique saddle path. Finally, in a third configuration, the two preceding equilibria coexist with a third equilibrium of full employment and missed inflation target:  $(k_{ind}, \Pi_{ind})$ . This equilibrium is undetermined and the other two are determined. These determined steady states are not unique. Therefore, they are only locally determined, not globally.

The question is that of the change from an dynamic time path to another, and in particular, which may explain the fall in secular stagnation starting from a full-employment equilibrium. So we are naturally interested in the credit crunch as an eventual cause of the crisis.

### 3.3 Credit crunch and secular stagnation

What happens if the constraint debt is tightened? Paradoxically, the capital stock of full employment is decreasing with the constraint of debt. Indeed, from equation (17) and assuming with no loss of generality that  $G = 0$ , it yields that in the neighborhood of the saddle point steady state defined as  $k_{FE} = \frac{s}{1+n-s(1-\delta)} (Ak_{FE}^{\alpha} - D)$ , the dynamics of

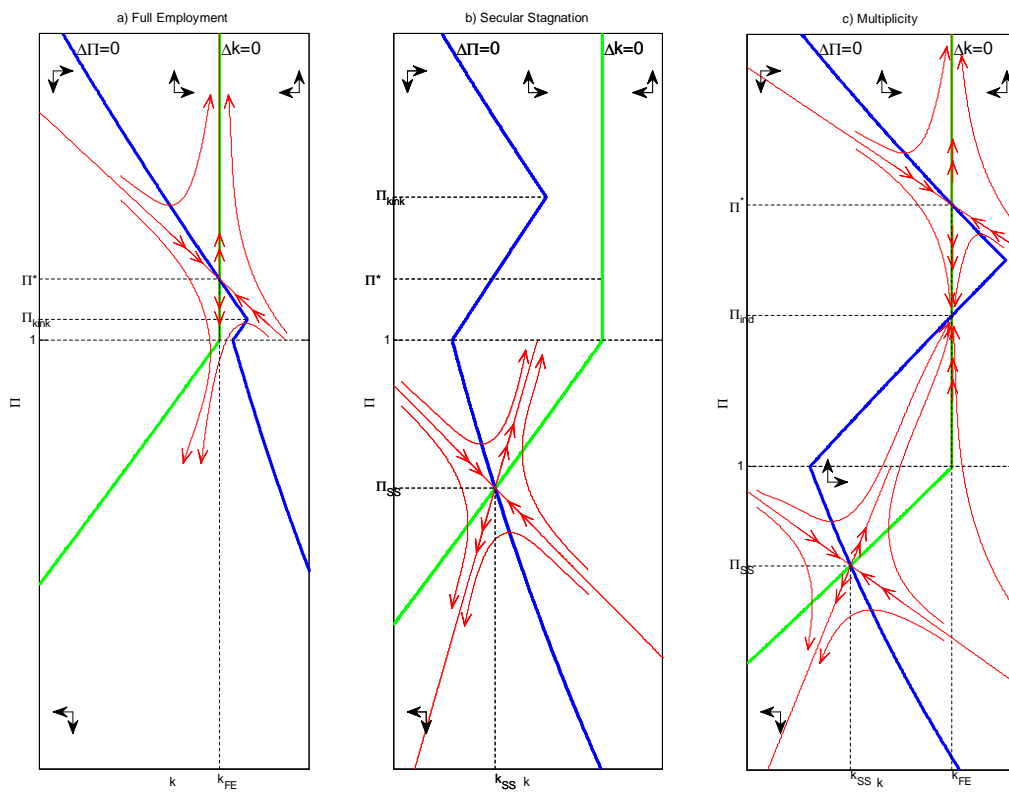


Figure 1: Features of the steady states

capital is as follows:  $dk_{t+1} = \varepsilon dk_t - \frac{s}{1+n} dD$ , where  $\varepsilon = \frac{s}{1+n} (A\alpha k_{FE}^{\alpha-1} + 1 - \delta) < 1$ . It then yields  $\frac{dk_{FE}}{dD} = -\frac{s}{1+n} \frac{1}{1-\varepsilon} < 0$ . As  $k_{FE} = \frac{D}{R_{eq}}$ , we deduce that at the steady state,  $\frac{dR_{eq}}{R_{eq}} > \frac{dD}{D}$ . These results come from the fact that the supply of savings is inelastic w.r.t the interest factor while the present value of the debt constraint (loan application) has an elasticity of 1 w.r.t the interest rate. A decrease in  $D$  automatically implies a decrease in the demand for credit that can only be adjusted by lower interest rates. Consequently, households are less indebted, increasing their future saving capacity and therefore the accumulation of savings.

Finally, it can be shown that if  $D < \left(\frac{A}{\delta + \frac{1+n}{s}}\right)^{\frac{1}{1-\alpha}}$ , then at the steady state, we have  $R_{eq} < 1$ , i.e. that the equilibrium interest rate ( $r_{eq} = R_{eq} - 1$ ) becomes negative, which can be problematic for monetary policy as outlined in the introduction. According to the Fisher equation, we know that  $R = \frac{1+i}{\Pi}$ , where  $i$  is the nominal interest rate and  $\Pi$  the inflation factor. Following a savings glut in the economy, monetary policy can be conducted toward the limit to zero and thereby induce a type of long-term stagnation in employment equilibrium. The nominal wage rigidity combined with positivity constraint of nominal policy interest rate is a key element to display a secular stagnation situation. The following lemma gives a necessary and sufficient condition for the existence of such a equilibrium (Walrasian disequilibrium).

**Lemma 1** *A secular stagnation equilibrium exists and is locally determined iff  $R_{eq} < 1 \iff D < \left(\frac{A}{\delta + \frac{1+n}{s}}\right)^{\frac{1}{1-\alpha}}$ . If  $R_{eq} < \frac{1}{\Pi^*} \iff \Pi_{kink} > \Pi^*$ , then the secular stagnation equilibrium is the unique equilibrium.*

**Proposition 2** *All things being equal, any sufficiently large decrease in  $D$  can bring back the economy in secular stagnation.*

Figure 2 illustrates the fall in secular stagnation after a credit tightening at time  $t = 0$ . Starting from a position of full employment (the capital level is normalized to unity), if the credit crunch is sufficiently large, then the equilibrium interest rate is sufficiently negative and the conventional monetary policy can no longer be conducted actively. In this case, the secular stagnation is the unique equilibrium of the economy and the economy plunges into recession with an underemployment of labor, thus associated with a production lower than its potential level, and deflation. The first period of the shock, as the capital is predetermined, it does not change,  $k_0 = k_{FE}$ . However, as soon as the second period it

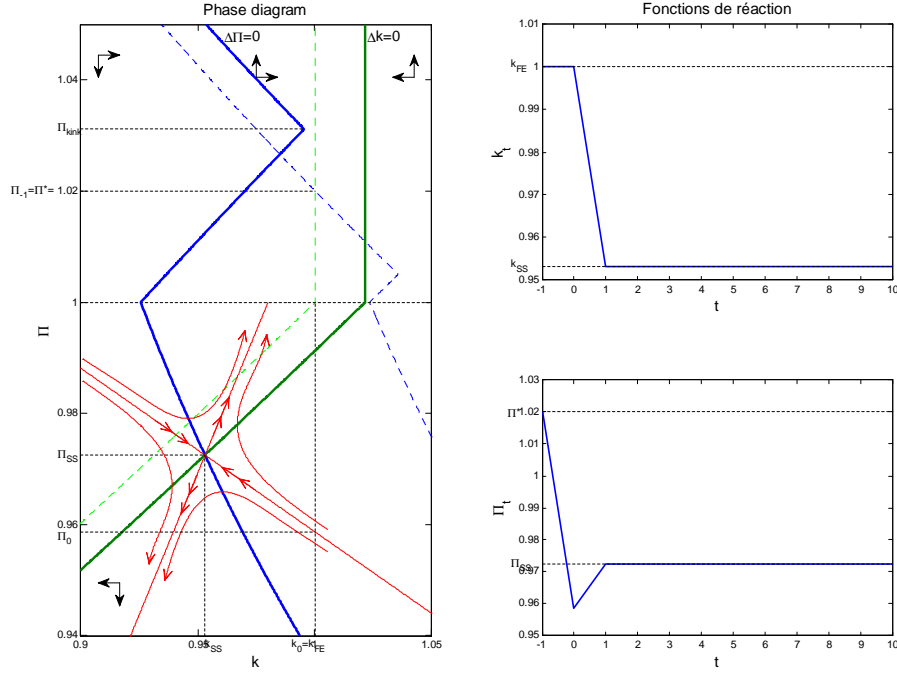


Figure 2: Dynamics of Secular Stagnation

reaches its steady state level of secular stagnation<sup>2</sup>,  $k_t = k_{SS} \forall t \geq 1$ . The fall into secular stagnation is very fast. We also observe an overshooting of the level of deflation just after the shock. Indeed, as the level of capital already installed can not be adjusted initially, there is an excessive supply which results in a higher deflation:  $\Pi_0 < \Pi_{SS}$ . Deflation then reaches its lower steady state level one period after the shock,  $\Pi_t = \Pi_{SS} \forall t \geq 1$ . Formally, it can be shown from the dynamics (21) that  $\Pi_{SS} = \frac{S(k_{FE}, \Pi_0)}{(1+n)D} = \frac{S(k_{SS}, \Pi_{SS})}{(1+n)D}$ . Therefore, as  $S'_k > 0$  and  $S'_\Pi > 0$  if  $\Pi < 1$ ,  $k_{FE} > k_{SS}$  yields  $\Pi_0 < \Pi_{SS} (< 1)$ . Note also that the secular stagnation equilibrium, as in Eggertsson and Merhotra (2014), and unlike Krugman and Eggertsson (2012), is persistent as long as the credit crunch lasts. From this point of view, active policies against the credit crunch are crucial in the fight against the secular stagnation.

Assume now such a policy is efficient so that it had brought back confidence in the financial markets. What's going on if the credit constraint returns to its original position? Of course we return to the initial situation where the only determined equilibrium is characterized by full employment. Nevertheless, as illustrated in Figure 3, capital returns

<sup>2</sup>The local dynamics is associated with an eigenvalue equal to zero (see Appendix).

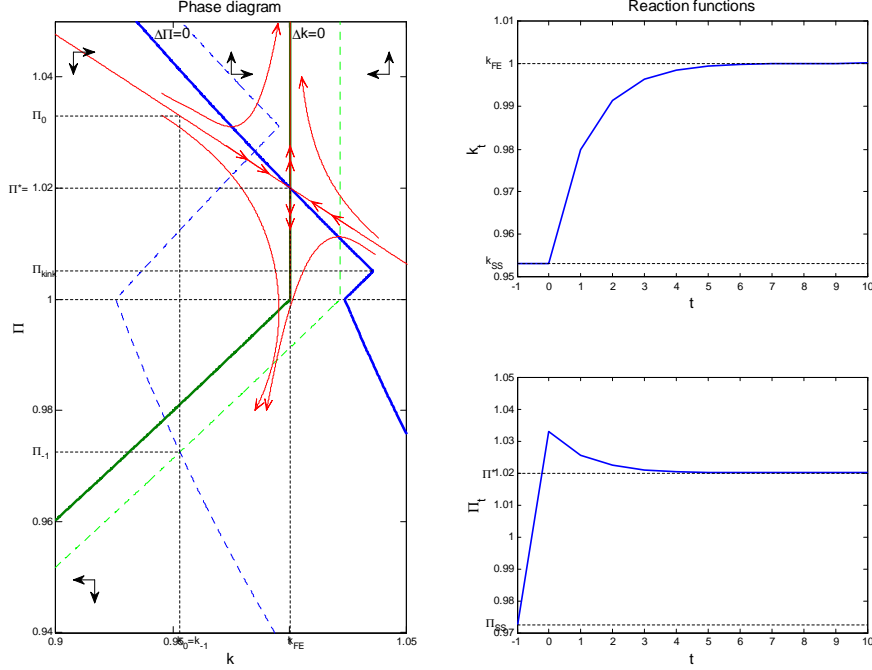


Figure 3: Dynamics of recovery

to its original level only after 7 periods. In other words, the fall into secular stagnation appears significantly faster than the recovery. This observation suggests that the economic policies against the secular stagnation must be made within the shortest possible time, preferably even before secular stagnation appears.

We observe again that the dynamics of inflation is characterized by an overshooting. However, in this case the monetary policy is active and an increase in  $\phi_{\Pi}$  can mitigate its importance. To see this, consider first the autonomous dynamics of capital characterized by  $k_{t+1} = \frac{1}{1+n} \mathcal{S}(k_t, 1)$  in the neighborhood of the full employment equilibrium:  $\frac{k_{t+1} - k_{FE}}{k_{FE}} = \eta \frac{k_t - k_{FE}}{k_{FE}}$  where  $\eta < 1$ . The stable manifold as illustrated in Fig. is then characterized by the following equation  $\frac{\Pi_t - \Pi^*}{\Pi^*} = -\frac{\eta}{\phi_{\Pi} - \eta} \frac{k_t - k_{FE}}{k_{FE}}$  (see Appendix), where  $\lim_{\phi_{\Pi} \rightarrow \infty} \left( -\frac{\eta}{\phi_{\Pi} - \eta} \right) = 0 \iff \lim_{\phi_{\Pi} \rightarrow \infty} (\Pi_t - \Pi^*) = 0 \forall t$ . The characterization of the dynamics of secular stagnation with capital accumulation is important not only to determine the speed of converge towards the secular stagnation steady state and its dynamic features (overshooting of deflation for example), but also to highlight the asymmetry.

It is worth noting that the existence of a secular stagnation equilibrium is not due only to the effects of the financial crisis. In particular, without going further into the details



of the model, a decrease in the growth of the labor force ( $n$  decrease) and an increase in the life expectancy (that can be associated with an increase in  $\beta$  then with an increase in the saving rate  $s$ ) also participate to explain the secular stagnation. According to Larry Summers in 2013 in the Financial Times, the secular stagnation may well have become the "new normal". In addition to the stabilization of financial markets, any economic policy that could be effective in the fight against the secular stagnation must be considered. We can think to monetary and fiscal policies, but also to structural policies designed to make the labor market more flexible.

## 4 Economic policy

As secular stagnation can exist only as wages are downwardly rigid, we can naturally think that promoting growth and employment goes through increasing the flexibility of the labor market. However, in secular stagnation, it has a paradoxical impact as a decrease in the wage rigidity  $\gamma$  tends to reduce the production and the employment. One may be surprised by this result. Indeed, when there is no rigidity, i.e.  $\gamma = 0$ , the production is always equal to its potential. However, this result can be easily explained. In secular stagnation, a stronger nominal wage flexibility results in recessionary effects because it generates deflationary pressures, and therefore, as monetary policy is constrained by the ZLB, an increase in the real interest rate ( $R = \frac{1}{\Pi}$ ). The demand, and then the effective production, are reduced at equilibrium. Paradoxically, a higher nominal wage flexibility yields an increase in the real wage. Increasing the labor market flexibility, unless it becomes total, has then counterproductive effects for the economy. Monetary and fiscal policies must then be considered in details.

### 4.1 Monetary policy

Suppose that the economy is characterized by a unique deflationary secular stagnation equilibrium as shown in Figure 1b ( $\Pi_{kink} > \Pi^*$ ). To get out of such a equilibrium, the monetary authorities can choose a policy consisting in increasing the inflation target  $\Pi^*$ . In this case, we see that this increase in the inflation target also increases  $\Pi_{kink}$  but less than proportionately:  $\frac{d\Pi_{kink}}{d\Pi^*} = 1 - \frac{1}{\phi_\pi} < 1$ . Starting from a situation characterized by kink  $\Pi_{kink} > \Pi^*$ , two configurations can then happen. Firstly, the increase of  $\Pi^*$  is not sufficient

and thus  $\Pi_{kink}$  kink remains below  $\Pi^*$ . In this case, the secular stagnation equilibrium is unique and the unchanged monetary policy is ineffective. Secondly, the upward of  $\Pi^*$  is sufficient such that  $\Pi^*$  becomes greater than  $\Pi_{kink}$ . In this case, as shown in Figure 4, the full employment equilibrium appears. This does not mean that monetary policy will necessarily be effective. It is observed in fact that the secular stagnation balance still exists. Therefore, nothing indicates that inflation expectations will automatically jump on the high saddle path converging to  $\Pi^*$ . It should be emphasized two properties. Firstly, the secular stagnation equilibrium is locally determined. Secondly, an infinity of other inflation levels  $\Pi \in (\Pi_{ss}, \Pi_{sup})$  bring on potentially convergent trajectories towards  $\Pi_{ind} < \Pi^*$ , where  $(k_{FE}, \Pi_{ind})$  is a locally stable equilibrium<sup>3</sup>. From this point of view, in line with Benhabib et al. (2001), we can consider that the Taylor rule involves a risk of destabilization even though its primary ambition is stabilizing.

In such a configuration, the anchoring of inflation expectations of private agents to conduct towards the right equilibrium is a difficult task for monetary authorities. For inflation targeting to be effective, it is crucial in particular that the central bank be credible enough (Woodford, 2004). While private agents do not believe the central bank when it announces a new inflation target, it is likely that inflation obtained is not equal to the target. The credibility of the central bank is then directly related to its ability to achieve its target in the past. In secular stagnation, the central bank can not by definition carry out target (monetary policy is inactive). Such property suggests that central bank has to react quickly enough to avoid deflationary trap.

## 4.2 Fiscal policy

It is easy to see that a fiscal policy will have inflationary effects and if they are strong enough they can afford to exit from secular stagnation. However, it is worth noting that such a policy will harm the long-term potential (proposition 3).

**Proposition 3** *The fiscal policy reduces the level of capital associated with full employment:  $\frac{dk_{FE}}{dG} < 0$ .*

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<sup>3</sup>In the appendix, we check that the normal equilibrium is associated locally to a higher eigenvalue to the unit and a lower eigenvalue, while the second balance is associated with two lower eigenvalues to unity.

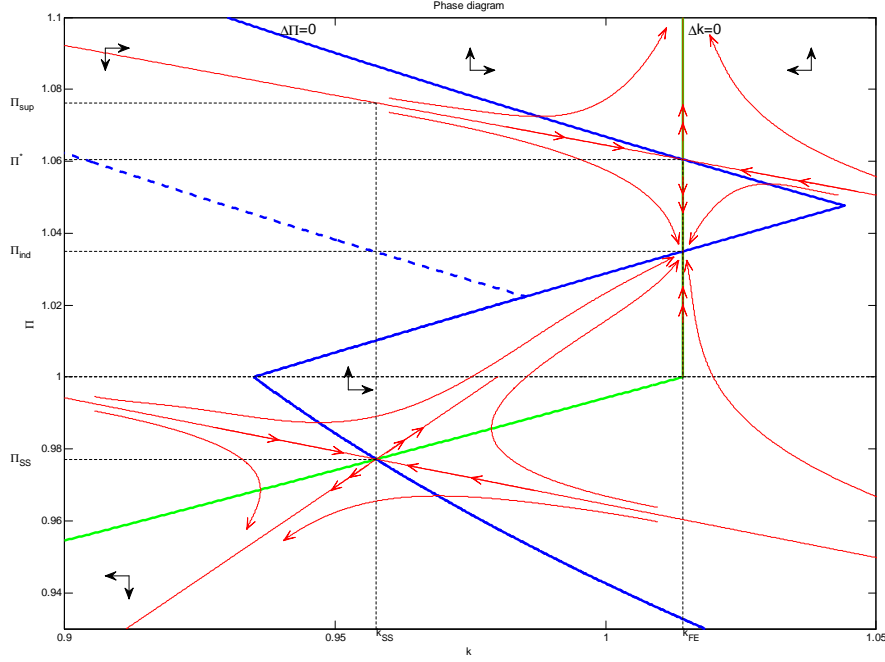


Figure 4: Secular stagnation, monetary policy and global indeterminacy

**Proposition 4** *There exist  $G_{\text{sup}} = AD^\alpha - \left(\frac{1+n}{s} + \delta\right) D$  and  $G_{\text{inf}} = A(D\Pi^*) - \frac{1+n-s(1-\delta)}{s} D\Pi^*$ ,  $G_{\text{sup}} \geq G_{\text{inf}}$ , such that:*

- $G < G_{\text{inf}}$  yields that the secular stagnation equilibrium is unique and then globally determined,
- $G > G_{\text{sup}}$  yields that the full employment equilibrium is unique and then globally determined.

This proposition sheds light on two threshold levels of public spending. If the fiscal stimuli are "too small" such that  $G < G_{\text{inf}}$ , the economy stays in secular stagnation (Fig. 6). Even if economic growth is enhanced and unemployment reduced, there remains some unemployment and additional public spendings are required to leave the secular stagnation regime. It is the case if  $G > G_{\text{sup}}$ . However, we can observe on Fig. 5 that either the long-run multiplier is higher than the short-term one if the fiscal stimuli are not too large, or the reverse if they are too large. Indeed, as exhibited on Figs 7 and 8, taxation are also associated with a reduction in the incentives to save. There is then a trade-off between the exit of secular stagnation and the accumulation of capital. In Fig. 7

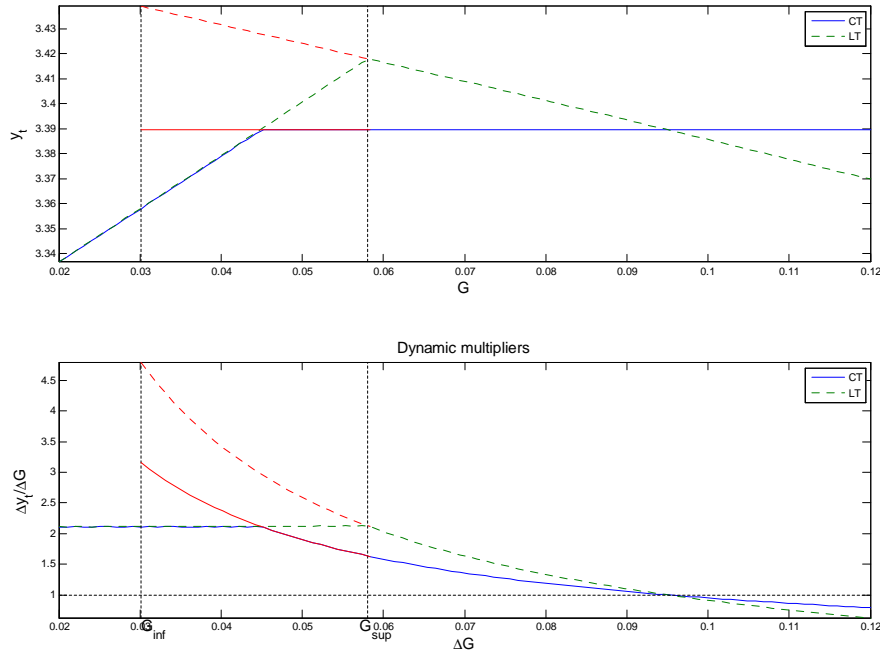


Figure 5: Impact of fiscal policy: production and multipliers (short and long run)

taxation is sufficiently small so that capital accumulation is promoted compared with the initial equilibrium with no public spendings. Therefore, after period 0, the capital stock increases and the fiscal multiplier exhibits an increased shape. By contrast, if taxation is too large, incentives to save are lowered so that there is less capital at the full employment equilibrium compared with the initial secular stagnation equilibrium. The fiscal multiplier exhibits a decreasing shape. Note also that if  $G_{\text{sup}} < G < G_{\text{inf}}$ , the equilibrium is not determined. The economy can jump out the secular stagnation to converge towards the full employment equilibrium (in red on Fig. 5), or stays in.

## 5 Conclusion

The secular stagnation hypothesis invites to reconsider the traditional macroeconomic analysis and therefore the design of economic policies. In this article, we have developed a capital accumulation model that integrates two types of market imperfections which affect respectively the credit market (rationing) and the labor market (nominal rigidity). The lessons of this model are many.

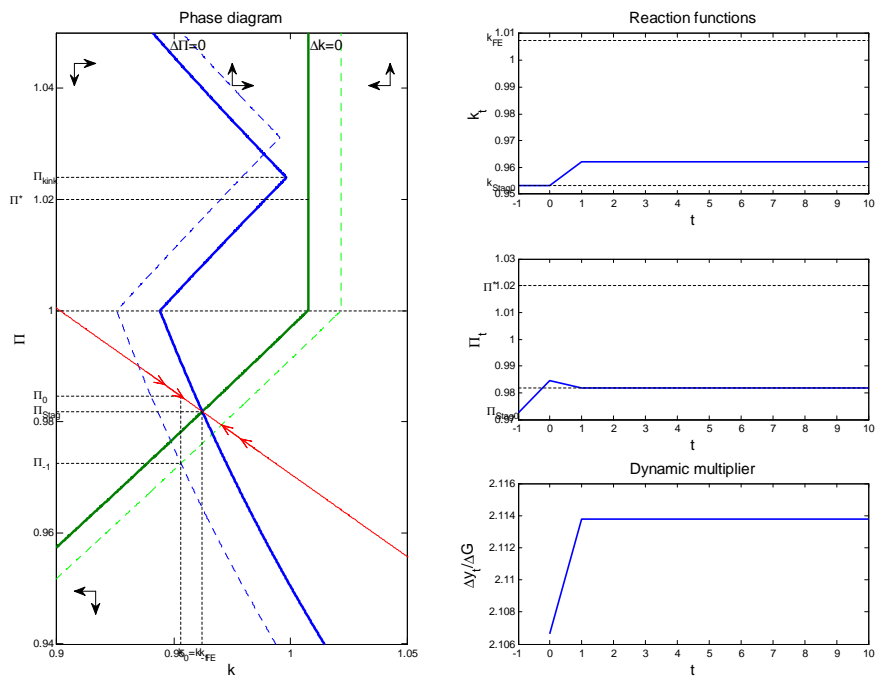


Figure 6: Fiscal policy: Impact of a "too small" fiscal stimulus

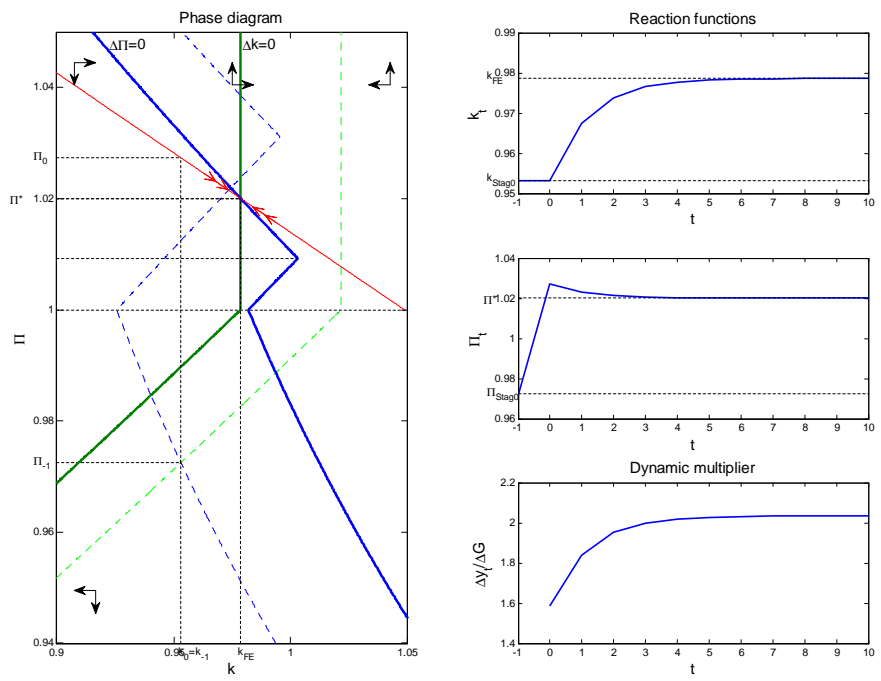


Figure 7: Fiscal policy: impact of a large stimulus

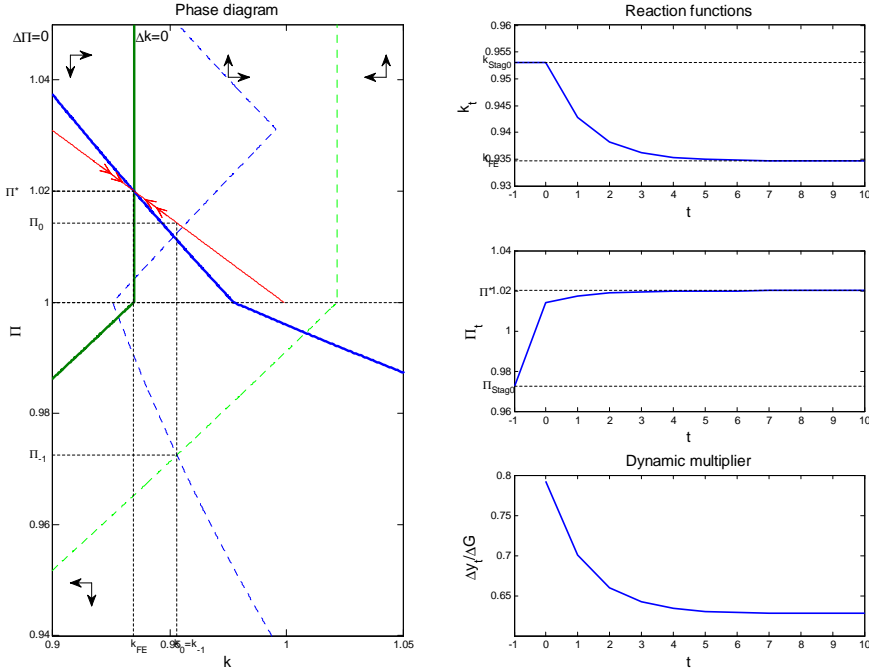


Figure 8: Fiscal policy: Impact of a "too large" stimulus

The emergence of a nominal rate close to zero ("zero lower bound") raises fears of a loss of effectiveness of "conventional" monetary policy based mainly on the setting of a official interest rate. In a context where the rate of inflation and the interest rate of full employment equilibrium are negative, macroeconomic dynamics can lead to permanent unemployment trajectories characterizing a regime of secular stagnation.

Then, to avoid ZLB, there is an urgent need to create inflation but also to avoid the "bubbles" on speculative assets (Tirole, 1985). Such a perspective might require special regulation (Gali, 2014). The existence of a deflationary equilibrium calls to question the merits of monetary policy rules too focused on inflation (Benhabib et al., 2001).

The model that we developed also teaches us to beware of deflationary effects of growth of potential output policies. The right policy mix is to support structural policies in a sufficiently accommodative monetary policy.

Reducing the savings to raise the real interest rate (for example, improving access to credit) is also an interesting idea but the impact on potential GDP is negative. There is a clear choice between exit the secular stagnation and depress the potential. An interesting solution is to finance infrastructure policy, education and R & D (increase of  $A$ ) by

public borrowing (up  $R_{eq}$ ). Indeed, a strong investment policy (public or private) can satisfy the twin objectives: supporting the aggregate demand and developing the potential production.

Regarding the weakening of long-term growth, population aging can let fear a drop in productivity including obsolescence of human capital (Sala-i-Martin, 1995; Cross et al 2012.). At the same time, the development of the "silver" economy could be a new source of growth with new needs (Tabata and Hashimoto, 2010) and specific R & D issues. In the long term, the scarcity of land resources (energy reserves, environmental stress) in the context of a large global population are also a major concern of the weakening of potential income.

Two extensions of this model require special attention.

First, it would be useful to introduce an hysteresis effect due to a shock on the production or demand. The hysteresis is characterized by the persistence of the consequences while the causes have disappeared. The cost of time may result in disqualification of unemployed workers and destruction of unused productive capital. The potential then naturally weakens. Staying in the crisis is particularly harmful and needs to quickly close the output gap to avoid the accumulation of negative effects, although paradoxically negative impact productivity have potentially favorable inflationary effects.

Second, this model passes over the international dimension by focusing only on domestic markets. Bernanke (2015) asserts that the opportunity to profitable investments outside national borders reduces the relevance of the secular stagnation hypothesis. Nevertheless, it is undeniable that the opening of borders has profoundly changed the macroeconomic forces. Questioning the perverse effect of globalization in terms of secular stagnation remains interesting. The permanent reduction in the overall demand for domestic goods caused a deindustrialization (significant drop in potential in some areas), imported unemployment and a deterioration in the current account implying a massive capital inflows in a context of strong currency by comparison with emerging markets and of low interest rates. Furthermore, the analysis in an open economy invites to question the interactions of monetary policies and in particular their impact on exchange rates. Indeed, a strong rise in the value of national currency against other currencies has consequences in terms of imported inflation.

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## Appendix: Study of the trajectories in the neighborhood of steady states

**Case 1.** Full employment and satisfied inflation target ( $i \geq 0, l = 1$ ) :

The equation (19) can be written  $\frac{D}{k_{t+1}} = \frac{1+i^*}{\Pi^{*\phi_\pi}} \frac{\Pi_t^{\phi_\pi}}{\Pi_{t+1}}$ . After log-linearization of this equation and the equation (??), we obtain:

$$\tilde{\pi}_{t+1} = \phi_\pi \tilde{\pi}_t + \eta \tilde{k}_t \quad (25)$$

and

$$\tilde{k}_{t+1} = \eta \tilde{k}_t \quad (26)$$

where  $\eta = \frac{\alpha A k^{\alpha-1} + 1 - \delta}{A k^{\alpha-1} - \frac{D+G}{k} + 1 - \delta}$ . In matrix form, the local dynamics can be expressed as follows:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} \phi_\pi & \eta \\ 0 & \eta \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

where  $\lambda_1 = \eta$  and  $\lambda_2 = \phi_\pi$  are the two eigenvalues ( $\det(M) = \eta\phi_\pi = \lambda_1\lambda_2$  and  $\text{tr}(M) = \phi_\pi + \eta = \lambda_1 + \lambda_2$ ). In the neighborhood of the steady state, we have  $\eta < 1$  and  $\phi_\pi > 1$ , the full employment steady state with satisfied inflation target is a saddle point.

**Case 2.** Full employment with unsatisfied inflation target ( $i = 0, l = 1$ ) :

In this case, the equation (??) and its linearized form (26) are still valid. By contrast, we have  $\Pi_{t+1} = \frac{k_{t+1}}{D}$  and then:

$$\tilde{\pi}_{t+1} = \tilde{k}_{t+1} = \eta \tilde{k}_t \quad (27)$$

In this case, the local dynamics is characterized by the following system:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} 0 & \eta \\ 0 & \eta \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

where  $\lambda_1 = \eta < 1$  and  $\lambda_2 = 0$  are the two eigenvalues. In this case the dynamics are undetermined.

**Case 3.** Secular stagnation equilibrium ( $i = 0, l < 1$ ) :

We deduce from (??):

$$\tilde{y}_t = \nu \tilde{k}_{t+1} - \frac{(1-\delta)s}{1+n} \nu \tilde{k}_t \quad (28)$$

where  $\nu = \frac{1}{1 + \frac{s}{1+n} \frac{D+G}{k} - \frac{(1-\delta)s}{1+n}}$ . Furthermore as  $y_t^s = A k_t^\alpha l_t^{1-\alpha}$ , we deduce  $\tilde{y}_t = \alpha \tilde{k}_t + (1-\alpha) \tilde{l}_t$ . Profit maximization gives  $(1-\alpha) k_t^\alpha l_t^{-\alpha} = w_t$  and  $l_t = \left(\frac{w_t}{1-\alpha}\right)^{-\frac{1}{\alpha}} k_t$ . We find  $\tilde{l}_t = \frac{-1}{\alpha} \tilde{w}_t + \tilde{k}_t$ , and:

$$\begin{aligned} \tilde{y}_t &= \alpha \tilde{k}_t + (1-\alpha) \left( \frac{-1}{\alpha} \tilde{w}_t + \tilde{k}_t \right) \\ &= \tilde{k}_t - \frac{1-\alpha}{\alpha} \tilde{w}_t \end{aligned} \quad (29)$$

Moreover, given the rigidity of nominal wages expressed by  $w_t = \gamma w_t \frac{1}{\Pi_t} + (1-\gamma)(1-\alpha) k_t^\alpha$ , the log-linear form can be written as follows  $\tilde{w}_t = \frac{\gamma}{\Pi} (\tilde{w}_t - \tilde{\pi}_t) + (1-\gamma)(1-\alpha) \alpha \frac{k^\alpha}{w} \tilde{k}_t$ , where  $\frac{k^\alpha}{w} = \frac{1-\frac{\gamma}{\Pi}}{(1-\gamma)(1-\alpha)}$ . We find:

$$\tilde{w}_t = \alpha \tilde{k}_t - \frac{\frac{\gamma}{\Pi}}{1 - \frac{\gamma}{\Pi}} \tilde{\pi}_t \quad (30)$$

By introducing equation (30) in (29), we obtain:

$$\tilde{y}_t = \alpha \tilde{k}_t + \frac{1-\alpha}{\alpha} \frac{\frac{\gamma}{\Pi}}{1 - \frac{\gamma}{\Pi}} \tilde{\pi}_t$$

From (28), we find:

$$\tilde{k}_{t+1} = \left( \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n} \right) \tilde{k}_t + \frac{1-\alpha}{\alpha\nu} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} \tilde{\pi}_t$$

Moreover, as  $\Pi_{t+1} = \frac{k_{t+1}}{D}$ , we have:

$$\tilde{\pi}_{t+1} = \tilde{k}_{t+1},$$

and we identify the dynamics in its matrix form:

$$\begin{pmatrix} \tilde{\pi}_{t+1} \\ \tilde{k}_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{1-\alpha}{\alpha\nu} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} & \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n} \\ \frac{1-\alpha}{\alpha\nu} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} & \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n} \end{pmatrix} \begin{pmatrix} \tilde{\pi}_t \\ \tilde{k}_t \end{pmatrix}$$

$\det(M) = 0$  et  $tr(M) = \frac{1-\alpha}{\alpha\nu} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} + \frac{\alpha}{\nu} + \frac{(1-\delta)s}{1+n}$ . If  $\delta = 1$ , then we deduce immediately that the two associated eigenvalues are  $\lambda_1 = \frac{1-\alpha}{\alpha\nu} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}} + \frac{\alpha}{\nu}$  and  $\lambda_2 = 0$ . A necessary and sufficient condition for a saddle-point equilibrium is  $\lambda_1 > 1$ . This condition is satisfied iff:

$$\nu < \alpha + \frac{1-\alpha}{\alpha} \frac{\frac{\gamma}{\Pi}}{1-\frac{\gamma}{\Pi}}$$

This condition is equivalent to the observation in the space  $(Y, \Pi)$  of a slope of demand  $(\frac{1}{\nu})$  which is greater than that of supply. This condition is always checked when the stagnation secular equilibrium exists (see Lemma 1).