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# A two-pillar DSGE monetary policy model for the euro area<sup>☆</sup>

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## ABSTRACT

The current financial crisis has revived the interest for monitoring both monetary and credit developments. Over the past two decades, consistent with the adoption of inflation targeting strategies by a growing number of central banks and the development of New Keynesian models for which monetary aggregates are largely irrelevant, money and credit have been progressively neglected in the conduct of monetary policy. A striking exception has been the Eurosystem, which has implemented a strategy known as the “two-pillar monetary policy strategy” giving a prominent role for money. In this paper, we develop a small optimizing model based on Ireland (2004), estimated on euro area data and featuring this two-pillar strategy. We evaluate an ECB-style cross-checking policy rule in a DSGE model with real balance effects of money. We find some evidence that indeed money plays a non-trivial role in explaining the euro area business cycle. This provides a rationale for the central bank to factor in monetary developments but also raises some issues regarding the reliability of M3 as an appropriate monetary indicator. We find some evidence that the ECB has systematically reacted to a filtered measure of money growth but weak evidence it has reacted more aggressively during excess money growth periods.

## 1. Introduction

The current financial crisis has reminded both academics and central bankers of the crucial role played by money and credit in economic and financial developments. Such a renewed interest stems from two facts. First, central banks too narrowly focused on short-term price developments and neglecting money and credit growth may overlook the built-up of financial imbalances. In the current crisis episode for instance, this built-up was associated with an housing price bubble, fueled by financial innovations in the mortgage market and reflected in the balance sheets of financial institutions, thus in both money and credit developments. The latter could have informed the policy maker about growing financial risks, would have them been closely monitored. Second, as a response to this crisis, central banks have massively expanded their balance sheets in order to stimulate their economies in a context of near zero interest rates, thereby expecting some real balance effects of money.

There are three main reasons for why money and credit have been neglected over the past two decades.

First, the long-term relationship postulated between monetary developments and inflation, and known as the quantity theory of

money, has been challenged over time. Despite Lucas' (1980) claim that the two quantity-theoretic propositions, namely that a given change in money growth induces an equal change in price inflation and an equal change in nominal rate, “possess a combination of theoretical coherence and empirical verification shared by no other propositions in monetary economics”, several authors, like McCallum (1984) and Whiteman (1984), have challenged this view and shown that these links were in fact regime dependent. In a recent contribution, Sargent and Surico (2010) show, based on a DSGE model embedding a money growth rule, that an aggressive policy rule can prevent the emergence of persistent movement in money growth and that such a policy regime is also characterized by low low-frequency correlation between inflation and money growth and between interest rate and money growth. In addition, Goodhart (1975) shows that “any observed statistical regularity will tend to collapse once pressure is placed upon it for control purposes.” The Goodhart's law therefore attribute the breakdown of former low frequency correlation between money growth and inflation to the implementation of monetary targeting strategies in the 1970s. Finally, structural shifts in velocity, due for instance to financial innovations, may also account for a reduced correlation between money and inflation. This point is made for example by Lucas (1988) or Orphanides and Porter (2000) for the US and more recently by Bordes et al. (2007) for the euro area.

Second, and as a consequence of the former point, the chronic instability of the relationship between money growth and inflation as well as doubt on the controllability of monetary aggregates in a context of a rapid pace of financial innovations have contributed to

<sup>☆</sup> The views expressed in this paper do not necessarily reflect the opinion of the Banque de France. We thank Harris Dellas, Behzad Diba, Michel Juillard, François Langot, Edward Nelson, Celine Poilly, Michael Woodford and participants to seminars at the Banque de France and CEPREMAP for their very helpful comments. All remaining errors would be ours.

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the decline of monetary targeting strategies. These monetary policy regimes have been progressively replaced by inflation targeting strategies since the beginning of the 1990s. In this new set up, money and credit can still have an information role, which is limited at best as a kind of add-on or cross-check. The focus however is displaced from low-frequency correlations to high-frequency ones, inflation targeting central banks tending to limit their assessment of risks to price stability to their forecast horizon. Consequently, their inflation target is generally defined over a two or three-year horizon for which the correlation between money growth and inflation is usually weaker and the information content of money dominated by a wide set of economic indicators.

Third, New Keynesian models have become predominant, both in academia and central banks, for monetary policy purposes. In standard New Keynesian frameworks, such as those developed by [Clarida et al. \(1999\)](#) or [Woodford \(2003\)](#), optimal monetary policy can be formulated as an interest rate reaction function or policy rule, which is consistent with the current practice of central banks, but without any reference to money. Money is usually appended to these models through a money demand equation. As monetary policy is implemented by fixing the short-term interest rate, it becomes essentially endogenous. This feature has led some model builders to drop money completely from the models used to study monetary policy.

There are however few attempts to allow for a special role for money, even in the context of New Keynesian frameworks. Monetary aggregates can for example play an important role in the context of imperfect information as indicator variable to deal with data uncertainty (see for instance [Coenen et al., 2005](#)), model uncertainty and persistent misperception of key economic variables such as the output gap (see for instance [Beck and Wieland, 2008](#)). A even more active role for money is acknowledged in the context of financial crises for which several authors recommend the switch from an interest rate rule to a money base rule (see [Christiano and Rostagno, 2001](#) or [Christiano et al., 2009](#)). In the vicinity of the zero lower bound for nominal interest rate, this case is even more stringent (see by example [Orphanides and Wieland, 2000](#)) and provides a rationale for the implementation of quantitative easing policies.

In this paper, we develop a DSGE model in which money enters into the structural equations of an otherwise standard New Keynesian model. Our main objective is to assess the extent to which money plays an active role in business cycle fluctuations in the euro area and is factored in by the European Central Bank (ECB hereafter) in the context of its two-pillar monetary policy strategy. Indeed, since its inception, the ECB has given a key role to monetary aggregates and their counterparts to assess the risks to price stability over the medium term in the euro area. So far, the two-pillar strategy implemented by the Eurosystem has received little support in the literature contrary to the inflation targeting framework. [Christiano et al. \(2006\)](#) and [Beck and Wieland \(2007\)](#) are the main contributions in this area. Contrary to [Woodford \(2007\)](#) who argues that there is no compelling reason to assign a prominent role to monetary aggregates in the conduct of monetary policy, we defend such a role by noting, first, that a monetary pillar offers an effective anchor for the price level and therefore would remove the long-term uncertainty about the price level associated to inflation targets and, second, by showing that indeed money plays an effective role in the economy.

Our “two-pillar” DSGE monetary policy model attempts to reflect both the essence of the Eurosystem’s monetary policy framework and the role of money in the conduct of monetary policy. To analyze this role, we elaborate on a model developed by [Ireland \(2004\)](#), which is a small scale DSGE model with sticky prices and monopolistic competition but without capital accumulation. In our setup, we first allow money to have an effective influence on both output and inflation and, second, the central bank to react to a filtered money growth indicator but not necessarily in a linear way. Unlike [Ireland](#)

(2004) and [Andrès et al. \(2006\)](#), we estimate our model resorting to Bayesian techniques rather than by maximum likelihood methods. By doing so, we impose less constraints on key parameters, in particular on the parameter measuring the effects of real balances on output and inflation. Indeed, the two above mentioned papers impose a non-negativity constraint on that parameter, which artificially leads to a zero coefficient estimate. As a consequence, these authors conclude that real balances have a limited, if any, role in explaining business cycle fluctuations. Two recent contributions by [Favara and Giordani \(2009\)](#) and [Canova and Menz \(2010\)](#) also argue that the cross-restrictions imposed by Ireland force estimates of the impact of money on other variables to zero, supporting our main findings.

By contrast, we show that money plays a non-trivial role in explaining the euro area business cycle. This provides a rationale for the central bank to factor in monetary developments but also raises some issues regarding the reliability of M3 as an appropriate monetary indicator. We also find some evidence that the ECB has systematically reacted to a filtered measure of money growth but weak evidence it has reacted more aggressively during excess money growth periods.

The paper proceeds as follows. Section 2 of the paper presents the main features of the ECB’s strategy and monetary analysis and the way we model it. Then, Section 3 details our two-pillar DSGE model for the euro area. Section 4 provides the model resolution and estimation. Finally, Section 5 discusses the role of money according to our estimates.

## 2. The European Central Bank and its monetary analysis

In this section, we describe how we model the ECB’s decision rule. First, we present the main features of the ECB monetary analysis and second we propose a formalization of the ECB’s policy rule.

### 2.1. A quick review of the role of money in the strategy

The most distinctive feature of the European Central Bank’s monetary policy framework has become known as the two-pillar strategy for assessing risk to price stability. In this very specific framework, money is given an important role, which, since the inception of the ECB, has been signalled by the announcement of a reference value for the growth of a broad monetary aggregate (M3).

The two-pillar strategy was reviewed in May 2003. While confirming the use of the two-pillar framework, the ECB’s Governing Council also emphasized that the “monetary analysis” (the former “first pillar” of the strategy) will mainly serve as a mean of cross-checking, from a long-term perspective, the indication stemming from the “economic analysis” (the former “second pillar”). In addition, and to underscore the long-term nature of the reference value, the Governing Council decided to discontinue the practice of an annual review. In practice, the reference value has not been reviewed since the inception of the ECB (it has remained unchanged at 4.5% since December 1998). This decision was interpreted by most observers as a downgrading of the role of money.

What is then the real role of money in the ECB’s monetary policy strategy and how is it factored in practice?

As far as the strategy is concerned, the role given to money acknowledges the fact that monetary growth and inflation are closely related in the medium to long run. Indeed, empirical studies carried out at the euro area level seem to confirm the monetarist statement according to which “inflation is always and everywhere a monetary phenomenon” ([Friedman, 1963](#)). In a recent contribution, [Bordes and Clerc \(2007\)](#) try to set out the need to announce a monetary growth reference value in the context of a two-pillar small backward-looking macroeconomic model. Their main point is that, contrary to the assumption usually made in New Keynesian frameworks, the central bank’s influence over the nominal interest rate does not operate in the

same way in the short term and in the long term. In the short term, the central bank can influence the nominal interest rate by increasing the quantity of money. This increases real balances and lowers the real interest rate and, consequently, the nominal interest rate through a liquidity effect. The liquidity effect however does not come into play in the medium to long run and, consequently, the central bank cannot influence the real interest rate. Its only means of action with regard to the nominal rate is to influence inflation expectations. This can be done through the announcement of a money supply growth target, which is derived in a way as to ensure the consistency between short-term and long-term inflation expectations. A similar argument is made by Nelson (2008). In this context, the reference value helps to reduce long-term price level uncertainty and acts as an error-correction mechanism ensuring the trend stationarity of the price level. Similarly, in the context of inaccurate estimates or imperfect knowledge regarding unobservables such as the output gap, Woodford (2007) and Beck and Wieland (2007) both argue that the ECB's computation of excess liquidity (i.e. deviation of actual M3 from the reference value), the cross-checking and finally the possibility to change interest rates in response to sustained deviations of long-run money growth are similar to the commitment to error-correction and therefore can have some stabilization properties.

In practice, money also seems to play a crucial role in the setting of monetary policy in the euro area. As an illustration, the ECB's President, J. C. Trichet, recently declared: "I consider the monetary pillar has been probably decisive when we decided to increase rates in December 2005, against the advice of the OECD, the IMF, and a number of observers", (Financial times, 17 May 2007). However, implementing "monetary analysis" in the euro area has proven difficult and very challenging over the recent years as the economy was hit by several shocks (see Papademos and Stark, 2010 for a comprehensive overview of monetary analysis and its main challenges): financial instability in the aftermath of the stock market collapse in 2000 and since Summer 2007, exceptionally high economic and geopolitical uncertainty between 2001 and 2003. In addition, there are some signs that a money demand shock occurred at the beginning of the 2000s in the euro area, as illustrated by an apparent structural shift in the trend velocity of money as evidenced by Bordes et al. (2007). Certainly, the uncertainties surrounding the assumed underlying trend in M3 income velocity have led the ECB to downplay the role of the reference value, which, indeed, has not been referred to in any introductory statement by the ECB's President since December 2002.

## 2.2. A two-pillar monetary policy rule

The aim of this subsection is to identify a monetary policy rule featuring the ECB's two-pillar approach, as clarified in the context of the review of the monetary policy strategy carried out in May 2003. The main aspects that we try to encapsulate in our setup are:

- a role for money in the setting of euro area policy rates. The reason is twofold: first, as our model will allow real balances to appear both in the IS and the Phillips curves, it may be optimal for the central bank to adjust key policy rates with respect to monetary developments; second, monetary analysis is one of the two pillars of the strategy. As such, it complements the information stemming from the economic analysis, usually summarized by both the inflation and the production stabilization objectives in standard versions of the Taylor rule.
- a medium to long-term orientation, as initially signalled by the announcement of a reference value for M3. Indeed, the Governing Council of the ECB seeks to exploit the long-term relationship between monetary growth and inflation, that is to say tries to see through the noise in monetary data to recover those underlying trends which are relevant for monetary policy decisions.

- finally, as money is not an intermediate target in the ECB's strategic framework, strong monetary developments or deviations of M3 growth from the reference value should not trigger a mechanistic policy reaction. Therefore, in contrast with Ireland (2004), Christiano et al. (2006) or Bordes and Clerc (2007), who model the policy rule as an augmented Taylor rule embedding actual monetary growth and assume a systematic response of the central bank to monetary developments, we allow for a non-linear or an asymmetric response of the central bank. In "normal time", when the monetary analysis carried by the Governing Council of the ECB does not signal risk to price stability over the medium run, the central bank may not or little react to on-going monetary developments. By contrast, when too strong monetary developments bear some risks to future price stability, i.e. in the presence of "excess liquidity", the central bank may react much stronger to money growth so as to bring prices to the appropriate or targeted price level path.

In order to reflect these different aspects of the two-pillar strategy, we first allow money growth to enter into a standard Taylor rule, in addition to the inflation and production stabilization objectives. However, we assume that the central bank, in contrast to the private sector – households in particular – focuses on a filtered measure of money growth. As in Beck and Wieland (2007), we assume that the central bank regularly tests whether filtered money growth still hovers around its long-term average. More specifically, we suppose that the central bank checks the following inequality:

$$\frac{1}{N_c} \sum_{k=1}^{N_c} \frac{|\widehat{\mu}_{t-k}^f - \bar{\mu}_t^f|}{\sigma_{\mu,t}} \geq \kappa_c \quad (1)$$

where

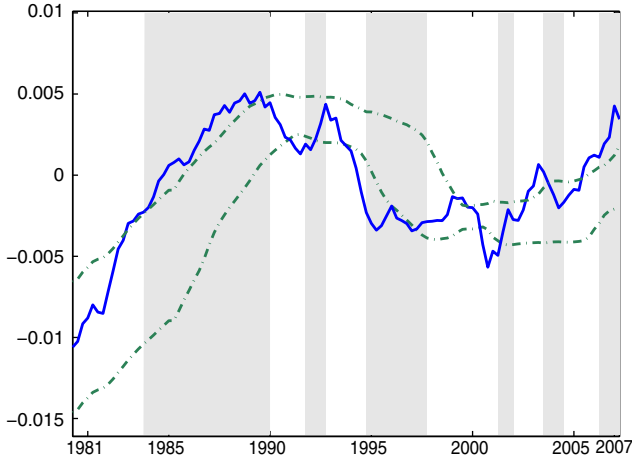
$$\widehat{\mu}_t^f = \widehat{\mu}_{t-1}^f + \delta (\widehat{\mu}_t - \widehat{\mu}_{t-1}^f) \quad (2)$$

is the filter used to approximate long-run values of money growth.  $\widehat{\mu}_t$  stands for (linearly de-trended) real money growth. The smoothing parameter  $\delta$  could be chosen so as to select a frequency at which long-run money growth is highly correlated with long-run inflation. As suggested in Gerlach (2004), we choose  $\delta = 0.15$ .  $\kappa_c$  corresponds to the critical value considered by the central bank.  $\bar{\mu}_t^f = \frac{1}{20} \sum_{k=1}^{20} \widehat{\mu}_{t-k}^f$  is the mean of the filtered money growth computed over the last 20 periods (i.e. the last five years in our model), while  $\sigma_{\mu,t}$  stands for the standard deviation of  $\widehat{\mu}_{t-k}^f$  computed over the same period. Therefore, the central bank assesses monetary developments using a time-varying window in order to capture medium to long-run shifts in monetary trends or velocity.

If the central bank obtains successive signals of a sustained deviation of filtered money growth from its medium to long-term average (i.e. if  $|\widehat{\mu}_{t-k}^f - \bar{\mu}_t^f| \geq \kappa_c \sigma_{\mu,t}$  on average over the last  $N_c$  periods), it responds by adjusting its key policy rate stronger than would have been the case otherwise. Therefore, the central bank sets the nominal interest rate according to the following augmented Taylor-type rule:

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1 - \rho_r) \left[ \rho_y \widehat{y}_{t-1} + \rho_\pi \widehat{\pi}_{t-1} + \rho_1 1_1 \widehat{\mu}_{t-1}^f + \rho_2 1_2 \widehat{\mu}_{t-1}^f \right] + u_{r,t} \quad (3)$$

where  $\widehat{y}_{t-1}$  is the lagged output,  $\widehat{\pi}_{t-1}$  the lagged inflation,  $1_1$  and  $1_2$  are two dummy variables such that:  $1_1 = 1$  if  $|\widehat{\mu}_{t-k}^f - \bar{\mu}_t^f| < \kappa_c \sigma_{\mu,t}$  on average over the last  $N_c$  periods,  $1_1 = 0$  otherwise;  $1_2 = 1 - 1_1$ .



**Fig. 1.** Filtered money growth and thresholds for  $(\kappa_c, N_c) = (1.1, 5)$ . The filtered money growth is in thick line. The smoothing parameter,  $\delta$ , is set to 0.15. The corridor in dashed lines defines the expected range of evolution of the filtered money growth given its last five year average and standard deviation. The shaded area stands for regime 2 corresponding to situation with excessive money growth (outside the corridor filtered money growth) for more than one year.

Therefore, the central bank seeks to distinguish between “normal” periods (or state 1) and inflationary ones (state 2), the state of the economy depending upon monetary dynamics. In this setup, we expect that  $\rho_2 > \rho_1 \geq 0$ . Despite this state-contingent rule, the model has a unique steady state.<sup>1</sup> As an illustration, Fig. 1 displays the corridor corresponding to the “normal” state, for a choice of  $(\kappa_c, N_c) = (1.1, 5)$ . According to this set of parameters, the euro area economy was during 59 periods out of 109 in the strong monetary growth state between 1982 and 2007.

### 3. A Two-Pillar DSGE model for the euro area

The rest of our model is based on the paper by Ireland (2004), who develops a small structural model of the monetary policy business cycle. We only give the main characteristics of this model and refer the reader to Ireland (2004) for further details. A key feature of this model is that it allows real balances to appear both in the IS and the Phillips curves. This direct effect of money on output and inflation could provide a justification to include a measure of money growth into the monetary policy rule as suggested above. But contrary to Ireland (2004) who add price rigidities à la Rotemberg (1982), we follow Christiano et al. (2005) and assume that prices are set on a staggered basis as in Calvo (1983). This is done so as to bring nominal rigidities into the model in a way that is consistent with and that provides micro-foundations to the two-pillar Phillips curve's representation proposed by Gerlach (2004).

Our economy consists of a representative household, a representative finished goods-producing firm, a continuum of intermediate goods-producing firms indexed by  $i \in [0, 1]$  and a monetary authority featuring the European Central Bank already described.

#### 3.1. Households

The representative household chooses real consumption,  $c_t$ , government bonds,  $B_t$ , hours worked,  $h_t$  and real money,  $m_t$  to maximize his intertemporal utility function:

$$E \sum_{t=0}^{\infty} \beta^t a_t \{u[c_t, m_t / e_t] - \eta h_t\}$$

<sup>1</sup> This property follows from the assumption that  $m_t$  is stationary and thus  $\bar{m} = 1$ .

where  $\beta \in ]0, 1[$  is the discount factor and  $\eta$  is a positive parameter.  $a_t$  is a preference shock affecting stochastic discount factor while  $e_t$  is a shock on preference for money. We also call this latter shock a velocity shock because it is responsible for shifts in the money demand equation. The household maximizes this objective subject to the following budget constraint:

$$P_{t-1}m_{t-1} + B_{t-1} + P_t w_t h_t + D_t \geq P_t m_t + \frac{B_t}{r_t} + P_t c_t$$

Where  $P_t$  denotes the nominal price of finished good at time  $t$ . The household's sources of funds consist of money carried into period  $t$ ,  $P_{t-1}m_{t-1}$ , the value of nominal zero-coupon Bonds,  $B_{t-1}$ , the labor income  $P_t w_t h_t$  where  $w_t$  is the real wage and the nominal dividend payments,  $D_t$ , received from the intermediate goods-producing firms. The use of these funds includes consumption,  $P_t c_t$ , purchasing of newly-issued bonds of  $B_t/r_t$ , where  $r_t$  is the gross nominal interest rate decided by the central bank, and the money,  $P_t m_t$  to be carried into period  $t + 1$ .

In this setup, the main difference with the standard New Keynesian assumptions is that money yields utility and that money and consumption may be non-separable in the utility function of the representative household. The first assumption is a shortcut to capture cash-in-advance without imposing constraint as shown by Feenstra (1986). The latter assumption implies that real balances may enter into the IS curve and into the New Keynesian Phillips Curve, i.e. opens up a channel through which money can affect both output and inflation.

The preference and money demand shocks,  $a_t$  and  $e_t$ , follow first order autoregressive processes as in Ireland (2004):

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \epsilon_{a,t} \quad (4)$$

$$\ln(e_t / e) = \rho_e \ln(e_{t-1} / e) + \epsilon_{e,t} \quad (5)$$

Where  $\rho_a$  and  $\rho_e$  are the persistence of shocks,  $\epsilon_{a,t}$  and  $\epsilon_{e,t}$  i.i.d zero-mean gaussian random variables with  $\sigma_a$  and  $\sigma_e$  standard deviations and  $e$  is the steady-state of the shock  $e_t$ .

#### 3.2. Firm behavior and price setting

Concerning the productive sector, our setup is similar to that of Smets and Wouters (2003). We assume that the final sector is perfectly competitive and products final goods with a Dixit-Stiglitz technology. This leads to an imperfect competitive market in the intermediary sector. Moreover, intermediary firms set their prices on a staggered basis à la Calvo. When they can not reset their prices, they can partially index their prices on last inflation rate. To close the final-goods market, we assume that aggregate demand is reduced to consumption so that output and consumption are assumed to be equal (market clearing condition).

##### 3.2.1. Finished goods-producing firms

At each period, the representative finished goods-producing firm uses  $y_t(i)$  units of each intermediate good  $i \in [0, 1]$ , purchased at the nominal price  $P_t(i)$  to manufacture  $y_t$  units of the finished good according to the constant-returns-to-scale technology describes by

$$y_t = \left( \int_0^1 y_t(i)^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)} \quad (6)$$

where  $\theta > 1$ . The finished goods-producing firm chooses  $y_t(i)$  to maximize its profits. This maximization leads to the following demand to the intermediate firm,  $i$ :

$$y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\theta} y_t \quad (7)$$



which reveal that  $\theta$  measures the constant price elasticity of demand for each intermediate good  $i \in [0,1]$ . The price  $P_t$  is defined by:

$$P_t = \left( \int_0^1 P_t(i)^{1-\theta} di \right)^{1/(1-\theta)}$$

### 3.2.2. Intermediate goods-producing firms

Even if we do not include capital in our model, the representative household owns the intermediate goods-producing firms, earns dividends and chooses to maximize the profits according to its own pricing-kernel as in [Smets and Wouters \(2003\)](#). Hence, because money matters in the utility function, households modify optimal price decision depending on the evolution of real balances. That is the reason why money finally enters in the Phillips curve.

Each intermediate goods-producing firm hires  $h_t(i)$  units of labor from the representative household to manufacture  $y_t(i)$  units of intermediate good  $i$  according to the following production function:

$$y_t(i) = z_t F(h_t(i)) \quad (8)$$

where  $F$  is increasing and concave and  $z_t$  is the aggregate productivity shock, which is assumed to follow a first order autoregressive process:

$$\ln(z_t / z) = \rho_z \ln(z_{t-1} / z) + \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim N(0, \sigma_z) \quad (9)$$

where  $1 > \rho_z > 0, z > 0$ . In equilibrium, this supply side disturbance acts as a shock to the Phillips curve.

The intermediate goods-producing firm sells its output in a monopolistically competitive market and sets nominal prices on a staggered basis, as in [Calvo \(1983\)](#). More formally, the firm resets its price with probability  $1 - \alpha_p$ , independently of the time elapsed since the last price adjustment. The remaining firms, with probability  $\alpha_p$ , set prices according to the following rule:

$$P_T(i) = \pi^{1-\gamma_p} \left[ \frac{P_{T-1}}{P_{T-2}} \right]^{\gamma_p} P_{T-1}(i) = \Gamma_T P_{T-1}(i). \quad (10)$$

that is to say that a firm that cannot optimally adjust its price sets it as a convex combination of past inflation and steady state inflation  $\pi$ . This framework implies that lagged inflation will enter into the linearized Phillips curve. Note that setting the inflation persistence parameter,  $\gamma_p$ , to zero would result in the standard Calvo model (i.e. a setup in which non-optimally adjusting firms would simply change their prices at the pace of steady state inflation).

The firms that do adjust their prices at date  $t$  do so by maximizing the expected discounted value of current and future profits converted into household's utility. Profits at some future date  $t+j$  are affected by the choice of price at time  $t$  only if the firm has not received another opportunity to adjust between  $t$  and  $t+j$ . Let  $P_t^*$  denotes the price chosen at date  $t$  and  $y_{t,T}^*(i)$  the production of good  $i$  at date  $T$  if the firm has not reset its price since date  $t$ . The firm's pricing decision problem then involves picking  $P_t^*$  to maximize

$$\Pi_t = E_t \sum_{T=t}^{\infty} (\beta \alpha_p)^{T-t} \frac{\lambda_T}{\lambda_t} \left\{ \Gamma_{t,T} \frac{P_t^*(i)}{P_T} y_{t,T}^*(i) - w_T F^{-1}(y_{t,T}^*(i) / z_t) \right\} \quad (11)$$

with respect to (7):

$$y_{t,T}^*(i) = \left( \Gamma_{t,T} \frac{P_t^*(i)}{P_T} \right)^{-\theta} y_T \quad (12)$$

where  $\lambda_t / P_t$  in Eq. (11) measures the marginal utility value to the representative household of an additional euro in profits received during period  $t$  and  $w_t$  measures the real wage paid by the firm.  $\Gamma_{t,T}$  is the coefficient which embodies the updating process of the prices.

For all  $t=0, 1, 2, \dots$  the first-order conditions for this problem are:

$$E_t \sum_{T=t}^{\infty} (\beta \alpha_p)^{T-t} \lambda_T \left[ \Gamma_{t,T} \frac{P_t^*(i)}{P_T} \frac{P_t}{P_T} - \mu \frac{w_T}{z_t} (F^{-1})' \left( \frac{y_{t,T}^*(i)}{z_t} \right) \right] = 0 \quad (13)$$

where  $\mu$  is the mark-up rate given by  $\mu = \frac{\theta}{\theta-1}$ .

### 3.3. Equilibrium and money

First we derive the first order conditions of economic agents in our economy, then we compute the steady-state and finally we log-linearize the equations around the steady state through a first order Taylor approximation as in [Uhlig \(1999\)](#) and [Woodford \(2003\)](#). Once log-linearized, the first-order conditions yield the following reduced form equations:

$$\hat{y}_t = E_t(\hat{y}_{t+1}) - \omega_1 (\hat{r}_t - E_t(\hat{\pi}_{t+1})) + \omega_2 (\hat{m}_t - E_t(\hat{m}_{t+1})) - \omega_2 (1 - \rho_e) \hat{e}_t + \omega_1 (1 - \rho_a) \hat{a}_t \quad (14)$$

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \beta E_t(\hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t) + \kappa_p \left( \omega_p + \frac{1}{\omega_1} \right) \hat{y}_t - \kappa_p \frac{\omega_2}{\omega_1} (\hat{m}_t - \hat{e}_t) - \kappa_p (1 + \omega_p) z_t \quad (15)$$

$$\hat{m}_t = \gamma_1 \hat{y}_t - \gamma_2 \hat{r}_t + \gamma_3 \hat{e}_t \quad (16)$$

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r) \left[ \rho_y \hat{y}_{t-1} + \rho_\pi \hat{\pi}_{t-1} + \rho_1 1_1 \hat{\mu}_{t-1}^f + \rho_2 1_2 \hat{\mu}_{t-1}^f \right] + u_{r,t} \quad (17)$$

$$\hat{\mu}_t^f = \hat{\mu}_{t-1}^f + \delta (\hat{m}_t - \hat{m}_{t-1} - \hat{\mu}_{t-1}^f) \quad (18)$$

where  $X_t = \log \frac{X_t}{\bar{X}}$  is the log-deviation of  $X$  to its steady state value. The model includes four first-order autoregressive shocks: a preference shock  $a_t$ , a velocity shock  $e_t$ , a productivity shock  $z_t$  and a monetary policy shock  $u_{r,t}$ .

Eq. (14) represents a forward-looking IS curve. It allows changes in real balances to directly affect the dynamics of output. All of the parameters, with the possible exception of  $\omega_2$ , ought to be non-negative. The parameter  $\omega_2$  measures the effect of real balances on aggregate output. This parameter is negatively proportional to the cross derivative of utility function (in consumption and money). Thus, some authors (cf. [Ireland \(2004\)](#) or [Andr s et al. \(2006\)](#)) test the non-separability hypothesis by testing whether this parameter is equal to zero or not ( $\omega_2 \neq 0$ ). If it is strictly positive, then money and consumption are complements as assumed in the mainstream literature; if it is negative, then the marginal utility of consumption is decreasing with respect to real balances.<sup>2</sup> Eq. (15) is a forward-looking Phillips curve that allows changes in real balances to also have an impact on the dynamics of inflation when  $\omega_2 \neq 0$ . As in Ireland, it is worth pointing out, first, that real money balances enter into the IS curve if and only if they enter into the forward-looking Phillips curve and, second, that what really matters for the dynamics of output and inflation are fluctuations in real balances once shifts in velocity have been factored in. Indeed, output and inflation in Eqs. (14) and (15) can be written as functions of  $(\hat{m}_t - \hat{e}_t)$ . Eq. (16) is a money demand function with income elasticity  $\gamma_1$  and interest semi-elasticity  $\gamma_2$ . Eq. (17) is the non-linear augmented Taylor rule, featuring the two-

<sup>2</sup> As an example, we can think of the non separable utility function  $u(c, m) = \frac{c^{1-\sigma_1}}{1-\sigma_1} \frac{m^{1-\sigma_2}}{1-\sigma_2}$ . The condition  $\omega_2 < 0$  corresponds to a value  $\sigma_2 > 1$ , and a strong concavity of the utility function with respect to money. See for instance [Gabriel et al., \(2007\)](#).

pillar monetary policy reaction function of the ECB. Finally, Eq. (18) links filtered money growth to the evolution of money.

#### 4. Resolution and estimation

##### 4.1. Resolution

Since we allow for a non-linear monetary policy reaction function to money, solving the model by standard linear method is impossible. We thus slightly adapt the methodology developed by Uhlig (1999) to approximate the solution function. To get the gist of the resolution method, we consider that agents do not forecast the next state of the economy but solve the two models associated with each possible state of the economy (i.e. normal vs excessive money growth) independently. This approach differs from a Markov-Switching model since endogenous variables completely determine the shift between the two regimes.

More formally, we elaborate on Uhlig (1999) and suppose that the log-linearized equilibrium relationship can be written as:

$$E_t(FX_{t+1} + G_t X_t + H_{t-1} X_{t-1} + RZ_t + SZ_{t-1}) = 0 \quad (19)$$

where the vectors  $X_t$  and  $Z_t$  are given by:

$$X_t = \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{r}_t \\ \hat{\mu}_t^f \end{bmatrix}, \quad Z_t = \begin{bmatrix} \hat{e}_t \\ \hat{a}_t \\ \hat{z}_t \\ \hat{u}_{r,t} \end{bmatrix}$$

and where the matrices  $F$ ,  $G_t$ ,  $H_{t-1}$ ,  $R$  and  $S$  are the matrices collecting the coefficients.  $G_t$  and  $H_{t-1}$  are time-dependent and respectively take two values  $g_1$ ,  $g_2$  and  $h_1$ ,  $h_2$  depending on the state of the economy in  $t$  and  $t-1$ .

We consider two different models, one associated with the policy reaction coefficient  $\rho_1$  (state 1, model 1) and the other with the policy reaction coefficient  $\rho_2$  (state 2, model 2). Owing to Uhlig, we can compute the transition matrices  $p_1$ ,  $q_1$ ,  $l_1$ , respectively  $p_2$ ,  $q_2$ ,  $l_2$ , which correspond to the version of the model in state 1, respectively in state 2.

The solutions of model 1 and model 2 are then given by:

$$X_{t+1} = p_1(X_t + l_1 Z_t) + q_1 Z_{t+1}, \quad (20)$$

$$X_{t+1} = p_2(X_t + l_2 Z_t) + q_2 Z_{t+1}, \quad (21)$$

To solve the model described by Eq. (19), we assume that the transition matrices only depend on the state of the economy at time  $t$  and not on previous periods; precisely, if the state at time  $t$  is 1, transition equation is given by (20), if the state at time  $t$  is 2, transition equation is given by (21), depending on past filtered money growth.

##### 4.2. Estimation

Contrary to Ireland (2004) or Andr s et al. (2006), we did not choose to compute the maximum of likelihood as such computation hardly converges toward a global maximum. Indeed, in our first attempts, we found the likelihood function ill-behaved, with multiple local peaks and large flat areas. Both Ireland (2004) and Canova and Menz (2010) report similar difficulties in estimating such a model resorting to Maximum Likelihood techniques. Therefore, as in Schorfheide (2000) or Smets and Wouters (2003), we applied Bayesian techniques to estimate the model. We refer to An and Schorfheide (2007) and Villaverde (2010) for a detailed review on Bayesian inference. The latter reviews the different arguments in favor

of Bayesian approach : the sparsity in the data, the flexibility of the DSGE models, and the fact that, in such models, likelihood is often flat in certain directions and may have multiple maxima and minima. The limits of the Maximum Likelihood inference stems from the lack of informative content of the data to precisely identify all the structural parameters of the model. Bayesian approach is an objective protocol to complete available information given by the sample with economic *a priori* on the structural parameters. These *a priori* on the parameters deform the likelihood function of the model in some directions. This approach accounts for a significative methodological march with respect to the minimization of the distance between impulse response functions of the model and those of a VAR, as for instance in Christiano et al. (2005). In particular, using Bayesian approach allows us to compare rigorously different models through their marginal densities as proposed by Geweke (1998).

Since our approximative solution is time dependent and backward-looking, we adapt the Kalman filter and the computation presented by Hamilton (1994) to obtain the likelihood recursively. Then, the sample log-likelihood conjugated with the prior distributions of parameters provide us with the posterior kernel distribution of a set of parameters. We compute the posterior kernel thanks to a Random Walk Metropolis Hastings Algorithm.<sup>3</sup>

We approximate the log-marginal density of the data by the Laplace Approximation and use it to compare different models (Geweke, 1998). Moreover, we assess the ability of these different models to replicate euro area stylized facts by comparing the theoretical and empirical autocovariances. While the log-marginal density provides us with a single value to rank alternative models, we can discuss the limits and the differences between different models by comparing the autocovariances.

##### 4.3. Data

To estimate the parameters of our DSGE model, we use data over the period 1980Q2–2007Q2 on four key macroeconomic variables for the Euro Area as a whole<sup>4</sup>: real GDP per capita, the growth of real money M3 per capita, CPI inflation rate and the 3-month short-term nominal interest rate. These data are extracted from the Euro Area Wide Model database (Fagan et al., 2001).<sup>5</sup> We also use the labor force data to normalize output and money growth. We use Eurostat data and linear projection (for 2006–2007) to update the labor force data. All the data are linearly detrended before the estimation.

##### 4.4. Assumptions and priors

In our benchmark estimation, four parameters are calibrated. The long-run nominal interest rate and the long-term inflation rate,  $\pi$ , are calibrated to 1.018 and 1.009 which correspond to their average values over our sample. This implies a discount factor,  $\beta$ , equal to 0.991. We fix the technology production function such that the labor income share in total output is 60%, so that  $\omega_p = 0.43$ . Finally, we set the slope of the Phillips curve  $\kappa_p = 0.05$ .<sup>6</sup> We directly estimate  $\sigma = 1/\omega_1$ , as prior is easier to define for this parameter.

<sup>3</sup> We reject the first 100 000 iterations before keeping around 160 000 iterations from the MCMC algorithm. We check the convergence of the distribution by looking at real-time moments of the chain.

<sup>4</sup> The countries taken into account in the aggregation are the 16 members of the euro area: Belgium, Germany, Ireland, Greece, Spain, France, Italy, Cyprus, Luxembourg, Malta, The Netherlands, Austria, Portugal, Slovenia, Slovakia and Finland.

<sup>5</sup> The aggregation is based on the so called "Index method" and most of the weights used for aggregating countries data are the constant GDP at market prices (PPP) for 1995.

<sup>6</sup> Precisely, we calibrate  $\alpha_p = 0.63$  because of difficulties in estimation of this parameter, which gives for a standard value of  $\theta = 8$ ,  $\kappa_p = 0.05$ .

**Table 1**

Estimation results. The first three columns define the a priori distribution. Model 1 corresponds to the model without money. Model 2 allows real balances in the model. Model 3 allows the monetary authority to react to filtered money growth. Model 4 and 5 introduce non-linearity in the monetary policy response to filtered money growth. Marginal density stands for the likelihood of the model according to the data (see Geweke, 1998). The last two lines report the mean of  $\rho_2 - \rho_1$  and the percentage of draws for which  $\rho_{ho_1} > \rho_2$  in the Monte Carlo Markov Chain.

Parameters ( $\kappa_c \cdot N_c$ )	Prior distributions			Posterior modes				
	Law	Mean	Standard deviation	Model 1	Model 2	Model 3	Model 4	Model 5
				$\omega_2 = 0$	$\rho = 0$	$\rho_1 = \rho_2$	$\rho_1 \neq \rho_2(1,1,5)$	$\rho_1 \neq \rho_2(1,4,4)$
<i>General parameters</i>								
Discount factor, $\beta$	Calibrated	0.991	–	0.991	0.991	0.991	0.991	0.991
Intertemporal elasticity of substitution, $\sigma = 1/\omega_1$	Normal	1.5	0.38	2.382	1.337	1.814	1.917	1.903
Cross relation money-consumption, $\omega_2$	Normal	0.1	10	–	– 4.885	– 2.843	– 3.123	– 3.095
Income elasticity of money demand, $\gamma_1$	Normal	0.014	0.010	0.013	0.016	0.025	0.019	0.017
Interest elasticity of money demand, $\gamma_2$	Normal	0.72	0.1	0.732	0.724	0.695	0.702	0.707
Steady State interest rate, $r$	Calibrated	1.018	–	1.018	1.018	1.018	1.018	1.018
Calvo indexation $\gamma_p$	Beta	0.24	0.1	0.147	0.135	0.108	0.079	0.100
Calvo probability $\alpha_p$	Calibrated	0.63	–	0.63	0.63	0.63	0.63	0.63
Curvature of production function $\omega_p$	Calibrated	0.43	–	0.43	0.43	0.43	0.43	0.43
<i>Persistences</i>								
Velocity shock, $\rho_e$	Beta	0.5	0.2	0.906	0.943	0.931	0.918	0.924
Preference shock, $\rho_a$	Beta	0.5	0.2	0.980	0.976	0.989	0.980	0.984
Productivity shock, $\rho_z$	Beta	0.5	0.2	0.937	0.872	0.888	0.908	0.905
Monetary policy shock, $\rho_c$	Beta	0.5	0.2	0.281	0.346	0.567	0.506	0.504
<i>Standard deviations</i>								
Velocity shock, $\sigma_e$	Inverse gamma	0.1	$\infty$	0.023	0.022	0.021	0.019	0.020
Preference shock, $\sigma_a$	Inverse gamma	0.1	$\infty$	0.009	0.009	0.009	0.009	0.009
Productivity shock, $\sigma_z$	Inverse gamma	0.1	$\infty$	0.013	0.013	0.014	0.014	0.014
Monetary policy shock, $\sigma_r$	Inverse gamma	0.1	$\infty$	0.002	0.002	0.002	0.002	0.002
<i>Monetary policy rule</i>								
Weight of past interest rate, $\rho_r$	Beta	0.5	0.2	0.766	0.776	0.651	0.673	0.687
Weight of output, $\rho_y$	Normal	0.125	0.05	0.111	0.147	0.134	0.115	0.104
Weight of inflation, $\rho_\pi$	Normal	1.7	1	1.535	1.091	0.677	0.736	0.776
Weight of money growth (normal times regime), $\rho_1$	Normal	1.2	0.5	–	–	1.327	1.095	1.222
Weight of money growth (excess liquidity regime), $\rho_2$	Normal	1.2	0.5	–	–	1.327	1.569	1.519
Marginal density				1895	1913.9	1943.1	1945.7	1942.9
Mean on the MH simulations $\langle \rho_2 - \rho_1 \rangle$							0.474	0.30
Share of draws where $\rho_1 > \rho_2$							0.06%	4.9%

As in Smets and Wouters (2003), the standard errors of the innovations are assumed to follow inverse gamma distributions (see Table 2). We choose Beta distributions for shock persistence parameters with mean 0.5 and standard error 0.2 as well as for the backward component of the Taylor rule. Indeed this kind of distribution covers the whole interval  $[0,1]$ . For the same reason, the prior for the indexation of non-optimized prices is set to follow a Beta distribution with mean 0.24 and standard error 0.1, which is consistent with Ireland's results for the US economy. Other priors follow Gaussian distributions. The prior for  $\gamma_1$  is consistent with the estimates of Ireland, slightly smaller than in the literature.<sup>7</sup> We choose a positive prior mean for  $\omega_2$ , as suggested in the literature but a very large standard error equal to 10 to allow for the possibility of  $\omega_2$  being positive or negative. Because the introduction of a non-linear Taylor rule is novel in the literature, priors for  $\rho_1$  and  $\rho_2$  are assumed to be equal. The mean of the prior equals 1.2. This prior is slightly higher than in Andrès et al. (2006) but is consistent with the results of Poilly (2007).

#### 4.5. Results of the estimations

We provide the results of the Bayesian estimation in Tables 1 and 2. We complete several estimates, which are summarized in Table 1. The first set of estimates corresponds to the model with a separable utility

function (first column), i.e. a situation in which money does not have an active role in the business cycle. This model corresponds to the standard New Keynesian model as presented in Woodford (2003) and our results are in line with the estimations of Ireland (2004) for the US and Andrès et al. (2006) for the Euro area. The second model corresponds to a non separable utility function and a simple Taylor rule. Hence, we consider a model in which money may matter for the business cycle but the central bank decides to not directly react to monetary developments. Column 2 of Table 1 summarizes the results. The obtained estimates are stable with respect to the first model and we find a very negative value for  $\omega_2$ . In the remaining versions, we consider augmented Taylor rules taking into account filtered money growth ( $\rho \neq 0$ ) and eventually introduce a non-linear reaction to it. First, we consider that central bank can react to monetary development but not in a state-contingent way (column 3,  $\rho_1 = \rho_2$ ). In this model, we notice that the main parameters of the model except those of the Taylor rule remain stable. As far as the Taylor rule is concerned, we find a strong reaction of the central bank to filtered money growth. This reaction to money growth induces a weaker reaction to inflation ( $\rho_\pi = 0.677$ ). Then, we introduce a state contingent-policy rule and consider a set of different values for  $(\kappa_c, N_c)$ , mainly for robustness checks (the two last columns of Table 1). In these two last cases, we find a higher response to monetary growth in the excess liquidity regime than in the normal time regime. For the sake of clarity, we mainly develop further analysis for the first set of parameters  $(\kappa_c, N_c) = (1,1,5)$  and we refer to this model as the state-contingent model.

Fig. 2 shows the posteriors obtained thanks to Metropolis Hastings algorithm in this last case (model 4) and provides some insights on the quality of the estimates. Precisely, we correctly estimate the persistences

<sup>7</sup> We point out that with a prior centered around 1, the obtained estimates are quite similar. Brief reminder on the results of Ireland (2004)  $\gamma_1 = 0.01$  for the US, Andrès et al. (2006)  $\gamma_1 \in [0.04; 0.118]$  for the EA, these authors show that the value of this parameter is lower than other studies and depends on the filtering of the data.



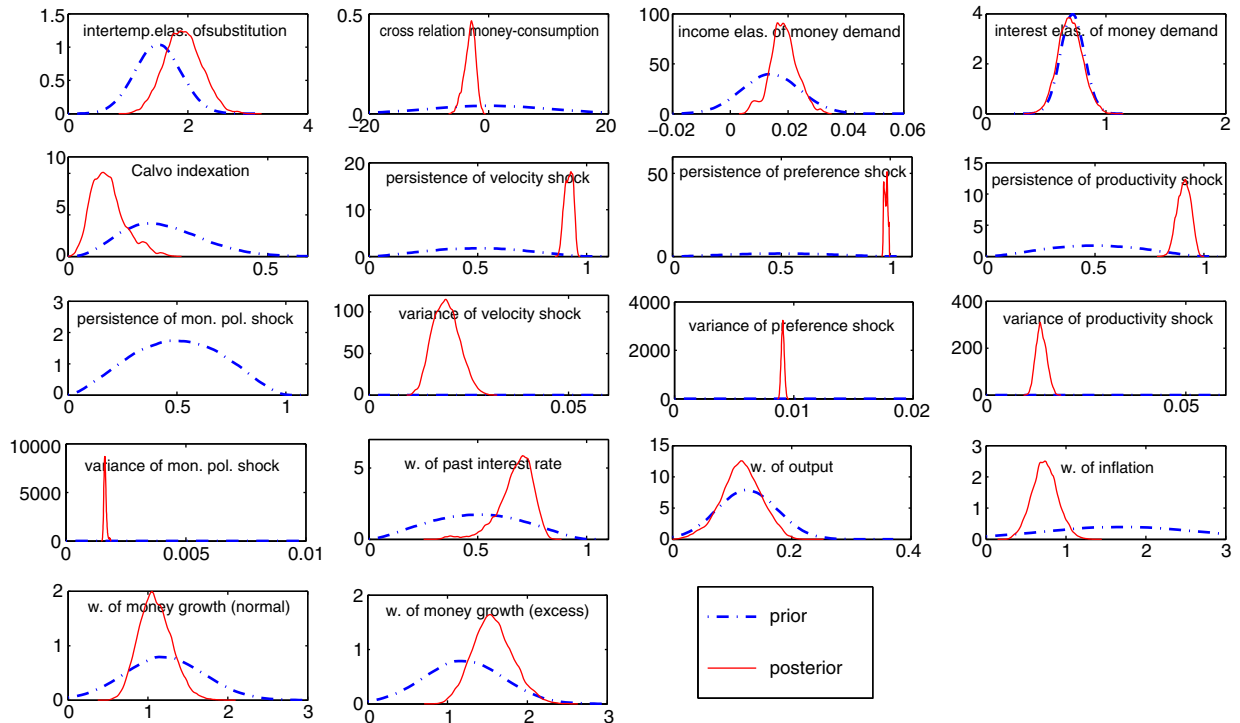
**Table 2**

Estimation results in details. The first three columns define the a priori distribution. We report here the results of the Metropolis Hastings algorithm for the Model 4, i.e. with money in the utility function and potential non-linear response of interest rate to filtered money growth.

Parameters	Prior distributions posterior			Posterior distribution (from MCMC)				
	Law	Mean	Standard deviation	Mode	Mean	5%	Median	95%
<i>General parameters</i>								
Discount factor, $\beta$	Calibrated	0.991	–	0.991	0.991	0.991	0.991	0.991
Intertemporal elasticity of substitution, $\sigma = 1/\omega_1$	Normal	1.5	0.38	1.884	1.917	1.413	1.910	2.4363
Cross relation money-consumption, $\omega_2$	Normal	0.1	10	–2.798	–3.123	–4.842	–3.019	–1.756
Income elasticity of money demand, $\gamma_1$	Normal	0.014	0.01	0.019	0.019	0.010	0.018	0.027
Interest elasticity of money demand, $\gamma_2$	Normal	0.72	0.1	0.711	0.702	0.537	0.700	0.876
Steady state interest rate $r$	Calibrated	1.018	–	1.018	1.018	1.018	1.018	1.018
Calvo indexation $\gamma_p$	Beta	0.24	0.1	0.079	0.101	0.043	0.095	0.187
Calvo probability $\alpha_p$	Calibrated	0.63	–	0.630	0.630	0.630	0.630	0.630
Curvature of production function $\omega_p$	Calibrated	0.43	–	0.430	0.430	0.430	0.430	0.430
<i>Persistences</i>								
Velocity shock, $\rho_e$	Beta	0.5	0.2	0.923	0.918	0.887	0.919	0.9470
Preference shock, $\rho_a$	Beta	0.5	0.2	0.984	0.980	0.968	0.980	0.9932
Productivity shock, $\rho_z$	Beta	0.5	0.2	0.922	0.908	0.857	0.909	0.9565
Monetary policy shock, $\rho_\varepsilon$	Beta	0.5	0.2	0.564	0.506	0.262	0.496	0.782
<i>Standard deviations</i>								
Velocity shock, $\sigma_e$	Inverse Gamma	0.1	$\infty$	0.018	0.019	0.014	0.019	0.025
Preference shock, $\sigma_a$	Inverse gamma	0.1	$\infty$	0.009	0.009	0.009	0.009	0.009
Productivity shock, $\sigma_z$	Inverse gamma	0.1	$\infty$	0.013	0.014	0.012	0.014	0.016
Monetary policy shock, $\sigma_r$	Inverse gamma	0.1	$\infty$	0.002	0.002	0.002	0.002	0.002
<i>Monetary policy rule</i>								
Weight of past interest rate, $\rho_r$	Beta	0.5	0.2	0.653	0.673	0.524	0.687	0.780
Weight of output, $\rho_y$	Normal	0.125	0.05	0.114	0.115	0.058	0.116	0.170
Weight of inflation, $\rho_\pi$	Normal	1.7	1	0.665	0.736	0.473	0.734	1.009
Weight of money growth (normal times regime), $\rho_1$	Normal	1.2	0.5	1.046	1.095	0.776	1.081	1.456
Weight of money growth (excess liquidity regime), $\rho_2$	Normal	1.2	0.5	1.475	1.569	1.176	1.556	2.003

and the variances of the shocks as the posterior distribution is narrower than the prior distribution meaning that data are really informative concerning these parameters. This graph also shows that we well identify

a negative sign of the cross relation money-consumption,  $\omega_2$ . Nevertheless, the estimates for elasticities in the money demand equation are not completely convincing along with the intertemporal elasticity of



**Fig. 2.** Priors and posteriors for a state contingent policy rule (Model 4). The dashed blue lines correspond to the *a priori* distributions of the deep parameters, while the red lines show the posterior distributions resulting from the Metropolis Hastings algorithm.

substitution,  $\sigma = 1/\omega_1$ , and the weight of output in the Taylor rule  $\rho_y$ . The other coefficients of the rule are well estimated.

## 5. Does money matter?

This section presents our main findings. First, we show that money matters and that monetary policy has systematically reacted to money growth. Second, we assess the main consequences of this departure from the standard literature on the ability of our model to track economic fluctuations.

### 5.1. A role for money

The first question we want to address is the relevance of monetary developments to explain euro area data. As can be seen in Table 1, the log marginal posterior difference between model 2 ( $\rho = 0$  only) and model 1 ( $\omega_2 = 0$  and  $\rho = 0$ ) is 18.9. Therefore, to choose model 1 over model 2, we need a prior probability over model 1  $1.6 \times 10^8$  ( $\exp(18.9)$ ) times larger than our prior probability over model 2. This evidence supports the assumption that money matters for the business cycle in the euro area.

As regards the parameters of interest, the response of aggregate demand ( $\omega_1 = 1/\sigma$ ) to changes in the real rate is significant and broadly in line with the estimates provided by Andrès et al. (2006) for the euro area but far above the estimates provided by Ireland for the United States. As far as money demand is concerned, the elasticity with respect to interest rates ( $\gamma_2 = 0.7$ ) is broadly in line with other estimate in the literature, but the income elasticity is very small ( $\gamma_1 = 0.01/0.02$ ) in sharp contrast with standard estimates for the euro area, generally slightly above 1. However, this may reflect the instability of money demand functions estimated on euro area data and evidenced over the last five years.<sup>8</sup>

The posterior mode of  $\omega_2$ , the parameter measuring the importance of real balances in the IS and Phillips curves, is negative and significant, whatever the version of the model. Therefore, this estimate associates an increase in real balances with a decrease in output and an increase in inflation. Though this result may seem at odd with the related literature, in particular Ireland (2004) and Andrès et al. (2006), few points are worth mentioning. First, both Ireland and Andrès and co-authors, who basically replicate Ireland's approach to euro area data, constrain that parameter to be positive; this technical assumption combined with the fact that the maximum is obtained in zero casts doubts on the effective positivity of  $\omega_2$ . As a consequence, these authors conclude that real balances have a limited, if any, role in explaining business cycle fluctuations. Second, the sign of  $\omega_2$  ultimately depends upon the properties of the utility function of the representative household and therefore can take either a positive or a negative sign. We leave open that possibility by assuming a flatter as possible prior distribution for  $\omega_2$ . Thus, we let data decides which function of money is dominant between the mean of transaction function and the store of value. A negative sign would imply that money acts as a substitute to consumption and that the store value effect is dominant. In such a case, a rise in inflation increases the nominal interest rate and leads to a decrease in money demand. As consumption and money are substitutes, consumption rises and leisure falls. Work effort then rises and with it output.

We turn now to the monetary policy rule. As money seems to matter to understand the dynamic of euro area data and as the Eurosystem's monetary strategy rely on a monetary pillar, it may be optimal for the European Central Bank to systematically take into account a measure of money growth in its reaction function.<sup>9</sup> In the

version 3 of our model (see Table 1, column 6), we suppose a constant and systematic reaction to the filtered money growth,  $\mu_t^f - 1$ . We set the value of the smoothing parameter  $\delta$  to 0.15. This implies that the half of a one-off rise in actual money growth would disappear after 4.5-quarters.<sup>10</sup> The weight of money growth in the policy rule is estimated around  $(1 - \rho_r)\rho = (1 - 0.651) * 1.33 = 0.46$ . Andrès et al. (2006) finds a slightly smaller result (0.35) for the euro area over a similar sample period. However, they suppose that the ECB reacts to actual money growth whereas we assume it reacts to the deviations of filtered money growth to its long-term average.

The introduction of monetary growth into the Taylor rule has a sizeable impact on the key parameters of the rule. For instance, the weight of the inflation decreases and becomes smaller than in standard Taylor-type rules without money. The gain of adding money growth into monetary policy rule is clearly identified by the log-marginal density. The log marginal likelihood difference between models with an augmented Taylor rule (model 3) and with a simple Taylor rule is 29.2. This result implies that we need a prior probability over model 2  $4.8 \times 10^{12}$  times larger than our prior probability over model 3 in order to reject the fact that including money into the policy reaction function improves the fit to the data.

Does the ECB react stronger to high monetary growth or put another way to "excess liquidity"? To answer this question, we estimate our model including a non-linear state-dependent monetary policy reaction function that factors in filtered money growth. Although we assume the same priors for the both coefficients, the mode of the posterior distribution indicates that  $\rho_1 = 1.01$  and  $\rho_2 = 1.57$  for the parameters  $(\kappa_c, N_c)$  set to (1.1, 5). In the last two lines of Table 1, we report the mean of the posterior distribution of  $\rho_2 - \rho_1$  and the ratio of draws where  $\rho_1$  is higher than  $\rho_2$  in the Monte Carlo Markov Chain. According to the posterior distribution, this difference is positive for 99.94% of posterior draws (see Table 1). Besides, there is a difference of 2.6 between the log-data density of this setup and the previous one without threshold, which shows that, for this choice of parameters  $(\kappa_c, N_c)$ , to chose model 3 over model 4, we need a prior probability over model 3  $13.6 = \exp(2.6)$  times larger than our prior probability over model 4. This would tend to show that, indeed, the European Central Bank tends to react more aggressively when money growth moves away from its long-term average.<sup>11</sup> Finally, the fact that  $\rho_1$  is strictly different from zero is consistent with the fact that money matters for the euro area economy. Though  $\rho_1$  is significantly different from  $\rho_2$ , such difference doesn't seem to matter a lot as far as the fit to the data is concerned.

### 5.2. Stabilizing properties of our augmented Taylor Rule

This subsection is devoted to the analysis of the ability of our model to replicate the euro area economy dynamics. To this aim, we study the impulse response functions to a velocity shock, a preference shock, a productivity shock and a monetary policy shock in different cases. We also consider the autocovariances in our analysis. Since the marginal likelihood is higher for model 4, we consider that the estimates obtained for the parameters in this model are the most relevant. We want to underline the impact of taking money into account into monetary policy rule and we consider alternative policy rules without money. The considered monetary policy rules are those with monetary growth of model 4, a monetary policy rule estimated on the euro area over the period 93–04 by Gerdesmeier et al. (2007), and a Taylor rule without money estimated when we fix the other parameters of the model.<sup>12</sup>

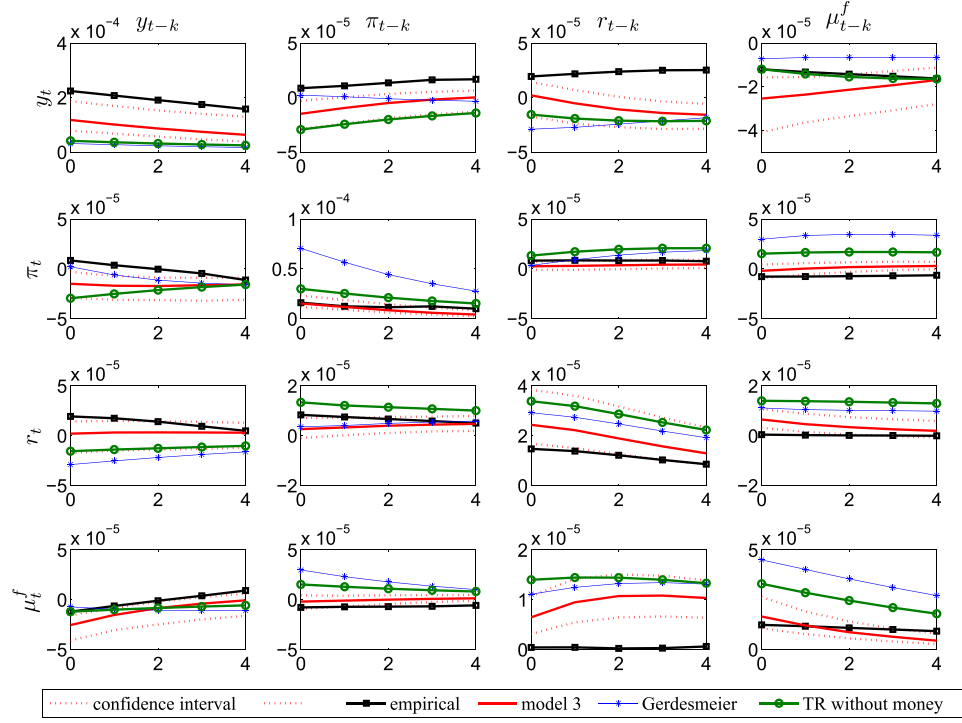
<sup>8</sup> Indeed, our estimates are very sensitive to the assumptions made on the priors of the parameters entering the money demand function.

<sup>9</sup> See the pre-version of Beck and Wieland (2007). Even though they integrate money as a proxy of expected inflation, their result tends to prove that including money growth in Philipps Curve implies a monetary policy depending on money growth.

<sup>10</sup> See Gerlach (2004) for more discussion about the value of  $\lambda$ .

<sup>11</sup> This result depends on the choice of  $(\kappa_c, N_c)$ , but remains in most of likely cases.

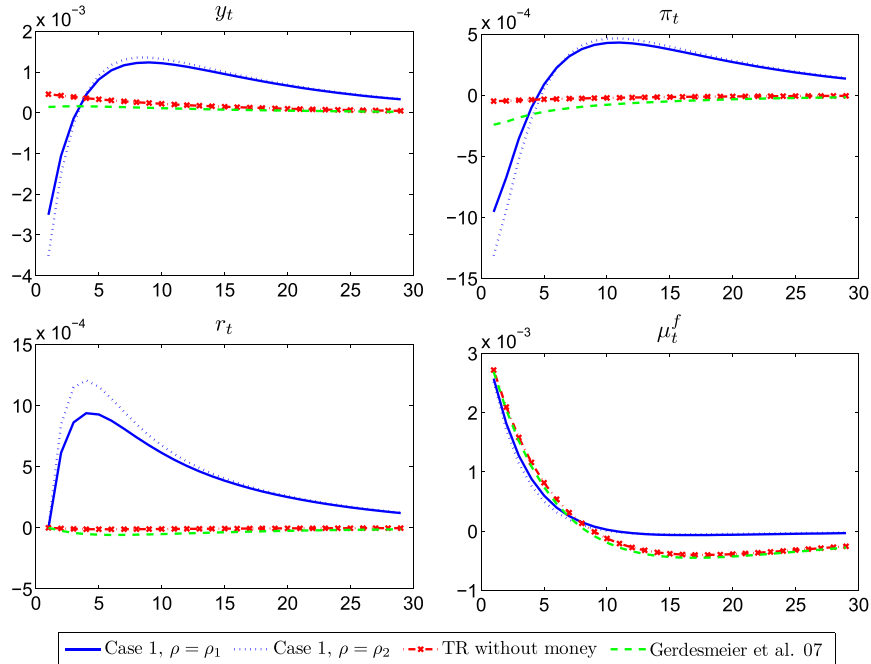
<sup>12</sup> The coefficients for the Taylor rule given by Gerdesmeier et al. (2007) are  $\rho_r = 0.9$ ,  $\rho_y = 0.83$  and  $\rho_\pi = 1.5$ , while the simulated coefficients for the Taylor rule without money are  $\rho_r = 0.74$ ,  $\rho_y = 0.088$  and  $\rho_\pi = 1.24$ .



**Fig. 3.** Comparison of empirical and theoretical autocovariances. These figures display the covariances between the variables in line and the lagged variables in column. In red, we plot the autocovariances predicted by the model with augmented Taylor rule (model 4) and in dashed red lines, the confidence intervals. In blue line with stars, the theoretical autocovariances of a model with a calibrated Taylor rule following Gerdesmeier et al. (2007). In green with circles, the autocovariances predicted by the model with a simple Taylor rule (model 2). In any cases, the model includes money in the utility function. Finally in black with full squares, the empirical autocovariances computed on the studied sample.

Concerning the impulse response functions, the introduction of money in the policy rule affects the response of the economy to a money demand shock (see Fig. 4). A velocity shock leads to a less than proportional increase in filtered monetary growth  $\mu_t^f$ , so that money

adjusted for velocity shifts decreases. In a “standard” Taylor rule, the substitution effect between consumption and money adjusted from velocity shift implies a positive effect on  $y$ . The impact on inflation is the combination of two effects: first, the marginal propensity to



**Fig. 4.** Impulse response function to a velocity shock ( $\epsilon_v$ ). In thick blue, we plot the impulse response functions (IRF) for the model with augmented Taylor rule (model 4) in regime 1 (no excess of liquidity) and in dashed blue line, in regime 2 (of excess liquidity). In discontinuous red line with crosses, we display the IRF for the model 2, i.e. with a simple Taylor rule. And in dashed green, we plot the IRF if the monetary authority is forced to follow the Taylor rule estimated by Gerdesmeier et al. (2007).

consume increases implying the fall of real wage and finally a downward pressure on prices, second the inflationary effect stemming from the widening of the output gap. The former effect is dominating in this case, and inflation reacts negatively. When money growth is accounted for by the central bank, the substitution effect between money adjusted from velocity shift and consumption is more than compensated by the surge in the anticipated interest rate due to the inclusion of money growth in the Taylor rule. The overall impact on output is therefore negative too. Then, the central bank increases its nominal interest rate, since  $\mu_t^f$  enters with a one-period lag into the Taylor-type rule, while both output and inflation increase due to the mechanical decrease of money growth. The growth effect is preponderant in the response of inflation. For this kind of shock, the response to money growth implies a higher volatility of both output and inflation, and a higher standard quadratic loss for the central bank. This result is consistent with [Pool \(1970\)](#), suggesting that a central bank that would react to money demand shocks without adjusting the money supply to keep its key interest rate unchanged would indeed generate greater output and inflation volatility.

The responses to a preference shock are quite similar when money is taken into account or not in the Taylor Rule (see [Fig. 5](#)). This kind of shock leads to an intertemporal change in the structure of preferences. Then, consumption and money rise, inflation rises due to the widening of the output gap. A productivity shock leads to standard responses for supply shocks (see [Fig. 6](#)). Consumption reacts positively and inflation is decreasing owing to downward pressures on marginal costs. In the case of the [Gerdesmeier et al. \(2007\)](#). Taylor rule, the response of output is lower but strictly positive. Indeed, due to the higher weight of output in the Taylor rule, expectations of consumption are lower implying a lower increase of current output. The responses to a preference shock and a productivity shock are consistent with other studies like [Smets and Wouters \(2003\)](#).

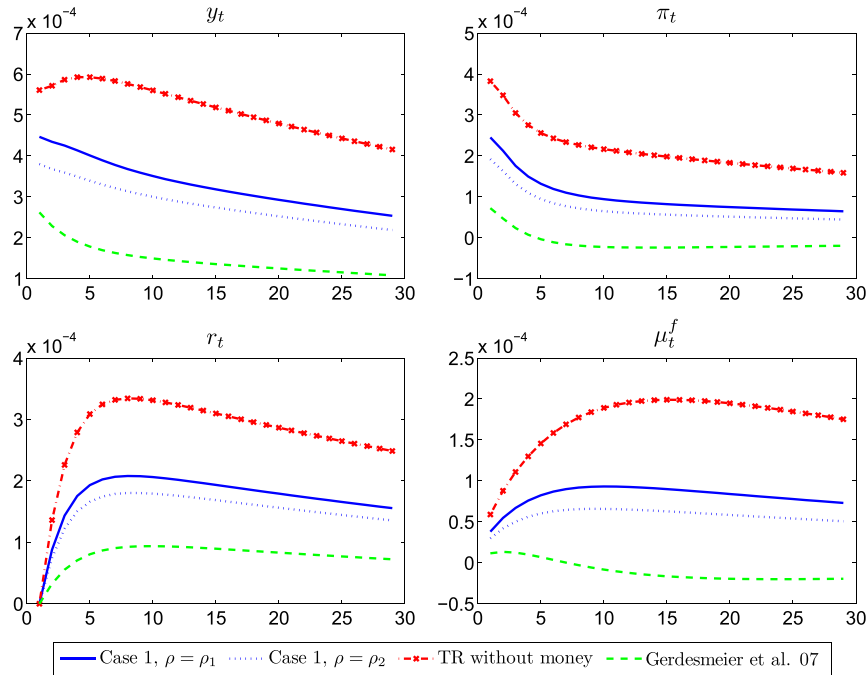
In the case of a monetary policy shock, money is decreasing as a consequence of the money demand curve (see [Fig. 7](#)). The substitution effect implies that consumption is increasing, while intuition would expect a fall due to the negative relation between consumption and

interest rate. Contrary to the previous case, the growth effect on inflation is largely dominated by the decrease of marginal costs.

Finally, including money in the policy rule is always stabilizing for inflation and money growth except when the economy is facing a velocity shock. However, the inclusion of money generally increases the volatility of output. Consequently, as far as autocovariances are concerned, the ECB two-pillar strategy seems to reduce the volatility of inflation, of money growth and of interest rate but increases the volatility of output (see [Fig. 3](#)). In terms of welfare, taking money into account in the Taylor rule may be welfare improving if the weight of stabilizing inflation and interest rate is predominant in the CB's objective function. Besides, [Fig. 3](#) indicates that the persistence of our model is consistent with real data, whereas some covariances are not perfectly replicated.

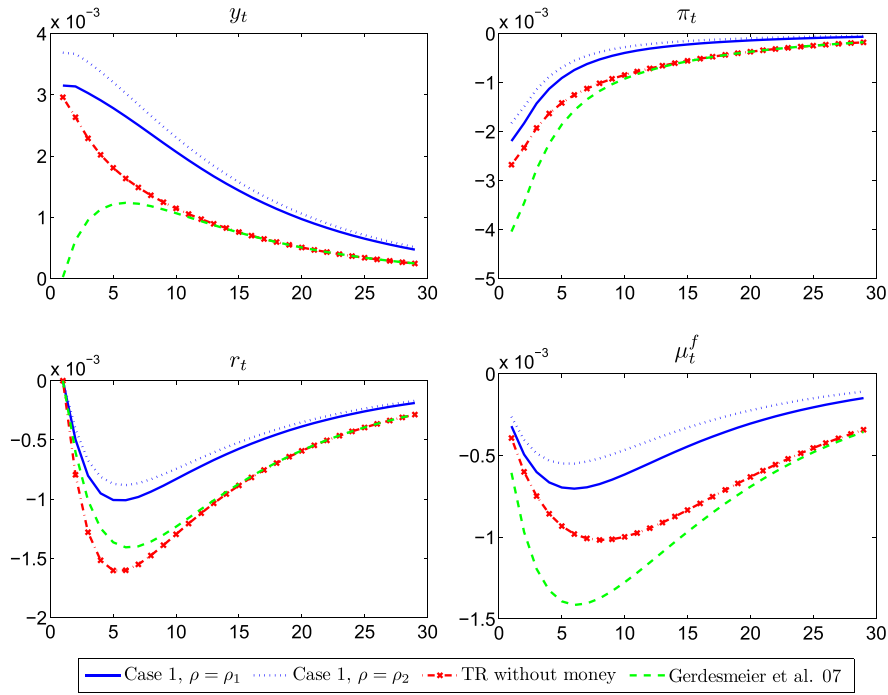
## 6. Conclusion

In this paper, we provide a rationale for the “two-pillar” monetary policy strategy of the ECB by constructing a small DSGE model that allows real balances to play an active role in explaining the euro area business cycle. We find some evidence that money plays indeed a significant role and enters in particular both in the IS and Phillips curves. In such a context, it would be optimal for the central bank to factor in monetary development into its monetary policy decisions. However, we also find some evidence that broad money and consumption are substitutes rather complements in the euro area. This result, as well as the impulse functions provided in the appendix, are consistent with the recent findings by [Giannone et al. \(2009\)](#) who, in the context of a large VAR model, evidence that in response to an exogenous monetary tightening, real output slows down while broad money aggregates (both M2 and M3) increase. This result stems from the fact that short-term interest rates represent more the return than the opportunity cost of broad money. Consequently, these results cast some doubt on the relevance of M3 as the appropriate monetary indicator for the conduct of monetary policy in the euro area and



**Fig. 5.** Impulse response function to a preference shock ( $\epsilon_a$ ). In thick blue, we plot the impulse response functions (IRF) for the model with augmented Taylor rule (model 4) in regime 1 (no excess of liquidity) and in dashed blue line, in regime 2 (of excess liquidity). In discontinuous red line with crosses, we display the IRF for the model 2, i.e. with a simple Taylor rule. And in dashed green, we plot the IRF if the monetary authority is forced to follow the Taylor rule estimated by [Gerdesmeier et al. \(2007\)](#).



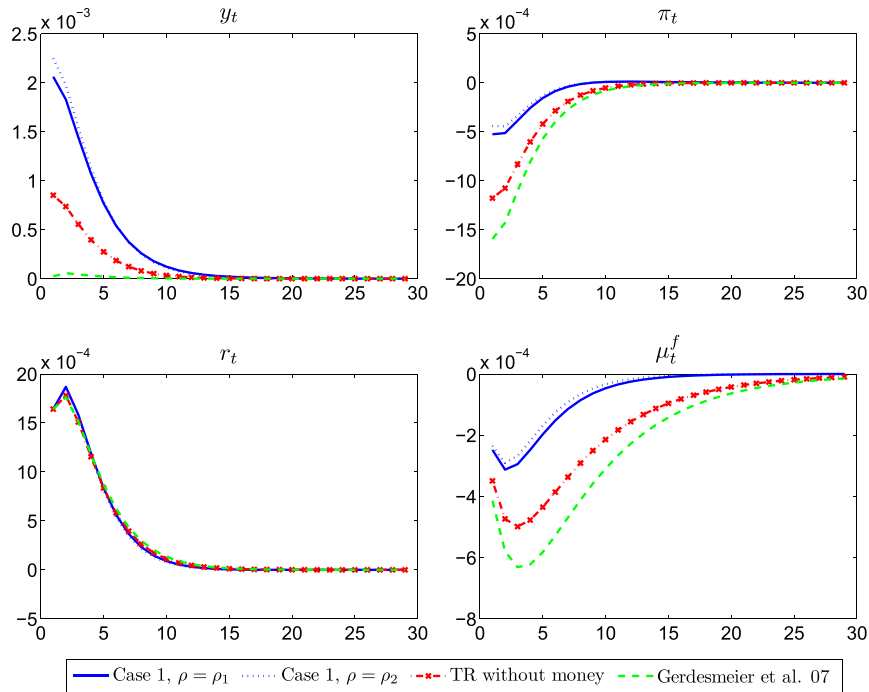


**Fig. 6.** Impulse response function to a productivity shock ( $\varepsilon_z$ ). In thick blue, we plot the impulse response functions (IRF) for the model with augmented Taylor rule (model 4) in regime 1 (no excess of liquidity) and in dashed blue line, in regime 2 (of excess liquidity). In discontinuous red line with crosses, we display the IRF for the model 2, i.e. with a simple Taylor rule. And in dashed green, we plot the IRF if the monetary authority is forced to follow the Taylor rule estimated by [Gerdesmeier et al. \(2007\)](#).

would call for considering rather narrower aggregates such as M1 for instance.

In addition, we find some evidence that the ECB has reacted in a systematic way to monetary developments but weak evidence that it has reacted in a non-linear way, that is more aggressively to excess money growth. One reason may be that, in practice, the reference

value has remained unchanged since the inception of the ECB in a context where M3 growth has systematically exceeded the reference value. As a consequence, we only find slight support for the presence of shifts between two monetary policy regimes in the euro area. Another reason is that what we call money in our set-up might not correspond exactly to what is meant and measured by money in



**Fig. 7.** Impulse response function to a monetary policy shock ( $u_r$ ). In thick blue, we plot the impulse response functions (IRF) for the model with augmented Taylor rule (model 4) in regime 1 (no excess of liquidity) and in dashed blue line, in regime 2 (of excess liquidity). In discontinuous red line with crosses, we display the IRF for the model 2, i.e. with a simple Taylor rule. And in dashed green, we plot the IRF if the monetary authority is forced to follow the Taylor rule estimated by [Gerdesmeier et al. \(2007\)](#).

reality and by the ECB, in particular because of the absence of a banking sector in our model. Alternatively, this might also reflect some concerns of the ECB regarding the reliability of M3. Indeed, the ECB disregarded M3 developments in the aftermath of the 2000 stock market crash, arguing M3 was distorted by portfolio shifts.

Finally, our findings call for further extensions, in particular to better understand the role of money and the reasons for why it may act as a substitute to consumption. The literature usually puts forward two main explanations: first, the existence of frictions and imperfections on credit markets (see for instance [Attanasio, 1995](#)); second asset market segmentation (as in [Alvarez et al., 2003](#) or [King and Thomas, 2007](#)). These items are high in the research agenda particularly in the context of the current financial crisis which has illustrated the need to better account for money, credit and asset markets.

## Appendix A

The linearized model and the steady state

$$\begin{aligned}\hat{y}_t + \omega_2(1-\rho_e)\hat{e}_t - \omega_1(1-\rho_a)\hat{a}_t &= E_t(\hat{y}_{t+1}) - \omega_1(\hat{r}_t - E_t(\hat{\pi}_{t+1})) \\ &+ \omega_2(\hat{m}_t - E_t(\hat{m}_{t+1})) \\ \hat{\pi}_t - \gamma_p\hat{\pi}_{t-1} &= \beta E_t(\hat{\pi}_{t+1} - \gamma_p\hat{\pi}_t) \\ &+ \kappa_p\left(\omega_p + \frac{1}{\omega_1}\right)\hat{y}_t - \kappa_p\frac{\omega_2}{\omega_1}\hat{m}_t + \kappa_p\frac{\omega_2}{\omega_1}\hat{e}_t - \kappa_p(1 + \omega_p)\hat{z}_t \\ \hat{m}_t &= \gamma_1\hat{y}_t - \gamma_2\hat{r}_t + \gamma_3\hat{e}_t \\ \hat{r}_t &= \rho_r\hat{r}_{t-1} + (1-\rho_r)\left[\rho_y\hat{y}_{t-1} + \rho_\pi\hat{\pi}_{t-1} + \rho_1\hat{\mu}_{t-1}^f + \rho_2\hat{\mu}_{t-1}^f\right] + u_{r,t} \\ \hat{\mu}_t &= \hat{m}_t - \hat{m}_{t-1} \\ \hat{a}_t &= \rho_a\hat{a}_{t-1} + \epsilon_{a,t} \\ \hat{e}_t &= \rho_e\hat{e}_{t-1} + \epsilon_{e,t} \\ \hat{z}_t &= \rho_z\hat{z}_{t-1} + \epsilon_{z,t} \\ \hat{u}_{r,t} &= \rho_e\hat{u}_{r,t-1} + \epsilon_{r,t}\end{aligned}$$

where

$$\begin{aligned}\beta r &= \pi \\ ru_2 &= (r-1)eu_1 \\ \omega_1 &= \frac{-u_1}{cu_{11}} \\ \omega_2 &= -\frac{(m/e)u_{12}}{cu_{11}} \\ \gamma_1 &= \left(\frac{cr\omega_2}{m\omega_1} + \frac{r-1}{\omega_1}\right)\gamma_2, \\ \gamma_2 &= \frac{r}{(r-1)(m/e)}\left[\frac{u_2}{(r-1)eu_{12} - ru_{22}}\right], \\ \gamma_3 &= 1 - (r-1)\gamma_2 \\ \omega_p &= -\frac{F''(n)F(n)}{(F'(n))^2} \\ \kappa_p &= \frac{(1-\alpha_p)(1-\beta\alpha_p)}{\alpha_p(1 + \vartheta\omega_p)}\end{aligned}$$

and where  $u_1 = u_1(c, m/e)$ ,  $u_2 = u_2(c, m/e)$ ,  $u_{11} = u_{11}(c, m/e)$ ,  $u_{12} = u_{12}(c, m/e)$ , and  $u_{22} = u_{22}(c, m/e)$ .

## Appendix B. Supplementary data

Supplementary data to this article can be found online at [doi:10.1016/j.econmod.2011.01.010](https://doi.org/10.1016/j.econmod.2011.01.010).

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