Regulation and Ruin Theory

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Controlling the Probability of Failure

THE STANDARD THEORETICAL APPROACH UNDERLYING insurance regulation originates in actuarial methods and, more specifically, in ruin theory. It's important to outline this theory, and then to discuss its limits, because it's the one that most insurance practitioners or regulators have in mind when thinking about insurance regulation, partly, of course, because a number of them have been trained as actuaries.

Broadly speaking, this approach posits that the aim of prudential regulation is to ensure that the probability of ruin of insurance companies is below some given "acceptable" value. The second assumption is that the main tool available to the regulator to reach this aim is to set a mandatory solvency margin, namely the minimum amount of a firm's own equity that may be used as a buffer. While this approach is traditional and classic in the insurance industry, it's interesting to note that a similar view recently became influential in banking, with the rise of "Value-at-Risk" methods to manage market and credit risks. These methods may involve very high-tech modeling tools, but they hinge on the same simple principles.

Let us illustrate this approach through the stylized example of an insurance company characterized by the following simplified balance sheet:

Assets A	Reserves R
	Equity E

At this stage, we neglect asset risk and thus assume that the assets of this company are composed of riskless investments, with a rate of return that we normalize to zero. We assume that the company doesn't underwrite new risks. The profits and losses of the company are determined by the runoff, or the difference between the current estimates of future claims, namely the reserves R, and their eventual costs.

Defining $(1 + \tilde{x})$ as the random variable that characterizes the ratio final cost/reserves, failure occurs at the end of the reference period when final cost $R(1 + \tilde{x})$ exceeds the value of assets:

$$R(1 + \tilde{x}) > A = R + E.$$

Subtracting R from both sides and then dividing by R, we see that failure is characterized by the stochastic event

$$\tilde{x} > E/R.$$

Therefore, if the probability distribution of the ratio $\tilde{x}$, Φ, has been estimated by statistical methods, the probability of failure can be estimated as

$$\Pr(\tilde{x} > E/R) = 1 - \Phi(E/R).$$

If m represents the 99 percent quantile of the probability distribution of $\tilde{x}$ (see Fig. 1), we see that imposing a minimum margin requirement limits the probability of failure below 1 percent:

$$\frac{\text{Equity}}{\text{Premiums}} \geq m \Leftrightarrow \Pr(\text{Failure}) \leq 1 - \Phi(m) = 1.$$

Ruin theory essentially aims to solve more-sophisticated, dynamic versions of this model in order to estimate the minimum solvency margin needed to obtain a survival probability of at least 99 percent (for example) over a given, possibly long, time horizon.

Let us now account for the riskiness of assets as follows. Consider a situation where the insurer has invested in two

DREAMSTIME

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types of assets: riskless assets, with a rate of return normalized to zero; and risky assets, with a random rate of return $\tilde{r}$. The balance sheet becomes

Riskless assets A_0	Reserves R
Risky assets A_1	Equity E

In this case, failure occurs when

$$A_0 + A_1(1 + \tilde{r}) < R(1 + \tilde{x}).$$

Since $A_0 + A_1 = R + E$, this is equivalent to

$$\tilde{y} < -E,$$

where $\tilde{y} = A_1\tilde{r} - R\tilde{x}$ denotes the net operating profit (the difference between financial income and runoff). Assuming that $\tilde{y}$ can be approximated by a normal distribution with mean 0 and variance σ_y^2, the probability of failure is approximated by $N(-E/\sigma_y)$, where N is the distribution of a standard normal variable. Since the 1 percent lower quantile of this distribution is approximately −2, the probability of failure will be less than 1 percent if E is at least equal to $2\sigma_y$. Now σ_y can be computed easily:

$$\sigma_y = \sqrt{A_1^2\sigma_r^2 + R^2\sigma_x^2},$$

where σ_r^2 denotes the variance of risky assets' returns and σ_x^2 denotes the variance of the loss ratio, and assuming that these two risks are independent. Therefore, the way to limit the probability of failure to a predetermined threshold is to impose a minimum capital requirement whose computation is reminiscent of the U.S. RBC:

$$\text{Equity} \geq \sqrt{4A_1^2\sigma_r^2 + 4R^2\sigma_x^2}.$$

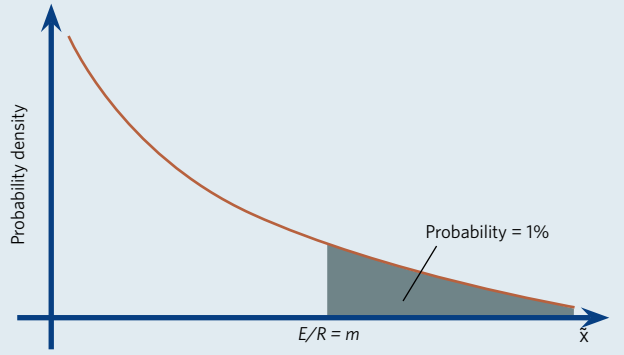
The assumption underlying such ratios, that technical and financial risks are independent, seems to be a heroic one. Casual evidence suggests that distressed insurance companies tend to experience financial and operational difficulties simultaneously. Thus, assuming that these risks are positively correlated seems more realistic. This correlation is, of course, related to the fact that both risks are driven by common factors, namely the organizational inefficiency of the company and the poor quality of its governance.

Practical Limits of Ruin Theory

Even if the actuarial approach provides theoretical foundations for margin requirements and RBC-type formulas, these formulas don't seem particularly good at predicting failures or financial distress in practice. Several scholars (Cummins et al. 1995, 1999; Grace et al. 1993) have studied the predictive power of the RBC and FAST scores for forecasting the failure or financial distress of U.S. insurance companies. All of these studies have concluded that the predictive power of these techniques is very weak. (Note that this does not necessarily imply that these scores are "wrong." It may simply be that statistical

FIGURE 1

Margin requirement for a probability of ruin of 1%.



models of insurance failures are bound to be rejected because the sample of failed insurance companies is too small.)

Other methods, based on cash-flow simulations, seem to work better. In any case, it's fair to say that no simple method is available for predicting the financial distress of insurance companies. It seems unrealistic to assume that supervisory agencies can implement a universal formula for limiting the probability of failures of insurance companies to an exogenous maximum.

Conceptual Limits of Ruin Theory

► *Ruin theory doesn't explain why regulating the probability of ruin is desirable.* Most corporate-finance textbooks start out with the well-known irrelevance result of Modigliani and Miller (1958). This result states that, without any friction on capital markets, the

debt-to-equity ratio of a firm should have no impact on its total value. Indeed, the capital structure affects only the way the total value of the firm is split between shareholders and debt holders.

If we apply the Modigliani-Miller theorem to insurance companies, it's not obvious why capping the probability of ruin should create any value. Asking the shareholders of an insurance company to pledge assets to cover the insurance liabilities in excess of collected premiums should be neutral in a frictionless world. Policyholders should, indeed, be willing to pay higher insurance premiums because this pledge reduces the insurer's probability of default. But this benefit would be exactly offset by the cost of newly committed capital, at least if there is no arbitrage opportunity.

The assumption of perfect, frictionless capital markets that underlies this Modigliani-Miller irrelevance result isn't satisfied in practice. Corporate-finance theory has put forward several reasons why capital structure matters. But ruin theory is, in general, developed in stylized, ideal models where none of the imperfections that may justify the relevance of capital structure are present. In other words, ruin theory studies prudential regulation in models where prudential regulation is pointless!

► *Ruin theory doesn't tell us why capital requirements are the best way to cap the probability of ruin.* Even if one takes for granted that the probability of failure of insurance companies has to be regulated, it's still not clear why the best control for this probability is the capital of insurance companies. To motivate this point, let's slightly enrich the elementary model of ruin theory used in this article by modeling the insurance portfolio more explicitly.

Consider an insurer with equity E and a portfolio of N independent and identically distributed risks. Denote by $\tilde{S}_i$ the random variable representing the loss derived from risk i, during the relevant time period, and for $i = 1,\ldots, N$. We assume that each $\tilde{S}_i$ has a mean μ (normalized to 1) and a standard deviation σ. Each risk is covered by a premium $1 + \rho$, where $\rho > 0$ represents the loading factor (net of reinsurance premiums).

The probability of ruin P_r is thus

$$\Pr(E + N(1+\rho) < \tilde{S}_1 + \tilde{S}_2 + \cdots + \tilde{S}_N) = \Pr(S_1 + \tilde{S}_2 + \cdots + \tilde{S}_N - N > E + N\rho).$$

By virtue of Chebyshev's inequality, one has

$$Pr \leq \frac{N\sigma^2}{(E+N\rho)^2} = \frac{1}{\beta^2}, \text{ where } \beta = \frac{E+N\rho}{\sqrt{N}\sigma}.$$

Chebyshev's inequality states that the probability that a zero-mean random variable $\tilde{Y}$ exceeds some threshold α is less than $\text{var}(\tilde{Y})/\alpha^2$. Here we take $\tilde{Y} = S_1 + \tilde{S}_2 + \cdots + \tilde{S}_N - N$ and $\alpha = E + N\rho$. Since individual losses are independent, the variance of $\tilde{Y}$ equals $N\sigma^2$.

Chebyshev's inequality yields a very conservative upper bound for the probability of ruin, used here for illustrative purposes only. In ruin theory, β is usually referred to as the security coefficient. Increasing β amounts to reducing the probability of ruin. This expression shows that there are many ways to raise β. Increasing E by means of capital requirements is, of course, one of them, but there are other methods, like increasing N or ρ or reducing σ.

Thus, instead of imposing capital requirements, why not require insurance companies to load their premiums sufficiently, buy a sufficient amount of reinsurance to reduce σ, or even hold sufficiently large portfolios?

Any sensible practitioner or economist has an obvious answer to this. Modification of the underwriting policy, by raising either the tariffs or the size of the portfolio, has to be carried out very cautiously. Otherwise, for reasons of informational asymmetry in particular, the complex adverse effects of such strategies on the nature of underwritten risks may well overcome the benefits.

Similarly, reinsurance reduces the volatility of losses σ, but also the expected profit ratio (net loading factor) ρ, because some fraction of the premiums is used to reward reinsurers' risk taking. Whether these two effects result in an increase or a decrease of the probability of ruin depends on the design and pricing of the reinsurance treaty.

On the whole, it's hard to believe that regulators would be able to use such alternative tools to control for the probability of ruin and to account properly for their adverse effects. The information-collection and technical skills required to achieve this are too important. But are things really different for capital requirements?

► *Ruin theory neglects the market's response to regulation.* Let us again assume that it's desirable to limit the occurrence of ruin due to bad luck. Let us further assume that the only tool available to the regulator to achieve this aim is a capital requirement. Ruin theory still misses the point that such a tool has to be handled very carefully in order to deliver an appropriate outcome. This approach views insurance companies as "black boxes'" transforming premiums into random variables.

But insurance companies are firms that respond optimally to their economic environment and business conditions. Imposing a capital requirement affects the cost of one of the crucial inputs of the insurance production function: capital. Therefore, any analysis of the impact of capital requirements should take into account the response of insurance companies to those new production costs.

The theoretical study of this response has been carried out in the banking sector by, among others, Kim and Santomero (1988) and Rochet (1992). These studies show that ill-designed capital requirements may lead to "regulatory arbitrage" by banks, namely activities aimed at reducing their regulatory capital requirement while actually increasing their risk of failure. This explains why the Basel Committee has expended a lot of effort reforming the Cooke ratio toward a more risk-based approach.

Regarding insurance, this suggests that the U.S. system of RBC, aimed at reflecting the riskiness of assets and insurance portfolios better than the European solvency margin, may deal better with regulatory arbitrage. However, it's not clear how a one-size-fits-all regulation, implemented by a regulator that cannot possibly know as much about firms as the firms themselves, could really be risk based and hence not distort insurers' strategies toward inefficient portfolios. ●