“Stock Market Volatility and Learning” by K. Adams, A. Marcet & J.P. Nicolini

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• Nothing exotic or non-standard in the model.
• Avoid freeing up too many parameters.
• An determined attempt to take learning models into the mainstream of macro.
General comments
• Show that during the (slow) process of convergence to rational expectations, an OLS learning model generates data that closely match the behavior of actual stock markets.
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- Show that during the (slow) process of convergence to rational expectations, an OLS learning model generates data that closely match the behavior of actual stock markets.
- Mechanism: beliefs affect prices and prices affect beliefs $\Rightarrow$ possibility of momentum effect
- Key result: this effect is strong during the transition to R.E., yet weak close to it.
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- OLS learning rule is intuitive, has desirable properties, yet remains arbitrary. At the very least, the exogeneity of the gain sequence is problematic.
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• No attention paid to the fact that, in this model, the fundamental value of the asset can be perfectly computed by all investors
Under the hood
• No dividend uncertainty: everyone knows $D_t = 1$ for all $t$ (⇒ focus on learning about *prices*).
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• Timing: observe $p(t)$, compute $p^E(t + dt)$, observe $p(t + dt)$ etc.
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• Interest rate: $r = \delta$
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- Timing: observe $p(t)$, compute $p^E(t + dt)$, observe $p(t + dt)$ etc.
- Interest rate: $r = \delta$
- No rational bubbles (e.g., Ramsey model)
Under risk neutrality

\[
\frac{1 + p^E}{p} = r
\]

Introduction

Comments

Under the hood

Simplifications

Pricing equation

Rational expectations

Irrational expectations

Two important results

Learning rule

More results

Moral

Fundamentals

Conclusion
Rational expectations

\[
1 + \frac{p^E}{p} = r
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Rational expectations

\[ \frac{1 + \dot{p}^E}{\dot{p}} = r \]

• Perfect foresight on prices: \( \dot{p}^F = \dot{p} \).
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- Perfect foresight on prices: \( \dot{p}^F = \dot{p} \).
- If there are no bubbles, the RE price is \( p = 1/r \).
- Investors rationally expect 
  \[
  \frac{\dot{p}^E}{p} = 0
  \]
Irrational expectations

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- Assume that people irrationally expect
  \[ \dot{p}^E / p = \beta, \quad \beta < r. \]
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Assume that people irrationally expect

$$\frac{1 + \dot{p}^E}{p} = r$$

• Then $p = 1/(r - \beta)$. 

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Irrational expectations

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- Assume that people irrationally expect
  \[ \dot{p}^E / p = \beta, \quad \beta < r. \]

- Then \( p = 1 / (r - \beta) \).
- Actual price growth is
  \[ \dot{p} / p = p \dot{\beta} = \dot{\beta} / (r - \beta). \]
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- Then \( p = 1/(r - \beta). \)
- Actual price growth is
  \[ \frac{\dot{p}}{p} = p \dot{\beta} = \dot{\beta}/(r - \beta). \]

- Hence both the level and rate of change of price-growth expectations determine actual growth. \textbf{General result}
Two important results

\[ \frac{\dot{p}}{p} = p\dot{\beta} = \frac{1}{r - \beta} \dot{\beta} \]

- Price growth generated by the learning model exceeds the fundamental growth rate (\(\dot{p} / p > 0\)) if growth expectations are increasingly bullish (\(\dot{\beta} > 0\)).
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- If the current price is high (\(\beta\) close to \(r\)), then even a small degree of bearishness (\(\dot{\beta} < 0\)) can cause a huge price collapse. When the current price is low, \(\dot{\beta}\) does not have a large influence on prices.
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But these are simply **results 1 and 4** in the paper!
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- They can be for sure be extended to a stochastic discount factor.
- They are general.
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• Hence

$$\dot{\beta} = -\theta' \beta, \quad \theta' = \frac{\theta}{1 - \theta \frac{1}{r - \beta}}.$$
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- In the *long run*, the gain \( \theta \) is small, so \( \theta' \approx \theta > 0 \): convergence to RE \((\beta \to 0)\).
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  \[ r > \beta > r - \theta. \]
- If this inequality is verified, anything that throws \( \beta \) off its (zero) SS RE value is reinforced in the short run by self-referential learning: *momentum, bubbles.*
• The paper assumes $\beta_0 = 0$ and relies on small dividend shocks to move $\beta$ away from zero right away (big initial gain).
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• The alternative (followed here) is to eliminate shocks and allow $\beta_0 \neq 0$. 
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• What the stripped-down model teaches us more generally about learning and stock prices?
  ▶ Short- and long-run stability properties of learning rules may differ a lot.
  ▶ Crucial role of decreasing gain.
  ▶ The shape of the learning rule is key to understanding how small dividend shocks interact, on impact, with learning to produce big movements in prices.
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• Moral: to understand price crashes, one needs a model in which the fundamental i) must be learned and ii) can crash.
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• Moral: use economic arguments instead (TVC condition probably rules out equilibria with \( \dot{p}^E / p = \beta > r \)).
Conclusion
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• Generalize H-J bounds to learning economies?