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SECULAR TRENDS IN WEALTH AND HETEROGENEOUS CAPITAL: LAND IS BACK... AND SHOULD BE TAXED

Odran Bonnet, Guillaume Chapelle, Alain Trannoy, and Etienne Wasmer

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Secular trends in Wealth and Heterogeneous Capital:

Land is back...and should be taxed *

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October 2, 2019

Abstract

The increase in wealth-to-income ratios in the second half of XXth century has recently received much attention. We decompose the trend in physical capital and housing, further decomposed into structures and land. In four out of five major countries analyzed, the positive trend in capital-income ratio arises from housing and specifically from its land component.

We therefore revisit the question of wealth inequality and taxation in adopting a Georgist perspective (from Henry George, 1879) subsequently endorsed by prominent economists. We introduce land and housing structures in Judd's optimal taxation framework. We show that an optimal taxation implies a property tax on land and no tax on capital.

When the range of property taxes is politically constrained, taxing the product of housing rents is not optimal, even with additional taxes on "imputed rents". Rent taxes are however less distortive than a capital tax. The distortion depends on the share of housing structures and how they react to the tax on rents. However, a tax on rents complemented by a subsidy on structures investments in rental housing units does almost as well as a land tax. As a side result, we find that Judd's result of no second best capital taxation extends to a larger range of parameters at the steady-state.

*Key words: Capital, Wealth, Housing, Land, Optimal tax, First best, second best. JEL codes: D91, O11, R14. This work is a development of the May 14, 2014 discussion paper 2014-07 of the Economics Department in Sciences Po (<http://ideas.repec.org/p/spo/wpecon/infodl12441-30nstiku669g1br6616n7mc2oq.html>), itself a translation of a first note in French (LIEPP's discussion paper 25, April 2014). We would like to thank, among many others, Philippe Aghion, Jess Benhabib, Alex Citanna, Robert Gary-Bobo, Luigi Guiso, Zvi Hercowitz, Ricardo Lagos, Claudio Michelacci, Abdoulaye Ndiaye, Luigi Pacello, Eric Rasmusen, Pietro Reichlin, Peter Rupert, Philippe Weil, Pablo Winant and Mike Woodford for discussions, useful suggestions and comments, as well as the seminar participants in numerous seminars including EIEF, Copenhagen Business School, Queen's College London, LISER, CREM in Paris Dauphine, Roy seminar in Paris, Sciences Po seminar, Leuven, Toulouse and Canazei. Etienne Wasmer and Guillaume Chapelle acknowledge support from ANR-11-LABX-0091 (LIEPP) and ANR-11-IDEX-0005-02. Alain Trannoy, Etienne Wasmer and Guillaume Chapelle also acknowledge support from ANR-17-CE41-0008 (ECHOPPE)

1 Introduction

In the recent years, a discussion arose around the trends in capital over income. In *Capital in the 21st Century*, Thomas Piketty carefully documents the strong secular rise of the capital/income ratio, especially in France, but also provides evidence of a similar trend in other countries (Piketty, 2013, 2014). Starting from this finding, the author argues for a widespread policy of increased wealth and capital taxation.

Our paper reconsiders the evolution of wealth and capital over the last decades and details the relative evolution of physical capital vs. housing. Housing alone explains the rise of wealth relative to income. The paper also details the relative evolution of housing structures vs land: the latter explains the rise of housing wealth relative to physical capital. The paper then investigates the normative implications of these trends. It brings back land, a long forgotten factor in modern public economics, to the heart of the theory, adopting the so-called Georgist view. Henry George's manifesto (George, 1879) *Progress and Poverty* argued within "the single tax movement" that a tax on land rent would allow to redistribute the return of the common heritage to benefit all individuals.

We extend a Judd-type model of capitalist and workers, a well-suited framework to study redistributive capital taxation, in introducing land, housing structures, housing consumption and housing rental market. In this setup, we show that it is enough to tax land. This is an old idea, revived here to understand in which cases and at which level land taxes can be achieved. When land can indeed be taxed, taxing productive capital would be almost useless. The upshot is that a wealth tax taxing uniformly all three kinds of wealth, land, structures and capital at the same rate is not recommended. The convergence to an optimal zero-tax rate on capital is fully consistent with the seminal Chamley-Judd result, see Judd (1985); Chamley (1986), and we explore its range of applicability.

Accounting for fixity of land and the consumption of its service by households, we quantify the positive rate for the property tax of land to reach the social planner's objective. The tax rate has to be set so as to reduce the inequalities of income, consumption and welfare between capitalists and workers-tenants. This tax will allow to compensate the wage-earners for the fact that they have no property right on capital and land. A uniform tax on capital would instead be misleading as it does not take into account the heterogeneity of capital.

A tax on the returns to housing capital (housing rents) can be implemented in the first best, but not alone. In the absence of tax on land, the rent tax distorts the allocation of land across types of agents through a classical tax-wedge effect; adding a tax on imputed rents (taxing owners-occupiers and landlords) corrects for this distortion, but still affects investments in residential structures which

is then sub-optimal. A tax on rent therefore requires the addition of i) a tax on land differentiated across the use of land - less on rental land and more on home-occupied land, and ii) further, requires a specific subsidy on investments on rented structures. In the absence of this specific combination of tax/subsidies structures, the first best cannot be achieved¹.

However, a land tax, whether homogeneous or differentiated, may not be available at any level. This may be due to the fact that not all countries have a land register (as in most developing countries); or because the first best property tax rate is not attainable because there is a cap on property tax rate (as in California); or finally because it is not possible for political reasons to tax implicit (imputed) rents. We therefore compare a set of second-best tax arrangements and confirm that taxation of capital is the most distortive.

We then explore the second best Ramsey logic where the social planner acts under the rationality constraints of agents, in the spirit of Judd (1985), Benhabib and Szóke (2019), Lansing (1999), Straub and Werning (2014, 2018), when a land tax is not available. We first qualify Judd's result of no taxation of productive capital in the limit, following the vein of Straub and Werning (2018). We then show the conditions under which this negative Judd result remains valid when a rent tax is available. We show that a rent tax (excluding imputed rents) may be second best optimal when the tax revenue is redistributed to tenants. We provide the optimal rent tax formula given by an inverse elasticity rule à la Ramsey. The optimal rate is decreasing in the supply elasticity of rental housing by landowners. The distortion introduced by a rent tax is lower than the distortion introduced by a capital tax and we offer an interpretation of this result thanks to the results obtained by Diamond and Mirrlees (1971a) in a static framework. Finally, the existence of a fixed factor, land, extends the range of parameters under which a steady-state with no capital tax is socially desirable.

Our paper is organized as follows. Section 2 discusses the evolution of the capital labor ratio, its components and its measurement. Section 3 introduces the optimal taxation framework in a model of workers and capitalists à la Judd augmented with housing (land and structure). Section 4 provides welfare analysis in the first best and in various second best exercises, and numerical results add up to our understanding. Section 5 develops a literature review and concluding comments are gathered in Section 6.

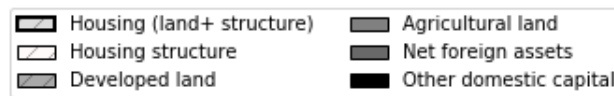
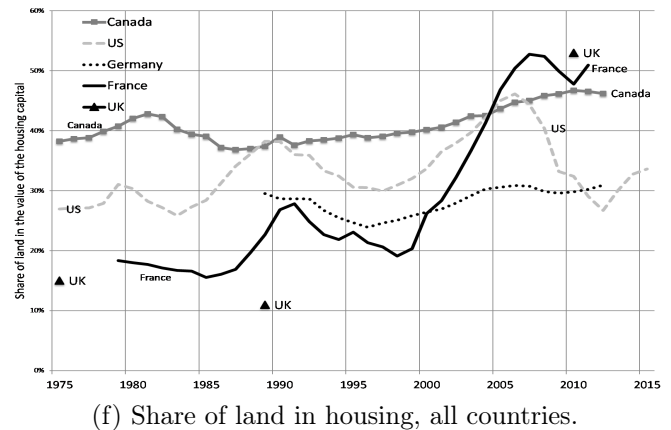
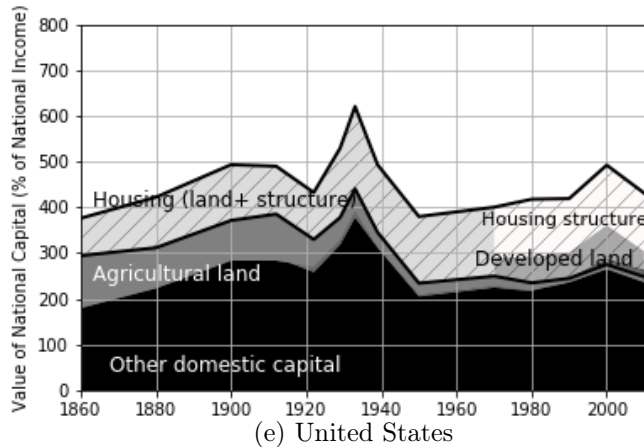
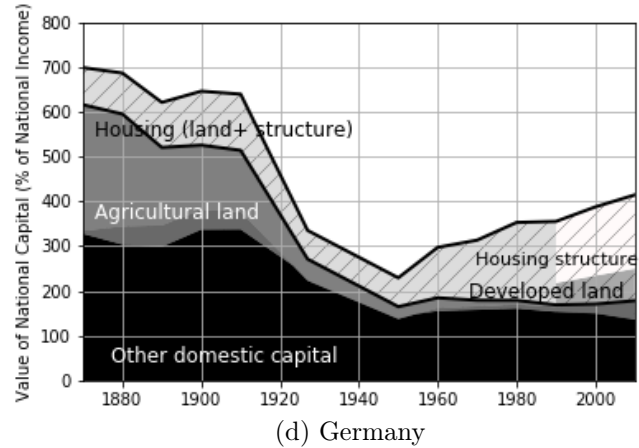
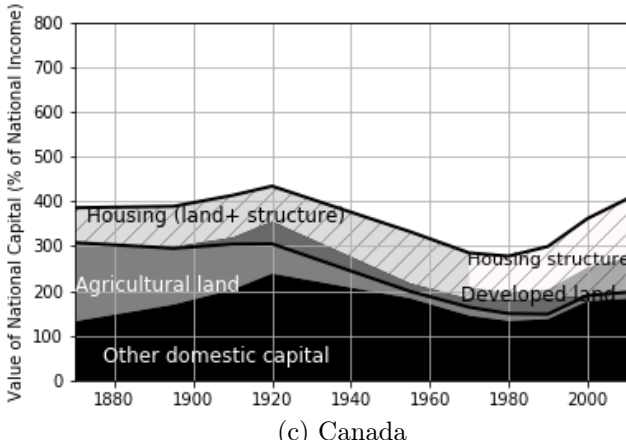
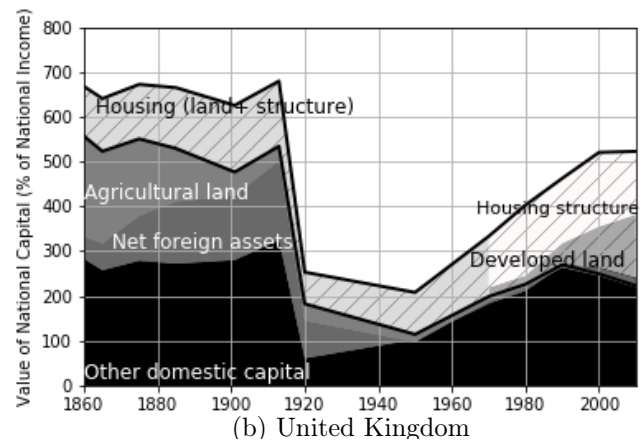
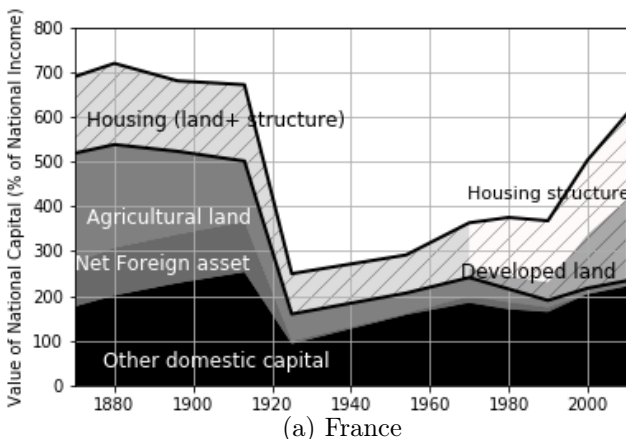
¹This set of instruments provides an illustration of Lipsey and Lancaster (1956) analysis.

2 Wealth: measurement and decomposition

2.1 Trends in aggregate wealth

Figure 1 represents the historical evolution of wealth to income ratios in five countries, using Piketty's decomposition into housing, agricultural land, net foreign assets and other domestic asset². It appears that housing became, in the XXth century, a major component of wealth and explains most of the upwards trends in France and the UK. This can be seen in inspecting the evolution of all series below the black area, which represent all but housing (agricultural land, net foreign assets and other capital). These wealth-to-income ratios would drop if one excluded housing wealth. This is in particular due to the decline in agricultural rent in France as shown in panel *a*) in Figure 1. In the UK, Canada and Germany (panels *b*) to *d*)), excluding housing similarly leads to very small secular increases in wealth-to-income ratios or to stable evolutions. In the absence of housing, instead of a U-curve, one would be observed a L-curve with a flat level of capital in the second half of the XXth century. In the US (panel *e*), the wealth to income ratio net of housing wealth was the same in 1770 as it was in 2010, and there is neither a long run trend nor a recent increase of this ratio. We do not however claim that housing should be excluded from the analysis of wealth to income ratios, but rather than the implication for taxation should take into account that housing structures and land are the primary drivers of the recent trends in wealth accumulation. It is also important to note that Piketty and Zucman (2014) not only acknowledged the role of housing, but also decompose the effects of prices versus real quantities in the development of K/Y .

²In the data in Figure 1, the measurement of housing wealth is the sum of two elements, structures and developed land with constructs, and Appendix A describes the main method as well as a comparison with alternative methods used in national accounts.



Figure's notes. Sources panels a) to e): Capital in the 21st Century, Figures 4.6 & Chart 3.2- see piketty.pse.ens.fr/capital21c and author's calculation. In Canada, net foreign assets are negative, explaining the small part above agricultural rent that is covered by housing wealth. Sources panel f): France, Canada and Germany: Piketty (<http://piketty.pse.ens.fr/en/capital21c>). United States: Davis and Heathcote (2007) (<https://www.lincolnst.edu/subcenters/land-values/price-and-quantity.asp>), last consultation November 2015. United Kingdom: Knoll et al. (2017).

Figure 1: The role of land and housing in the secular variations in the wealth-to-income ratio. **France, US, Canada, UK and Germany.**

2.2 Trends in the land component of wealth

A second observation derived from Figure 1 is the specific role of developed land in the trends. Housing prices can be decomposed into a land component and construction costs (or alternatively the price of “structures”). The land component is “almost” a fixed factor: it is not easily reproducible or at least relatively inelastic in the short run due to geographical or legal barriers; while housing structures are the outcome of current or past residential investments and are more elastic. Therefore, before proceeding with a normative analysis, it is worth separating out the role of each respective component of housing.

To the original Piketty series, we added a decomposition of housing into land and structures when available, typically after the 1970’s. Following the methodology in Davis and Heathcote (2007), one uses a perpetual inventory method similar to the one used to recover capital stocks. This decomposes the evolution of each component (land and structure). It is further illustrated in panel f) which presents the developed land leverage (the share of land in the value of housing wealth). In all countries, the share of land in housing has been rising over the last 4 decades. Nowadays, it represents about 50% of the housing wealth in Canada, France and the UK. We can therefore conclude that occupied land is the main factor responsible for the rise of housing wealth and therefore in the wealth-to-income ratios. This is especially visible in the two countries where the wealth-to-income ratio increased most since the 1950’s, France and the UK.

Our analysis rejoins that of other works on the overall trends of land. In particular, similar conclusions on the importance of land price dynamics for a sample of 14 OECD countries were found in Knoll et al. (2017). Most of the appreciation in housing prices comes from an appreciation of land prices while construction costs only went through a moderate increase³.

Finally, the reasons behind the rise of residential land value are beyond the scope of this paper and have been discussed convincingly in a growing stream of literature having notably documented the large values of urban land (Albouy (2016); Albouy et al. (2018)) which might be partially explained by land use regulation (Albouy and Ehrlich (2018); Glaeser et al. (2005)).

In conclusion of this empirical introduction, the main facts relevant for theory are: i) capital is

³We do not take a stance on the reasons for the increasing importance of land. It might be caused by the lack of innovation in the transportation sector (Knoll et al., 2017), a concentration of individuals within the national territory in major agglomerations, it could be financial shocks and innovations as in Garriga et al. (2019) who study the decorrelation of US housing prices and rents due to financial shocks. It can finally be due to land regulation interacting with growing housing demand. The causes of these dynamics are left for further research but our empirical findings rejoin those of Knoll et al. (2017), Geerolf (2018), Grossmann and Steger (2016) and Borri and Reichlin (2016). Finally, even if our focus is on the capital stock, it is worth noting that similar remarks can apply to trends in capital income: Cette et al. (2019) recently emphasized that the decline in labor share could partially be explained by the rise of real estate income.

highly heterogeneous; ii) in particular, the recently observed rise in the capital to GDP ratio is mostly due to a rise in housing (and not to an accumulation of the physical stock of capital and structures); iii) land, a quasi-fixed asset plays an important role in these developments.

The next two Sections will therefore introduce developed land and investment in housing structures to a standard model of physical capital and labor. It will be used to study differentiated taxation of wealth, since taxing physical capital and land through a unique wealth tax could be sub-optimal given the heterogeneity in capital and sources of wealth.

3 A model of optimal taxation with housing and physical capital

This Section develops a framework to understand the sharing of returns to wealth, productive capital and land. Judd (1985)'s model is particularly well suited to discuss redistributive optimal taxation of capital because its structure yields the highest incentives to tax capital for redistributive purposes: i) there are agents with no savings - that is, no access to credit markets - and consuming only their labor income and the possible transfers; and ii) there are agents with the ability to transfer wealth across periods owning a combination of productive capital.

We start from Judd's redistribution framework of a two-class economy with workers and capitalists producing one composite good. We extend it with housing, as a combination of land and structure. The agents with no access to savings market consume housing and therefore are both workers and tenants. The agents with access to capital market own physical capital, invest and replace depreciation, own land, consume part of it for themselves and obtain rents from workers/tenants for the rest; they also invest in structures.

The capitalists are the sole landlords of the economy; whereas workers are tenants⁴. The model is thus a model of a representative working class with no assets and a representative capitalist class holding all assets, land and physical capital, in the spirit of Judd (1985). We will consider throughout this Section, for expositional reasons, the case where the mass of workers and capitalists are the same. In Appendix C we discuss the extension to the general case of individual agents of different mass.

We start from the social planner's program referred to as the best situation. Next, we look at how a decentralized equilibrium with appropriate taxes reaches the first best of the social planner. We then study the case in which the optimal tax structure cannot be implemented; we refer to this situation as constrained social optimum. Finally, we discuss convergence issues along the way of

⁴This is a highly simplified, though a quite good representation of the actual world. It is true that housing property is more widespread than financial asset, but land ownership is more concentrated: according to Wolff (2017), the wealthiest 10% owned 82% of undeveloped land which is a buffer stock of developed land.

Straub and Werning (2018) and extend some of their results to housing.

3.1 Setup

Time is discrete, indexed by t . Agents consume each period a representative consumption good and a composite housing service. The housing service associates both land and housing structures. There are two types of agents, a representative capitalist and a representative agent.

3.1.1 Capitalists

The class owning all assets is called generically capitalists. Following the traditional exposition and notations, capitalists consume an amount C_t and a housing service denoted by H_t . The utility function is $\sum_{t=0}^{\infty} \beta^t U(C_t, H_t)$ where $0 < \beta = (1 + \rho)^{-1} < 1$ is a discount factor and we denote the marginal utility as respectively $U_C(t)$ and $U_H(t)$ when inputs are at time t . The preferences for each good are supposed to satisfy the Inada conditions, that is, the first units of consumption (respectively housing service) have an infinite marginal utility for all consumption baskets. Inada conditions also impose that the limit of marginal utility will go to 0 in infinity which will insure that transversality conditions to be satisfied.

Capitalists own both a capital good K_t and the total quantity of land $\bar{\mathcal{L}}$, assumed to be fixed; they also have the property of housing structures. Capitalists allocate part of land \mathcal{L}_t to their own consumption, and devote the rest to the rental housing sector, in quantity $\mathcal{L}_t - \bar{\mathcal{L}}$. Capital is used to produce a generic good with the mass of labor of size 1, thanks to a constant returns to scale production technology denoted $F(K_t, 1) = f(K_t)$. Capitalists inherit a stock of capital K_0 , which depreciates at rate δ ; they invest a quantity I_t so that

$$K_{t+1} = K_t(1 - \delta) + I_t.$$

Following the traditional exposition and notations, capitalists consume an amount C_t . The production of housing service is denoted by

$$H_t = H(\mathcal{L}_t, S_t)$$

where S_t is the housing structure of the housing units consumed. This production of service follows Lancaster (1966) and combines different inputs (land and structures) that enters into utility as a bundle. This is similar to Becker's home production theory, where in Becker (1965), this is a combination

of time and resources. Unless specified, the function $H(., .)$ will be assumed to be constant-returns to scale and subject to Inada conditions. It means that camping (land but not structure) or piles of bricks alone (structures but no land) provides no utility⁵.

We denote by $H'_{\mathcal{L}}(t)$ and $H'_S(t)$ the derivatives at time t of the marginal product of housing service of each argument. S_t is another capital good and its law of motions is similar to that of investment with depreciation rate δ_S :

$$S_{t+1} = S_t(1 - \delta_S) + I_t^S$$

Similarly, they invest in the structures for rented units denoted by s_t and therefore:

$$s_{t+1} = s_t(1 - \delta_s) + I_t^s$$

In what follows and to simplify exposition, we assume that $\delta_s = \delta_S$.

The produced good $f(K_t)$ can be transformed into consumption C_t, c_t or into new capital K_{t+1} , into new structures S_{t+1} or s_{t+1} or into government consumption G_t . We assume for simplicity that the marginal rate of transformation between these different components is 1.

3.1.2 Workers

The class generically called workers supplies its labor to the capitalist in fixed quantity and rents the housing service h_t from it. Their consumption is denoted by c_t and the housing service is supplied according to the production function similar to H_t , denoted h_t with

$$h_t = h(l_t, s)$$

where $l_t = \bar{\mathcal{L}} - \mathcal{L}_t$. As before, we denote by $h'_s(t)$ and $h'_l(t)$ the marginal product of each input. Formally, $h'_{\mathcal{L}}(t) = -h'_l < 0$ ⁶. Finally, workers' utility is the one period consumption of goods and housing service, $u(c_t, h_t)$.

⁵Our modelling of housing follows Muth (1969) and assumes a weakly separable utility function between the composite commodity from one side and land and structure in the other side (see also the discussion in McDonald (1981). This is in line with recent empirical evidence provided in Combes et al. (2017) and Epple et al. (2010) who find that the housing production function is close to a Cobb Douglas.

⁶For this housing service s_t , as for that of the capitalist defined above, we will think of their production as a constant returns to scale function with Inada conditions applying there too. This specification is sometimes used in the macroeconomic literature. See for instance the recent paper by Garriga et al. (2019)

3.2 Social planner's program

3.2.1 Social planner's objective

The social planner wants to maximize a weighted average of the utility of each agent, where the weight of capitalists/landowners is given by $1 \geq \gamma \geq 0$. A weight of zero implies that the social planner only cares about those without property rights on land and capital (we will call this situation Rawlsian). A weight equal to 1 is the utilitarian case. We will mostly consider intermediate cases with $0 < \gamma < 1$ with low values of γ , consistently with the literature. γ can also be interpreted as the mass of capitalists relative to workers. We prove in Appendix that all subsequent results hold if this mass is introduced in the objective functions of the social planner.

The social planner resource constraint is

$$f(K_t) + (1 - \delta)K_t + (1 - \delta_S)(S_t + s_t) = c_t + C_t + S_{t+1} + s_{t+1} + K_{t+1} \quad (1)$$

Hence, taxes affecting the decentralized equilibrium will be purely redistributive and have the purpose of raising the consumption of goods and housing of the worker.

3.2.2 Social planner's optimization

We assume that $U(., .)$ and $u(., .)$ are increasing concave in each argument. The budget constraint per period of the social planner is concave as well as its objective function. The maximization problem of the social planner can then be replaced by the following Lagrangian:

$$\begin{aligned} \max_{c_t, C_t, S_{t+1}, s_{t+1}, \mathcal{L}_t, K_{t+1}} \sum_t \beta^t \{ & u(c_t, h(\bar{\mathcal{L}} - \mathcal{L}_t, s_t)) + \gamma U(C_t, H(\mathcal{L}_t, S_t)) \} \\ & + \beta^t \lambda_t \{ f(K_t) + (1 - \delta)K_t + (1 - \delta_S)(S_t + s_t) \\ & - c_t - C_t - S_{t+1} - s_{t+1} - K_{t+1} \} \end{aligned}$$

subject to three transversality conditions on each stock:

$$\beta^t U'_C(t) M_{t+1} \rightarrow 0 \quad (2)$$

for $M_{t+1} = K_{t+1}, S_{t+1}, s_{t+1}$. In what follows, to avoid multiple indices, we add the time period when relevant in parenthesis, e.g. $R^{Kgross}(t+1)$ is the gross return to capital producing returns next period

and defined as:

$$R^{Kgross}(t+1) = f'(K_{t+1}) + 1 - \delta$$

We then obtain the first order conditions for each period:

$$\partial C_t, c_t \quad \lambda_t = \gamma U'_C(t) = u'_c(t) \quad (3)$$

$$\partial \mathcal{L}_t \quad \gamma U'_H H'_L(t) = u'_h h'_l(t) \quad (4)$$

$$\partial S_{t+1} \quad \lambda_t = \beta \lambda_{t+1} (1 - \delta_S) + \gamma \beta U'_H H'_S(t+1) \quad (5)$$

$$\partial s_{t+1} \quad \lambda_t = \beta \lambda_{t+1} (1 - \delta_S) + \beta u'_h h'_s(t+1) \quad (6)$$

$$\partial K_{t+1} \quad \lambda_t = \beta \lambda_{t+1} R^{Kgross}(t+1) \quad (7)$$

This notably implies the following steady-state relations:

$$\partial C_t, c_t \quad \lambda = \gamma U'_C = u'_c \quad (8)$$

$$\partial \mathcal{L}_t \quad \gamma U'_H H'_L = u'_h h'_l \quad (9)$$

$$Euler \quad S_{t+1} \quad \beta^{-1} = \frac{U'_H H'_S}{U'_C} + 1 - \delta_S \quad (10)$$

$$Euler \quad s_{t+1} \quad \beta^{-1} = \frac{u'_h h'_s}{u'_c} + 1 - \delta_S \quad (11)$$

$$Euler \quad K_{t+1} \quad \beta^{-1} = R^{Kgross} = f'(K) + 1 - \delta \quad (12)$$

The first two equalities are the result of intra-period optimization. The first one states that the planner wants to equalize the marginal utilities of consumption across the two types of agents, up to the social weight γ , and the second one has a similar interpretation in terms of the marginal utility of the housing service with respect to land. They also imply that the marginal rate of substitution between land as a producer of housing service and consumption are equal across agents: $\frac{U'_H H'_L}{U'_C} = \frac{u'_h h'_l}{u'_c}$, that leads also to determine the ratio of marginal utilities across agents and fix it to γ :

$$\frac{u'_h h'_l}{U'_H H'_L} = \frac{u'_c}{U'_C} = \gamma \quad (13)$$

In addition, there are three intertemporal first-order conditions, the Euler equations of the problem. The Euler equation on capital classically states that the net return on capital at the social optimum has to be equal to the discount rate of agents. The earlier equations on structures have similar interpretations: one invests on structure up to the point that the net return of structure given by the marginal rate of substitution ($U'_H H'_S/U'_C$ or $u'_h h'_s/u'_c$) net of depreciation will be equal to the

discount rate. It stems from these three Euler equations that the net rate of return in investing in productive capital or in the two types of structures must be the same. Combining (10), (11) and (12), one further gets that the marginal productivity of capital in the case when the depreciation rate is the same for capital and structures and in the more general case, the result holds up to differences in depreciation rates:

$$\frac{U'_H H'_S}{U'_C} + (1 - \delta_S) = \frac{u'_h h'_s}{u'_c} + (1 - \delta_S) = f'(K) + (1 - \delta) = \beta^{-1} \quad (14)$$

The three equalities in equation (14) together with equation (13) and the resource constraint (1) define five conditions for six endogenous variables: $c, C, K, S, s, \mathcal{L}$. The conditions are independent of the social weight γ . Hence, they define an efficient allocation set of dimension 1 and correspond to what we hereafter refer to as the "first best". The particular solution chosen by the social planner solution depends of γ and is fixed thanks to either equation (3) or (4).

Finally, the marginal rate of transformation between space (land) and structures must be equal across agents regardless of the weight given to the capitalist:

$$\frac{H'_S}{H'_\mathcal{L}} = \frac{h'_s}{h'_l} \quad (15)$$

and combining equations (13) and (15), the marginal rate of substitution between consumption and structures are equalized across agents:

$$\frac{U'_C}{U'_H H'_S} = \frac{u'_c}{u'_h h'_s}. \quad (16)$$

3.3 The decentralized equilibrium with taxes

3.3.1 Setup

We discuss the distortions generated by several tax schemes. First, we investigate the simplest form of capital taxation relying on a single tax on land $\tau_{\bar{\mathcal{L}}}$ on productive capital τ_K owned by the capitalist. In the welfare discussion, we also consider a tax on housing rents paid by landlords τ_H .

In next sub-sections, we also consider a tax on imputed rents for landlords living in their own property (denoted by τ_{HI}), and finally turn to a living tax on the housing consumption of both capitalists and workers (τ_{liv}). Finally, we discuss how these taxes might be combined with a tax or a subsidy on housing investments, i.e on structures for capitalists (τ_S) or workers (τ_s). For convenience and to avoid repetitions, all taxes are introduced simultaneously in Appendix B.3, and our

Propositions are simple cases of the general case but with start here with taxes on capital, rents and land. Capitalists do not work, so their income is the sum of the net return on physical capital, net rents and other taxes that may affect them, and they pay a market wage w_t . Markets are perfectly competitive, and we define the wage w_t as usual as:

$$w_t = f(K_t) - f'(K_t)K_t \quad (17)$$

The net, after-tax return on capital is

$$R_t^{Knet} = (1 - \tau_{K,t})R_t^{Kgross} \quad (18)$$

where taxes on capital at time t are $\tau_{K,t}$.

We also use the notation R_t^{Hgross} for the gross rent on land and $\tau_{H,t}$ for the tax rate of rents so that $R_t^{Hnet} = R_t^{Hgross}(1 - \tau_{H,t})$ and capitalists therefore receive a rent income $h_t R_t^{Hnet}$.

In addition, we allow for taxes of land. The tax on land may differ for each type of land but for the moment we introduce a unique tax is $\tau_{\bar{\mathcal{L}},t}$ and the income from taxation of capitalist's land is $T_t^{\mathcal{L}} = \tau_{\bar{\mathcal{L}},t}\bar{\mathcal{L}}$.

Let $T_t^K = \tau_{K,t}R_t^{Kgross}K$ be the tax revenue from capital, $T_t^H = \tau_{H,t}R_t^{Hgross}h_t$ the tax revenue from rents. The sum of these components are the total taxes T_t that will be transferred to the worker, with

$$T_t = T_t^K + T_t^H + T_t^{\mathcal{L}}$$

3.3.2 Decentralized agents' program

The objective of each agents (capitalist and workers) is concave as well as their resource constraints so that we can directly proceed with their respective Lagrangians. The capitalist optimizes over an infinite horizon, whereas the program of the worker is a static one, consuming the current disposable income and transfers into consumption and housing services. In the absence of government bonds

and thus other assets than housing and capital⁷, the capitalist solves:

$$\begin{aligned} & \max_{C_t, H_t, \mathcal{L}_t, K_{t+1}, s_{t+1}, S_{t+1}} \sum_t \beta^t U(C_t, H(\mathcal{L}_t, S_t)) \\ & + \beta^t \lambda_t^K \{ R_t^{Knet} K_t + R^{Hnet} h(\bar{\mathcal{L}} - \mathcal{L}_t, s_t) \\ & + (1 - \delta_S) S_t + (1 - \delta_S) s_t - T_t^{\mathcal{L}} \\ & - C_t - S_{t+1} - s_{t+1} - K_{t+1} \} \end{aligned}$$

subject to subject to $\beta^t U'_C(t) K_{t+1} \rightarrow 0$, and the program of the worker is:

$$\begin{aligned} & \max_{c_t, h_t} u(c_t, h_t) \\ & \text{subject to } c_t + h_t R_t^{Hgross} = w_t + T_t \end{aligned}$$

In this program, the worker expresses a demand for both h_t and c_t . It cannot chose separately land and structures. This specification of choices leads to the first order conditions:

Worker: intraperiod

$$\partial c_t \quad u'_c = \lambda_t^w u'_h(t) \quad (19)$$

$$\partial h_t \quad u'_h(t) = \lambda_t^w R_t^{Hgross}(t) \quad (20)$$

$$\Leftrightarrow u'_h(t) = u'_c R_t^{Hgross} \quad (21)$$

Capitalist: intraperiod

$$\partial C_t \quad U'_C(t) = \lambda_t^K \quad (22)$$

$$\partial \mathcal{L}_t \quad U'_H H'_\mathcal{L}(t) = \lambda_t^K R^{Hnet} h'_l(t) \quad (23)$$

Capitalist: intertemporal (Euler)

$$\partial S_{t+1} \quad \lambda_t^K (1 + \tau_{St}) = \beta \lambda_{t+1}^K (1 - \delta_S) + \beta U'_H H'_S(t+1) \quad (24)$$

$$\partial s_{t+1} \quad \lambda_t^K (1 + \tau_{st}) = \beta \lambda_{t+1}^K (1 - \delta_S) + \beta \lambda_{t+1}^K R_{t+1}^{Hnet} h'_s(t+1) \quad (25)$$

$$\partial K_{t+1} \quad \lambda_t^K = \beta \lambda_{t+1}^K R_{t+1}^{Knet} \quad (26)$$

The intra-period conditions state that the opportunity cost of getting more housing service is the

⁷One interesting way to rewrite the capitalist choice is to introduce wealth defined as $A_t = K_t + (H_t + h_t)$. In this case the resource constraint is such that: $C_t + A_{t+1} - \bar{H} = R_t^{Kgross} (1 - \tau_{K,t}) K_t + R_t^{Hgross} (1 - \tau_{H,t}) (H_t + h_t) - H_t R_t^{Hgross} (1 - \tau_{H,t})$. If the net return on the two assets were identical, then the equation simplifies to $C_t + A_{t+1} - \bar{H} = R_t^{Knet} A_t - H_t R_t^{Knet}$. It has an interesting implication for our discussion because it precisely shows why housing is a particular asset. Landowners have to sacrifice the rent corresponding to their own land use, the last term of the right hand side of equality, otherwise absent from the standard asset accumulation equation when housing is not modeled.

forgone utility of consumption $R_t^{Hgross} u'_c$ for the worker or $R_t^{Hnet} U'_C$ for the capitalist with an extra term for differential tax on the land allocated to housing service h_t or H_t . In the steady-state and after re-arrangement of the different terms detailed in Appendix, one obtains:

Intraperiod allocations

$$\frac{u'_h h'_t}{U'_H H'_C} (1 - \tau_H) = \frac{u'_c}{U'_C} \quad (27)$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{U'_H H'_S}{U'_C} \quad (28)$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + (1 - \tau_H) \frac{u'_h h'_s}{u'_c} \quad (29)$$

$$\partial K \quad \beta^{-1} = R^{Kgross} (1 - \tau_K) \quad (30)$$

Anticipating the results of next Section, it appears, in comparing the decentralized equilibrium in equations (10)-(12) and equation (16) with the social planner's allocations reported above, that taxes on physical capital will affect capital accumulation, while taxes on rent will both distort intra-period allocation of consumption and residential investments, whereas land tax is not distortive. Moreover, the land tax can even be differentiated across types of lands to alleviate the effect of other distortive taxes. The next subsection will further elaborate on the welfare comparison of these taxes.

4 Welfare discussion

4.1 Achieving the social planner's objective with a unique tax on land

To achieve the first best level of capital, the first order conditions of the decentralized equilibrium must coincide with those of the social planner. Further, since there is a large number of welfare solutions, the particular value of γ chosen leads to select the preferred first best solution of the social planner. Collected taxes are used to redistribute income and achieve the desired level of relative marginal utility for each agent.

As regards to capital, Euler equation (12) for the social planner is the target for the corresponding decentralized equilibrium equation (30) and implies that $\tau_K = 0$. A necessary condition to reach a first best is therefore that the tax on capital is zero⁸.

⁸With a tax on investment $\tau_{I,t}$ similar to tax on structures and explored in Appendix, the dynamic Euler equation on capital would become $\lambda_t^K (1 + \tau_{I,t+1}) = \beta \lambda_{t+1}^K [R_{t+1}^{Knet} + (1 - \delta) \tau_{I,t+1}]$. One would then have $1 - \tau_K = 1 + \tau_I \beta \delta$. In the steady-state, if one taxes returns on capital with a positive tax τ_K , one would need to subsidize its investment with a negative tax rate τ_I hence the perfect substitutability between the two tax instruments. Out of the steady-state, the

Similarly, the presence of a strictly positive tax on housing rents τ_H would distort equation (27) relative to the first best in equation (13). Hence, $\tau_H = 0$ too. This avoids a wedge between the value of land between homeowners and tenants. One could argue that also taxing homeowners on "imputed rents" would remove this wedge. This is true, but a positive tax on actual rents and on imputed rents will not be first best in the general case because it distorts investment decisions. This is explored in Propositions 2 and 3 that follow.

Finally, and most importantly here, of the absence of housing taxes is again a necessary condition, but a particular social optimum is not necessarily decentralized in absence of additional taxes and transfers. To do so, one needs to adjust the level of the tax of land to transfer and equalize the ratio of the social marginal utilities of the capitalist and the worker to the desired value of utility desired given a particular social weight γ . This level of land tax is studied in our first Proposition.

Proposition 1 (Optimal land tax): *With a land tax, a first best can be achieved with the combination of*

1. a zero tax on returns on capital: $\tau_K = 0$,
2. a zero tax on housing rents: $\tau_H = 0$,
3. a single tax on land fully redistributed to workers, with revenue $T^{\mathcal{L}} = \tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}}$.
4. In the particular (utilitarian) case $\gamma = 1$ where the utilities of capitalist and worker are separable and identical, the tax revenue is used to collect the returns to capitalists assets (both housing and capital) and redistributed them to reach the desired level by the social planner:

$$\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}} = hR^{Hgross} + Kf'(K) - \frac{1}{2}[f(K) + \delta K + \delta_S(s + S)] \quad (31)$$

5. In the general case $\gamma < 1$, there is no such explicit formula, but a similar formula exists in special cases.

- With an exponential negative sub-utility functions (CARA) $\ln u(c) = -\nu c$ and $\ln U(C) = -\nu C$

$$\tau_{\bar{\mathcal{L}}}\bar{\mathcal{L}} = hR^{Hgross} + Kf'(K) - \frac{1}{2}[f(K) + \delta K + \delta_S(s + S)] - \frac{\ln \gamma}{2\nu} \quad (32)$$

two tax instruments are however not perfect substitutes, because the tax on returns $\tau_{K,t+1}$ taxes the current capital stock K_t and therefore past investments, while $\tau_{I,t+1}$ taxes the new investments. Since we focus on steady-state, we leave this aside and only focus on taxes on returns.

- With instead a CCRA function for the sub-utility of consumption for both the worker and capitalist $U(C) = \frac{C^{(1-\sigma)}}{1-\sigma}$ and $u(c) = \frac{c^{(1-\sigma)}}{1-\sigma}$, the optimal land tax for any social welfare weight for the capitalist is given by

$$\tau_{\bar{\mathcal{L}}} = hR^{Hgross} + Kf'(K) - f(K) + \frac{f(K) - \delta K - \delta_S(s + S)}{1 + \gamma^{-\sigma}} \quad (33)$$

Proof: Given the inelastic supply of land, the homogeneous land tax is not distorting the intra-period choices. $T^{\mathcal{L}}$ enters the government constraint resource and leads, for all values of γ , to

$$\tau_{\bar{\mathcal{L}}} = T^{\mathcal{L}} = R^{Hgross} h \left(1 + \frac{c - C - w - K + R^{Kgross} K}{2R^{Hgross} h} \right) \quad (34)$$

This revenue is used to transfer to the worker the full amount of rents, plus a correcting term reflecting the amount of redistribution beyond rental costs to reach optimal levels of consumption. The correcting term in equation (34) is easier understood in the case where $\gamma = 1$: this implies, when the utility functions of capitalists and workers are identical and separable, that $c = C$ and $h = H$. In that case, the right hand side is half of the difference in gross income between the capitalist/landlord and the worker/tenant, plus housing expenses. After a few easy steps using in particular equation (17) and the definition of returns on capital, one obtains equation (31) in the Proposition. Equations (32) and (33) follow the same logic. See Appendix B.4.3 for further details. ■

In the utilitarian case of 3) above, the simple formula for the optimal property tax implies an equal sharing rule of total resources of the economy net of depreciation, desired by the social planner. The social planner uses the tax on land to compensate workers for the extra-returns on capital and land of the capitalists and equalize their consumption. The product of these taxes is used to redistribute to the worker so as to insure the optimal level of consumption and housing service and needs to be positive to redistribute to workers⁹. The other two formulas apply to the case with $\gamma < 1$ in the CARA (point 4 above) and CRRA (point 5 above) cases. A similar logic of compensation for unequal property rights applies. As expected, the optimal taxed increases as γ goes down from 1 towards 0.

⁹The optimal tax may be negative if the capital share in production is small and the capitalist pays large replacement costs of capital and structures. This is due to the fact that the capitalist and the worker are given an equal demographic weight. In Appendix Section C we relax this and the introduction of demographic weights never affect the first order conditions of the decentralized equilibrium and of the social planner, only the resource constraints and the level of income and consumption of individual agents.

4.2 Policy constraints on land taxes and rent taxes

Many political constraints prevent the first best from being implemented. As put explicitly by Chari et al. (2016), it may not be possible to confiscate a fraction of capital owned by capitalists once and for all. As discussed above, a one-time expropriation of physical capital is not necessary if the social planner could tax land, especially depending on its use. A land tax plays a similar role to the one-time taxation of physical capital as its base is inelastic.

Our analysis so far has shown that the tax on land is indeed crucial to get a first best. What we don't know yet is the distance between the first best and a world where the tax on land cannot be set at its optimal value. This is an important case to study since many political economy reasons limit the use of land taxes, either directly, or indirectly due to the inability to implement them.

The most obvious restriction is the absence of land register. Only 50 countries out of 200 have one according to van der Molen (2003). The other constraint are more political in nature. First it appears that the property tax is the most "hated" tax, as coined in Cabral and Hoxby (2012) in the US, and in Sweden (see Nordblom et al. (2006)). California is famous for the cap on the property tax (proposition 13, June 6, 1978).

It is sometimes claimed that taxes on imputed rents might restore efficiency. We will discuss the range of validity of this proposition later, but even when this is true, it may also be difficult to implement a tax on imputed rent. This is due to the fact that the actual value of the rent is not observed, homeowners may have a greater reluctance to accept it.

Along a related line of reasoning, it may be much easier to tax actual rents, since this is an income that is actually observed, despite its distortive aspect. And for the sake of realism, one may want to explore the possibility of positive tax rents: it is generally treated by the tax authority as a regular income and therefore subject to an income tax.

Overall, we are now going to study the efficiency aspects of a positive tax on rents, and see under which conditions and with which additional tax instruments one can restore efficiency, in a Lipsey-Lancaster's approach (Lipsey and Lancaster (1956)) to the decentralisation of the first best. We start with the case where structures are built up and are given, a situation we referred to as inelastic structures. Next we go on by considering the general case.

4.3 Efficiency with a positive tax on rents and inelastic structures: a differentiated tax on land and a tax on "imputed rents"

Proposition 1 states that the social planner can achieve its desired outcome by only taxing land, but this solution requires to give up on taxation of rents because it is distortive: inspection of equations (27) and (29) implies that a positive tax on rents τ_H will introduce a wedge from the intra-period optimality condition, e.g. equation (13).

It is however possible to restore this condition to be exactly that of the first best, in introducing a differentiated tax on land depending on land use, denoted respectively by $\tau_{\mathcal{L}}$ for the tax on capitalists land and τ_l the tax on landlords land, and denoting their difference by $\Delta\tau_{\mathcal{L}} = \tau_{\mathcal{L}} - \tau_l$. In this case, the social planner has one extra degree of freedom to reach its desired, first best allocation, and this is true for any social weight.

An alternative and common way to restore efficiency of that particular condition is to introduce a tax on imputed rents τ_{HI} , that is on the "rent equivalent" that owner-occupiers serve to themselves.

However, we will now prove that these two solutions will work only in the absence of structures in the housing service: if s, S have a non-zero elasticity in H, h , the tax on rent also affects the return to housing investments, and in particular may lead to sub-optimal investments in rental structures s in equation (11).

We proceed in two steps. We first abstract away, in this sub-section, from the distortions on s, S in the Euler equations. This corresponds to the case where housing service has no structures, only land. In sub-section 4.4.2, we add new instruments that potentially correct for the distortions of investments in structures and discuss their complexity.

So, in the absence of Euler equation on structures, one can show (see Appendix B.3 and B.4.2) that the intra-period allocation of land of landlords in the decentralized equilibrium is given by:

$$\begin{aligned} &\text{Intraperiod allocations} \\ &\frac{u'_h h'_l}{U'_H H'_{\mathcal{L}}} \left(1 - \tau_H + \tau_{HI} \frac{H'_{\mathcal{L}}}{h'_l} + \frac{\Delta\tau_{\mathcal{L}}}{R^{Hgross} h'_l} \right) = \frac{u'_c}{U'_C} \end{aligned} \quad (35)$$

while the first order condition on capital is unchanged.

This leads to a formula for the first-best taxes on land and rents summarized in Proposition 2, since efficiency requires that the term in brackets must be equal to 1.

Proposition 2 (Distortions and efficient land allocation with rent taxes): *In the absence of structures, that is, when $H = H(\mathcal{L})$ and $h = h(l)$, a positive tax on rent τ_H does not distort the*

intra-period allocation if and only if it is compensated by:

1. *either a positive differential tax on land $\Delta\tau_{\mathcal{L}} = \tau_{\mathcal{L}} - \tau_l$, with:*

$$\Delta\tau_{\mathcal{L}} = \tau_H R^{Hgross} h'_l \quad (36)$$

2. *a tax on imputed rents τ_{HI} such that*

$$\tau_{HI} = \tau_H \frac{h'_l}{H'_{\mathcal{L}}} \quad (37)$$

3. *if instead the social planner set for simplicity an equal rate for the actual and imputed rents taxes, that is if $\tau_H = \tau_{HI}$, efficiency only arises if the housing service is linear in land;*
4. *if the service is not linear in land, the equality $\tau_H = \tau_{HI}$ implies, in the absence of the differentiated land tax, a deviation of \mathcal{L} from the first best. The direction of the variation from the first best depends on the marginal product of land in the housing service. In particular, if tenants live in smaller housing units, then $\frac{H'_{\mathcal{L}}}{h'_l} < 1$ and the quantity of land allocated to tenants is too small relative the first best, under separability of housing and consumption in utility.*

These two options remove the distortion effects of the tax on rents, either by taxing more the land used by landlords than the tax on rented land, and the differential tax on land evaluated at its marginal benefits for housing services must be equal to the amount of tax on rents; or by taxing the housing services of homeowner at a higher level as tenants.

4.4 The case with taxes on rent and with structures

The previous analysis indicated that the intraperiod allocation of land is distorted with positive taxes on rents, but that the distortion could be corrected by additional taxes on imputed rents. However, as already alluded to in the text, taxes on rent and imputed rents also affect the two Euler equations on structure. We start by showing the nature of this inefficiency, and then derive a new set of instruments to correct it.

4.4.1 The under-provision of structures

Appendix B.3 shows that with a tax on rent and on imputed rents, the Euler equations are modified as follows:

Intertemporal allocations (Euler)

$$\partial_S \beta^{-1} = 1 - \delta_S + \frac{U'_H H'_S}{U'_C} - \tau_{HI} R^{Hgross} H'_S \quad (38)$$

$$\partial_s \beta^{-1} = 1 - \delta_s + (1 - \tau_H) \frac{u'_h h'_s}{u'_c} \quad (39)$$

It is easy to see that the tax on imputed rents reduces the demand for investments in homeowner structures S in equation (38), although this may not be a primary concern for social planner interested only in workers (a zero value for γ). However, in equation (39) the tax on rents reduces the demand for structures of tenants to a sub-optimal level: landlords under-invest in the walls and other landlord-provided equipment, leading to a lower quality of the housing service. To summarize:

Proposition 3 (Distortions with structures): *There are several deviations from the first best with taxes on rents with imputed rents.*

1. *A single tax on actual rents also distorts the investment in housing structures as well as the intraperiod allocation of space.*
2. *When the social planner tries to restore the intraperiod allocations of housing between homeowners and tenants with positive imputed rents, this further distorts the investment in structures S of the capitalist.*
3. *The outcome of a tax is generally to reduce investments in structures.*
 - *Pose now $\mathbf{s} = (s/l)$ the structure quantity per unit of land. The rent tax including imputed rent entails a new equilibrium with underprovision of \mathbf{s} with respect to the first best*
 - *Pose $\mathbf{S} = \frac{\mathcal{S}}{\mathcal{L}}$ the capitalist-structure quantity per unit of land. We further assume that the housing production function is Cobb-Douglas. Then, the rent tax including imputed rent with compensation for the land distortion considered in Proposition 2.2 leads to a new equilibrium with underprovision of \mathbf{S} with respect to the first best.*

This Proposition is rather pessimistic about the ability of the social planner to use a tax on rents. There are some words of caution here however. First, the social planner does not need to add the

imputed rent tax, if it can use a differential tax on land to correct the distortions on the intraperiod allocations of housing between homeowners and tenants. In that case, the only inefficiency left is that on the Euler equation on tenants' structures. We now show that these specific inefficiencies may be corrected with an additional instrument, a set of subsidies on structures. See Appendix B.4.4 for the details of the proof of the underprovision of structures investments.

4.4.2 Introducing taxes or subsidies on housing structures

We introduce the notations for taxes on structures that are paid by the capitalist (since the capitalist owns all the structures). The tax can further be differentiated across usages: the structures S_t devoted to its own consumption, or the structures s_t devoted to rental. At this stage, we do not restrict the sign of the taxes: they can be negative to allow for subsidies if needed, e.g. to favor investment and possibly correct for distortions from other taxes.

Investment in the structures of S_t or s_t may be taxed through flat tax. The tax may be negative for instance to subsidy renovation, including environmental concerns.

To simplify and to treat both taxes/subsidies symmetrically, we introduce the notations τ_{St} and τ_{st} the tax rates per unit invested, the product of the tax equals:

$$T_t^{s,S} = \tau_{St} [S_{t+1} - S_t(1 - \delta_S)] + \tau_{st} [s_{t+1} - s_t(1 - \delta_S)]$$

In the steady-state, the tax basis is therefore positive and respectively $\delta_S S$ and $\delta_S s$. We will assume that the taxes on capitalist's structures is non-negative but that the tax on rented structures may be a subsidy¹⁰.

Proposition 4 (Restoring the first best with subsidies on structures):

1. *In the absence of a tax on rents, a tax on structures is like a tax on capital: it distorts intertemporal choices in a way than cannot be alleviated. The first best requires a zero tax on investments on structures:*

$$\tau_S = \tau_s = 0$$

2. *With a positive tax on rents, the social planner can compensate the distortion on structures s induced by the tax on rents by subsidizing structures at the same rate, with the rule $\tau_s = -\tau_H < 0$, but it cannot restore the first best on housing consumption choices only with that*

¹⁰An alternative specification is to tax gross investments net of depreciation, that is to have in the steady-state the opposite sign for the tax base. This only changes the sign of the tax rate and we keep the current formulation.

specific subsidy. This requires a differential property tax again, using the same formula as that in Proposition 2.

3. Symmetrically, a tax on imputed rents leads to a distortion on the structures of capitalists. It similarly requires a subsidy of their investments in structures, $\tau_S < 0$.

Proof: Appendix B.3 derives the first order conditions of agents in the general case. With the taxes considered in this sub-section, these conditions are:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_L} \left(1 - \tau_H + \tau_{HI} \frac{H'_L}{h'_l} + \frac{\Delta\tau_L}{R^{Hgross} h'_l} \right) = \frac{u'_c}{U'_C} \quad (40)$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{1}{1 + \tau_S} \frac{U'_H H'_S}{U'_C} - \frac{\tau_{HI}}{1 + \tau_S} R^{Hgross} H'_S \quad (41)$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{1 - \tau_H}{1 + \tau_s} \frac{u'_h h'_s}{u'_c} \quad (42)$$

and the comparison with the social planner's objective leads to the results. In particular on point 3) above, one requires from equation (41) and the comparison with the social planner in equation (10) that $\tau_S = -\tau_{HI} \frac{U'_C R^{Hgross}}{U'_H}$. See Appendix B.4.5 for further details. ■

This set of propositions links the two subsidies on structures and the tax on rents. They show that the first best is possible even with taxes on rents, but only if one creates new subsidies that are complex and in some cases, raise the complexity and therefore the costs for the fiscal administration. From this perspective, a tax on rents is less problematic. Therefore, this section should not be seen as a full defence of the tax/subsidies combinations that exist in some countries where landlords receive subsidies to develop new rental units devoted to low rents rental, but it captures some of the rationale for it¹¹.

It is finally easily shown that the previous analysis regarding a rent tax including imputed rent tax can be generalized to a so-called living tax. Such a tax exists in France and Germany. It is a tax paid by both workers and capitalists, on their respective housing units. In our framework, this tax is not particularly compelling since it immediately reduces the living standard of the worker and requires higher taxes to reach the desired level of consumption. We explore this in Appendix B.4.6 .

¹¹In France for instance, various successive schemes such as Scellier, Pinel and other bills have been passed over the decades that are inspired from the logic of compensating the taxes on rents by promoting residential investment. See in particular Bono and Trannoy (2012); Chapelle et al. (2018).

4.5 Simulations

In the remainder of the text, we will study the impact on social welfare with a tax on land that can be differentiated according to its use (rental or occupancy), and compare it to the cases where are only available, respectively, a tax on rents, a tax on returns to capital, a tax or subsidy to investments in structures s, S , a tax on imputed rents and some of their combinations. We therefore study the comparative statics of the decentralized equilibrium. We define the social welfare as the sum of weighted utility of agents for a given weight γ . When it reaches a maximum with a given level of taxes and subsidies, this is equivalent to a *constrained optimum of the social planner*. All our results are analyzed at the steady state.

4.5.1 Simulation: functional forms and parameters

These simulation exercises are however only illustrative and not meant to be a proper calibration. In particular, we explore the following context: the utility is linearly separable in consumption of the good and housing.

The utility functions are assumed to be separable and homogeneous across agents, with iso-elastic subutility. The housing production functions are also identical across agents and Cobb-Douglas, as well the production function of goods. The inverse of decay rate for housing corresponds to 50 years. The depreciation rate is higher for machines (10 years). The shares of land and structures in the production of housing service are assumed to be equal, which corresponds roughly to the decomposition in Figure 1 (at least for France and the UK, the share of land being lower in the US and Canada). The share of capital is equal to $1/3$. The share of housing, that includes shelter costs and various additional services associated with housing, is conservative with only 15% of total consumption. To sum up:

$$\begin{aligned}
 u(c, h) &= \frac{c^{1-\sigma_c}}{1-\sigma_c} + a_h \frac{h^{1-\sigma_h}}{1-\sigma_h} \\
 h(\bar{\mathcal{L}} - \mathcal{L}, s) &= (\bar{\mathcal{L}} - \mathcal{L})^{a_{\mathcal{L}}} s^{1-a_{\mathcal{L}}} \\
 U(C, H) &= \frac{C^{1-\sigma_C}}{1-\sigma_C} + a_H \frac{H^{1-\sigma_H}}{1-\sigma_H} \\
 H(\mathcal{L}, S) &= \mathcal{L}^{a_{\mathcal{L}}} s^{1-a_{\mathcal{L}}} \\
 f(K) &= K^{a_K}
 \end{aligned}$$

and benchmark numerical values: $\beta = 0.95$, $\delta_S = \delta_s = 0.02$, $\delta = 0.1$, $\sigma_C = \sigma_c = \sigma_H = \sigma_h = 0.5$, $a_h = a_H = 0.15$, $a_K = 1/3$, $a_{\mathcal{L}} = 0.5$.

4.5.2 First best tax on land vs. taxes on rents and structures

A first exercise, reported in Figure 2 is to compare the first best taxation on land with alternative tax profiles. We focus only on low values of γ (0 and 0.25) for which the social planner wants to increase all types of taxes. The horizontal axis in the top panel is the respective tax rate. In particular, the tax on land is relative to price of properties: it is defined as $\hat{\tau}_{\bar{L}}$ in

$$\hat{\tau}_{\bar{L}} = T^{\mathcal{L}} \frac{1 - \beta}{R^{Hnet}} \quad (43)$$

The first best tax on land (curve a on the legend, the curve with upwards triangles) clearly dominates taxation schemes with no tax on land. A very low tax rate is enough to raise welfare fast, because the tax base is large and inelastic. Another first best tax scheme is to tax rents, compensated by a subsidy on structures of tenants and a differential tax on land. This is represented by the curve with stars (ii, in the legend). The scale for this tax is the tax on rent (or the subsidy on structures since both are equal). Here, the tax base is therefore smaller, requiring higher taxes.

In contrast, the other, constrained efficient taxes require much larger tax rates to raise welfare and some of them are either negative or inverted U-shape, due to accumulated distortions. These taxes are in decreasing order of progression of welfare: a tax on housing rents compensated by a subsidy on tenant structures s (iii, on the legend, inverted triangles); a tax on both rents and imputed rents (iv, line with crosses), a tax on housing rents (v, dashed line); and a tax on structures of capitalists S (vi, circles), can be compared.

The most effective scheme away from the first best, is to therefore combine a tax on rents and a subsidy to rental housing structures: it does well initially but the remaining distortion on the intratemporal allocation of land accumulates as taxes grow and a maximum is reached after a tax rate of 60%. Taxing rents without subsidies on structures becomes rapidly inefficient. Taxing rents and imputed rents, is relatively less effective in comparison with the second best taxation of rents combined with structures subsidy.

In the bottom panels, we report the social welfare function as a function of the total tax revenue T relative to GDP, defined here using the consumption approach, $GDP = C + c + R^{Hgross}(H + h)$. The tax on land and the optimal differentiated tax on land to correct for the tax on rents are now superposed, as both are first best (a and b in the legend). For the pure tax on rents, the curve is backward bending on this graph. This is a Laffer effect of reducing the tax product above a certain distortive level. There is no such thing for the land tax which can push up the tax revenues and thus the transfers to the desired levels. The other taxation schemes contain this Laffer effects too.

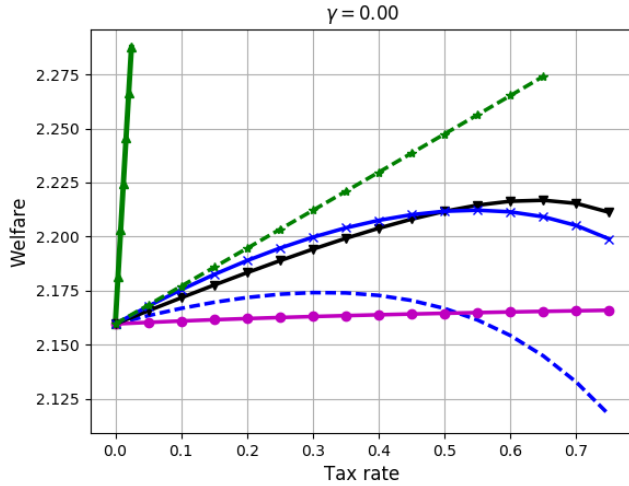
These simulations are illustrative and should be confirmed by a proper calibration. They however indicate that for this range of parameters values, a tax on rents complemented by a subsidy on structures of rental housing units does as well as a land tax, but with higher tax rates and a complicated transfers/taxes scheme.

4.5.3 Impact of the response of structures: tax on capital vs tax on rents

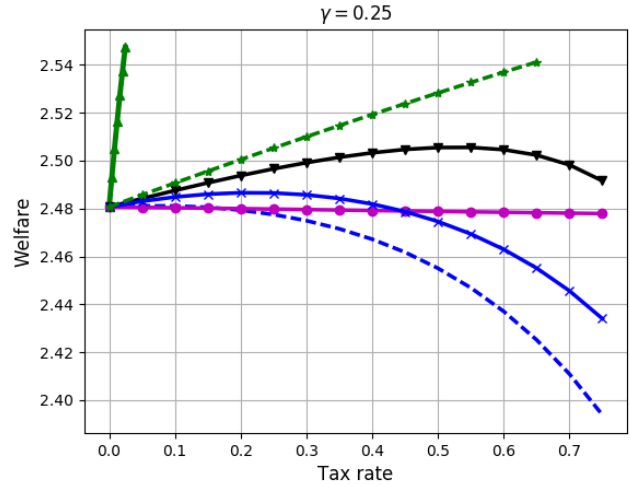
The negative welfare effect arises from the response of structure investment to taxation of rents, as this reduces the profitability of structures in the rental market which is detrimental to the welfare of workers. To see the magnitude of this effect, we report in Figure 3 the effect of taxation on rents with a full response of s, S and as a comparison, the same exercise when structures are maintained fixed at the initial level. It can be seen that, with $\gamma = 0$, the tax on rents that maximizes social welfare in this constrained optimum is around 0.7 when structures are fixed constant and only 0.3 when structures adjust and the level reached is much below because of the induced disinvestment in structures s . With $\gamma = 1/4$, the optimal tax on rents moves from 0.6 when structures are fixed constant to slightly above 0 when they adjust and again, taxes on rents lead to a much lower social welfare after a while.

A last exercise, reported in Figure 4, top panels, compares the taxation of capital and of rents in the K-H economy (without structures): without ambiguity, the tax on rents τ_H dominates that of capital τ_K , although again, this is because the distortive effect of τ_H on structures is not taken into account here. Last, in the absence of housing (the K economy), the effect of a tax on capital is always negative, as capital stock decreases as taxation increases. Here we only focus on the case $\gamma = 0$.

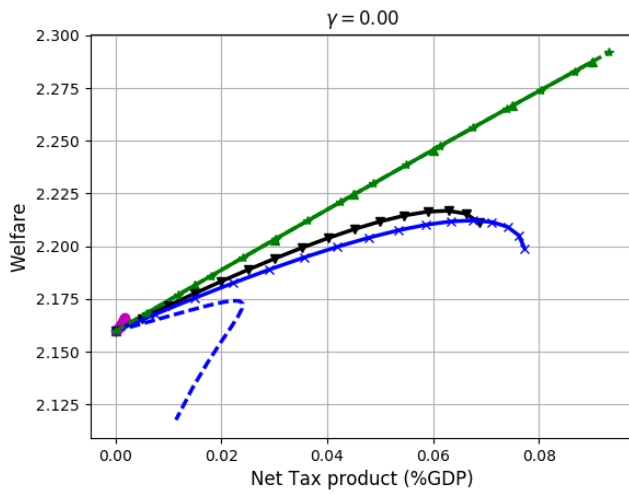
For completeness, a graph superposing to the exercise in Figure 2 the additional tax on capital is reported in Appendix Figure D.1. It also illustrates the massive distortive effect of capital taxation. Note however that if capital must be taxed, it is better to also tax rents and imputed rents, to reduce the distortion in the returns to investments across types of capital (tenant structures and capital) and in land allocation. This is illustrated in Appendix Figure D.1 by lines vii and viii in the legend. See for instance Skinner (1996) on this point. In Appendix Figure D.2, one further compares taxes on capital K , on rents R^H and on homeowners structures S , showing again that the distortion of physical capital taxation is larger than taxation of structures: structures are attached to a fixed factor (land) and returns on structures are less elastic to taxation than returns on physical capital.



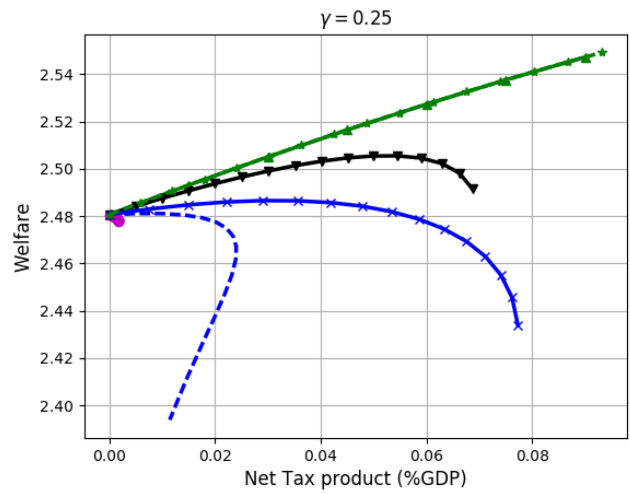
(a) Welfare and tax rate, $\gamma = 0$



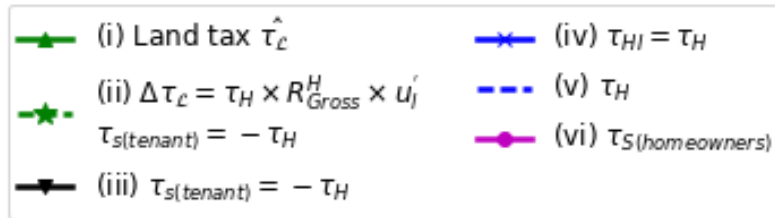
(b) Welfare and tax rate, $\gamma = 0.25$



(c) Welfare and tax Product, $\gamma = 0$

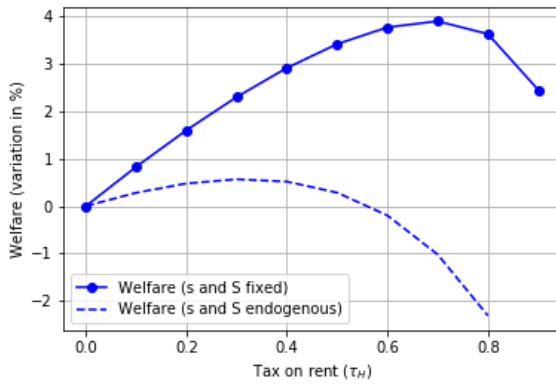


(d) Welfare and tax Product, $\gamma = 0.25$

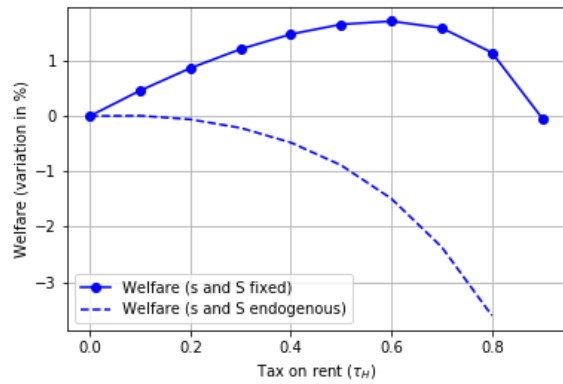


Figure's notes. Variation in the social welfare function $u(c) + \gamma U(C)$ and the tax revenue in the decentralized equilibrium, for different values of γ , the social welfare weight (respectively 0 and 1/4). Comparison between the first best policy (i), a homogeneous tax on land $\hat{\tau}_L$ (plain green line with triangles up), or its variant (ii) with a positive tax on rents, a differentiated tax on land and a subsidy to housing structures of tenants (discontinued green line with stars), and second best policies: (iii) a tax on rents compensated by a subsidy on residential investments $\tau_s < 0$ (plain black line with triangles down), (iv) a tax on rent equal to a tax on imputed rents $\tau_H = \tau_{HI}$ (plain blue line with cross), (v) a tax on rents τ_H alone (discontinued blue line); (vi) a tax on homeowners structures $\tau_S > 0$ (plain purple line with squares). Top panels: x-axis is the respective tax rates; bottom panels: x-axis is the respective total tax revenue.

Figure 2: The relative impact of different tax schemes on the social welfare function (economy with physical capital, housing, land and structures).



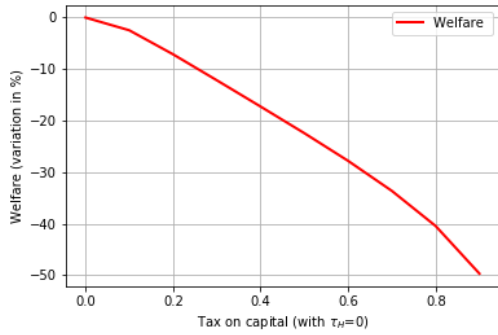
(a) $\gamma = 0$



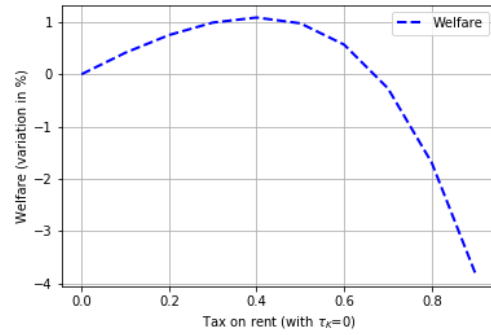
(b) $\gamma = 0.25$

Figure's notes. Variation in the social welfare function $u(c) + \gamma U(C)$ in the decentralized equilibrium. Comparison between tax on rents τ_H when structures S and s react to it (dashed line) and the counterfactual (circles) in which they are maintained fixed at the pre-tax level $\tau_H = 0$ for different values of γ , the social welfare weight (respectively 0 and 1/4).

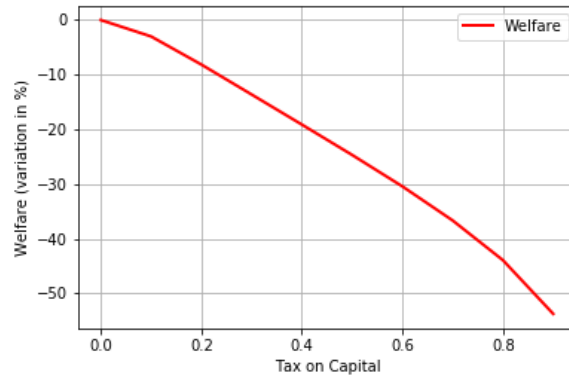
Figure 3: The role of housing structures on the impact of taxes on rents (economy with physical capital, housing, land and structures, vs. economy with only physical capital and land).



(a) Tax on capital in the K-H economy



(b) Tax on rent in the K-H economy



(c) Capital tax in the K economy

Figure's notes. Variation in the Rawlsian social welfare function $u(c) + \gamma U(C)$ with $\gamma = 0$ in the decentralized equilibrium. Comparison between the tax on capital τ_K (K-H model, top left chart), the tax on rents τ_H (K-H model, top right chart) and the tax on capital in the absence of housing (K-model, bottom chart).

Figure 4: Comparing the tax on physical capital and tax on rents in an economy with physical capital and land or only capital).

4.6 Ramsey-second best results and the convergence of multipliers

So far we show that the ability of the social planner to redistribute is limited by the elastic response of investments in physical capital as well as that of residential investments. Given the order of magnitude of the gains from land taxation observed in the simulation exercises, the social planner would have more to gain from implementing a first best tax on land owners than in trying to reach a constrained optimum with taxes on rents and subsidized on residential investments.

Nevertheless, in this Sub-Section we will investigate the second best optima in the Ramsey sense. At this stage we have not been able to derive additional results when structures are added since they add two dynamic (Euler) equations to a system that is already complex to solve dynamically, and we assume from now on that $H_t = \mathcal{L}_t$ and $h_t = l_t$.

4.6.1 Definition and issues

The optimal taxation problem in the absence of the first best solutions is complex. It involves convergence problems emphasized in Straub and Werning (2018) that we interpret as a non-concavity of the constraints faced by the social planner. We will however attempt to provide some partial results in line with those already discussed.

We study a Ramsey problem of choosing among two distortive tax instruments: either a constant tax rate on “new” capital or a constant tax rate on housing rents. With the revenues, the government finances redistribution from capitalists to workers. We exclude from the outset a lump-sum tax on capitalists, because we want to rule out confiscation of capital. The social decision maker maximizes social welfare under the following constraints: the resource constraint of the economy for each period, the FOCs of the capitalist (Euler, intra-period allocation between consumption and housing, transversality) and the FOC of the worker. An important dimension here is the level of the intertemporal elasticity of substitution in consumption.

4.6.2 Restatement of Judd’s result without housing in the K-model (no housing)

In the original Judd’s model in the absence of housing, the social planner’s program reads:

$$\begin{aligned} & \underset{c_t, C_t, K_{t+1}}{Max} \sum_{t=0}^{\infty} \beta^t [u(c_t) + \gamma U(C_t)] \\ s.t. \quad & c_t + C_t + K_{t+1} = f(K_t) + (1 - \delta)K_t \\ & \beta U'_C(C_t)(C_t + K_{t+1}) - U'_C(C_{t-1})K_t = 0 \end{aligned}$$

with a transversality constraint $\beta^t U'_C(C_t) K_{t+1} \rightarrow 0$ and an initial stock of capital K_0 given. The multiplier of the resource constraint equation is λ and μ denotes the multiplier of the Euler equation. As discussed in Straub and Werning (2018), in their framework, this is necessary and sufficient to have σ_C below unity to insure that μ converges. We will show that a similar condition holds here, although the cutoff point is no longer 1 but a larger value.

Judd's result in the version given by Straub and Werning (2018) can be summarized in the next Lemma, proved in Appendix¹²:

Lemma 1: *Straub and Werning (2018): Suppose quantities and multipliers converge to an interior steady state, i.e., C_t , c_t and K_t converge to positive values and μ_t converges. Then the tax on capital is zero in the limit.*

We provide next an intermediate result that complements Judd's statement as it appears in the above proposition. We focus to a case already considered by Straub-Werning and assume that the capitalist's utility derived from consumption is iso-elastic, that is, there exist σ_C , the inverse of which being the intertemporal elasticity of substitution (IES) with $\sigma_C > 0$ and $\sigma_C \neq 1$ such that $U(C) = \frac{C^{1-\sigma_C}}{1-\sigma_C}$.

A useful notation captures the distance to the social planner's first best in terms of the ratio of marginal utilities: let the quantity $\tilde{\gamma}_c$ captures this, with $\tilde{\gamma}_c = \gamma \frac{U'(C)}{u'(c)}$. This ratio of marginal utility weighted by social welfare weights, 1 for the worker and γ for the capitalist, should be equal to 1 in the first best. In a second best analysis, the consumption of capitalists should be higher than the consumption of workers and then typically $\tilde{\gamma}_c < 1$, and the lower $\tilde{\gamma}_c$, the farther the second best is from the first best. Even if $\tilde{\gamma}_c$ is related to one of the primitives of the model, γ , it is clearly endogenous.

Lemma 2: *Suppose quantities converge to an interior steady state. Then the multiplier μ_t converges to $\mu = \frac{\gamma(1-\tilde{\gamma}_c)}{\tilde{\gamma}_c(1-\sigma_C)}$ and is positive iff $(1-\tilde{\gamma}_c)(1-\sigma_C) > 0$. More specifically, if $\tilde{\gamma}_c < 1$ then the convergence of multipliers (to a positive number) occurs if and only if $\sigma_C < 1$.*

The convergence of the Euler condition multiplier arises when three conditions are met: $\sigma_C < 1$ (intertemporal elasticity of substitution larger than 1), the capitalist consumes more than the worker and the social welfare weight γ of the capitalist is lower than that of the worker. The condition $\tilde{\gamma}_c < 1$ is always satisfied at the second best for a redistributive social planner (with $\gamma < 1$).

¹²Note that we have treated each agent in the social planner program as representative, the social planner is free to choose a γ that corrects for the different number of agents. Indeed, we make all proofs with a mass of capitalists that can be treated as a parameter m . This mass will simply be isomorphic to the social weight of capitalists and therefore ex post does not matter.

We are now ready to characterize the second best optimum with housing but we will do it within the frame of the above proposition. In particular, we will first restrict our attention to the case of $\sigma_C < 1$ before proving additional results in the extension to the case $\sigma_C > 1$.

4.6.3 Second best taxation in the K-H-model (with housing)

We specify the preferences of both agents to be separable, with $U(C_t, H_t) = U_C(C_t) + U_H(H_t)$ and $u(c_t, \bar{H} - H_t) = u_c(c_t) + u_h(h_t)$. We still assume $U_C(C) = \frac{C^{1-\sigma_C}}{1-\sigma_C}$ and $u_c(c) = \frac{c^{1-\sigma_c}}{1-\sigma_c}$. At this stage, $U_H(\cdot)$ and $u_h(\cdot)$ remain general instead, we simply denote by $\sigma_H(H) = -HU_H''/U_H'$ and $\sigma_h(h) = -hu_h''/u_h'$ the (non-constant) coefficients of relative risk aversion of the utility for housing of respectively homeowners and tenants.

The separability assumption allows us to simplify the proofs of the next propositions. Consider the planner program with housing:

$$\max_{c_t, C_t, H_t, K_{t+1}, \tau_{H,t}} \sum_{t=0}^{\infty} \beta^t [u(c_t, \bar{H} - H_t) + \gamma U(C_t, H_t)]$$

such that:

$$c_t + C_t + K_{t+1} = f(K_t) + (1 - \delta)K_t = 0, \text{ multiplier } \lambda$$

$$\beta U'_C(t)(C_t + K_{t+1} - R_t^{Hgross}(1 - \tau_{H,t})(\bar{H} - H_t) - U'_C(t-1)K_t) = 0, \text{ multiplier } \mu$$

$$R_t^{Hgross} u'_c - u'_h = 0, \text{ multiplier } \eta_1$$

$$R_t^{Hgross} (1 - \tau_{H,t}) U'_C(t) - U'_H(t) = 0, \text{ multiplier } \eta_2$$

$$\tau_{H,t} \cdot \Phi = 0, \text{ multiplier } \Phi \geq 0$$

$$\beta^t U'_C(C_t) K_{t+1} \rightarrow 0$$

We introduce another indicator of distance to the social planner's first best, in the housing dimension this time: $\tilde{\gamma}_h = \gamma \frac{U'_H(H)}{u'_h(h)}$. As for consumption, we expect $\tilde{\gamma}_h$ to be lower than 1 at the second best optimum. The main result of this section is in the next Proposition.

Proposition 5 (Optimal capital tax in the Ramsey problem with housing): *Assume that the following instruments are available to the decision maker: a tax on capital, a lump sum benefit to workers and a tax on rents. Consider an economy where the preferences of both the capitalist and workers are separable, with a CCRA sub-additive utility of consumption. Suppose that quantities converges to an interior steady state and that $\sigma_C < 1$, $\tilde{\gamma}_c < 1$ and $\tilde{\gamma}_h < 1$, then the optimal tax on capital is 0 and the optimal tax on rents is positive in the limit. Consequently the stock of capital in the second best remains equal to the stock of capital in the first best.*

The proposition amounts to proving that both the numerator and denominator of the multiplier of Euler's equation in the social planner's program are positive, as displayed in Appendix D, equation (D.22). A value $\sigma < 1$ is sufficient for this multiplier to be positive. We offer the following interpretation of the result. In a static setting, Diamond and Mirrlees (1971a,b) obtains a very robust result in optimal taxation: they show that it is better not to distort production and therefore not to tax intermediate goods. However, depending on the context, it may be second best optimal to tax consumption. According to Chari et al. (2016), these results cannot be directly applied to a dynamic setting because of variations in labor productivity in time, that depends on the capital stock. Nevertheless, the intuition conveyed by the Diamond-Mirrlees papers still holds in our context, specifically in the steady state. Housing is a consumption good and under some conditions it can be optimal to tax it, while it is not optimal to tax capital because it is an intermediate good. In addition, the fact that developed land is in fixed supply in our model leads to simpler proofs.

Another proposition delivers the optimal tax formula which is an inverse elasticity rule à la Ramsey:

Proposition 6 (Ramsey-optimal rent tax): *The optimal rent tax is given by*

$$\frac{\tau_H}{1 - \tau_H} = \frac{1 - \tilde{\gamma}_c}{\epsilon_s}$$

where ϵ_s the supply elasticity of rental housing land with respect to net rent. The supply elasticity with respect to net rent should be equal in absolute terms to the demand elasticity with respect to gross rent at the equilibrium of the rental market.

Note also that in the simplest case of the same constant relative risk aversion function for the sub-utility of housing as for consumption ($\sigma_C = \sigma_h = \sigma_H$), one obtains in addition that $\epsilon_s = \frac{1}{\sigma_H}$, and the formula is $\frac{\tau_H}{1 - \tau_H} = \sigma_H(1 - \tilde{\gamma}_c)$.

Assume $\sigma_h, \sigma_H > 0$. In this case, one does not require $\sigma_C > 1$ to obtain a convergent program: σ only need to be lower than a cutoff elasticity, defined from Appendix equation (D.22). In that equation, the numerator is positive and the denominator is positive if and only if:

$$\sigma_C < \sigma_C^* = \frac{1}{1 - \frac{hRH_{net}}{c} \left(\frac{1}{\sigma_h} + \frac{h}{H} \frac{\sigma_H}{\sigma_h} \right)} \geq 1 \quad (44)$$

Proposition 7 (Range of convergence of Ramsey solutions with housing): *Even if $\sigma_C \geq 1$, as long as $\sigma_C < \sigma_C^*$ where $\sigma_C^* \geq 1$, the convergence of multipliers still holds, as in Lemma*

2. Instead, consistent with Straub and Werning (2018), in the absence of housing consumption that is when the share of housing $\frac{hR^{Hnet}}{c}$ is close to zero, one finds $\sigma_C^* = 1$ in the limit case in which preferences are such that the housing consumption share of workers tends to zero relative to the consumption of the composite goods.

Our second best optimal taxation result in Proposition 6 holds for a large set of parameters, in particular with an IES smaller than 1, within a range that can be large. The intuition for the extension in the range is the following: when $\sigma_C > 1$, the substitution effect of taxation is smaller than the income effect of taxation. In the absence of housing, capital taxation would normally increase savings, leading to the divergence of multipliers as soon as $\sigma_C < 1$. Adding up land and a new tax instrument related to land, we can partially relax this effect and the set of parameters leading to interior solutions of second best taxation is therefore larger.

To fix ideas, if $\sigma_h = \sigma_H = 1$, the share of housing in consumption of tenants is 0.125 and the ratio of aggregate housing of each agents is $h/H = 2^{13}$, one has $\sigma^* = 1/(1 - 0.125/0.875 * 3) = 1.75$. Our result can be brought closer to to the interpretation of Chari et al. (2016) according to which the result of Straub and Werning (2018) of heavy taxation of capital when IES is lower than 1 is to some extent an illustration of an incomplete set of fiscal instruments. They show that with a moderate consumption taxation in the second period, we do not need to tax capital anymore.

One can provide a further result in the particular case of Cobb-Douglas preferences, $U(c, h) = \phi \log h + (1 - \phi) \log c$, and on the production side, the Cobb-Douglas function $Y = K^{a_K} L^{1-a_K}$. In this case it is possible to show an explicit formula for the optimal second best tax of rents. We can state (see Appendix D.2 for the proof) that:

Corollary: *There is a unique second best optimal tax rate τ_H obtained as the root of the second degree equation. The tax rate is decreasing with the social welfare weight γ of the capitalist which means that the upper bound of the optimal range of tax rate is 50%.*

The optimal tax on rents is implicitly given by

$$\tau_H = \frac{1}{2} - \frac{\gamma}{2} \frac{1 - \phi}{\phi} \frac{1}{1 + \frac{\mathcal{K}^*}{\phi w^*} + \frac{\tau_H}{1 - \tau_H} \frac{1 - \phi}{\phi} \frac{\mathcal{K}^*}{w^*}} \quad (45)$$

where $\mathcal{K}^* = \beta K^*/(1 - \beta)$ is obtained from the stationary value of capital K^* . With reasonable calibration values described in Section 3.4, we will obtain a tax rate on rents of 40%. In Appendix

¹³Away from the assumptions of equal weights of workers and capitalists, h/H is replaced in the proofs by $h/(Hm)$ where m is the mass of capitalists. Hence, this back-of-the-envelope calculations corresponds to a ratio of the land consumed by agent over capitalists equal to 2.

we introduce an arbitrary mass of capitalists $m < 1$ and the term γ becomes $m\gamma$ in equation (45). In both cases, the Rawlsian program leads to the highest possible taxation of $\tau_H = 1/2$, while in the utilitarian case, the rate is below 50%.

5 Literature review

This paper touches different topics that we all discuss in this Section. First, it is a paper on wealth and capital accumulation. The rise of the capital/income ratio and its implications on inequalities derived in Piketty (2014) have generated many contributions that responded to various challenges of the main thesis: Krusel and Smith (2014), Rognlie (2016), Acemoglu and Robinson (2015), Stiglitz (2015a,b), Mankiw and Summers (2015), Weil (2015), Auerbach and Hassett (2015), Jones (2015), Kopczuk (2015), Hildenbrand (2016). There have been several directions in these works: critics related to the inability of the model to account for varying rates of return on capital especially in the limits of infinite accumulation; critics of the fact that the net elasticity of substitution between capital and labor is below 1 in most studies while for the share of capital in national income and the capital-to-wealth ratio to co-move positively it should be above 1; critics on the data used to assess the explicit and the simple theory that risk-adjusted returns to capital are greater than GDP growth ($r > g$) leads to inflation of capital over GDP; critics of the lack of differentiation between wealth and capital.

A particular line of discussion was the recognition that earnings of capital relative to GDP had not evolved as much as the ratio capital to GDP. We notably documented this in detail in our previous work (Bonnet et al., 2014) in France with the apparent inconsistency of the data with the “first law of capitalism”: the positive co-movement between the capital share and the capital to income ratio is not visible in the data. This suggested that an important ingredient was either missing or at least deserved more emphasis. This missing piece that we discussed in that paper is the heterogeneity of capital, that naturally appears when one explicitly distinguish wealth from capital¹⁴. The discussions on capital have always led to the greatest economic controversies. The most famous of which was between the two Cambridge, in the 1960s. The debate was about possible inconsistencies and tautologies in the measurement of the capital stock and its earnings, as well as the nature of capital and the difference, if any, with pure wealth. Housing in particular has a market value (wealth) that may be disconnected from its returns especially for homeowners.

Second, this paper is part of a very dynamic literature on capital taxation for redistributive rea-

¹⁴We follow the recent analysis of China in Piketty et al. (2019) where authors report wealth to GDP ratios (Figures 3, 4, 5, 7a to d, etc, pages 49 to 56), and not capital to GDP ratios.

sons¹⁵. In the early work of Diamond and Mirrlees (1971b), capital as an intermediate good should not be taxed under linear tax schedules. Atkinson and Stiglitz (1976) favor non-linear labor taxation instead. In the Ramsey framework, Chamley (1986) argues that future capital investments should not be taxed in the long-run, within the range of convergent paths, as well as Judd (1985) in a second-best convergent paths with workers without assets. Under the assumption of uninsurable shocks, Aiyagari (1994) corrects the overaccumulation of capital for precautionary motives with a capital tax instead. With asymmetries of information on individual's characteristics, Golosov et al. (2003) establish a similar result under non-linear tax schemes. A subsequently central mechanism to the literature pioneered by this article is that wealth raises leisure and reduces work incentives. Judd's paper was subsequently revisited by several authors. Lansing (1999)'s finding is that under log preferences, optimal capital taxes could be positive forever at some interior steady state. Reinhorn (2019) shows that it is the only way in which an interior steady state can violate the zero tax result in Judd's framework. Out of the steady-state, Straub and Werning (2018) discuss non-convergent paths in Judd's analysis. In particular, with low elasticities of intertemporal substitution, savings is too important and taxing capital must lead to long-term expropriation, while with higher elasticities, taxes tend to zero, but from simulations, the convergence is very slow. In a more general setting Basetto and Benhabib (2006) and Benhabib and Szóke (2019) establish that with more heterogeneity in wealth, a positive capital tax rate can prevail in the steady state, without considering knife-edge preferences. The last authors consider with CRRA preferences and constant elasticity of substitution (CES) production functions and an uneven distribution of wealth. They show that in the framework of neo-classical growth model, agents in the bottom or middle part of the distribution may prefer the redistributive effects associated to a tax on capital forever even if it entails large inefficiency costs. It would be interesting to see whether our results convey to this more general setting. Chien and Wen (2019) build an infinite-horizon Aiyagari-type model and find there zero taxation of capital in the long-run. In the transition path, the level of optimal capital tax also depends on the elasticity of intertemporal substitution. In our paper, we limit our analysis to the long-run, and have no precautionary saving nor leisure that could leads to similar results as the literature above. In particular, we do not address the complex dynamics paths, although we showed that with a fixed factor, the range of parameters needed to converge for the land market and the intertemporal substitution may be different.

Third, our paper is about the heterogeneity of capital and in particular the role of land as a fixed factor generating rents. The question of optimal land and housing taxation used to be central. A

¹⁵See for instance Ndiaye (2017)

seminal contribution can be viewed in Henry George (1879)'s book, *"Progress and Poverty"*, advocating for a single tax on land. This contribution was adapted to the urban economics framework with the famous Henry-George Theorem developed in Arnott and Stiglitz (1979) which states that, at an optimal city size, a land rent tax is the only tax needed; it is sufficient to finance local public goods. This theorem was recently reformulated in a dynamic and macroeconomics setting by Mattauch et al. (2013). The authors derive an optimal public investment formula in terms of the land rent. An important assumption in their work, and a difference with the urban literature, is that residential land do not enter the utility function. Prominent economists subsequently shared the view that land should be taxed. William Vickrey wrote in Vickrey (1996): *"removing almost all business taxes, including property taxes on improvements, excepting only taxes reflecting the marginal social cost of public services rendered to specific activities, and replacing them with taxes on site values, would substantially improve the economic efficiency of the jurisdiction."*; a quote from Milton Friedman supports the land tax: *"In my opinion, the least bad tax is the property tax on the unimproved value of land, the Henry George argument of many, many years ago."*; while a manifesto of economists (William Vickrey, Jacques Thisse, Tibor Scitovsky, James Tobin, Richard Musgrave, Franco Modigliani, Zvi Griliches, William Baumol, Robert Solow among others) wrote a Letter to Gorbachev in 1990 *"It is important that the rent of land be retained as a source of government revenue. While the governments of developed nations with market economies collect some of the rent of land in taxes, they do not collect nearly as much as they could, and they therefore make unnecessarily great use of taxes that impede their economies—taxes on such things as incomes, sales and the value of capital."*

Fourth, the introduction of land in the optimal taxation literature was addressed in Stiglitz (2015a). He shows that, when land is only a productive input, its taxation would increase the consumption of workers. In his setting there is only one consumption good and land does not provide a housing service. The model leaves aside residential land to focus on land consumed by firms. Eerola and Määttänen (2013) on the other hand, addresses specifically the question of housing taxation. They develop a model with a representative agent that derives utility from non-housing consumption, leisure and housing which is only composed by its structure and has no land component. This key assumption leads to the conclusion that "in the first-best, the tax treatment of business and housing capital should always be the same", and that in "the second-best, in contrast, the optimal tax treatment of housing capital depends on the elasticities of substitution between non housing consumption, housing, and leisure". The model is also silent on the role of residential land. Our optimal taxation of land and rents in the second best rejoins these recent approaches but accounts for the fact that land is consumed by households through housing services and thus enters the utility function.

Fifth, different models with an intergenerational structures provide additional insights. In Overlapping Generation Models (OLG), land and housing represent savings vehicles. For example, Buiter (2010) insists on the fact that housing (composed by land and structure) is not wealth. He finds that individual consumption is unchanged if housing prices increase in line with the fundamentals (hence the exclusion of bubbles). Regarding taxation, OLG models leave more room for positive optimal tax rates on capital (Conesa et al., 2009) because of potential dynamic inefficiencies. However, this does not mean that land and housing should not be treated differently. One interesting point was raised in Deaton and Laroque (2001) which introduces land in the consumption function of a standard OLG growth model and shows that land markets introduce large distortions making the case for its nationalization. As such an extreme scenario might be difficult to realize, complementary discussions on land taxation might be required. A first answer can be found in Chamley and Wright (1987) who analyzes the fiscal incidence of a land tax but as in Stiglitz (2015a) and Eerola and Määttänen (2013), land does not enter the consumption function. Kim and Lee (1997) suggest that a land tax is likely to create dynamic inefficiency distorting the no arbitrage condition rely on the same assumption. More recently, Nakajima (2010) developed an OLG model taking into account the specific tax treatment of housing income and the difference across occupation status. In such a framework when imputed rents are not taxed, the author finds that Conesa et al. (2009) results may not hold and that what is called “malleable” capital should almost not be taxed. The intuition is similar as Eerola and Määttänen (2013): housing and productive capital should not be taxed differently *provided that land is not a component of housing*. When investigating the fiscal incidence in this framework, housing is only composed by the structure and land is ignored with the exception of Skinner (1996) who develops an OLG model where housing comes in fixed supply and can thus be assimilated to land. This work does not discuss what should be the optimal tax rates of different types of capital but documents the large welfare cost generated by the preferential tax treatment of housing when productive capital is taxed. We go beyond Skinner (1996) and argue that due to the fixity of land, returns to homeowners housing structures can be taxed more than capital, and instead that residential structures must be subsidized.

6 Conclusion

One upon a time, land was a central piece of the classical analysis. But it has been moved at the periphery of modern theory of prices. Several recent evolutions however point towards the need to reinstate land: housing represents between 20 and 30% of consumer expenditures and at least 40% of

household wealth. In most places, prices of developed land surge, in particular in metropolitan areas. Reintroducing land and land price leads to the natural conclusion that land tax must be positive and large. This idea does not need to be confined to urban economics where it already plays a key role, as beautifully advocated for the funding of local public goods with the so-called Henry-George theorem. The importance of land taxes goes beyond the microeconomics of cities and naturally embraces the issue of redistribution at the macro level with implications on long-term growth. We therefore see Piketty's contribution as important to trigger a revival on this land issue. Our reading of his empirical findings is that the growth of the wealth-income ratio is mainly a housing phenomenon, whereas, instead, the ratio of physical capital to income is fluctuating around 2 without any neat trend in every country, despite evidence of accrued concentration.

Our first part documents that capital is heterogeneous and the recent inflation of housing wealth matters more than before: since a large part of housing reflects the underlying fixed factor (land). Our second set of conclusions is normative. In terms of optimal taxation, it is crucial to distinguish between produced capital and housing (physical capital and structures) and land. Indeed, taxing land (or property) enables to make transfers from landowners to workers/tenants and to increase the income of the latter. However, taxing physical capital or housing structures through a tax on rents has distortive effects. The quantitative exercises in this paper suggests that taxing rents may increase the social welfare of a planner willing to redistribute. Distortive effects may be important, but can be alleviated with subsidies to housing investments. Absent these subsidies, a courageous effort to tax land leads to large welfare gains of the order of magnitude higher, say between 5 to 20%.

The analysis led us to conclude (more optimistically, or perhaps just less pessimistically) that long-run trends in wealth may eventually become good news for policy makers if they are able to implement non-distortive redistribution. Our analysis indicates that differentiating wealth taxes by type of wealth allows for a lighter taxation on physical capital than on housing structures of homeowners, themselves lighter than developed land.

Lastly, the Judd model used as a benchmark in this paper is a natural framework to reintroduce land and housing in dynamic public finance. It should be noted that it is an extreme "Piketty's world" since all productive capitals belong to the capitalist hands. We acknowledge that the analysis must be pursued to understand better the concentration of capital in some hands and not others, and how intergenerational forces affect the dynamics of wealth accumulation through capital and land.

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A The measurement of housing capital by statistical agencies: the example of France (INSEE)

Piketty and Zucman's measurement of capital "*follows the most recent international guidelines as set forth in the 2008 System of National Accounts*" Piketty and Zucman (2013). They use series of statistical agencies to measure their capital/income ratio. The measurement of capital, and in particular housing capital, follows a particular methodology which is summarized and highlight to justify the adjustments proposed here.

According to OECD (2001), the framework used in several countries is the perpetual inventory methodology (PIM) briefly described by Piketty and Zucman: "*The goal of the perpetual inventory method (PIM) is to approximate the current market value of a number of capital assets when it cannot be directly observed. The general idea is that this value can be approximated by cumulating past investment flows and making suitable price adjustments.*" Piketty and Zucman (2013).

Our main interest will be in the "*suitable price adjustments*" for the housing market, namely the price index used to evaluate the value of the volume of capital. The perpetual inventory methodology used by the French institute of statistics (INSEE) to provide an estimate of the national housing capital stock is described as follows.

A.1 Measurement of the stock (volume) of housing capital in a reference year

"*In France, housing capital is estimated through a first step of estimating the total stock and value of housing in a reference year (1988). INSEE then follows over time the evolution of the number of buildings from aggregate housing investments, deflated the housing construction index; and the evolution of land with constructs using the evolution of the surface area covered by housing units and the development of the surface area covered by houses. To get the year- by-year value of housing capital stock, the above- described volume is multiplied by*

the price index of existing housing. Furthermore, new buildings were also evaluated at the price of existing housing units. Hence, housing capital follows year-to-year evolutions of housing prices, by contraction."

The assessment of the housing capital starts indeed from an initial survey-based assessment in the reference year. Two main surveys of 1988 (French Housing Survey and Survey on building land¹⁶ were used to assess the housing capital stock divided between buildings and their underlying land.

From the surveys we calculate:

$$K_{1988}^{Housing} = K_{1988}^{Dwelling} + K_{1988}^{land}$$

where the housing capital which is broken down between dwellings and land.

A.2 The time evolution of the volume of housing capital

As we saw, housing capital is divided between land and dwellings, the evolutions of which are followed separately.

For dwellings, the stock in the following years is computed iteratively from the initial year¹⁷ taking into account depreciation (δ)¹⁸ and yearly capital increments (Gross Fixed Capital Formation (GFCF)) deflated by construction costs index (CCI)¹⁹:

$$Vol(K_{n+1}^{Dwelling}) = (1 - \delta)Vol(K_n^{Dwelling}) + \frac{GFCF_{n+1}}{CCI_{n+1}}$$

For land, the statistical agency similarly follows the changes in developed land on the national territory with respect to the reference year using an index of the surface developed (S):

$$Vol(K_{n+1}^{land}) = S_{n+1} \times K_{1988}^{land}$$

Finally, the volume of housing capital is just the addition of both series for each year at the price of the reference year, here 1988²⁰:

$$Vol(K_{n+1}^{Housing}) = Vol(K_{n+1}^{Dwelling}) + Vol(K_{n+1}^{land})$$

¹⁶In 1988, two surveys are available: the housing survey "Enquête Logement", (EL) and information gathered by tax authority on land price (from the IMO file from the Direction Générale des Impôts which provides for the last time in 1988 an evaluation of the price of building land) Baron (2008).

¹⁷For each of these years after 1988, the value of the stock of housing will be calculated step by step. Baron (2008).

¹⁸From the CCF rate (that is, the depreciation rate) we compute net capital. Baron (2008)

¹⁹To evaluate the net capital of 1988 at 2000s prices, we deflate using the construction cost index as a price index for buildings Baron (2008).

²⁰E.g., the volume of housing capital at the end of 1989 (at 1988 prices) is found adding the net capital flow of end 1989 evaluated at 1988 prices Baron (2008).

A.3 Pricing of the evolution of housing capital and decomposition

The volume of capital is then obtained multiplying its volume by the house price index²¹ (HP):

$$K_{n+1}^{Housing} = HP_{n+1} \times Vol(K_{n+1}^{Housing})$$

This value (used in *Capital in the 21st Century*) is then broken down into land and dwellings. This step appears to be the most important to understand our reasoning since it shows that housing capital is evaluated at the market price of year n+1. The Value of the structure alone can be recovered by multiplying the Volume of dwelling with the Construction Price Index:

$$K_{n+1}^{Dwelling} = Vol(K_{n+1}^{Dwelling}) \times CCI_{n+1}$$

The developed Land Capital is the residual:

$$K_{n+1}^{land} = K_{n+1}^{Housing} - K_{n+1}^{Dwelling}$$

B Theory Appendix: Social planner's allocation and decentralized equilibrium with structures

B.1 Steady-state in the K-H-S-s model

Here we report in the most general case the first order conditions of the social planner and of the decentralized equilibrium as in the text. Given the Inada conditions assumed in the text, all partial derivatives are strictly between 0 and infinity.

B.2 Social planner's objective

We have:

$$\partial C_t, c_t \quad \lambda_t = \gamma U'_C(t) = u'_c(t) \tag{B.1}$$

$$\partial \mathcal{L}_t \quad \gamma U'_H H'_L(t) = u'_h h'_l(t) \tag{B.2}$$

$$\partial S_{t+1} \quad \lambda_t = \beta \lambda_{t+1} (1 - \delta_S) + \gamma \beta U'_H H'_S(t+1) \tag{B.3}$$

$$\partial s_{t+1} \quad \lambda_t = \beta \lambda_{t+1} (1 - \delta_S) + \beta u'_h h'_s(t+1) \tag{B.4}$$

$$\partial K_{t+1} \quad \lambda_t = \beta \lambda_{t+1} R^{Kgross}(t+1) \tag{B.5}$$

²¹"The housing patrimony at the end of 1989 is obtained multiplying by the price index for the whole France " Baron (2008).

This notably implies the following steady-state relations:

$$\partial C_t, c_t \quad \lambda = \gamma U'_C = u'_c \quad (\text{B.6})$$

$$\partial \mathcal{L}_t \quad \gamma U'_H H'_L = u'_h h'_l \quad (\text{B.7})$$

$$\text{Euler } S_{t+1} \quad \beta^{-1} = \frac{U'_H H'_S}{U'_C} + 1 - \delta_S \quad (\text{B.8})$$

$$\text{Euler } s_{t+1} \quad \beta^{-1} = \frac{u'_h h'_S}{u'_c} + 1 - \delta_S \quad (\text{B.9})$$

$$\text{Euler } K_{t+1} \quad \beta^{-1} = R^{Kgross} = f'(K) + 1 - \delta \quad (\text{B.10})$$

B.3 Decentralized equilibrium

We start from the most general tax structure, to save on calculations later on. We therefore consider all settings considered in the main text as particular cases. As compared to the text, we also add a tax on investments in physical capital τ_I treated symmetrically with the taxes/subsidies on structures to show its substitutability with the tax on returns.

We consider τ_{St} and τ_{st} as (net) taxes (a negative value is a subsidy) per unit invested, and the product of the tax is equal to:

$$T_t^{s,S} = \tau_{St} [S_{t+1} - S_t(1 - \delta_S)] + \tau_{st} [s_{t+1} - s_t(1 - \delta_s)] \quad (\text{B.11})$$

By symmetry, we similarly introduce a tax on net investment,

$$T_t^I = \tau_{I,t} [K_{t+1} - K_t(1 - \delta)]$$

and net returns on capital are taxed too:

$$T_t^K = \tau_{K,t} [f'(K_t) + 1 - \delta]$$

and use the notation for the net return on capital as:

$$R_t^{Knet} = R_t^{Kgross} (1 - \tau_{K,t})$$

In the steady-state, the tax basis is therefore positive and respectively $\delta_S S$ and $\delta_s s$, and taxes on land are potentially differentiated, with $\tau_{\mathcal{L}}$ the tax on the land occupied by landlords, and τ_l the tax on land developed for tenants, and therefore a tax revenue on land equal to:

$$T_t^{\mathcal{L}} = \tau_{\mathcal{L}} \mathcal{L} + \tau_l (\bar{\mathcal{L}} - \mathcal{L})$$

opening the door for a correction in the allocation of land by landlords. When the tax rates are identical, we denote them (in the text or in this Appendix) as $\tau_{\bar{L}} = \tau_{\mathcal{L}} = \tau_l$.

Rents are also taxed at a rate $\tau_{H,t}$. A tax on imputed rents for the landlord may be possible. It is introduced as $\tau_{HI,t}$. Both lead to a revenue:

$$T_t^H = \tau_{H,t} R_t^{Hgross} h_t + \tau_{HI,t} R_t^{Hgross} H_t$$

and we use the notation for the net return on housing investments as:

$$R_t^{Hnet} = R_t^{Hgross} (1 - \tau_{H,t})$$

Finally, we allow for a living tax, paid by all residents (homeowners and tenants). The tax revenue is calculated at the rent or implicit rent (gross), and the tax rate is denoted by $\tau_{liv,t}$ and leads to revenue:

$$T_t^{liv} = \tau_{liv,t} R_t^{Hgross} (H_t + h_t)$$

Overall, all taxes sum up to deliver the total amount taxed T_t :

$$T_t = T_t^{liv} + T_t^H + T_t^K + T_t^I + T_t^{s,S}$$

while the total net transfer is instead:

$$Transfert_t = \tau_{liv,t} R_t^{Hgross} (H_t) + T_t^H + T_t^K + T_t^I + T_t^{s,S}$$

It is interesting to note that the living tax for the landlord is calculated in the same way as for the imputed rent. In what follows, we keep track of these two perfect substitute taxes and the sum $\tau_{liv} + \tau_{HI}$ appear in all FOC of the capitalist, as the sum of the living tax and the imputed rent tax for the landlord.

Then, the program of the capitalist is

$$\begin{aligned} & \max_{C_t, H_t, \mathcal{L}_t, K_{t+1}, s_{t+1}, S_{t+1}} \sum_t \beta^t \{U(C_t, H(\mathcal{L}_t, S_t))\} \\ & + \beta^t \lambda_t^K \left\{ R_t^{Knet} K_t + (1 + \tau_{I,t})(1 - \delta)K_t + R_t^{Hgross} (1 - \tau_{H,t})h_t(\bar{\mathcal{L}} - \mathcal{L}_t, s_t) \right. \\ & \qquad \qquad \qquad \left. - (\tau_{liv,t} + \tau_{HI,t}) R_t^{Hgross} H_t \right. \\ & \qquad \qquad \qquad \left. + (1 - \delta_S)(1 + \tau_{S_t})S_t + (1 - \delta_s)(1 + \tau_{s_t})s_t - T_t^{\mathcal{L}} \right. \\ & \qquad \qquad \qquad \left. - C_t - S_{t+1}(1 + \tau_{S_t}) - s_{t+1}(1 + \tau_{s_t}) - K_{t+1}(1 + \tau_{I,t}) \right\} \end{aligned}$$

subject to subject to $\beta^t U'_C(t) M_{t+1} \rightarrow 0$, for $M_t = K_t, S_t, s_t$.

The program of the worker is:

$$\begin{aligned} & \max_{c_t, h_t} u(c_t, h_t) \\ & \text{subject to } c_t + h_t R_t^{Hgross} (1 + \tau_{iv,t}) = w_t + T_t \end{aligned}$$

We have, denoting by λ_t^w the multiplier of the constraint of the worker, and λ_t^K the multiplier of the constraint of the capitalist, and $\Delta\tau_{\mathcal{L},t} = \tau_{\mathcal{L},t} - \tau_{l,t}$:

Worker: intraperiod

$$\partial c_t \quad u'_c = \lambda_t^w \tag{B.12}$$

$$\partial h_t \quad u'_h(t) = \lambda_t^w R_t^{Hgross}(t)(1 + \tau_{iv,t}) \tag{B.13}$$

$$\Leftrightarrow u'_h(t) = u'_c R_t^{Hgross}(1 + \tau_{iv,t}) \tag{B.14}$$

Capitalist: intraperiod

$$\partial C_t \quad U'_C(t) = \lambda_t^K \tag{B.15}$$

$$\partial \mathcal{L}_t \quad U'_H H'_\mathcal{L}(t) = \lambda_t^K \left(R_t^{gross}(1 - \tau_{H,t})h'_i(t) + \Delta\tau_{\mathcal{L}} + (\tau_{iv,t} + \tau_{HI,t})R_t^{Hgross} H'_\mathcal{L}(t) \right) \tag{B.16}$$

Capitalist: intertemporal (Euler)

$$\begin{aligned} \partial S_{t+1} \quad \lambda_t^K (1 + \tau_{St}) &= \beta \lambda_{t+1}^K (1 - \delta_s)(1 + \tau_{St}) + \beta U'_H H'_S(t+1) \\ &\quad - \lambda_{t+1}^K \beta (\tau_{iv,t+1} + \tau_{HI,t+1}) R_{t+1}^{Hgross} H'_S(t+1) \end{aligned} \tag{B.17}$$

$$\partial s_{t+1} \quad \lambda_t^K (1 + \tau_{st}) = \beta \lambda_{t+1}^K (1 - \delta_s)(1 + \tau_{st}) + \beta \lambda_{t+1}^K R_{t+1}^{Hgross} (1 - \tau_{H,t+1}) h'_s(t+1) \tag{B.18}$$

$$\partial K_{t+1} \quad \lambda_t^K (1 + \tau_{I,t+1}) = \beta \lambda_{t+1}^K [R_{t+1}^{Knet} + (1 - \delta)\tau_{I,t+1}] \tag{B.19}$$

or in the steady-state,

Worker: intraperiod

$$u'_h = u'_c R^{Hgross}(1 + \tau_{iv}) \tag{B.20}$$

Capitalist: intraperiod

$$\partial C_t \quad U'_C = \lambda^K \tag{B.21}$$

$$\partial \mathcal{L}_t \quad U'_H H'_\mathcal{L} = U'_C (R^{Hgross}(1 - \tau_H)h'_i + \Delta\tau_{\mathcal{L}} + (\tau_{iv} + \tau_{HI})R^{Hgross} H'_\mathcal{L}) \tag{B.22}$$

Capitalist: intertemporal (Euler)

$$\begin{aligned} \partial S \quad U'_C (1 + \tau_S) &= \beta U'_C (1 - \delta_s)(1 + \tau_S) + \beta U'_H H'_S \\ &\quad - U'_C \beta (\tau_{iv} + \tau_{HI}) R^{Hgross} H'_S \end{aligned} \tag{B.23}$$

$$\partial s \quad U'_C (1 + \tau_s) = \beta U'_C (1 - \delta_s)(1 + \tau_s) + \beta U'_C R^{Hgross} (1 - \tau_H) h'_s \tag{B.24}$$

$$\partial K \quad U'_C (1 + \tau_I) = \beta U'_C [R^{Knet} + (1 - \delta)\tau_I] \tag{B.25}$$

Re-arranging the terms, using in particular $R^{Hgross} = \frac{u'_h}{u'_c}(1 + \tau_{liv})^{-1}$, one obtains:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_L} \left(\frac{1 - \tau_H}{1 + \tau_{liv}} + \frac{\tau_{liv} + \tau_{HI}}{1 + \tau_{liv}} \frac{H'_L}{h'_l} + \frac{\Delta\tau_L}{(1 + \tau_{liv})R^{Hgross}h'_l} \right) = \frac{u'_c}{U'_C} \quad (\text{B.26})$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{1}{1 + \tau_S} \frac{U'_H H'_S}{U'_C} - \frac{\tau_{liv} + \tau_{HI}}{1 + \tau_S} R^{Hgross} H'_S \quad (\text{B.27})$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{1 - \tau_H}{1 + \tau_s} \frac{1}{1 + \tau_{liv}} \frac{u'_h h'_s}{u'_c} \quad (\text{B.28})$$

$$\partial K \quad \beta^{-1} = \frac{R^{Kgross}(1 - \tau_K) + (1 - \delta)\tau_I}{1 + \tau_I} \quad (\text{B.29})$$

The system can be compared to the social planner's allocation, characterized by the following equation in the steady-state. We have:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_L} = \frac{u'_c}{U'_C} = \gamma \quad (\text{B.30})$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{U'_H H'_S}{U'_C} \quad (\text{B.31})$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{u'_h h'_s}{u'_c} \quad (\text{B.32})$$

$$\partial K \quad \beta^{-1} = R^{Kgross} \quad (\text{B.33})$$

The system described in subsection 3.3.2 and in subsection 4.1 with *only taxes on rents and returns to capital and a homogenous tax on land* and in Proposition 1, is:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_L} (1 - \tau_H) = \frac{u'_c}{U'_C} \quad (\text{B.34})$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{U'_H H'_S}{U'_C} \quad (\text{B.35})$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + (1 - \tau_H) \frac{u'_h h'_s}{u'_c} \quad (\text{B.36})$$

$$\partial K \quad \beta^{-1} = R^{Knet} \quad (\text{B.37})$$

The system described in subsection 4.3 and Propositions 2 and 3, *with the same tax on rents and returns on capital but a differential tax on land by land use*, is given by:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_L} \left(1 - \tau_H + \frac{\Delta \tau_L}{R^{Hgross} h'_l} \right) = \frac{u'_c}{U'_C} \quad (\text{B.38})$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{U'_H H'_S}{U'_C} \quad (\text{B.39})$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + (1 - \tau_H) \frac{u'_h h'_s}{u'_c} \quad (\text{B.40})$$

$$\partial K \quad \beta^{-1} = R^{Knet} \quad (\text{B.41})$$

The system described in Section 4.4.2 and Proposition 4, *with the same tax on rents and returns on capital, a differential tax on land by land use and taxes or subsidies on residential structures s and S* is given by:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_L} \left(1 - \tau_H + \frac{\Delta \tau_L}{R^{Hgross} h'_l} \right) = \frac{u'_c}{U'_C} \quad (\text{B.42})$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{1}{1 + \tau_S} \frac{U'_H H'_S}{U'_C} \quad (\text{B.43})$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{1 - \tau_H}{1 + \tau_s} \frac{u'_h h'_s}{u'_c} \quad (\text{B.44})$$

$$\partial K \quad \beta^{-1} = R^{Knet} \quad (\text{B.45})$$

B.4 Sufficient conditions for first best efficiency

B.4.1 Physical capital

Comparison of social planner Euler equation on capital (B.33) and the general case of the Euler decentralized equilibrium (B.29) implies that the tax on returns on capital and the tax on investments are redundant and individually distortive. In other words, if one taxes returns on capital with a positive tax τ_K , one would need to subsidize its investment with a negative tax rate τ_I hence the perfect substitutability between the two tax instruments. If there is no subsidy nor a tax on private investment, $\tau_I = 0$, then the optimal taxation result applies: $\tau_K = 0$ reaches the first best.

B.4.2 Intra-period housing/consumption allocation

Comparison of the social planner's equation reflecting intra-period allocations, equation (B.30) and the corresponding decentralized equation (B.26) implies that in the decentralized equilibrium, the marginal rate of substitution of goods consumption and land consumption have to be equalized to that of the first best, which

is obtained when:

$$\frac{1 - \tau_H}{1 + \tau_{liv,t}} + \frac{\tau_{liv} + \tau_{HI}}{1 + \tau_{liv,t}} \frac{H'_L}{h'_l} + \frac{\Delta\tau_{\mathcal{L}}}{(1 + \tau_{liv,t})R^{Hgross}h'_l} = 1 \quad (\text{B.46})$$

that is:

- in the absence of a living tax ($\tau_{liv} = 0$) and of imputed rent tax ($\tau_{HI} = 0$), the tax on rents can and must be compensated by a differential tax on land such that

$$\Delta\tau_{\mathcal{L}} = \frac{\tau_H}{R^{Hgross}h'_l}$$

- In particular, the landlord occupying its land must be taxed at a higher rate than the landlord renting its land to tenants, to restore the intra-period allocation of income of tenants and landlords.
- Again, in the absence of a tax on rents, the first best is reached with a non-differentiated tax on land:

$$\Delta\tau_{\mathcal{L}} = 0$$

which does not exclude a tax on land itself. See infra.

- In the absence of a differential tax on land ($\Delta\tau_{\mathcal{L}} = 0$), the tax on rent could potentially be compensated by a negative living tax (that is, a subsidy) on landlords implicit rents, but this is not possible with a positive tax and in any event, this would distort the choice of structures S - see below.

B.4.3 Optimal tax product of land

In the first best described above, one can now calculate the optimal tax levied on land to reach the social optimum desired by the social planner. We study the case with $\tau_I = \tau_K = \tau_S = \tau_{liv} = \tau_H = 0$, one has that s, S, K reach their first best level.

We assume here separable utility functions and identical utility functions. To insure $\frac{u'_c}{u'_S} = \gamma$, the social planner must then fix an optimal level of transfers using the tax $\tau_{\mathcal{L}}$:

$$c + hR^{Hgross} = f(K) - Kf'(K) + \tau_{\mathcal{L}}\bar{\mathcal{L}} \quad (\text{B.47})$$

$$C - hR^{Hgross} = Kf'(K) - \delta K - \delta_S(s + S) - \tau_{\mathcal{L}}\bar{\mathcal{L}} \quad (\text{B.48})$$

with $\beta^{-1} = R^{Kgross} = f'(K) + 1 - \delta$ thus $K(\delta, \beta)$. In all cases:

$$c + C = f(K) - \delta K - \delta_S(s + S)$$

In the case $\gamma = 1$, the consumption are equalized, leading to:

$$c = C = \frac{f(K) - \delta K - \delta_S(s + S)}{2}$$

and, by difference of the two resource constraints (B.47) and (B.48):

$$2hR^{Hgross} = f(K) - 2Kf'(K) + \delta K + \delta_S(s + S) + 2\tau_{\bar{L}}\bar{L}$$

or

$$\tau_{\bar{L}}\bar{L} = hR^{Hgross} + Kf'(K) - \frac{1}{2}[f(K) + \delta K + \delta_S(s + S)] \quad (\text{B.49})$$

which is equation (31) in the text, In the general case with positive γ and CARA utility function, if $\ln u(c) = -\nu c$ and $\ln U(C) = -\nu C$, therefore:

$$c - C = -\frac{\ln \gamma}{\nu}$$

In that case, by difference of the resource constraints, for separable utility functions, identical and exponential negative functions (CARA) in the general case $\gamma \leq 1$:

$$\tau_{\bar{L}}\bar{L} = hR^{Hgross} + Kf'(K) - \frac{1}{2}\left[f(K) + \delta K + \delta_S(s + S) - \frac{\ln \gamma}{2\nu}\right] \quad (\text{B.50})$$

which is equation (32) in the text. The product of the land tax must grow as γ (share of the capitalist) goes down.

Still in the general case of $\gamma \leq 1$, equation (33) in the text is obtained in the case of CRRA sub-utility functions $U(C) = \frac{1}{1-\sigma}C^{(1-\sigma)}$. At the first best, $\gamma U'(C) = \gamma C^{-\sigma} = c^{-\sigma}$. Indeed, we have $\gamma \frac{C^{-\sigma}}{c^{-\sigma}} = 1$. or

$$\frac{C}{c} = \frac{1}{\gamma}^{\sigma}$$

Then

$$c + C = c\left(1 + \frac{C}{c}\right) = c\left[1 + \left(\frac{1}{\gamma}\right)^{\sigma}\right] \quad (\text{B.51})$$

Now, summing up equations (B.47) and (B.48), one obtains

$$c + C = f(K) - \delta K - \delta_S(s + S) \quad (\text{B.52})$$

Using (B.51) and (B.52) one gets,

$$c = \frac{f(K) - \delta K - \delta_S(s + S)}{1 + \left(\frac{1}{\gamma}\right)^{\sigma}} \quad (\text{B.53})$$

Now using equation (B.48), one obtains

$$\tau_{\mathcal{L}}\bar{\mathcal{L}} = hR^{HGross} + Kf'(K) - f(K) + c \quad (\text{B.54})$$

Replacing c by its expression in (B.53) in (B.54) one obtains the result in equation (33) in the text.

B.4.4 Proof of Proposition 3.3

On the suboptimal investment in s .

Let us defined as in the Proposition $\mathbf{s} = \frac{s}{l}$ the structure quantity per unit of land. The rent tax including imputed rent entails a new equilibrium with underprovision of \mathbf{s} with respect to the first best. Introducing a small tax τ_H , the new equilibrium must respect the Euler equation with respect to tenant structure equation (39) is defined by:

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H)R^{HGross}h'_s(s, l)$$

Using the constant return to scale property of the function h such that leads to $h(s, l) = lg(\mathbf{s})$ where $g(\mathbf{s})$ is an increasing concave function of land per structure. The Euler equation with respect to tenant structure now writes

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H)R^{HGross}g'(\mathbf{s})$$

Fully differentiating it and rearranging leads to

$$\frac{d\mathbf{s}}{d\tau_H} = \frac{g' \left(1 - (1 - \tau_H) \frac{dR^{HGross}}{d\tau_H R^{HGross}} \right)}{(1 - \tau_H)R^{HGross}g''} \quad (\text{B.55})$$

The denominator of the RHS of (B.55) is negative given the concavity of g . Now we can prove that the numerator is positive. Because the housing market is competitive, it is a standard result that we we have full shifting of the tax, that is $dR^{HGross} - dR^{Hnet} = d\tau_H R^{HGross}$ or

$$\frac{dR^{HGross}}{d\tau_H R^{HGross}} < 1$$

implying that the numerator of (B.55) is positive. It follows that $\frac{d\mathbf{s}}{d\tau_H} < 0$. ■

On the suboptimal investment in S . τ_H τ_{HI}

Similarly, denote by $\mathbf{S} = \frac{S}{\mathcal{L}}$ the capitalist-structure quantity per unit of land. We further assume that the housing production function is Cobb-Douglas. With the rent tax profiles including imputed rents tax considered in Proposition 2.2,

$$\tau_{HI} = \tau_H \frac{h'_t}{H'_\mathcal{L}} \quad (\text{B.56})$$

Therefore condition (35) becomes

$$\frac{U'_H}{U'_C} = \frac{u'_h h'_l}{u'_c H'_L} = R^{HGross} \frac{h'_l}{H'_L} \quad (\text{B.57})$$

We show that a new equilibrium is reached with underprovision of \mathbf{S} with respect to the first best. Introducing a small tax τ_{HI} , the new equilibrium must respect equation (38) rewritten here for convenience:

$$\beta^{-1} = 1 - \delta_s + H'_S \left(\frac{U'_H}{U'_C} - \tau_{HI} R^{HGross} \right) \quad (\text{B.58})$$

Combining equations (B.56) and (B.57) into (B.58) we obtain

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H) R^{HGross} H'_S \frac{h'_l}{H'_L} \quad (\text{B.59})$$

As in the previous proof on s , using that H is constant return to scale leads to rewrite equation (B.58) as

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H) R^{HGross} G'_S \frac{h'_l}{H'_L} \quad (\text{B.60})$$

with $H(S, \mathcal{L}) = \mathcal{L}G(\mathbf{S})$

and $G(\mathbf{S})$ is a concave function. Under the assumption that both h and H are Cobb-Douglas with the same elasticity ν , $h(s, l) = l^\nu s^{(1-\nu)} = lg(s)$ and $H(S, \mathcal{L}) = \mathcal{L}G(\mathbf{S})$ with $g(x) = G(x) = x^{1-\nu}$, one has then

$$h'_l = \nu(s/l)^{1-\nu} = \nu \mathbf{s}^{1-\nu}$$

$$H'_L = \nu(S/\mathcal{L})^{1-\nu} = \nu \mathbf{S}^{1-\nu}$$

$$G' = (1 - \nu) \mathbf{S}^{-\nu}$$

Using these expressions in (B.59) leads to

$$\beta^{-1} = 1 - \delta_s + (1 - \tau_H) R^{HGross} (1 - \nu) \frac{\mathbf{S}^{1-\nu}}{\mathbf{S}} \quad (\text{B.61})$$

Fully differentiate (B.61) gives after a few steps, one has

$$-R^{HGross} \mathbf{s} d\tau_H + (1 - \tau_H) \mathbf{s} dR^{HGross} - (1 - \tau_H) R^{HGross} \frac{\mathbf{S}}{\mathbf{S}} d\mathbf{S} + (1 - \tau_H) (1 - \nu) R^{HGross} d\mathbf{s} = 0$$

and then finally

$$\frac{d\mathbf{S}}{d\tau_H} = - \frac{\mathbf{S} \left(1 - (1 - \tau_H) \frac{dR^{HGross}}{d\tau_H R^{HGross}} \right)}{(1 - \tau_H) R^{HGross}} + (1 - \nu) \frac{\mathbf{S}}{\mathbf{s}} \frac{d\mathbf{s}}{d\tau_H} \quad (\text{B.62})$$

We already know that the first term of the RHS of (B.62) is negative as well as that the second term is negative

(see the proofs for the underprovision of s). Hence,

$$\frac{dS}{d\tau_H} < 0$$

■

B.4.5 Optimal taxation/subventions of structures

Still without living tax, comparisons of the decentralized equilibrium Euler equations on structures, equations (B.43) and (B.44) to the social planner counterparts (B.31) and (B.32), one obtains the optimal value of the tax/subsidy on S when

$$\tau_S = 0$$

The equivalent result on the structures of tenants s financed by landlords, the optimal value of the tax/subsidy on s is reached when

$$\tau_H = -\tau_s \tag{B.63}$$

that is, when a subsidy on investment in structures exactly compensate for the tax on rents.

B.4.6 Living tax with no other tax

To be exhaustive, one adds a discussion of the living tax in the particular case in which the tax on imputed rents is removed: $\tau_{HI} = 0$. In that case, the decentralized system becomes:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_L} \left(\frac{1}{1 + \tau_{iv,t}} + \frac{\tau_{iv}}{1 + \tau_{iv,t}} \frac{H'_L}{h'_l} \right) = \frac{u'_c}{U'_C} \tag{B.64}$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{U'_H H'_S}{U'_C} - \tau_{iv} R^{Hgross} H'_S \tag{B.65}$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{1}{1 + \tau_{iv}} \frac{u'_h h'_s}{u'_c} \tag{B.66}$$

It is easy to show that the living tax is distortive on S and reduces its return. It is also easy to see from equation (B.66) that the living tax reduces the returns on investment in s . In both cases, both marginal rates of substitution $\frac{U'_H H'_S}{U'_C}$ and $\frac{u'_h h'_s}{u'_c}$ must increase. Last, from equation (B.64), a positive living tax increases $\frac{u'_h h'_l}{U'_H H'_L} / \frac{u'_c}{U'_C}$ relative to the first best.

C Extension with demographic weights

In this Appendix, we verify that the first order conditions in the decentralized equilibrium and of the Pareto-optimum are independent of the assumption of a unique representative capitalist. We assume that there is a mass m of capitalists and keep as a normalization a mass 1 of workers. Only the resource condition will have m as an parameter, that shifts the consumption and income of capitalists.

The program of the capitalist is, using C_t, H_t for its per capita consumption, as well as S_t, s_t, K_t for the structures and capital stock per capitalist. To get the aggregates, one simply multiply by m and chose notations $\bar{K}, \bar{S}, \bar{s}, \bar{L}$ for respectively the aggregate stocks $\bar{K} = Km; \bar{S} = Sm, \bar{s} = sm, \bar{L} = mL$. For the stock of structures for tenants, the total stock \bar{s} is divided by the mass of tenants that is 1. Similarly, the land consumed by tenants is $l = \bar{\mathcal{L}} - \bar{L}$ where $\bar{\mathcal{L}}$ is the total stock of land. Finally, the returns on housing must be divided by m since capitalists share the product.

C.1 Decentralized program with demographic weight

The total marginal product of one additional unit of capital \bar{K}_t is $f'(\bar{K}_t)$. The amount invested is therefore m times the marginal amount of individual capital; that amount is split in each of the capitalists, so each capitalist receives for its marginal investment in K_t

$$mf'(\bar{K}_t)/m = f'(\bar{K}_t)$$

and pays for its own individual depreciation. It follows that one can define the gross return to capital as:

$$R_t^{Kgross} = f'(K_t m) + 1 - \delta$$

and

$$R_t^{Knet} = [f'(K_t m) + 1 - \delta] (1 - \tau_{K,t})$$

The program of the capitalist is:

$$\begin{aligned} & \max_{C_t, H_t, \mathcal{L}_t, K_{t+1}, s_{t+1}, S_{t+1}} \sum_t \beta^t \{U[C_t, H(\mathcal{L}_t, S_t)]\} \\ & + \beta^t \lambda_t^K \left\{ R_t^{Knet}(mK_t) + \tau_{I,t}(1 - \delta)K_t + R_t^{Hgross}(1 - \tau_{H,t})h_t(\bar{\mathcal{L}} - m\mathcal{L}_t, ms_t)/m \right. \\ & \quad \left. - (\tau_{iv,t} + \tau_{HI,t})R_t^{Hgross}H_t(\mathcal{L}_t, S_t) \right. \\ & \quad \left. + (1 - \delta_S)(1 + \tau_{St})S_t + (1 - \delta_s)(1 + \tau_{st})s_t - T_t^{\mathcal{L}} \right. \\ & \quad \left. - C_t - S_{t+1}(1 + \tau_{St}) - s_{t+1}(1 + \tau_{st}) - K_{t+1}(1 + \tau_{I,t}) \right\} \end{aligned}$$

subject to subject to $\beta^t U'_C(t)M_{t+1} \rightarrow 0$, for $M_t = \bar{K}_t, \bar{S}_t, \bar{s}_t$. Simply note that here, one needs to account for the fact that aggregate returns to capital depend on the total aggregate stock.

The program of the worker is still:

$$\begin{aligned} & \max_{c_t, h_t} u(c_t, h_t) \\ & \text{subject to } c_t + h_t R_t^{Hgross} (1 + \tau_{iv,t}) = w_t + T_t \end{aligned}$$

where the combinations entering the rental housing service $h_t = (l_t^*, \bar{s}_t^*)$ will be chosen at the equilibrium by landlords.

We have, denoting by λ_t^w the multiplier of the constraint of the worker, and λ_t^K the multiplier of the constraint of the capitalist, and $\Delta\tau_{\mathcal{L},t} = \tau_{\mathcal{L},t} - \tau_{l,t}$:

Worker: intraperiod

$$\partial c_t \quad u'_c = \lambda_t^w \tag{C.1}$$

$$\partial h_t \quad u'_h(t) = \lambda_t^w R_t^{Hgross}(t)(1 + \tau_{iv,t}) \tag{C.2}$$

$$\Leftrightarrow u'_h(t) = u'_c R_t^{Hgross} (1 + \tau_{iv,t}) \tag{C.3}$$

Capitalist: intraperiod

$$\partial C_t \quad U'_C(t) = \lambda_t^K \tag{C.4}$$

$$\partial \mathcal{L}_t \quad U'_H H'_C(t) = \lambda_t^K \left(R_t^{gross} (1 - \tau_{H,t}) h'_l(t) + \Delta\tau_{\mathcal{L}} + (\tau_{iv,t} + \tau_{HI,t}) R_t^{Hgross} H'_C(t) \right) \tag{C.5}$$

Capitalist: intertemporal (Euler)

$$\begin{aligned} \partial S_{t+1} \quad \lambda_t^K (1 + \tau_{St}) &= \beta \lambda_{t+1}^K (1 - \delta_S) (1 + \tau_{St}) + \beta U'_H H'_S(t+1) \\ &\quad - \beta (\tau_{iv,t+1} + \tau_{HI,t+1}) R_{t+1}^{Hgross} H'_S(t+1) \end{aligned} \tag{C.6}$$

$$\partial s_{t+1} \quad \lambda_t^K (1 + \tau_{st}) = \beta \lambda_{t+1}^K (1 - \delta_s) (1 + \tau_{st}) + \beta \lambda_{t+1}^K R_{t+1}^{Hgross} (1 - \tau_{H,t+1}) h'_s(t+1) \tag{C.7}$$

$$\partial K_{t+1} \quad \lambda_t^K (1 + \tau_{I,t+1}) = \beta \lambda_{t+1}^K [R_{t+1}^{Knet} + (1 - \delta) \tau_{I,t+1}] \tag{C.8}$$

or in the steady-state,

Worker: intraperiod

$$u'_h = u'_c R^{Hgross} (1 + \tau_{iv}) \quad (C.9)$$

Capitalist: intraperiod

$$\partial C_t \quad U'_C = \lambda^K \quad (C.10)$$

$$\partial \mathcal{L}_t \quad U'_H H'_\mathcal{L} = U'_C (R^{Hgross} (1 - \tau_H) h'_l + \Delta \tau_\mathcal{L} + (\tau_{iv} + \tau_{HI}) R^{Hgross} H'_\mathcal{L}) \quad (C.11)$$

Capitalist: intertemporal (Euler)

$$\begin{aligned} \partial S \quad U'_C (1 + \tau_S) &= \beta U'_C (1 - \delta_S) (1 + \tau_S) + \beta U'_H H'_S \\ &\quad - \beta (\tau_{iv} + \tau_{HI}) R^{Hgross} H'_S \end{aligned} \quad (C.12)$$

$$\partial s \quad U'_C (1 + \tau_s) = \beta U'_C (1 - \delta_s) (1 + \tau_s) + \beta U'_C R^{Hgross} (1 - \tau_H) h'_s \quad (C.13)$$

$$\partial K \quad (1 + \tau_I) = \beta [R^{Knet} + (1 - \delta) \tau_I] \quad (C.14)$$

Re-arranging the terms, using in particular $R^{Hnet} = (1 - \tau_H) \frac{u'_h}{u'_c} (1 + \tau_{iv})^{-1}$, one obtains:

Intraperiod allocations

$$\frac{u'_h h'_l}{U'_H H'_\mathcal{L}} \left(\frac{1 - \tau_H}{1 + \tau_{iv}} + \frac{\tau_{iv} + \tau_{HI}}{1 + \tau_{iv}} \frac{H'_\mathcal{L}}{h'_l} + \frac{\Delta \tau_\mathcal{L}}{(1 + \tau_{iv}) R^{Hgross} h'_l} \right) = \frac{u'_c}{U'_C} \quad (C.15)$$

Intertemporal allocations (Euler)

$$\partial S \quad \beta^{-1} = 1 - \delta_S + \frac{1}{1 + \tau_S} \frac{U'_H H'_S}{U'_C} - (\tau_{iv} + \tau_{HI}) R^{Hgross} H'_S \quad (C.16)$$

$$\partial s \quad \beta^{-1} = 1 - \delta_s + \frac{(1 - \tau_H)}{1 + \tau_s} \frac{1}{1 + \tau_{iv}} \frac{u'_h h'_s}{u'_c} \quad (C.17)$$

$$\partial K \quad \beta^{-1} = \frac{R^{Kgross} (1 - \tau_K) + (1 - \delta) \tau_I}{1 + \tau_I} \quad (C.18)$$

C.2 Social planner with demographic weights

The system can be compared to the social planner's allocation. Its program is now:

$$\begin{aligned} \max_{C_t, c_t, S_{t+1}, s_{t+1}, \mathcal{L}_t, K_{t+1}} \sum_t \beta^t \{ & u [c_t, h (\bar{\mathcal{L}} - m\mathcal{L}_t, ms_t)] + m\gamma U [C_t, H (\mathcal{L}_t, S_t)] \} \\ & + \beta^t \lambda_t \{ f(mK_t) + (1 - \delta) mK_t + (1 - \delta_S) m(S_t + s_t) \\ & - c_t - mC_t - mS_{t+1} - ms_{t+1} - mK_{t+1} \} \end{aligned}$$

subject to three transversality conditions on each stock:

$$\beta^t U'_C(t) M_{t+1} \rightarrow 0 \quad (C.19)$$

for $M_t = \bar{K}_t, \bar{S}_t, \bar{s}_t$. Again, to avoid multiple indices, we add the time period when relevant in parenthesis, e.g. $R^{Kgross}(t+1)$ is the gross return to capital producing returns next period and defined as:

$$mR^{Kgross}(t+1) = mf'(mK_{t+1}) + m(1 - \delta)$$

We then obtain the first order conditions for each period:

$$\partial C_t \quad m\lambda_t = \gamma mU'_C(t) \tag{C.20}$$

$$\partial c_t \quad \lambda_t = u'_c(t) \tag{C.21}$$

$$\partial \mathcal{L}_t \quad m\gamma U'_H H'_L(t) = mu'_h h'_l(t) \tag{C.22}$$

$$\partial S_{t+1} \quad m\lambda_t = m\beta\lambda_{t+1}(1 - \delta_S) + m\gamma\beta U'_H H'_S(t+1) \tag{C.23}$$

$$\partial s_{t+1} \quad m\lambda_t = m\beta\lambda_{t+1}(1 - \delta_S) + m\beta u'_h h'_s(t+1) \tag{C.24}$$

$$\partial K_{t+1} \quad m\lambda_t = m\beta\lambda_{t+1}R^{Kgross}(t+1) \tag{C.25}$$

This notably implies the following steady-state relations, m disappears and one is left with:

$$\partial C_t, c_t \quad \lambda = \gamma U'_C = u'_c \tag{C.26}$$

$$\partial \mathcal{L}_t \quad \gamma U'_H H'_L = u'_h h'_l \tag{C.27}$$

$$Euler \quad S_{t+1} \quad \beta^{-1} = \frac{U'_H H'_S}{U'_C} + 1 - \delta_S \tag{C.28}$$

$$Euler \quad s_{t+1} \quad \beta^{-1} = \frac{u'_h h'_s}{u'_c} + 1 - \delta_S \tag{C.29}$$

$$Euler \quad K_{t+1} \quad \beta^{-1} = R^{Kgross} = f'(K) + 1 - \delta \tag{C.30}$$

D Appendix: Proofs of propositions of the second best analysis

D.1 Without housing

For the sake of the completeness of the proof, we restate the optimization problem.

$$\max_{c_t, C_t, K_{t+1}} \sum_{t=0}^{\infty} \beta^t (u(c_t) + \gamma mU(C_t))$$

$$f(K_t) + (1 - \delta)K_t - c_t - mC_t - K_{t+1} = 0 ; \text{multiplier } \lambda_t$$

$$\beta U'(C_{t+1})(mC_{t+1} + K_{t+2}) - U'(C_t)K_{t+1} = 0 ; \text{multiplier } \mu_t \tag{D.1}$$

$$K_0 \text{ given}$$

$$\beta^t U'(C_t)K_{t+1} \rightarrow 0 \text{ transversality condition}$$

where the constraint of the capitalist has been introduced in the Euler equation. The first order conditions are:

$$\beta^t u'(c_t) = \beta^t \lambda_t \quad (\text{D.2})$$

$$m\gamma U'(C_t) + \mu_{t-1} U''(C_t)(mC_t + K_{t+1}) + mU'(C_t) = m\lambda_t + \mu_t U''(C_t)K_{t+1} \quad (\text{D.3})$$

$$\beta^{t+1} \lambda_{t+1}(f'(K_{t+1}) + (1 - \delta)) + \beta^t \mu_{t-1} U'(C_t) = \beta^t \mu_t U'(C_t) + \beta^t \lambda_t \quad (\text{D.4})$$

Proof of Lemma 2, Section 4.6.2: *Suppose quantities converge to an interior steady state. Then the multiplier μ_t converges to $\mu = \frac{\gamma(1-\tilde{\gamma}_c)}{\tilde{\gamma}_c(1-\sigma_C)}$ and is positive iff $(1 - \tilde{\gamma}_c)(1 - \sigma_C) > 0$. More specifically, if $\tilde{\gamma}_c < 1$ then the convergence of multipliers to a positive number requires $\sigma_C < 1$. Proof:* We deduce from (D.2)

$$u'(c_t) = \lambda_t$$

Then (D.3) becomes

$$m\gamma U'(C_t) - m\lambda_t + \mu_{t-1} U''(C_t)(mC_t + K_{t+1}) + mU'(C_t) = \mu_t U''(C_t)K_{t+1} \quad (\text{D.5})$$

or dividing by $U''(C_t)K_{t+1}$

$$\mu_t = \mu_{t-1} \left(\frac{mC_t + K_{t+1}}{K_{t+1}} + \frac{mU'(C_t)}{U''(C_t)K_{t+1}} \right) + \frac{m\gamma U'(C_t)}{U''(C_t)K_{t+1}} - \frac{m\lambda_t}{U''(C_t)K_{t+1}}$$

Using the power specification of the utility function,

$$U(C) = \frac{C^{1-\sigma_C}}{1-\sigma_C}$$

implying

$$\sigma_C = -\frac{U''(C)}{U'(C)}C$$

we obtain after trivial steps

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'(c_t)}{U'(C_t)} \right] \quad (\text{D.6})$$

Consider (D.6) when $\mu_t - \mu_{t-1} = 0$ at the limit. We check that the multiplier is positive. Starting from

$$\mu(\sigma_C - 1) - \gamma + \frac{u'(c)}{U'(C)} = 0$$

with μ being the limit of μ_t in infinity and defining

$$\frac{\gamma U'(C)}{u'(c)} = \tilde{\gamma}_c$$

we finally obtain

$$\mu = \frac{\gamma(1 - \tilde{\gamma}_c)}{\tilde{\gamma}_c(1 - \sigma_{CC})} > 0 \quad (\text{D.7})$$

■

D.2 With housing and a rent tax

The Ramsey problem we consider is the following:

$$\max_{c_t, C_t, K_{t+1}, H_t, \tau_{H,t}} \sum_{t=0}^{\infty} \beta^t (u_c(c_t) + u_h(\bar{H} - H_t) + \gamma m(U_C(C_t) + U_H(H_t)))$$

such that:

$$f(K_t) + (1 - \delta)K_t - c_t - mC_t - K_{t+1} = 0 \quad \text{multiplier } \lambda_t \geq 0 \quad (\text{D.8})$$

$$\beta U'_C(C_{t+1})(mC_{t+1} + K_{t+2} - R_{t+1}^{Hgross}(1 - \tau_{H,t+1})(\bar{H} - mH_{t+1})) - U'_C(C_t)K_{t+1} = 0 \quad \text{multiplier } \mu_t \quad (\text{D.9})$$

$$R_t^{Hgross} u'_c(c_t) - u'_h(\bar{H} - mH_t) = 0, \quad \text{multiplier } \eta_{1t} \quad (\text{D.10})$$

$$R_t^{Hgross}(1 - \tau_{H,t})U'_C(C_t) - U'_H(H_t) = 0, \quad \text{multiplier } \eta_{2t} \quad (\text{D.11})$$

$$\tau_{H,t} \geq 0, \quad \text{multiplier } \phi_t \geq 0 \quad (\text{D.12})$$

$$\beta^t U'_C(C_t)K_{t+1} \rightarrow 0 \quad (\text{D.13})$$

K_0 given

In the program, we have inserted from the start the budget constraint of the capitalist in the Euler equation. The Walras Law implies that we can omit to write down the budget constraint of the worker. In the following we make a small abuse of notations in omitting the subscripts for the subtility of consumption and housing both for the worker and capitalist. The FOCs are:

- With respect to c_t

$$\beta^t u'(c_t) + \beta^t \eta_{1t} R_t^{Hgross} u''(c_t) = \beta^t \lambda_t$$

or

$$\lambda_t = u'(c_t) + \eta_{1t} R_t^{Hgross} u''(c_t) \quad (\text{D.14})$$

With respect to C_t

$$\begin{aligned} & \beta^t m \gamma U'(C_t) + \beta^t \mu_{t-1} (U''(C_t)(mC_t + K_{t+1} - R_t^{Hnet}(\bar{H} - mH_t)) + mU'(C_t)) + \beta^t \eta_{2t} U''(C_t) R_t^{Hnet} \\ & = \beta^t m \lambda_t + \beta^t \mu_t U''(C_t) K_{t+1} \end{aligned}$$

or

$$\begin{aligned} m\gamma U'(C_t) + \mu_{t-1}(U''(C_t)(mC_t + K_{t+1} - R_t^{Hnet}(\bar{H} - mH_t)) + mU'(C_t)) + \eta_{2t}U''(C_t)R_t^{Hnet} \\ = m\lambda_t + \mu_t U''(C_t)K_{t+1} \end{aligned} \quad (D.15)$$

- With respect to K_{t+1}

$$\beta^{t+1}\lambda_{t+1}(f'(K_{t+1}) + (1 - \delta)) + \beta^t\mu_{t-1}U'(C_t) = \beta^t\mu_t U'(C_t) + \beta^t\lambda_t \quad (D.16)$$

- With respect to H_t

$$\beta^t(-mu'(\bar{H} - mH_t) + m\gamma U'(H_t)) + m\beta_t^t\mu_{t-1}\beta U'(C_t)(R_t^{Hnet}) + m\beta^t\eta_{1t}u''(\bar{H} - mH_t) - \beta^t\eta_{2t}U''(H_t) = 0$$

or

$$-mu'(\bar{H} - mH_t) + m\gamma U'(H_t) + m\mu_{t-1}\beta U'(C_t)(R_t^{Hnet}) + m\eta_{1t}u''(\bar{H} - mH_t) - \eta_{2t}U''(H_t) = 0 \quad (D.17)$$

- With respect to $\tau_{H,t}$

$$\beta^t(U'(C_t)R_t^{Hgross}(\bar{H} - mH_t)\mu_{t-1} - \beta^t R_t^{Hgross}U'(C_t)\eta_{2t} + \phi = 0$$

or

$$(\bar{H} - mH_t)\mu_{t-1} - \eta_{2t} + \phi = 0 \quad (D.18)$$

- with in addition, the usual Kuhn and Tucker condition.

$$\phi_t\tau_{H,t} = 0$$

Proof of Proposition 5 (Optimal capital tax in the Ramsey problem with housing), Section 4.6.3: *Assume that the following instruments are available to the decision maker, tax on capital, lump sum benefit to workers, tax on rents. Consider an economy where the preferences of both the capitalist and workers are separable, with a CCRA sub-additive utility of consumption. Suppose that quantities converges to an interior steady state and that $\sigma_C < 1$, $\tilde{\gamma}_c < 1$ and $\tilde{\gamma}_h < 1$. Then the optimal tax on capital is 0 and the optimal tax on rents is positive in the limit. Consequently the stock of capital in the second best remains equal to the stock of capital in the first best.*

Proof: The proof consists of exhibiting sufficient conditions for the Euler equation multiplier to be still positive. We express the other multipliers in terms of the Euler equation multiplier. The proof is in three steps.

Step 1: Expression of other multipliers in function of the Euler multiplier If a rental tax is second-best optimal for any t , then $\phi_t = 0$. Then from (D.18)

$$\eta_{2t} = (\bar{H} - mH_t)\mu_{t-1} \quad (\text{D.19})$$

If quantities and μ_t converge, then η_{2t} converges. Plugging (D.19) in (D.17) gives $-mu'(\bar{H} - mH_t) + m\gamma U'(H_t) + m\mu_{t-1}\beta U'(C_t)(R_t^{Hnet}) + m\eta_{1t}u''(\bar{H} - mH_t) - (\bar{H} - mH_t)\mu_{t-1}U''(H_t) = 0$ or $-m\eta_{1t}u''(\bar{H} - mH_t) = -mu'(\bar{H} - mH_t) + m\gamma U'(H_t) + \mu_{t-1}(mR_t^{Hnet}U'(C_t) - (\bar{H} - mH_t)U''(H_t))$

$$\eta_{1t} = \frac{-mu'(h_t) + m\gamma U'(H_t) + \mu_{t-1}(mR_t^{Hnet}U'(C_t) - h_tU''(H_t))}{-mu''(h_t)}$$

Using (D.11) we get

$$\eta_{1t} = \frac{-mu'(h_t) + m\gamma U'(H_t) + \mu_t(mU'(H_t) - h_tU''(H_t))}{-mu''(h_t)}$$

Factorising $U'(H_t)$ and defining

$$\frac{\gamma U'(H_t)}{u'(h_t)} = \tilde{\gamma}_h \quad (\text{D.20})$$

$$\eta_{1t} = U'(H_t) \frac{-\frac{\gamma m}{\tilde{\gamma}_h} + \gamma m + \mu_{t-1} \left(m - h_t \frac{U''(H_t)}{U'(H_t)} \right)}{-mu''(h_t)}$$

Finally denoting $\sigma_H(H_t) = -H_t \frac{U''(H_t)}{U'(H_t)} > 0$ we get

$$\eta_{1t} = \frac{U'(H_t)}{-mu''(h_t)} \left[-\frac{\gamma m}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) + \mu_{t-1} \left(m + \frac{h_t}{H_t} \sigma_H(H_t) \right) \right] \quad (\text{D.21})$$

If quantities and μ_t converge, then η_{1t} converges. From (D.14) we deduce that if η_{1t} converges, then λ_t converges.

Step 2 Expression of the Euler multiplier μ_t . Now we look at equation (D.15)

$$\begin{aligned} m\gamma U'(C_t) + \mu_{t-1}(U''(C_t)(mC_t + K_{t+1} - R_t^{Hnet}(\bar{H} - mH_t)) + mU'(C_t)) + \eta_{2t}U''(C_t)R_t^{Hnet} \\ = m\lambda_t + \mu_t U''(C_t)K_{t+1} \end{aligned}$$

Using the expression (D.14) for λ_t and the expression (D.19) for η_{2t} , the above expression becomes

$$\begin{aligned} m\gamma U'(C_t) + \mu_{t-1}(U''(C_t)(mC_t + K_{t+1} - R_t^{Hnet}h_t) + mU'(C_t)) + \mu_{t-1}U''(C_t)R_t^{Hnet}h_t \\ = mu'(c_t) + \mu_t U''(C_t)K_{t+1} + m\eta_{1t} R_t^{Hgross}u''(c_t) \end{aligned}$$

Since two terms vanish, we can write

$$\begin{aligned} & m\gamma U'(C_t) - mu'(c_t) + \mu_{t-1}(U''(C_t)(mC_t + K_{t+1}) + mU'(C_t)) - m\eta_{1t} R_t^{Hgross} u''(c_t) \\ & = \mu_t U''(C_t) K_{t+1} \end{aligned}$$

Dividing by $U''(C_t)K_{t+1}$ and noticing that only the last term of the LHS is different from the expression obtained without housing (see D.5) one gets using (D.6)

$$\mu_t - \mu_{t-1} = \frac{mC_t}{K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'(c_t)}{U'(C_t)} \right] - \frac{m\eta_{1t} R_t^{Hgross} u''(c_t)}{U''(C_t)K_{t+1}}$$

Now replacing η_{1t} by its expression in (D.21)

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'(c_t)}{U'(C_t)} \right] - \frac{R_t^{Hgross} u''(c_t) U'(H_t)}{U''(C_t) K_{t+1} u''(h_t)} \left[\frac{\gamma m}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) - \mu_{t-1} \left(m + \frac{h_t}{H_t} \sigma_H(H_t) \right) \right]$$

or

$$\begin{aligned} \mu_t - \mu_{t-1} &= \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'(c_t)}{U'(C_t)} \right] \\ &\quad - \frac{R_t^{Hgross} u''(c_t) U'(H_t)}{K_{t+1} u''(h_t)} \frac{C_t U'(C_t)}{C_t U''(C_t) U'(C_t)} \left[\frac{\gamma m}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) - \mu_{t-1} \left(1 + \frac{h_t}{mH_t} \sigma_H(H_t) \right) \right] \end{aligned}$$

In case of an isoelastic utility function on consumption, one gets introducing σ_C

$$\begin{aligned} \mu_t - \mu_{t-1} &= \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'(c_t)}{U'(C_t)} \right] \\ &\quad + \frac{C_t R_t^{Hgross} u''(c_t) U'(H_t)}{\sigma_C K_{t+1} u''(h_t) U'(C_t)} \left[\frac{\gamma m}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) - \mu_{t-1} \left(m + \frac{h_t}{H_t} \sigma_H(H_t) \right) \right] \end{aligned}$$

Putting $\frac{mC_t}{\sigma_C K_{t+1}}$ in factor and using (D.10)

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'(c_t)}{U'(C_t)} + \frac{u'(h_t) u''(c_t) U'(H_t)}{u''(h_t) u'(c_t) U'(C_t)} \left(\frac{\gamma}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) - \mu_{t-1} \left(1 + \frac{h_t}{mH_t} \sigma_H(H_t) \right) \right) \right]$$

Introducing σ_c and $\sigma_h(h_t) = -h_t \frac{U''(h_t)}{U'(h_t)}$

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[(\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'(c_t)}{U'(C_t)} + \frac{\sigma_c h_t U'(H_t)}{\sigma_h(h_t) c_t U'(C_t)} \left(\frac{\gamma}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) - \mu_{t-1} \left(1 + \frac{h_t}{mH_t} \sigma_H(H_t) \right) \right)) \right]$$

And using (D.11)

$$\mu_t - \mu_{t-1} = \frac{mC_t}{\sigma_C K_{t+1}} \left[\mu_{t-1}(\sigma_C - 1) - \gamma + \frac{u'(c_t)}{U'(C_t)} + \frac{\sigma_c h_t R^{Hnet}}{\sigma_h(h_t) c_t} \left(\frac{\gamma}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) - \mu_{t-1} \left(1 + \frac{h_t}{mH_t} \sigma_H(H_t) \right) \right) \right]$$

Consider an interior steady state. If the multipliers converge, at the limit $\mu_{t+1} - \mu_t = 0$. We are looking at the value of the limit value of the multiplier μ and we establish at which sufficient conditions it is positive

$$\mu(\sigma_C - 1) - \gamma + \frac{u'(c)}{U'(C)} + \frac{\sigma_c h R^{Hnet}}{\sigma_h c} \left[\frac{\gamma}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) - \mu \left(1 + \frac{h_t}{m H_t} \sigma_H \right) \right] = 0$$

or

$$\begin{aligned} \mu(\sigma_C - 1) - \gamma \frac{\tilde{\gamma}_c - 1}{\tilde{\gamma}_c} + \frac{\sigma_c h R^{Hnet}}{\sigma_h c} \left[\frac{\gamma}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) - \mu \left(1 + \frac{h_t}{m H_t} \sigma_H \right) \right] &= 0 \\ \gamma \frac{1 - \tilde{\gamma}_c}{\tilde{\gamma}_c} + \frac{\sigma_c h R^{Hnet}}{\sigma_h c} \frac{\gamma}{\tilde{\gamma}_h} (1 - \tilde{\gamma}_h) &= \mu \left[1 - \sigma_C + \frac{\sigma_c h R^{Hnet}}{\sigma_h c} \left(1 + \frac{h}{m H} \sigma_H \right) \right] \\ \mu &= \gamma \frac{\frac{1 - \tilde{\gamma}_c}{\tilde{\gamma}_c} + \frac{\sigma_c h R^{Hnet}}{\sigma_h c} \frac{1 - \tilde{\gamma}_h}{\tilde{\gamma}_h}}{1 - \sigma_C + \sigma_c \left(\frac{h R^{Hnet}}{c} \left(\frac{1}{\sigma_h} + \frac{h}{m H} \frac{\sigma_H}{\sigma_h} \right) \right)} \end{aligned} \quad (D.22)$$

or in the case where $\sigma_C = \sigma_c$

$$\mu = \gamma \frac{\frac{1 - \tilde{\gamma}_C}{\tilde{\gamma}_C} + \frac{\sigma_C h R^{Hnet}}{\sigma_h c} \frac{1 - \tilde{\gamma}_h}{\tilde{\gamma}_h}}{1 - \sigma_C + \sigma_C \left(\frac{h R^{Hnet}}{c} \left(\frac{1}{\sigma_h} + \frac{h}{m H} \frac{\sigma_H}{\sigma_h} \right) \right)} \quad (D.23)$$

We can then state the following lemma.

Appendix Lemma A-1. *Consider separable preferences conditions with CCRA subadditive utility of consumption. Suppose that there exists an interior steady state, $K, c, C, h, H > 0$. Assume $\sigma_C = \sigma_c < 1$, $\tilde{\gamma}_c < 1$ and $\tilde{\gamma}_h < 1$. Then the multipliers μ_t converge to a positive value.*

Step 3 Dynamic accumulation of capital at the steady state Now we move to equation (D.16)

$$\beta^{t+1} \lambda_{t+1} (f'(K_{t+1}) + (1 - \delta)) + \beta^t \mu_{t-1} U'(C_t) = \beta^t \mu_t U'(C_t) + \beta^t \lambda_t$$

which becomes $\frac{\lambda_{t+1}}{\lambda_t} (f'(K_{t+1}) + (1 - \delta)) = \frac{1}{\beta} + \frac{(\mu_t - \mu_{t-1})}{\beta \lambda_t} U'(C_t)$ If there is an interior steady state, we have shown that the multipliers μ_t converge with the stated conditions and step 1 shows that if μ_t converges, then λ_t also converges. We deduce that $f'(K_{t+1}) + (1 - \delta) = \frac{1}{\beta}$ and then $R^{Kgross} = \frac{1}{\beta}$.

The Euler equation of the capitalist at the steady state implies that $\tau_K = 0$ and then the stock of capital remains the same as in the first best. Making stock of the all previous steps, we can state Proposition 6 in the text. ■.

Proof of Proposition 6 in the paper, Section 4.6.3: *Under the assumptions stated in Lemma 1, the optimal rental tax at the steady state is given by*

$$\frac{\tau_H}{1 - \tau_H} = \frac{1 - \tilde{\gamma}_c}{\epsilon_s}$$

where ϵ_s the elasticity of the rental housing supply with respect to the net rent, being computed at the housing equilibrium.

Proof: We consider the stationary state where capital and wage rate are at their first best values by virtue of Proposition 4. Let us denote them respectively K^* and w^* . The Ramsey problem we have to solved is simpler than the original one given by conditions (D.8) to (D.11), since we do not have any more to consider explicitly condition (D.9), the Euler equation. The separability utility is specifically helpful here because the marginal utility of consumption of the capitalist does not depend on his consumption of housing. The conditions (D.10) and (D.11) still hold and it is therefore useful to express the problem using the indirect utilities functions of the capitalist and the worker. Let us denote them respectively $V(\tau_H)$ and $v(\tau_H)$. At the steady state, the decision maker maximizes the weighted sum of indirect utility functions

$$\begin{aligned} \text{Max}_{\tau_H, 0 < \tau_H < 1, T} W &= v(w^* + T^H - R^{Hgross}(\tau_H)h, h) + \\ &\quad \gamma m V \left(\frac{1 - \beta}{\beta} \frac{K^*}{m} + (1 - \tau_H) R^{Hgross}(\tau_H) (\bar{H}/m - H), H \right) \end{aligned}$$

under the budget constraint

$$T^H = \tau_H R^{Hgross}(\tau_H) h(R^{Hgross}(\tau_H)), \text{ multiplier } \lambda$$

It is somewhat simpler to work with the ad valorem tax $\widetilde{\tau}_H$ and the net-of-tax rent R^{Hnet}

$$R^{Hnet}(1 + \widetilde{\tau}_H) = R^{Hgross}$$

than to work with τ_H defined by

$$R^{Hnet} = R^{Hgross}(1 - \tau_H)$$

The two tax rates are related by

$$\widetilde{\tau}_H = \frac{\tau_H}{1 - \tau_H} \tag{D.24}$$

We define the elasticity of the rental supply with respect to to the net-of-tax rent R^{Hnet}

$$\epsilon_s = \frac{(\bar{H} - mH)' R^{Hnet}}{(\bar{H} - mH) R^{Hnet}}$$

or with $h(R^{Hnet}) = \bar{H} - mH(R^{Hnet}) = m \left(\frac{\bar{H}}{m} - H \right)$

$$\epsilon_s = \frac{h'(R^{Hnet}) R^{Hnet}}{h(R^{Hnet}) R^{Hnet}}$$

Now the objective function reads

$$W(T^H, \widetilde{\tau}_H) = v(w^* + T^H - R^{Hnet}(1 + \widetilde{\tau}_H)h, h) + \gamma m V \left(\frac{1 - \beta}{\beta} \frac{K^*}{m} + R^{Hnet} \frac{h(R^{Hnet})}{m}, H \right)$$

under the resource constraint

$$T^H = \widetilde{\tau}_H R^{Hnet}(\widetilde{\tau}_H) h(R^{Hnet}(\widetilde{\tau}_H))$$

Using the envelope theorem, the FOCs with respect to T and $\widetilde{\tau}_H$ are:

$$v'_c(\cdot) = \lambda$$

$$v'_c(\cdot)(-R^{Hnet}h - (1 + \widetilde{\tau}_H)R^{Hnet}h) + \gamma m V'_C(\cdot)(R^{Hnet} \frac{h}{m}) + \lambda(R^{Hnet}h + \widetilde{\tau}_H(R^{Hnet}h + R^{Hnet}h'R^{Hnet})) = 0$$

Eliminating λ and regrouping terms we obtain

$$v'_c(\cdot)(-(1 + \widetilde{\tau}_H)R^{Hnet}h + \widetilde{\tau}_H(R^{Hnet}h + R^{Hnet}h'R^{Hnet})) + \gamma V'_C(\cdot)(R^{Hnet}h) = 0$$

Dividing by $v'_c(\cdot)$ and using the definition of $\widetilde{\gamma}_c$

$$-(1 + \widetilde{\tau}_H)R^{Hnet}h + \widetilde{\tau}_H(R^{Hnet}h + R^{Hnet}h'R^{Hnet}) + \widetilde{\gamma}_c(R^{Hnet}h) = 0$$

Dividing by $R^{Hnet}h$ we obtain

$$-1 - \widetilde{\tau}_H + \widetilde{\tau}_H(1 + \epsilon_s) + \widetilde{\gamma}_c = 0$$

or

$$\widetilde{\tau}_H = \frac{1 - \widetilde{\gamma}_c}{\epsilon_s}$$

and using (D.24) we get

$$\frac{\tau_H}{1 - \tau_H} = \frac{1 - \widetilde{\gamma}_c}{\epsilon_s}$$

■

Proof of Corollary, Section 4.6.3

We consider the particular case of Cobb-Douglas preferences: $U(c, h) = \phi \log h + (1 - \phi) \log c$. The steady state situation is characterized by a capital level, a wage level and a return on capital that are denoted with an asterisk (*), and which are identical to that of the first best since the tax on capital is zero. There are three unknowns, R^{Hnet} , $\widetilde{\tau}_H$, T and three equations

The equilibrium condition on the housing market at the aggregate level

$$h_d(R^{Hnet}, T^H, \widetilde{\tau}_H) = h_s(R^{Hnet}, T^H, \widetilde{\tau}_H) \tag{D.25}$$

The state budget balance

$$T^H = hR^{Hnet}\widetilde{\tau}_H \quad (D.26)$$

The second-best optimal tax on rents

$$\widetilde{\tau}_H = \frac{1}{\epsilon_s}(1 - \widetilde{\gamma}_c) \quad (D.27)$$

We also use

$$\frac{\gamma U'(C)}{u'(c)} = \widetilde{\gamma}_c \frac{c}{C} \quad (D.28)$$

Step 1: computing the supply elasticity of renting housing.

The supply elasticity of renting housing is not constant. Starting from the equation of equilibrium on the market for the optimal tax $\widetilde{\tau}_H$ where h_s is the supply at the macro level

$$h_d(R^{Hgross}) = h_s(R^{Hnet})$$

Differentiating this expression

$$h'_d(R^{Hgross})dR^{Hgross} = h'_s(R^{Hnet})dR^{Hnet}$$

Since $R^{Hgross} = R^{Hnet}(1 + \widetilde{\tau}_H)$ then $dR^{Hgross} = dR^{Hnet}(1 + \widetilde{\tau}_H)$ dividing by dR^{Hgross} we get

$$h'_d(R^{Gross}) = h'_s(R^{Hnet}) \frac{1}{1 + \widetilde{\tau}_H}$$

Now multiplying by R^{Gross}

$$h'_d(R^{Gross})R^{Hgross} = h'_s(R^{Hnet})R^{Hnet}$$

And finally dividing by h_d

$$\frac{h'_d(R^{Gross})R^{Hgross}}{h_d} = \frac{h'_s(R^{Hnet})}{h_s}$$

We conclude that at the equilibrium

$$\epsilon_s = -\epsilon_d$$

and we know the price elasticity of the demand of housing with a Cobb-Douglas which is -1 . Then,

$$\epsilon_s = 1 \quad (D.29)$$

Step 2: Computing the tax formula.

We deduce from (D.27), (D.28), and (D.29) that

$$\widetilde{\tau}_H = 1 - \gamma \frac{c}{C} \quad (\text{D.30})$$

We also need the demands of the composite consumption good

$$\begin{aligned} c &= (1 - \phi)(w^* + T^H) \\ mC &= (1 - \phi)(A^* + R^{Hnet}\overline{H}) \end{aligned}$$

with $(\frac{1-\beta}{\beta})K^* = A^*$ and therefore

$$\frac{c}{C} = \frac{m(w^* + T^H)}{A^* + R^{Hnet}\overline{H}}$$

which allows to offer another expression for $\widetilde{\tau}_H$.

$$\widetilde{\tau}_H = 1 - \gamma \frac{m(w^* + T^H)}{A^* + R^{Hnet}\overline{H}} \quad (\text{D.31})$$

Step 3: Expression of the equilibrium rent R^{Hnet} .

We also need the expressions of the housing demands

$$h_d(R^{Hnet}, T^H, \widetilde{\tau}_H) = \frac{\phi(w^* + T^H)}{R^{Hgross}} = \frac{\phi(w^* + T^H)}{R^{Hnet}(1 + \widetilde{\tau}_H)} \quad (\text{D.32})$$

$$mH(R^{Hnet}, T^H, \widetilde{\tau}_H) = \frac{\phi(A^* + R^{Hnet}\overline{H})}{R^{Hnet}} \quad (\text{D.33})$$

The macro supply of renting housing

$$h_s(R^{Hnet}, T^H, \widetilde{\tau}_H) = \overline{H} - mH(R^{Hnet}, T^H, \widetilde{\tau}_H) = \overline{H} - \frac{\phi(A^* + R^{Hnet}\overline{H})}{R^{Hnet}} = \frac{R^{Hnet}\overline{H} - \phi A^* - \phi R^{Hnet}\overline{H}}{R^{Hnet}} =$$

$$h_s(R^{Hnet}, T^H, \widetilde{\tau}_H) = \frac{-\phi A^* + (1 - \phi)R^{Hnet}\overline{H}}{R^{Hnet}} \quad (\text{D.34})$$

$$h_s(R^{Hnet}, T^H, \widetilde{\tau}_H) > 0$$

if and only if

$$-\phi A^* + (1 - \phi)R^{Hnet}\overline{H} > 0$$

$$h_s(R^{Hnet}, T^H, \widetilde{\tau}_H) > 0 \text{ iff } R^{Hnet} > \frac{\phi}{1-\phi} \frac{A^*}{\overline{H}} \quad (\text{D.35})$$

Starting from $h_d = h_s$ and using (D.32) and (D.34) we get

$$\frac{\phi(w^* + T^H)}{R^{Hnet}(1 + \widetilde{\tau}_H)} = \frac{-\phi A^* + (1 - \phi)R^{Hnet}\overline{H}}{R^{Hnet}}$$

or

$$\phi(w^* + T^H) = -\phi A^*(1 + \widetilde{\tau}_H) + (1 - \phi)(1 + \widetilde{\tau}_H)R^{Hnet}\overline{H}$$

which leads to an expression of the equilibrium rent in function of T^H and $\widetilde{\tau}_H$

$$R^{Hnet} = \frac{\phi[(w^* + T^H) + A^*(1 + \widetilde{\tau}_H)]}{\overline{H}(1 - \phi)(1 + \widetilde{\tau}_H)} = \frac{\phi(w^* + T^H)}{\overline{H}(1 - \phi)(1 + \widetilde{\tau}_H)} + \frac{\phi}{1 - \phi} \frac{A^*}{\overline{H}} > \frac{\phi}{1 - \phi} \frac{A^*}{\overline{H}} \quad (\text{D.36})$$

and then the condition for a positive supply of renting housing (D.35) is satisfied.

Now we need to compute $R^{Hnet}\overline{H}$

$$R^{Hnet}\overline{H} = \frac{\phi}{1 - \phi} \frac{(w^* + T^H) + A^*(1 + \widetilde{\tau}_H)}{1 + \widetilde{\tau}_H} = \frac{\phi}{1 - \phi} \left[\frac{w^* + T^H}{1 + \widetilde{\tau}_H} + A^* \right] \quad (\text{D.37})$$

It is interesting to note that the value of land rents in this economy does not depend on the housing land stock \overline{H} . This is so because the price-elasticity of housing demand is unitary.

Using in (D.31)

$$\widetilde{\tau}_H = 1 - \gamma \frac{m(w^* + T^H)}{A^* + R^{Hnet}\overline{H}}$$

we obtain

$$\widetilde{\tau}_H = 1 - \gamma m \frac{(1 - \phi)(w^* + T^H)}{A^* + \frac{\phi(w^* + T^H)}{1 + \widetilde{\tau}_H}} \quad (\text{D.38})$$

Step 4: Elimination of T .

From the (D.26) and using the demand of housing of the worker (D.32) we get

$$\begin{aligned} T^H &= \frac{\phi(w^* + T^H)}{1 + \widetilde{\tau}_H} \widetilde{\tau}_H & (\text{D.39}) \\ T^H(1 + \widetilde{\tau}_H) &= \phi(w^* + T^H)\widetilde{\tau}_H \\ T^H(1 + \widetilde{\tau}_H) - \phi T \widetilde{\tau}_H &= \phi w^* \widetilde{\tau}_H \\ T^H(1 + \widetilde{\tau}_H - \phi \widetilde{\tau}_H) &= \phi w^* \widetilde{\tau}_H \end{aligned}$$

$$T^H = \phi w^* \frac{\widetilde{\tau}_H}{1 + \widetilde{\tau}_H(1 - \phi)} \quad (\text{D.40})$$

Other expressions are also useful

$$\frac{\phi(w^* + T^H)}{1 + \widetilde{\tau}_H} = \frac{T^H}{\widetilde{\tau}_H} \quad (\text{D.41})$$

and

$$\frac{w^* + T^H}{T^H} = \frac{1 + \widetilde{\tau}_H}{\phi \widetilde{\tau}_H} \quad (\text{D.42})$$

Dividing both numerator and denominator and then using (D.41), (D.42) and (D.40) we get

$$\widetilde{\tau}_H = 1 - \gamma m \frac{(1-\phi)(w^*+T^H)}{\frac{\phi(w^*+T^H)}{1+\widetilde{\tau}_H} \frac{1}{T^H} + \frac{A^*}{T^H}}$$

and after a few steps, $\widetilde{\tau}_H$ is the root of the equation of second degree

$$\widetilde{\tau}_H = 1 - \gamma m \frac{1 - \phi}{\phi} \frac{1 + \widetilde{\tau}_H}{1 + \frac{A^*}{\phi w^*} + \widetilde{\tau}_H \frac{1 - \phi}{\phi} \frac{A^*}{w^*}} \quad (\text{D.43})$$

Claim 1. *There is a unique second best optimal tax rate strictly between 0 and 1 (except for the Rawlsian case where it is 1) which is the root of the second degree equation (D.43). The tax rate is decreasing with the social welfare weight γ of the capitalist.*

Proof:

It involves a few steps.

Fact 1:

$$\text{For } \gamma = 0, \widetilde{\tau}_H^* = 1$$

$\widetilde{\tau}_H^* = 1$ then implies that $\tau_H^* = 1/2$

Fact 2:

$$\frac{d\widetilde{\tau}_H^*}{d\gamma} < 0$$

Indeed, we can write (D.43)

$$\Psi(\widetilde{\tau}_H) = \widetilde{\tau}_H + f(\widetilde{\tau}_H) - 1 = 0$$

with

$$f(\widetilde{\tau}_H) = \gamma m \frac{1 - \phi}{\phi} \frac{1 + \widetilde{\tau}_H}{1 + \frac{A^*}{\phi w^*} + \widetilde{\tau}_H \frac{1 - \phi}{\phi} \frac{A^*}{w^*}} > 0$$

$$\Psi'(t) = 1 + f'(\widetilde{\tau}_H)$$

$$f'(\widetilde{\tau}_H) \text{ is of the sign of } [1 + \frac{A^*}{\phi w^*} + t \frac{1-\phi}{\phi} \frac{A^*}{w^*}] - (1 + \widetilde{\tau}_H) \frac{(1-\phi)}{\phi} \frac{A^*}{w^*} = 1 + \frac{A^*}{\phi w^*} - \frac{(1-\phi)}{\phi} \frac{A^*}{w^*} = 1 + \frac{A^*}{w^*} > 0$$

$$\Psi'(\gamma) > 0$$

$$\text{Then } \frac{d\widetilde{\tau}_H^*}{d\gamma} = -\frac{\Psi'(\gamma)}{\Psi''(\widetilde{\tau}_H)} < 0$$

Fact 3. Except the Rawlsian case, $\widetilde{\tau}_H^* = 1$ cannot be solution. $1 + f(1) - 1 > 0$

Fact 4. Let us take the utilitarian case which gives the lowest tax by fact 2. $\widetilde{\tau}_H^* = 0$ cannot be solution because $f(0) - 1 < 0$ or $f(0) = m \frac{1-\phi}{\phi} \frac{1}{1 + \frac{A^*}{\phi w^*}} < 1$. Indeed, $\frac{1-\phi}{\phi} < \frac{1}{m}(1 + \frac{A^*}{\phi w^*})$, $\frac{1}{\phi} - 1 < (\frac{1}{\phi} \frac{A^*}{m w^*} + \frac{1}{m})$ since $\frac{A^*}{m w^*} > 1$ and $\frac{1}{m} > 1$.

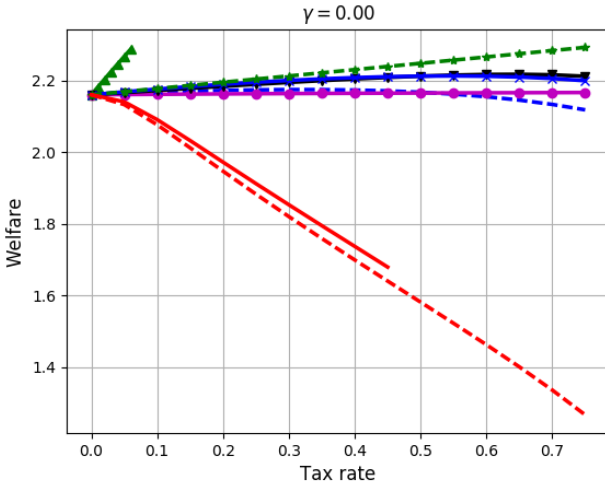
Fact 5. $\Psi(\widetilde{\tau}_H) > 0$. By the theorem of intermediate values, since f is continuous, there is at least a root between 0 and 1 and since f is increasing there is a unique tax rate which solves the equation. ■

Step 5. An explicit formula for the second best optimal tax rate.

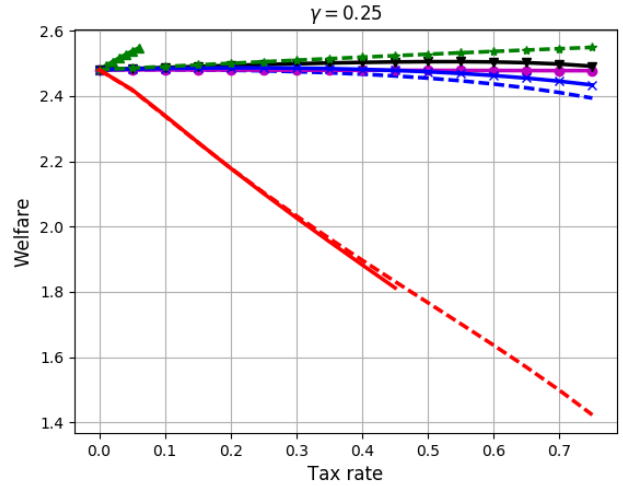
$$\widetilde{\tau}_H^* = \frac{-(1 + \frac{A^*}{w^*} + \gamma m \frac{(1-\phi)}{\phi}) + \Delta^{1/2}}{2[\frac{1-\phi}{\phi} \frac{A^*}{w^*}]} \quad (\text{D.44})$$

$$\text{and } \Delta = (1 + \frac{A^*}{w^*} + \gamma m \frac{1-\phi}{\phi})^2 - 4 \frac{1-\phi}{\phi} \frac{A^*}{w^*} \left[\gamma m \frac{1-\phi}{\phi} - [1 + \frac{A^*}{\phi w^*}] \right] > 0$$

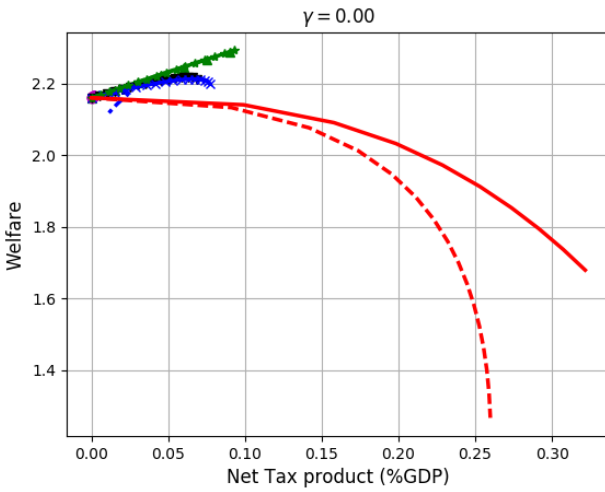
E Additional simulations comparing capital tax to various housing and land taxes/subsidies



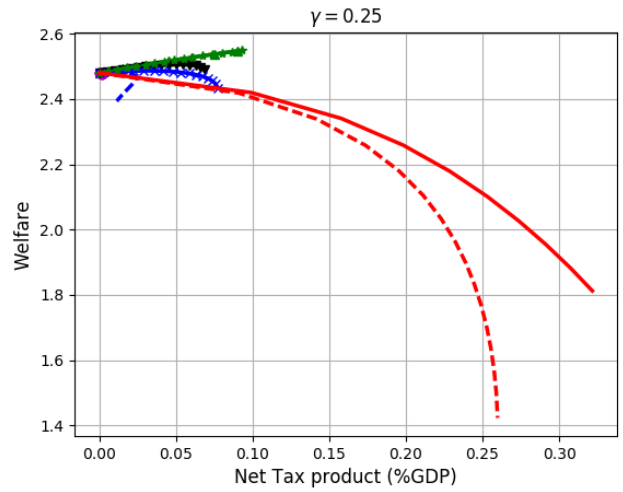
(a) Welfare and tax rate, $\gamma = 0$



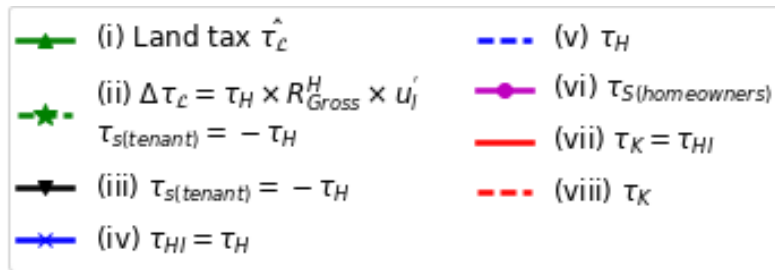
(b) Welfare and tax rate, $\gamma = 0.25$



(c) Welfare and tax Product, $\gamma = 0$

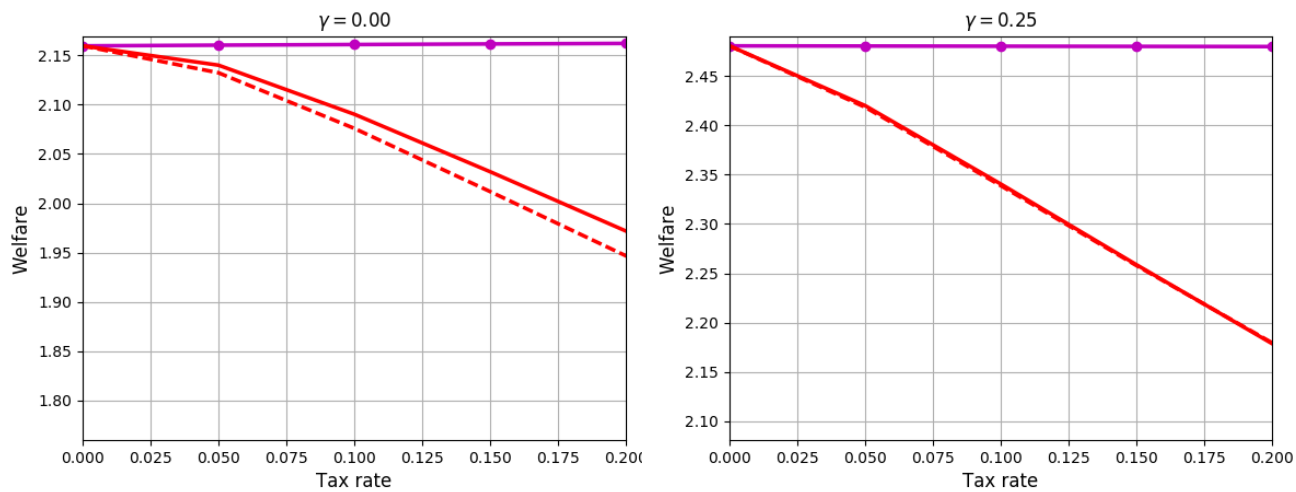


(d) Welfare and tax Product, $\gamma = 0.25$



K-H-s-S economy. Variation in the social welfare function $u(c) + \gamma U(C)$ and the tax revenue in the decentralized equilibrium, for different values of γ , the social welfare weight (respectively 0 and 1/4). Comparison between the first best policy (i), a homogeneous tax on land $\hat{\tau}_L$ redistributed to workers (plain green line with triangles up), or its variant (ii) with a positive tax on rents, a differentiated tax on land and a subsidy to housing structures of tenants (discontinued green line with stars), and second best policies: (iii) a tax on rents compensated by a subsidy on residential investments $\tau_s < 0$ (plain black line with triangles down), (iv) a tax on rent equal to a tax on imputed rents $\tau_H = \tau_{HI}$ (plain blue line with cross), (v) a tax on rents τ_H alone (discontinued blue line); (vi) a tax on homeowners structures $\tau_S > 0$ (plain purple line with squares); (vii) a tax on capital equalized to the tax on rents (plain red line); (viii) a pure tax on capital (dashed red line). Top panels: x-axis is the respective tax rates; bottom panels: x-axis is the respective total tax revenue.

Figure D.1: K-H-s-S economy. Variation in the social welfare function with capital taxation



(a) Welfare and tax rate, $\gamma = 0$

(b) Welfare and tax rate, $\gamma = 0.25$



K-H-s-S economy. Variation in the social welfare function $u(c) + \gamma U(C)$ for different values of γ , the social welfare weight (respectively 0 and 1/4). Comparison between a tax on capitalist's structure τ_S redistributed to workers, dotted line) and taxes on capital, τ_K (c, red) and capital and rent, $\tau_K = \tau_H$ (b, dashed).

Figure D.2: K-H-s-S economy. Variation in the social welfare function : capital Vs. structure taxation