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# Search, Project Adoption and the 

## Fear of Commitment

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# Search, Project Adoption and the Fear of Commitment* 

Vidya Atal ${ }^{\dagger}$ Talia Bar${ }^{\ddagger}$ and Sidartha Gordon ${ }^{\S}$

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#### Abstract

We examine project adoption decisions of firms constrained in the number of projects they can handle at once. Adoption requires a commitment for a period of uncertain duration, restricting the firm in subsequent periods. Capacity constraints create a "fear of commitment" - some positive return projects are not adopted. In the sequential move dynamic game, the second mover sometimes adopts projects that were rejected by the first, even when both firms are symmetric and equally informed. We study the effects of competition on the fear of commitment, and compare the jointly optimal adoption decision to the behavior of strategic non-cooperative firms.


[^0]Keywords: adoption, project selection, commitment, Markov perfect equilibrium
JEL Classification Codes: L10, L13, D21.

## 1 Introduction

Decision makers, such as researchers, firms and government agencies, often have to choose whether to engage in projects that require committing limited resources. While resources are committed to development or implementation of a project, they cannot be used in another project; or a firm might need to put on hold its search for other projects. Projects may include research, development or adoption of an invention, providing services to clients, acquisition of an innovative start-up, and introduction of a new product to the market. Such projects take time to complete, and tie a firm's resources for a period of time. When making a choice about a current project, the decision maker is typically not fully informed about the duration of the commitment to the project or about future opportunities. Committing limited resources (e.g., researchers' and other employees' time, physical space, lab equipment, etc.) to one project may result in the need to forgo future, perhaps more profitable, projects.

In our infinite horizon model, new project opportunities arise every period. The decision maker learns the expected return of the current period's project, but is uncertain about the return from future projects. We use dynamic programming to characterize the decision maker's adoption behavior. The optimal adoption policy is characterized by a reservation value, so that a project is adopted if its expected return exceeds the reservation value. Whereas unconstrained decision makers take on any project with a positive net expected return, constrained decision makers are more prudent. They reject some projects that have
positive returns, so as not to commit resources and risk missing out on better opportunities in the future. We refer to this tendency to wait for better projects as the "fear of commitment." The optimal reservation values will be higher (the decision maker will be more likely to reject a current project) when each project commitment is expected to last longer, and when the decision maker is more patient. The optimal adoption threshold is also higher when project returns are drawn from a more favorable distribution, because there is a higher probability that the return from a future project exceeds the return from the current project.

Repeated adoption decisions are often taken in competitive settings. Consider, for example, the companies Coca-Cola and PepsiCo. While best known for their cola drinks, each of these companies repeatedly adopted many innovative drinks. Coca-Cola company's page on Facebook, for example, states that the company has "a portfolio of more than 3,300 beverages, from diet and regular sparkling beverages to still beverages such as 100 percent fruit juices and fruit drinks, waters, sports and energy drinks, teas and coffees, and milk-and soy-based beverages, our variety spans the globe. ${ }^{1 "}$ Adopting a product requires a period of intense effort for the company. It involves conducting market research, designing the bottle and label, developing a marketing strategy, etc.

The sports drink Gatorade was developed in 1965 in the University of Florida for its football team the Gators. ${ }^{2}$ In 1992, after gaining increased popularity, Gatorade approached Coca-Cola about using Coke's distribution system. ${ }^{3}$ In 2000, after considering acquiring Quaker Oats, the company that owned Gatorade, Coca-Cola decided against the acquisition. As a headline in the Chicago Sun Times from November 22, 2000 states, "Coke backs off bid

[^1]to buy Quaker Oats - Company's decision could open door for rival PepsiCo ${ }^{4}$," in August 2001, PepsiCo acquired Quaker Oats for its Gatorade.

We study the optimal project adoption strategies of capacity constrained firms in a dynamic duopoly game. Every period, one project opportunity arises. The expected return from each period is randomly drawn from a known distribution. The first firm to consider each project is selected at random. If it does not adopt the project, the other firm will consider adopting it. When no commitment is required, the first firm to consider each project adopts any project with a positive return, and the second firm never adopts a project that was rejected by the first firm. But, when projects require commitment of resources for a certain period of time, some positive return projects are not adopted by either firm. The first firm will adopt projects that have a high enough return. Interestingly, there can be projects with intermediate levels of return that are rejected by the first firm, yet adopted by the second firm. This is true even though both firms are identical and have the same information about project returns. The intuition for this result is that, for projects with intermediate return, each firm prefers its rival to adopt the project, thus commit its resources leaving the non-adopting firm with less competition over future, perhaps more profitable projects. This result holds true whether the externality that the adopting firm imposes on the non-adopting firm is positive or negative, provided that the magnitude of the externality is smaller than the return from the project to the adopting firm.

To examine the effects of competition on the "fear of commitment," we compare the project adoption decisions of a firm who has no competition to that of a firm that faces a strategic competitor. We find that adoption thresholds are lower under competition. That

[^2]is, a duopolist adopts projects that this firm would not have adopted absent a competitor in the market.

We consider also the project adoption decisions of a firm that has a less strict capacity constraint and can adopt two projects at a time. When a single firm decides on project adoptions, we show that the less constrained firm has lower adoption thresholds (less fear of commitment) than the more constrained firm. Interpreting the two-projects capacity model as that of a joint venture of two firms with one project capacity each, we find that in duopoly competition, firms exhibit more fear of commitment than in a joint venture.

The rest of this paper proceeds as follows. In Section 2, we discuss related literature. In Section 3, we describe the basic model with a single decision maker. In Section 4, we develop the game between duopolists and analyze the strategic interaction between them. Section 5 studies the single decision maker with two-projects capacity constraint and compares the choices made in a joint venture to the choices made in the non-cooperative duopoly game. Section 6 concludes.

## 2 Related Literature

Our model contributes to the literature on sequential search, and more specifically, to the much smaller branch that studies the strategic interaction between a small number of searching agents. We consider first the decision maker version of our model, and its relation to job search models, and then describe the relation of our model to models of strategic search.

In our model, once a firm adopts a project, it commits resources to the development of the project until its random termination date. In early job search models (e.g., McCall, 1970;

Mortensen, 1970), workers cannot search for a job while employed, and staying employed is better than becoming unemployed. In recent years, Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002), have introduced models where workers continue searching for a better job while employed. Thus in leading job search models, commitment is either not binding or it is not assumed. In contrast, in our setting, the necessity to spend time and other resources to developing the newly found projects is a natural source of commitment. Binding commitments characterizes many situations firms face. For example, if a service provider (e.g., a consulting firm) takes on a client, the firm is typically committed (legally or to preserve his reputation) to complete the service promised; when adopting a project, firms might enter binding agreements with its employees, clients, or suppliers; or, while working on an existing project, the firms' researchers might not observe new project opportunities. Sometimes, at the start of a project, a large sunk investment is required which makes abandoning the project for another, even if it has a higher expected return, not worthwhile. In labor search models, the worker alternates between spells of unemployment and employment. Similarly, in our single firm model, the firm alternates between spells of search for a new project and development of a project it recently found.

An important difference between our model and previous search models is that unlike the job search case, the firm's payoff from a project is independent of the development duration: one can interpret it as the present value of the project. The firm is stuck with the development of a project until it is complete. Termination frees the firm to adopt an additional profitable project without forgoing the returns of the earlier project. Whereas a worker fears a layoff, our firm eagerly expects project completion. Being committed does not allow the firm to engage in new project opportunities. This assumption is appealing for
the applications we consider. For example, if the firm needs to use its resources to develop a new product that it will then market, the firm is better off when development is completed and the product can enter the market. When the project terminates, the firm's resources are free and can be used for other projects, and it will still benefit from the product that was developed and introduced to the market.

In our analysis of strategic interactions, our model also contributes to a sparse (but highly relevant to the field of industrial organization) literature that studies search behavior and project selection by strategic firms. Whereas the analysis of search and adoption by single decision makers is important in competitive settings (such as in labor markets) or for understanding monopolies, in many markets firms act strategically. A firm would take into account how its own search and adoption strategies would affect the opportunities available and the behavior of rivals. Our contribution to strategic search models lies in the form of the strategic interaction between our searching firms, which differs from other interactions that have been studied, and better applies to some strategic situations.

In the context of a game, in one category of models, known as patent or $\mathrm{R} \& \mathrm{D}$ races (Loury 1979, Lee and Wilde 1980, Reinganum 1983b), search intensity is the main strategic variable. In these models, there is essentially one object to be found, so that selectivity is not an issue. The firms' payoffs depend on which firm finds it first and how long the race lasts. The main insight from these models is that, in equilibrium, firms search more intensively than joint profit maximization would require.

In a surprisingly small second category of models, selectivity is taken as the main strategic variable. Our model belongs to this group. In these models, firms play reservation strategies, and a firm's selectivity is therefore measured by the threshold it uses for adop-
tion. ${ }^{5}$ Reinganum (1982, 1983a) considers a game where sequential search for a technology is undertaken simultaneously in a first stage. Firms do not observe the outcomes of the other firms' searches, but only their reservation thresholds. Once all firms have completed their search, firms compete in the goods market. The dates at which the different firms stop searching and the order in which they complete the search play no role. ${ }^{6}$ Daughety and Reinganum (2000) use a similar framework to study search for evidence by opposing parties in a trial. Lippman and Mamer (1993) enrich Reinganum's (1982) strategic search problem by introducing a timing dimension. They assume an extreme form of first-mover advantage whereby only the first adopter is rewarded. Hoppe (2000) extends the analysis to the case where the loser of this race may benefit from second-mover advantages. Interestingly, a firm's best response threshold is nonincreasing in the other firm's threshold in Reinganum (1982), increasing in Lippman and Mamer (1993) and decreasing in Hoppe (2000) when the second-mover advantage is strong enough. Our model differs from the ones discussed in this paragraph in several important ways.

First, what all of these models have in common is that the agents search in different

[^3]pools. The interaction comes from the effect that the other agent's search outcome has in a subsequent stage (competition on a goods market, trial, etc.) The value of what an agent finds depends on what the other agents find. However, what an agent can find does not depend on the other agent's behavior. In contrast, in our model, the selectivity of a firm at any point in time affects the distribution of projects faced by the other firm both in the same period and in subsequent periods in complicated ways. If firm 1 is approached first by an innovator in some period, and adopts his project, then on the one hand firm 2 will not receive any offer in the same period. But on the other hand, firm 1 might be committed for a certain period of time, which will leave firm 2 as a temporary monopolist in the market for innovation until either firm 1 completes developing its product or firm 2 adopts a project. The second difference is that, in our model, what each firm has found and adopted is common knowledge. Firms best respond not only to the other firm's thresholds strategies, but also to their current state. The third difference is that our firms alternate between periods of search and periods of development. This is a common setting in job search models, but, to the best of our knowledge, it has not been analyzed in a strategic search game.

In its application, our work relates to a literature in management and finance on capital allocation. This literature considers mostly the decision to finance a single project, and much of the work focuses on information asymmetry (see, for example, Harris and Raviv, 1996 and Zhang, 1997). In our paper, information is symmetric, and the focus is on the need to commit limited capacity. Hall and Lerner (2010) survey the literature on financing of innovation with a focus on financial market reasons for under-investment. Our assumption on firms' project capacity constraint is different from the standard capital financing constraint. It is not enough for firms to be able to borrow at a reasonable interest rate. Firms can be limited
in the number of projects they adopt even if they have cash reserves, or access to venture capital. Capacity constraints could arise due to a limited number of skilled scientists and engineers, limited physical space to run experiments, etc. The problem our paper analyzes has also some similarity to the problem of dynamic assignment of a single durable object to successive agents, considered by Bloch and Houy (2011). In their model, the firm prefers to learn an agent's type before assigning him an object. In our model, the type of the project is known and the project immediately dies if it is not adopted. Our focus is on the effect of competition and capacity constraints on the fear of commitment.

## 3 Basic Model

We begin by describing the model in the context of one decision maker - a single firm repeatedly deciding whether to adopt projects that arise sequentially. In the next section, we add strategic interactions between firms.

Consider a discrete time infinite horizon model. The decision maker maximizes the discounted sum of expected payoffs. The discount factor is $0<\delta<1$. A new project opportunity arises every period. If the decision maker is not yet committed, he can decide whether to adopt the project. A project requires a commitment of resources, which prevents the firm from engaging in another project at the same time. The duration of a project is random. To capture this, we assume that every period, conditional on being committed in the previous period, the commitment ends with a probability $p$. The decision-maker has resource constraints so that he can be committed to only one project at a time. Hence, the decision maker can only adopt a project if he is not currently committed to an earlier
project. If we think of a project as the acquisition of a start-up innovative firm, the duration of the commitment can include the time it takes to transfer knowledge from the innovator to the firm, time to develop the product, time to come up with a marketing strategy, etc.

A project's return $v$ is the expected discounted present value of benefits from the project at the time it is adopted. ${ }^{7}$ Project returns are identically and independently drawn from a known distribution. The cumulative distribution function for project returns is $F(v)$ on $[\underline{v}, \bar{v}]$, with $\underline{v} \leq 0$, and a finite $\bar{v}>0$. For all $v>0$, we assume $F$ is differentiable with a finite density $f(v)>0$. There may be an atom (a discontinuity in $F$ ) at zero, so as to allow for a positive probability that no project arises. The payoff in a period in which no project is adopted is zero.

### 3.1 A Single Decision Maker's Adoption Behavior

Let us denote the value function in any period for which the decision maker is not committed, before realization of the project's return, by $V_{0}$. Let $V_{1}$ denote the value function for the decision maker who is committed from an earlier period. If the decision maker is not committed, then when a project has a payoff of $v$, adopting a project has a payoff $v+\delta\left(p V_{0}+(1-p) V_{1}\right)$ and not adopting the project has a payoff $\delta V_{0}$. The decision maker maximizes payoff by choosing to adopt or not to adopt:

$$
\max \{\underbrace{v+\delta\left(p V_{0}+(1-p) V_{1}\right)}_{\text {adopt }}, \underbrace{\delta V_{0}}_{\text {don't adopt }}\} .
$$

[^4]The project is adopted if $v+\delta\left(p V_{0}+(1-p) V_{1}\right) \geq \delta V_{0}$. This decision rule defines a threshold for adoption:

$$
\begin{equation*}
v_{0}=\delta(1-p)\left(V_{0}-V_{1}\right) \tag{1}
\end{equation*}
$$

The value in the state without commitment is:

$$
\begin{align*}
V_{0} & =\int_{\underline{v}}^{v_{0}} \delta V_{0} f(v) d v+\int_{v_{0}}^{\bar{v}}\left[v+\delta\left(p V_{0}+(1-p) V_{1}\right)\right] f(v) d v \\
& =\delta V_{0}+\int_{v_{0}}^{\bar{v}}\left(v-v_{0}\right) f(v) d v \tag{2}
\end{align*}
$$

The value in the committed state is:

$$
V_{1}=\delta\left[p V_{0}+(1-p) V_{1}\right]
$$

Re-arranging, we have:

$$
\begin{equation*}
V_{1}=\frac{\delta p}{1-\delta(1-p)} V_{0} \tag{3}
\end{equation*}
$$

Substituting (3) into (1) and then into (2), and re-arranging, we obtain the following implicit definition of the adoption decision threshold $v_{0}$ :

$$
\begin{equation*}
\frac{1-\delta(1-p)}{\delta(1-p)} v_{0}=\int_{v_{0}}^{\bar{v}}\left(v-v_{0}\right) f(v) d v \tag{4}
\end{equation*}
$$

Lemma 1 There exists a unique solution to (4) in the range ( $0, \bar{v}$ ) which defines the adoption threshold.

Lemma 1 implies that an optimal threshold return exists and is unique. Unless $p=1$, this threshold is strictly positive, that is, when the decision maker can only commit to one project at a time and a project's expected duration is longer than one period, some positive return projects are not adopted.

The next proposition presents comparative statics results describing how the reservation value depends on the parameters $\delta$ and $p$.

Proposition 1 The reservation value $v_{0}$ is lower the more likely is the project to end (the higher p), i.e., the shorter is the expected commitment time and the more patient the decision maker is (the higher $\delta$ ).

The proposition is proved by implicit differentiation of (4). Intuitively, the threshold is higher when the commitment required for each project is expected to last longer, because adopting the project is more costly in terms of missed future opportunities. In the pharmaceutical industry, for example, where it takes about 10 years to bring a drug to market (see Nicholas, 1994), we would expect higher adoption thresholds than in areas with short duration projects, as perhaps is true in the context of software development. Conditional on the project being adopted, project return is expected to be higher in an industry in which project duration is longer.

The adoption threshold is also higher when the decision maker is more patient. This is because waiting for a potentially better project in the future is less costly to him. Combining the results for $\delta$ and $p$, we also conclude that if project opportunities arise more frequently, then the project adoption reservation value would be higher. This is because in a setting with more frequent project opportunities, the discount factor is higher (as there would be a lower discount rate per shorter period) and there is a smaller probability that the project terminates by the following period. Comparing a decision maker (or firm) that is more prolific or creative and can generate frequent projects to a less prolific one, we would expect the more prolific decision maker to adopt higher return projects.

We next compare the thresholds of adoption for two distributions of project returns, one of which is either more favorable in terms of first order stochastic dominance, or a mean preserving spread of the other. In the first case, the threshold for adoption is higher for the dominating distribution, because there is a higher probability that a better project will arise in the following periods. In the second case, the threshold for adoption is higher for the spread, because the option value of waiting for a better project is higher, under the riskier distribution.

Proposition 2 For a distribution of returns that either first order stochastically dominates another one or is a mean-preserving spread of another one or if both of these statements hold, the threshold level is at least as high. ${ }^{8}$

In research areas where projects tend to have higher values, decision makers will be less likely to commit to projects of relatively lower value. Similarly, in research areas where projects have the same expected values but more risky returns, decision makers will be less likely to commit to projects of relatively lower value, because the option value of waiting for a better project is higher.

## 4 Strategic Decision Makers

In this section, we examine the project adoption decisions in a duopoly setting. One firm's adoption decision can affect the payoffs of the other. We assume only one firm can adopt

[^5]each project. We examine project adoption strategies and the effects of competition on the fear of commitment.

### 4.1 The Game

We consider competition between two firms $A$ and $B$. Every period, a new project opportunity with a value $v$ drawn from the distribution $F(v)$ arises. One firm chosen at random (each firm is chosen with a probability $\frac{1}{2}$ ) is the first to make a decision to adopt the project or not. If the first firm adopts the project, the period ends. If the first firm does not adopt the project, the other firm can decide whether to adopt it. As we assumed before, a firm can adopt at most one project at a time and the commitment of resources ends each period with a probability $p$. The adopting firm obtains rights to a patent, or a lead time advantage resulting in the random return $v$. For a project with return $v$ to the adopting firm, we denote the return of the non-adopting firm by $\gamma v$, where $\gamma \in(-1,1)$. When $\gamma<0$, a firm suffers a negative externality if its rival adopts the project. This can be the case for example if the project results in an improvement of quality or reduction in cost for the adopting firm, which reduces profits and market share for the non-adopting firm. When $\gamma>0$, a firm enjoys a positive externality if the other firm adopts the project. A firm can benefit from its rival's adoption when there are network externalities, or technology spillover. The non-adopting firm can also benefit if, for example, the project involves the introduction of a new product, and the first adopting firm becomes a Stackelberg leader in the product market, while the non-adopting firm becomes a follower. Either way, we assume the return for the rival firm is smaller in magnitude than the return for the adopting firm, $|\gamma|<1$. If no firm adopts the
project, payoffs are zero for each firm.
We restrict attention to Markov strategies. A firm's decision will only depend on the current state. States of the world are denoted by $(i, j)$, where $i, j \in\{0,1\}$. In a state with $i=0$ firm $A$ is not committed and in a state with $i=1$ firm A is committed. Similarly, $j$ indicates the commitment of firm $B$.

We look for a symmetric Markov perfect equilibrium. The value function (before realization of the project's return and the choice of the first mover) in any state $(i, j)$ is denoted by $V_{i, j}^{A}$ for firm $A$ and $V_{i, j}^{B}$ for firm $B$. Because we focus on symmetric equilibria, $V_{i, j}^{A}=V_{j, i}^{B}=V_{i, j}$. Equilibrium strategies are characterized by threshold levels of return. The threshold for the non-committed firm in a state $(0,1)$ is denoted by $v_{0,1}$. In state $(0,0)$, the threshold for the firm that was offered the project first is $v_{0,0}^{1}$ and for the second firm (in case the first firm rejected the project), we denote the threshold by $v_{0,0}^{2}$.

### 4.2 Analysis

In state $(1,1)$, both firms are committed and no firm can adopt a project. Next period's state depends on whether each of the firms is freed from its previous commitment or not. The value in state $(1,1)$ is thus given by:

$$
\begin{equation*}
V_{1,1}=\delta\left[p^{2} V_{0,0}+p(1-p) V_{1,0}+p(1-p) V_{0,1}+(1-p)^{2} V_{1,1}\right] \tag{5}
\end{equation*}
$$

In state $(0,1)$, firm $B$ is committed and it cannot adopt a project. Hence, if firm $A$ does not adopt, no firm adopts. If a project with return $v$ arises, firm $A$ can either adopt it and immediately transition to the state $(1,1)$ where both firms are committed, or not adopt and transition in the next period to one of the states where it is not committed. Hence, firm $A$
will maximize:

$$
\max \{\underbrace{v+V_{1,1}}_{\text {adopt }}, \underbrace{\delta\left(p V_{0,0}+(1-p) V_{0,1}\right)}_{\text {don't adopt }}\}
$$

The threshold return for adoption is:

$$
\begin{equation*}
v_{0,1}=\delta\left(p V_{0,0}+(1-p) V_{0,1}\right)-V_{1,1} . \tag{6}
\end{equation*}
$$

Accounting for these thresholds of adoption for the non-committed firm, the value functions in states $(0,1)$ and $(1,0)$ are:

$$
\begin{align*}
V_{0,1} & =\int_{v_{0,1}}^{\bar{v}} v f(v) d v+\left(1-F\left(v_{0,1}\right)\right) V_{1,1}+F\left(v_{0,1}\right) \delta\left(p V_{0,0}+(1-p) V_{0,1}\right)  \tag{7}\\
V_{1,0} & =\int_{v_{0,1}}^{\bar{v}} \gamma v f(v) d v+\left(1-F\left(v_{0,1}\right)\right) V_{1,1}+F\left(v_{0,1}\right) \delta\left(p V_{0,0}+(1-p) V_{1,0}\right) . \tag{8}
\end{align*}
$$

In state $(0,0)$, one of the firms is chosen at random to consider the project first. If the first firm does not adopt the project, then the second firm faces the choice:

$$
\max \{\underbrace{v+\delta\left(p V_{0,0}+(1-p) V_{1,0}\right)}_{\text {adopt }}, \underbrace{\delta V_{0,0}}_{\text {don't adopt }}\} .
$$

The threshold level is:

$$
\begin{equation*}
v_{0,0}^{2}=\delta(1-p)\left(V_{0,0}-V_{1,0}\right) . \tag{9}
\end{equation*}
$$

Returning to the first firm's decision, if $v \geq v_{0,0}^{2}$ so that the second firm will adopt if it has the opportunity to do so, then the first firm faces the choice:

$$
\max \{\underbrace{v+\delta\left(p V_{0,0}+(1-p) V_{1,0}\right)}_{\text {adopt }}, \underbrace{\gamma v+\delta\left(p V_{0,0}+(1-p) V_{0,1}\right)}_{\text {don't adopt } \Rightarrow \text { rival adopts }}\} .
$$

The firm would adopt this project if the following condition holds:

$$
v \geq \frac{\delta(1-p)}{(1-\gamma)}\left(V_{0,1}-V_{1,0}\right):=\widetilde{v}
$$

If $v<v_{0,0}^{2}$ so that the second firm will not adopt the project even if the first firm to consider it did not, then the first firm faces essentially the same choice as that of the second firm after the first firm rejected a project. Because we are considering the range $v<v_{0,0}^{2}$, the first firm does not adopt such project. Thus, for $v<v_{0,0}^{2}$, neither firm adopts. The threshold of adoption for the first firm is therefore:

$$
\begin{equation*}
v_{0,0}^{1}=\max \left\{v_{0,0}^{2}, \widetilde{v}\right\} \geq v_{0,0}^{2} . \tag{10}
\end{equation*}
$$

For $v \geq v_{0,0}^{1}$, the first firm adopts. If $\left(V_{0,0}-V_{1,0}\right)<\frac{1}{(1-\gamma)}\left(V_{0,1}-V_{1,0}\right)$, then there is a range of project returns $v_{0,0}^{2} \leq v<v_{0,0}^{1}$ so that the first firm to be offered the project does not adopt, but the second firm does. The equation that defines the value function in state $(0,0)$ is given by:

$$
\begin{equation*}
V_{0,0}=\int_{v_{0,0}^{2}}^{\bar{v}} \frac{(1+\gamma)}{2} v f(v) d v+\delta F\left(v_{0,0}^{2}\right) V_{0,0}+\delta\left[1-F\left(v_{0,0}^{2}\right)\right] \frac{(1-p)\left(V_{0,1}+V_{1,0}\right)+2 p V_{0,0}}{2} . \tag{11}
\end{equation*}
$$

In the special case where $p=1$, each project only lasts one period. The commitment of resources is not a binding constraint. It follows immediately from the system of equations above that the thresholds of adoption satisfy $v_{0,0}^{2}=v_{0,1}=v_{0,0}^{1}=0$. That is, when $p=1$, a firm will adopt any project that has a positive expected return. From now on, we consider $p<1$, when commitment is necessary.

Proposition 3 For $p<1$, in a symmetric Markov perfect equilibrium, the thresholds for adoption in state $(0,0)$ satisfy $v_{0,0}^{1}>v_{0,0}^{2}$, so that in the range $v_{0,0}^{1}>v \geq v_{0,0}^{2}$, the first firm rejects the project, but the second firm adopts it. Additionally, $v_{0,0}^{1} \geq v_{0,1}$, (with a strict inequality for $p>0$ ), so that a firm is more selective when its rival is not committed.

From (10), we know that $v_{0,0}^{1} \geq v_{0,0}^{2}$. This means that the first firm to be offered a project never accepts one that would have been rejected by the second firm. When project returns are low, $v<v_{0,0}^{2}$, the project is not adopted. When project returns are high, $v \geq v_{0,0}^{1}$ the first firm to be offered the project accepts it. Interestingly, Proposition 3 implies that there are intermediate values of project returns for which the first firm to be offered the project passes on the opportunity. Yet, the second firm who is offered the same project, adopts it. These intermediate range project returns are high enough for the second firm to adopt, (knowing that its rival already decided not to adopt the project), but not high enough to induce the first firm to adopt. This is because the first firm knows that if it does not adopt the project, its rival will adopt it, committing resources and leaving the first firm in a better position to adopt projects in future periods.

To examine the effects of competition on the fear of commitment, we compare the adoption rates when two firms compete to those of the single decision maker. We show that the threshold return for a project to be adopted is lower when two firms compete than when there is a single decision maker. Thus, competition reduces the fear of commitment.

Proposition 4 Competition reduces the fear of commitment: $v_{0,0}^{1}<v_{0}$.

The comparison in Proposition 4 is done under the assumption that the project return has the same distribution in the single decision maker's problem and in the game. If the firm enjoys a higher return from any project when it is alone in the market, the distribution of returns in the decision maker's problem may dominate that in the duopoly case. As we have shown in Proposition 2, this would imply an even higher threshold of adoption. So the result of Proposition 4 holds even if the decision maker earns more from each project compared to
the duopolist.

## 5 Two Projects Capacity Constraint

We consider now a single decision maker's project adoption rule when the decision maker has enough resources to work on two, but not more, projects at the same time. We compare the decision maker's choice in the two projects capacity model to the single project capacity constraint. Also, this version of the model allows us to compare project adoption decisions of strategic non-cooperative firms with the decision of a joint venture. We first set up the dynamic programming equations for the (single decision maker) two projects capacity case. We assume that when a project of value $w$ arises, its adoption results in a payoff of $w(1+\gamma)$ to the decision maker. This payoff is the sum of payoffs in the game we solved in the previous section. We can then compare the equilibrium project adoption behavior to the optimal adoption behavior of the joint venture. We think of this joint venture as a coordinated adoption decision of the projects, but no collaboration among firms in the product market. We also consider the special case $\gamma=0$ which can be interpreted as the optimal decision for a single decision maker with two-projects capacity. Examining this case allows us to compare the adoption thresholds of a single decision maker with one or two projects capacity.

We denote the value functions of the decision maker with two projects capacity with $W_{i}$, to distinguish from that of single project capacity values $V_{i}$. The index in the value $W_{i}$ refers to the number of projects to which the decision maker is committed, $i \in\{0,1,2\}$. In state 2 , when committed to two projects, the decision maker cannot adopt a project. The
optimal choices in states 0 and 1 are given by thresholds of adoption $w_{0}$ and $w_{1}$, respectively. Similar to the analysis in the previous sections, we write the system of dynamic programming equations that characterizes optimal decisions of the two-projects capacity decision maker.

In state 2, the decision maker is already committed to two projects. The value in state 2 is given by:

$$
\begin{equation*}
W_{2}=\delta\left[p^{2} W_{0}+2 p(1-p) W_{1}+(1-p)^{2} W_{2}\right] \tag{12}
\end{equation*}
$$

In state 1 , if a project with return $w$ arises, the decision maker will maximize:

$$
\max \{\underbrace{w(1+\gamma)+W_{2}}_{\text {adopt }}, \underbrace{\delta\left(p W_{0}+(1-p) W_{1}\right)}_{\text {don't adopt }}\}
$$

The threshold return for adoption satisfies the following condition:

$$
\begin{equation*}
(1+\gamma) w_{1}=\delta\left(p W_{0}+(1-p) W_{1}\right)-W_{2} \tag{13}
\end{equation*}
$$

The value functions in state 1 is:

$$
\begin{equation*}
W_{1}=\int_{w_{1}}^{\bar{v}} v(1+\gamma) f(v) d v+\left(1-F\left(w_{1}\right)\right) W_{2}+F\left(w_{1}\right) \delta\left(p W_{0}+(1-p) W_{1}\right) . \tag{14}
\end{equation*}
$$

In state 0 , the decision maker faces the choice:

$$
\max \{\underbrace{w(1+\gamma)+\delta\left(p W_{0}+(1-p) W_{1}\right)}_{\text {adopt }}, \underbrace{\delta W_{0}}_{\text {don't adopt }}\} .
$$

The threshold level satisfies:

$$
\begin{equation*}
(1+\gamma) w_{0}=\delta(1-p)\left(W_{0}-W_{1}\right) \tag{15}
\end{equation*}
$$

The equation that defines the value function in state 0 is given by:

$$
\begin{equation*}
W_{0}=\int_{w_{0}}^{\bar{v}}(1+\gamma) v f(v) d v+\delta\left[1-F\left(w_{0}\right)\right]\left((1-p) W_{1}+p W_{0}\right)+\delta F\left(w_{0}\right) W_{0} . \tag{16}
\end{equation*}
$$

Equations (12)-(16) define the solution to the optimal decision of the firm who can adopt at most two projects.

We reduce the system to a system of two equations in only the thresholds $w_{0}$ and $w_{1}$. We first show that the decision maker is less likely to adopt a project when she has already committed resources to one project, i.e., $w_{1}>w_{0}$.

Proposition 5 For a decision maker who can commit to adopt at most two projects, the fear of commitment is greater when one project is underway than when it is not committed, $w_{1}>w_{0}$.

This result is intuitive, if the firm has not yet adopted any project, adopting the current project will tie resources but still leave an opportunity to adopt another promising project in the next period. However, for a firm that is already committed, adopting now means that it cannot adopt until the commitment to one of the projects ends. ${ }^{9}$

In the next two subsections, we use the system derived above to examine how the decisions of firms with different capacity constraints compare (the case $\gamma=0$ ), and also how the adoption decisions of strategic players compare to that of a joint venture.

### 5.1 Adoption Decision and Project Capacity Constraint

We compare the adoption strategy of a firm that has only one project capacity to that of a less constrained firm that has a 2-projects capacity. In Section 3.1, we derived the

[^6]adoption threshold $v_{0}$ of the single project capacity firm. The analysis in the beginning of this section, when specialized to the case $\gamma=0$, describes the adoption thresholds $w_{1}$ and $w_{0}$, that a 2-projects capacity decision maker has when he is committed to one project, or when he is not committed to any project, respectively. Our next proposition establishes that the single project decision maker has a higher threshold of adoption compared to the threshold of adoption of the 2-projects capacity firm, even when this firm has already committed resources to one project. Intuitively, the reason for this is that for the two projects capacity firm, when all its resources are committed, the expected time until at least one of the commitments is relieved is shorter than for the single project capacity firm who has committed all of its resources as the probability that one of two projects ends is larger than the probability that one project ends.

Proposition 6 For a decision maker who has a capacity of one project, the fear of commitment is greater than for a firm with two projects capacity, even when the two projects capacity firm has already committed resources to one project, i.e., $v_{0}>w_{1}$.

An implication of Proposition 6 is that the average return of adopted projects is larger for a firm that has a more stringent project capacity constraint, because the fear of commitment leads the constrained firm not to adopt some relatively low return projects that the less constrained firm would adopt.

### 5.2 Joint Decision versus the Non-cooperative Game

The analysis in this section accounts for the payoff externality parameter, $\gamma \in(-1,1)$, as in the game analyzed in Section 4. It allows us to compare the adoption strategies of firms
in the non-cooperative game to the behavior of a decision maker that jointly decides on the adoption rule of both firms. This joint decision maker has a 2-projects capacity. There are two comparisons of interest. First, we compare $w_{1}$ to $v_{0,1}$. That is, we compare the adoption threshold of the 2-projects capacity firm when it is committed to one project but is free to adopt another to the adoption threshold of a firm that is not committed but has a rival that is committed. Second, we compare $w_{0}$ to $v_{0,0}^{2}$. That is, we compare the threshold of the 2-projects capacity firm when it is free of commitments, to the adoption threshold above which at least one of the competing firms in the game adopts when both firms are free of commitment.

Proposition 7 A non-cooperatively competing duopolist has a stronger fear of commitment (higher thresholds of adoption) compared to the jointly optimal decision maker, $w_{1}<v_{0,1}$ and $w_{0}<v_{0,0}^{2}$.

To prove this proposition, we first argue that the joint decision maker can obtain at least as high a value in state 0 as the sum of values of both firms in the game in state $(0,0)$, i.e., $W_{0} \geq 2 V_{0,0}$. This is true because the decision maker can mimic the adoption strategies in the equilibrium of the game. We then derive the given inequalities on thresholds from the systems of dynamic programming equations. Proposition 7 suggests that competition between firms with capacity constraints on project adoptions results in firms setting too high a bar for adoption, compared to what would be optimal for them under a joint decision making.

## 6 Concluding Remarks

Project adoption often requires firms to commit limited resources, preventing them from adopting other projects while they are committed. Because more promising projects could arise during the time a firm is committed to a project it adopted earlier, constrained firms have a tendency not to adopt some profitable projects. We referred to this phenomenon as a "fear of commitment."

We first considered a single decision maker that could adopt at most one project at a time. We characterized conditions under which the fear of commitment was stronger. The constrained firm has a reservation value such that it adopts only projects that exceed this value. The reservation value is higher when project commitment is expected to be longer, when the firm is more patient, and if project opportunities arise more frequently. As a result, in industries with long project durations, like the pharmaceutical industry, the expected return from adopted projects would be higher compared to industries where projects do not require long commitments. Also, a more prolific firm (that has frequent potential projects) would adopt higher return projects. The reservation value is also higher for firms that face a first order stochastically dominating distribution of project returns.

We compared firms with different project capacities. Compared to a firm with one project capacity, a less constrained, two-projects capacity firm has less fear of commitment even during a period when it is already committed to one project and has resources available only for one other project.

In a strategic environment, project adoption by one firm can change the profitability of a rival, as well as the rival's opportunity to adopt. Not adopting a project allows a rival
to adopt it, while adopting a project will increase the rival's opportunity to adopt future projects. We show that firms sometimes prefer their rivals to adopt a project, so as to lessen future competition on projects. Our analysis suggests that there is a range of project returns for which the first firm to consider the project does not adopt it, but the second firm does. This may explain why we sometimes observe firms adopting projects that rivals, who had an opportunity to adopt before, did not.

Our analysis suggests that firms that face a strategic rival will tend to adopt projects of lower expected value than firm who do not face competition: competition reduces the fear of commitment. But had the firms been able to jointly make adoption decisions, their adoption thresholds would have been lower than in the non-cooperative equilibrium.

In attempt to maintain tractability and a simple exposition, we made certain simplifying assumptions. In our model, if the firm is committed to a project, it cannot adopt another project until the commitment ends. In reality, it is likely that firms can at some cost be released from a previous commitment. Another way to look at this is to think of the firm making a fixed sunk investment every time it adopts a project. This cost needs to be incurred again if the firm switches to another project. We analyzed the case in which these fixed costs are high. More generally, when the fixed cost is not too high, if a new project of high enough return arises, the firm might find it worthwhile to abandon an old project and adopt the new. Thus, firms also have thresholds of adoption in states where their resources have already been committed. Assuming it is sufficiently costly to be released from a previous commitment, the fear of commitment would still be present, and we expect the results to be qualitatively similar.

In analyzing strategic interactions, our model assumes sequential decisions. One can make
alternative assumptions on the nature of the game. For example, firms might simultaneously decide on project adoption, or one firm might be a leading firm and always have the first opportunity to decide on adoption, or the order of sequential move might be endogenous as well. We expect that some of our results will carry over to such variations of the model, but others will not. In particular, our result in Proposition 3 that the first firm may reject projects that the second firm adopts, obviously relies on the sequential play assumption. Instead, in the simultaneous game, in a symmetric equilibrium, there may be a range of intermediate project returns for which firms randomize the decision to adopt.

We considered the strategic behavior of the adopting firms, but not of projects. If, however, project opportunities arise when independent innovators propose them, they might also act strategically so as to extract surplus from the adopting firms. The game played each period might take the form of an auction or a bargaining game in such circumstances. These extensions might be interesting to pursue in future work.

## A Appendix: Proofs

Proof of Lemma 1. From (4), we know that $v_{0}$ is the solution to $g(x)=0$ where $g($. takes the following form:

$$
\begin{equation*}
g(x)=\frac{[1-\delta(1-p)]}{\delta(1-p)} x-\int_{x}^{\bar{v}}(v-x) f(v) d v \tag{17}
\end{equation*}
$$

Evaluating $g(x)$ at $x=0$ and at $x=\bar{v}$, we find that:

$$
g(0)<0 \text { and } g(\bar{v})>0 .
$$

Therefore a solution exists. Now, differentiating the function $g(x)$, we find that:

$$
g^{\prime}(x)=\frac{[1-\delta(1-p)]}{\delta(1-p)}+[1-F(x)]>0
$$

This implies that there exists a unique solution to (17) in the range $(0, \bar{v})$.
Proof of Proposition 1. Consider $g\left(v_{0}, \delta, p\right)$ as defined in (4) only taking into account the parameters $\delta$ and $p$ as arguments of the function. Implicit differentiation of $g\left(v_{0}, \delta, p\right)=0$ shows that:

$$
\begin{aligned}
& \frac{\partial g}{\partial v_{0}} \frac{d v_{0}}{d p}+\frac{\partial g}{\partial p}=0 \Rightarrow \frac{d v_{0}}{d p}=-\frac{\partial g}{\partial p} / \frac{\partial g}{\partial v_{0}} \\
& \frac{\partial g}{\partial v_{0}} \frac{d v_{0}}{d \delta}+\frac{\partial g}{\partial \delta}=0 \Rightarrow \frac{d v_{0}}{d \delta}=-\frac{\partial g}{\partial \delta} / \frac{\partial g}{\partial v_{0}}
\end{aligned}
$$

From the proof of Lemma 1, we know that $\frac{\partial g}{\partial v_{0}}>0$. Hence,

$$
\begin{aligned}
\operatorname{sign}\left(\frac{d v_{0}}{d p}\right) & =\operatorname{sign}\left(-\frac{\partial g}{\partial p}\right) \\
& =\operatorname{sign}\left[-\frac{v_{0}}{\delta(1-p)^{2}}\right]<0
\end{aligned}
$$

We conclude that the reservation value is lower the higher the probability that the project ends, or the shorter is the expected time the decision maker will be committed when taking on a project.

Similarly,

$$
\begin{aligned}
\operatorname{sign}\left(\frac{d v_{0}}{d \delta}\right) & =\operatorname{sign}\left(-\frac{\partial g}{\partial \delta}\right) \\
& =\operatorname{sign}\left[\frac{v_{0}}{\delta^{2}(1-p)}\right]>0 .
\end{aligned}
$$

Thus, the reservation value is higher the more patient the decision maker is.
Proof of Proposition 2. The threshold return level solves $g\left(v_{0}\right)=0$ which we defined in (17) in the proof of Lemma 1. Using integration by parts, we rearrange the function $g$
and write it as:

$$
\begin{equation*}
g\left(v_{0}\right)=\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0}-\int_{v_{0}}^{\bar{v}}[1-F(v)] d v . \tag{18}
\end{equation*}
$$

Consider two cumulative distribution functions $F^{a}$ and $F^{b}$, defined on the interval $(-\infty,+\infty)$, which contains their compact supports $\left[\underline{v}^{a}, \bar{v}^{a}\right]$ and $\left[\underline{v}^{b}, \bar{v}^{b}\right]$. For all $v<\underline{v}^{i}$, let $F^{i}(v)=0$ and for all $v>\bar{v}^{i}$, let $F^{i}(v)=1$. Let $g_{a}(v)$ and $g_{b}(v)$ be defined on $(-\infty,+\infty)$ as in (18) for the corresponding distribution $F^{a}$ or $F^{b}$. From Lemma 1, we know that these functions are both increasing in $v$. Denote $v_{0}^{a}$ and $v_{0}^{b}$ the solutions to $g_{a}(v)=0$ and $g_{b}(v)=0$, respectively.

First, consider the case where $F^{b}$ first order stochastically dominates $F^{a}$, i.e., $F^{b}(v) \leq$ $F^{a}(v)$ for all $v \in(-\infty,+\infty)$, with a strict inequality for at least some $v$. Then $g_{b}(v) \leq g_{a}(v)$ for all $v$, with a strict inequality for some $v$. In particular, $g_{b}\left(v_{0}^{a}\right) \leq g_{a}\left(v_{0}^{a}\right)=0$. Because the function $g_{b}$ is increasing, this implies that $v_{0}^{a} \leq v_{0}^{b}$, which is the desired conclusion. Note that if the inequality $F^{b}(v) \leq F^{a}(v)$ is strict for some $v>v_{0}^{a}$, we obtain a strict inequality $v_{0}^{a}<v_{0}^{b}$. If $F^{b}(v)=F^{a}(v)$ for all $v>v_{0}^{a}$, then $v_{0}^{a}=v_{0}^{b}$.

Second, consider the case where $F^{b}$ is a mean preserving spread of $F^{a}$, i.e.

$$
\begin{equation*}
\int_{-\infty}^{v}\left[F^{b}(v)-F^{a}(v)\right] d v \geq 0 \tag{19}
\end{equation*}
$$

for all $v$ (with a strict inequality for some $v$ ) and

$$
\begin{equation*}
\int_{-\infty}^{+\infty}\left[F^{b}(v)-F^{a}(v)\right] d v=0 \tag{20}
\end{equation*}
$$

Then,

$$
\begin{aligned}
g_{b}\left(v_{0}^{a}\right) & =g_{b}\left(v_{0}^{a}\right)-g_{a}\left(v_{0}^{a}\right) \\
& =\int_{v_{0}^{a}}^{+\infty}\left[F^{b}(v)-F^{a}(v)\right] d v \\
& =\int_{-\infty}^{+\infty}\left[F^{b}(v)-F^{a}(v)\right] d v-\int_{-\infty}^{v_{0}^{a}}\left[F^{b}(v)-F^{a}(v)\right] d v \\
& =-\int_{-\infty}^{v_{0}^{a}}\left[F^{b}(v)-F^{a}(v)\right] d v \leq 0 .
\end{aligned}
$$

The first equality holds by definition of $v_{0}^{a}$, the fourth by (20) and the final inequality by (19). Because the function $g_{b}$ is increasing, the inequality $g_{b}\left(v_{0}^{a}\right) \leq 0$ implies that $v_{0}^{a} \leq v_{0}^{b}$, which is the desired conclusion. Note that if the inequality (19) is strict for some $v>v_{0}^{a}$, we obtain a strict inequality $v_{0}^{a}<v_{0}^{b}$. If it holds as an equality for all $v>v_{0}^{a}$, then $v_{0}^{a}=v_{0}^{b}$.

Proof of Proposition 3. Let us define

$$
\begin{equation*}
\widetilde{v}=\frac{\delta(1-p)}{(1-\gamma)}\left(V_{0,1}-V_{1,0}\right) \tag{21}
\end{equation*}
$$

By (10), we know that:

$$
v_{0,0}^{1}=\max \left\{v_{0,0}^{2}, \widetilde{v}\right\} .
$$

It immediately follows that $v_{0,0}^{1} \geq v_{0,0}^{2}$, and that $v_{0,0}^{1}>v_{0,0}^{2}$ iff $\widetilde{v}>v_{0,0}^{2}$. We need to show $\widetilde{v}>v_{0,0}^{2}$.

We first reduce the system (5)-(11) that defines the equilibrium to a system of three equations in the three unknowns: $\widetilde{v}, v_{0,1}$ and $v_{0,0}^{2}$.

Lemma 2: In equilibrium, if $p<1$, then the following three conditions hold:

$$
\begin{equation*}
\frac{[1-\delta(1-p)]}{\delta(1-p)} \widetilde{v}=\int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v \tag{22}
\end{equation*}
$$

$$
\begin{gather*}
\frac{[1-\delta(1-p)] v_{0,1}}{\delta(1-p)}=\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v+\frac{p\left(v_{0,0}^{2}-v_{0,1}\right)}{\delta(1-p)^{2}}  \tag{23}\\
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^{2}=\binom{\int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v-\frac{(1-\gamma)}{2} \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v}{-\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v+(1-\gamma) \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v} \tag{24}
\end{gather*}
$$

## Proof of Lemma 2:

From (7) and (6), we find that:

$$
\begin{align*}
V_{0,1} & =\int_{v_{0,1}}^{\bar{v}} v f(v) d v+\left(1-F\left(v_{0,1}\right)\right) V_{1,1}+F\left(v_{0,1}\right) \delta\left(p V_{0,0}+(1-p) V_{0,1}\right)  \tag{25}\\
& =V_{1,1}+v_{0,1}+\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v
\end{align*}
$$

From (8), (6) and (21), we have:

$$
\begin{aligned}
V_{1,0} & =\int_{v_{0,1}}^{\bar{v}} \gamma v f(v) d v+\left(1-F\left(v_{0,1}\right)\right) V_{1,1}+F\left(v_{0,1}\right) \delta\left(p V_{0,0}+(1-p) V_{1,0}\right) \\
& =V_{1,1}+\int_{v_{0,1}}^{\bar{v}} \gamma v f(v) d v+F\left(v_{0,1}\right)\left[\delta\left(p V_{0,0}+(1-p) V_{1,0}\right)-V_{1,1}\right] \\
& =V_{1,1}+\int_{v_{0,1}}^{\bar{v}} \gamma v f(v) d v+F\left(v_{0,1}\right)\left[v_{0,1}-(1-\gamma) \widetilde{v}\right] \\
& =V_{1,1}+v_{0,1}+\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v-(1-\gamma)\left[\widetilde{v}+\int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v\right]
\end{aligned}
$$

Combining with (25), we find that:

$$
V_{1,0}=V_{0,1}-(1-\gamma)\left[\widetilde{v}+\int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v\right]
$$

Rearranging, and using (21)we obtain (22) :

$$
\frac{[1-\delta(1-p)]}{\delta(1-p)} \widetilde{v}=\int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v
$$

We substitute (5) into (6) to find that:

$$
\begin{aligned}
v_{0,1} & =\delta\left(p V_{0,0}+(1-p) V_{0,1}\right)-V_{1,1} \\
& =\delta(1-p)\left[p V_{0,0}-p V_{1,0}+(1-p) V_{0,1}-(1-p) V_{1,1}\right] \\
& =p \delta(1-p)\left(V_{0,0}-V_{1,0}\right)+\delta(1-p)^{2}\left(V_{0,1}-V_{1,1}\right) .
\end{aligned}
$$

Substituting (9) and (25),.we find that:

$$
v_{0,1}=p v_{0,0}^{2}+\delta(1-p)^{2}\left[v_{0,1}+\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v\right]
$$

Rearranging, we obtain (23) :

$$
\frac{[1-\delta(1-p)] v_{0,1}}{\delta(1-p)}=\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v+\frac{p\left(v_{0,0}^{2}-v_{0,1}\right)}{\delta(1-p)^{2}} .
$$

Next, we subtract $\delta(1-p) V_{1,0}$ from both sides of (8) and rearrange using (6) and (21) to find that:

$$
\begin{align*}
{[1-\delta(1-p)] V_{1,0} } & =\delta p V_{0,0}+\int_{v_{0,1}}^{\bar{v}}\left(\gamma v-v_{0,1}+\delta(1-p)\left(V_{0,1}-V_{1,0}\right)\right) f(v) d v  \tag{26}\\
& =\delta p V_{0,0}+\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v-(1-\gamma) \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v
\end{align*}
$$

Lastly, we subtract $\delta V_{0,0}$ form both sides of (11) and substitute (9) and (21) to find that:

$$
\begin{align*}
(1-\delta) V_{0,0} & =\binom{\int_{v_{0,0}^{2}}^{\bar{v}} \frac{(1+\gamma)}{2} v f(v) d v-\left[1-F\left(v_{0,0}^{2}\right)\right] \delta V_{0,0}}{+\delta\left[1-F\left(v_{0,0}^{2}\right)\right] \frac{(1-p)\left(V_{0,1}+V_{1,0}\right)+2 p V_{0,0}}{2}} \\
& =\int_{v_{0,0}^{2}}^{\bar{v}} \frac{(1+\gamma)}{2} v f(v) d v-\left[1-F\left(v_{0,0}^{2}\right)\right] \delta(1-p)\left[V_{0,0}-\frac{\left(V_{0,1}+V_{1,0}\right)}{2}\right] \\
& =\int_{v_{0,0}^{2}}^{\bar{v}} \frac{(1+\gamma)}{2} v f(v) d v-\left[1-F\left(v_{0,0}^{2}\right)\right] \delta(1-p)\left[\left(V_{0,0}-V_{1,0}\right)-\frac{\left(V_{0,1}-V_{1,0}\right)}{2}\right] \\
& =\int_{v_{0,0}^{2}}^{\bar{v}}\left[\frac{(1+\gamma)}{2} v-v_{0,0}^{2}+\frac{(1-\gamma)}{2} \widetilde{v}\right] f(v) d v \\
& =\int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v-\frac{(1-\gamma)}{2} \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v . \tag{27}
\end{align*}
$$

Using (9), we have:

$$
\begin{aligned}
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^{2} & =[1-\delta(1-p)]\left(V_{0,0}-V_{1,0}\right) \\
& =(1-\delta) V_{0,0}-\left[[1-\delta(1-p)] V_{1,0}-\delta p V_{0,0}\right]
\end{aligned}
$$

Substituting (27) and (26), we obtain (24) :

$$
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^{2}=\binom{\int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v-\frac{(1-\gamma)}{2} \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v}{-\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v+(1-\gamma) \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v} .
$$

This completes the proof of lemma 2 .

Lemmas 3 and 4 take care of the special cases $p=1$ and $p=0$. In Lemmas 5 and 6 , we consider intermediate values $p \in(0,1)$.

Lemma 3: In equilibrium, $v_{0,0}^{2}=v_{0,1}=\widetilde{v}=v_{0,0}^{1}$ iff $p=1$.

Proof of Lemma 3: If $p=1$, then the system (5)-(11) immediately implies that $v_{0,0}^{2}=v_{0,1}=v_{0,0}^{1}=0$, and by its definition, also, $\widetilde{v}=0$.

Suppose, $v_{0,0}^{2}=v_{0,1}=\widetilde{v}=v_{0,0}^{1}$ and $p<1$. Then substituting $v_{0,1}$ for $v_{0,0}^{2}$ and $\widetilde{v}$ into (24), we have

$$
\begin{aligned}
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1} & =\frac{(1-\gamma)}{2} \int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v \\
& <\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v=\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1} \text { from }(22)
\end{aligned}
$$

which is a contradiction. This completes the proof of Lemma 3.
In the next lemma, we consider the case $p=0$.
Lemma 4: When $p=0$, we have $\widetilde{v}=v_{0,1}>v_{0,0}^{2}$.
Proof of Lemma 4: Let us define the function $g($.$) as in equation (17). When p=0$,
from (23) and (22), we have

$$
g(\widetilde{v})=g\left(v_{0,1}\right)=0 \Rightarrow \widetilde{v}=v_{0,1} .
$$

Substituting it into (24), we have

$$
\begin{aligned}
g\left(v_{0,0}^{2}\right) & =\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^{2}-\int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v \\
& =-\frac{(1-\gamma)}{2} \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v+(1-\gamma) \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v \\
& =\left[-\frac{(1-\gamma)}{2}\right] \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\gamma \int_{\widetilde{v}}^{\bar{v}}(v-\widetilde{v}) f(v) d v .
\end{aligned}
$$

Because $g$ is a strictly increasing function, if $g\left(v_{0,0}^{2}\right)=0$, then $g\left(v_{0,0}^{2}\right)=0=g(\widetilde{v})$ implies that $\widetilde{v}=v_{0,1}=v_{0,0}^{2}$ which, as we showed earlier, holds iff $p=1$. A contradiction! If
$g\left(v_{0,0}^{2}\right)>0$, then $g\left(v_{0,0}^{2}\right)>0=g(\widetilde{v})$ implies that $\widetilde{v}<v_{0,0}^{2}$, which in turn implies that:

$$
\int_{\widetilde{v}}^{\bar{v}}(v-\widetilde{v}) f(v) d v>\int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v
$$

Also, as $\gamma>-1$, we have $\gamma>-\frac{(1-\gamma)}{2}$. Hence, $g\left(v_{0,0}^{2}\right)<0$. A contradiction again! Therefore, it must be the case that $g\left(v_{0,0}^{2}\right)<0=g(\widetilde{v}) \Leftrightarrow \widetilde{v}=v_{0,1}>v_{0,0}^{2}$. This completes the proof of Lemma 4.

Lemma 5: In equilibrium, for $0<p<1$ it must be that $\widetilde{v} \neq v_{0,1}$.
Proof of Lemma 5: Suppose $\widetilde{v}=v_{0,1}$. By (22) and (23), $\widetilde{v}=v_{0,1}$ iff

$$
\begin{aligned}
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1} & =\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v \Leftrightarrow \\
p\left(v_{0,0}^{2}-v_{0,1}\right) & =0
\end{aligned}
$$

Therefore, if $\widetilde{v}=v_{0,1}$ then, (because $p>0$ ) it must be that $v_{0,0}^{2}=v_{0,1}=\widetilde{v}=v_{0,0}^{1}$, which, by Lemma 3, holds iff $p=1$, a contradiction! This completes the proof of lemma 5.

We next rule out the possibility that $\widetilde{v}<v_{0,1}$, establishing that $\widetilde{v}>v_{0,1}$.
Lemma 6: If $0<p<1$, then in equilibrium, $\widetilde{v}>v_{0,1}$.
Proof of Lemma 6: We already know by Lemma 5 that $\widetilde{v} \neq v_{0,1}$. Suppose, by contradiction, that $\widetilde{v}<v_{0,1}$. By (22) and (23), $\widetilde{v}<v_{0,1}$ iff

$$
\begin{aligned}
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1} & >\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v \Leftrightarrow \\
p\left(v_{0,0}^{2}-v_{0,1}\right) & >0 \Leftrightarrow \\
p & >0 \text { and } v_{0,1}<v_{0,0}^{2} .
\end{aligned}
$$

Because the function $g$ is increasing, this implies $g\left(v_{0,0}^{2}\right)>g\left(v_{0,1}\right)>g(\widetilde{v})$.

However, by the definition of $g$, and from (23) and (24), we have

$$
\begin{aligned}
& g\left(v_{0,0}^{2}\right)-g\left(v_{0,1}\right) \\
& =\binom{\left[\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^{2}-\int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v\right]}{-\left[\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}-\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v\right]} \\
& =\left[-\frac{(1-\gamma)}{2} \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v+(1-\gamma) \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v\right] \\
& -\left[\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}-\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v\right] \\
& =\binom{-\frac{(1-\gamma)}{2} \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\gamma \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v}{+\int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}} \\
& =\binom{-\frac{(1-\gamma)}{2} \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\gamma \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v}{+\frac{[1-\delta(1-p)]}{\delta(1-p)} \widetilde{v}-\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1}} \text {, from (22) } \\
& =-\frac{(1-\gamma)}{2} \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\gamma \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\frac{[1-\delta(1-p)]}{\delta(1-p)}\left(v_{0,1}-\widetilde{v}\right) \text {. }
\end{aligned}
$$

The last term is positive (with a negative sign in the front) because $\widetilde{v}<v_{0,1}$. We argued that $v_{0,1}>\widetilde{v}$ implies that $v_{0,0}^{2}>v_{0,1}>\widetilde{v}$, which in turn implies that:

$$
\int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v>\int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v .
$$

Also, $\gamma>-\frac{(1-\gamma)}{2}$ always because $\gamma \in(-1,1)$. Hence,

$$
\left[\left(-\frac{(1-\gamma)}{2}\right) \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v-\gamma \int_{v_{0,1}}^{\bar{v}}(v-\widetilde{v}) f(v) d v\right]<0
$$

which means that $\left[g\left(v_{0,0}^{2}\right)-g\left(v_{0,1}\right)\right]<0$. A contradiction! Therefore, it cannot be the case that $v_{0,0}^{2}>v_{0,1}$ and thus $v_{0,1}>\widetilde{v}$ is not possible. Therefore, for all $p>0$, it has to be the case that $\widetilde{v}>v_{0,1}$. This completes the proof of Lemma 6 .

To complete the proof of Proposition 3, note that $\widetilde{v}>v_{0,1}$ holds true iff $p>0$ and $v_{0,1}>v_{0,0}^{2}$. To argue this, recall that $\widetilde{v}$ is the solution to $g(\widetilde{v})$ (defined in (18)) which is an increasing function. Therefore, $\widetilde{v}>v_{0,1}$ if and only if $g\left(v_{0,1}\right)<0$ which holds true if and only if:

$$
\begin{aligned}
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,1} & <\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,1}\right) f(v) d v \Leftrightarrow \\
p\left(v_{0,0}^{2}-v_{0,1}\right) & <0 \Leftrightarrow \\
p & >0 \text { and } v_{0,1}>v_{0,0}^{2} .
\end{aligned}
$$

We know from Lemma 6 that for all $0<p<1, \widetilde{v}>v_{0,1}$, and we have just shown that $v_{0,1}>v_{0,0}^{2}$, together these facts imply that $v_{0,0}^{1}=\widetilde{v}>v_{0,0}^{2}$.

Combining with our finding in Lemma 4 for the case $p=0$, we conclude that, for all $0 \leq p<1$, we have $v_{0,0}^{1}=\widetilde{v} \geq v_{0,1}>v_{0,0}^{2}$.

Proof of Proposition 4. From the proof of Lemma 1, we know that $v_{0}$ is the solution to $g\left(v_{0}\right)=0$, where $g($.$) is an increasing function defined in (17).$

From the proof of Proposition 3, we know that $v_{0,0}^{1}=\widetilde{v}>v_{0,1}$ for all $p \in(0,1)$ and hence we can rewrite (22) as follows:

$$
\begin{aligned}
\frac{[1-\delta(1-p)]}{\delta(1-p)} v_{0,0}^{1} & =\int_{v_{0,1}}^{\bar{v}}\left(v-v_{0,0}^{1}\right) f(v) d v \\
\text { or, } g\left(v_{0,0}^{1}\right) & =-\int_{v_{0,1}}^{v_{0,0}^{1}}\left(v_{0,0}^{1}-v\right) f(v) d v<0=g\left(v_{0}\right)
\end{aligned}
$$

This implies that $v_{0,0}^{1}<v_{0}$.
Proof of Proposition 5. Note that when $p=1$, we have $w_{1}=w_{0}=0$.
Consider $p<1$. Using (13), we can re-write (14) as follows:

$$
\begin{equation*}
W_{1}=(1+\gamma) \int_{w_{1}}^{\bar{v}}\left(v-w_{1}\right) f(v) d v+\delta\left(p W_{0}+(1-p) W_{1}\right) \tag{28}
\end{equation*}
$$

Similarly, using (15), we can re-write (16) as follows:

$$
\begin{equation*}
W_{0}=(1+\gamma) \int_{w_{0}}^{\bar{v}}\left(v-w_{0}\right) f(v) d v+\delta W_{0} \tag{29}
\end{equation*}
$$

Substituting the two equations above into (15) and re-arranging, we get:

$$
\begin{align*}
(1+\gamma) w_{0} & =\delta(1-p)\left[\begin{array}{c}
(1+\gamma) \int_{w_{0}}^{\bar{v}}\left(v-w_{0}\right) f(v) d v \\
-(1+\gamma) \int_{w_{1}}^{\bar{v}}\left(v-w_{1}\right) f(v) d v+(1+\gamma) w_{0}
\end{array}\right] \\
\text { or, } \frac{[1-\delta(1-p)]}{\delta(1-p)} w_{0} & =\int_{w_{0}}^{\bar{v}}\left(v-w_{0}\right) f(v) d v-\int_{w_{1}}^{\bar{v}}\left(v-w_{1}\right) f(v) d v .
\end{align*}
$$

Again, substituting (12), (15) and (28) into (13), we get:

$$
\begin{aligned}
(1+\gamma) w_{1} & =\delta\left(p W_{0}+(1-p) W_{1}\right)-\frac{\delta}{\left[1-\delta(1-p)^{2}\right]}\left[p^{2} W_{0}+2 p(1-p) W_{1}\right] \\
& =\frac{\delta(1-p)^{2}}{\left[1-\delta(1-p)^{2}\right]}(1-\delta) W_{0}-\left[1-\frac{2 p}{\left[1-\delta(1-p)^{2}\right]}\right](1+\gamma) w_{0} \\
& =\frac{\delta(1-p)^{2}}{\left[1-\delta(1-p)^{2}\right]}(1+\gamma) \int_{w_{0}}^{\bar{v}}\left(v-w_{0}\right) f(v) d v-\left[1-\frac{2 p}{\left[1-\delta(1-p)^{2}\right]}\right](1+\gamma) w_{0}
\end{aligned}
$$

Substituting (30) and re-arranging, we get:

$$
w_{1}=\frac{\delta(1-p)^{2}}{\left[1-\delta(1-p)^{2}\right]} \int_{w_{1}}^{\bar{v}}\left(v-w_{1}\right) f(v) d v+\frac{p}{\left[1-\delta(1-p)^{2}\right]} w_{0}
$$

or,

$$
\begin{equation*}
\frac{[1-\delta(1-p)]}{\delta(1-p)} w_{1}=\int_{w_{1}}^{\bar{v}}\left(v-w_{1}\right) f(v) d v+\frac{p}{\delta(1-p)^{2}}\left(w_{0}-w_{1}\right) . \tag{31}
\end{equation*}
$$

As we have done in the previous proofs, we can re-write (30) and (31) as follows:

$$
\begin{align*}
& g\left(w_{0}\right)=-\int_{w_{1}}^{\bar{v}}\left(v-w_{1}\right) f(v) d v  \tag{32}\\
& g\left(w_{1}\right)=\frac{p}{\delta(1-p)^{2}}\left(w_{0}-w_{1}\right) \tag{33}
\end{align*}
$$

where $g($.$) is an increasing function defined as in (17).$
For all $p<1$, if $w_{0} \geq w_{1}$, then $g\left(w_{1}\right) \geq 0>g\left(w_{0}\right)$ which implies $w_{1}>w_{0}$, a contradiction! Hence, $w_{1}>w_{0}$ for all $p<1$.

Proof of Proposition 6. Note that we assume project values are drawn from the same distribution in either the two projects or the single project capacity model. From the proof of Lemma 1 , we know that $v_{0}$ is the solution to a strictly increasing function $g\left(v_{0}\right)=0$. From the proof of Proposition 5, we know that $w_{1}>w_{0}$ for all $p<1$ and by (33) in the proof of Proposition 5, we know that $0>g\left(w_{1}\right)$. It follows that:

$$
g\left(v_{0}\right)=0>g\left(w_{1}\right)>g\left(w_{0}\right) .
$$

This implies that $v_{0}>w_{1}>w_{0}$.
Proof of Proposition 7. We follow three steps to prove this proposition.
Step 1: We argue that the optimal project adoption strategy of the 2-projects capacity firm yields a value that is at least as large as the sum of equilibrium values of the two firms in the game, $W_{0} \geq 2 V_{0,0}$. The reason why this is true is that the 2-projects capacity decision maker can mimic the adoption behavior of the strategic firms by setting thresholds:

$$
w_{0}=v_{0,0}^{2} \text { and } w_{1}=v_{0,1} .
$$

Under this assumption, a project is implemented by the decision maker if and only if one of the firms would have adopted the project in the game. Under this adoption strategy, it can
be shown that:

$$
\left\{\begin{array}{c}
W_{1}=\left(V_{0,1}+V_{1,0}\right) \\
W_{0}=2 V_{0,0} \\
W_{2}=2 V_{1,1}
\end{array}\right.
$$

solves the system of equations (12), (14) and (16) that defines values for given thresholds in the 2-projects capacity problem. Because the 2-projects capacity decision maker can achieve at least $2 V_{0,0}$ in state 0 , it must be that in the optimal strategy, $W_{0} \geq 2 V_{0,0}$.

Step 2: We have shown in Proposition 3 that $v_{0,0}^{2}<v_{0,0}^{1}=\widetilde{v}$. This implies that,

$$
\int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v>\int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v
$$

and hence from (27), we have:

$$
\begin{aligned}
(1-\delta) 2 V_{0,0} & =2 \int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v-(1-\gamma) \int_{v_{0,0}^{2}}^{\bar{v}}(v-\widetilde{v}) f(v) d v \\
& >2 \int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v-(1-\gamma) \int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v \\
& =(1+\gamma) \int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v .
\end{aligned}
$$

Suppose, by contradiction, that $w_{0} \geq v_{0,0}^{2}$. Then,
$(1-\delta) 2 V_{0,0}>(1+\gamma) \int_{v_{0,0}^{2}}^{\bar{v}}\left(v-v_{0,0}^{2}\right) f(v) d v \geq(1+\gamma) \int_{w_{0}}^{\bar{v}}\left(v-w_{0}\right) f(v) d v=(1-\delta) W_{0}$, by
But we argued that $W_{0} \geq 2 V_{0,0}$, a contradiction! Hence, it must be that $w_{0}<v_{0,0}^{2}$.
Step 3: We argue that because $v_{0,0}^{2}>w_{0}$, it must follow that $v_{0,1}>w_{1}$. By contradiction, if it was the case that $v_{0,1} \leq w_{1}$, then, using the definition of $g($.$) in (17) in the proof of$

Lemma 1, we have:

$$
\begin{aligned}
g\left(v_{0,1}\right) & \leq g\left(w_{1}\right) \text { because } g(.) \text { is increasing } \\
& \Rightarrow \frac{p\left(v_{0,0}^{2}-v_{0,1}\right)}{\delta(1-p)^{2}} \leq \frac{p\left(w_{0}-w_{1}\right)}{\delta(1-p)^{2}} \text { by (23) and (33) } \\
& \Rightarrow\left(v_{0,0}^{2}-v_{0,1}\right) \leq\left(w_{0}-w_{1}\right) \\
& \Rightarrow\left(v_{0,0}^{2}-w_{0}\right) \leq\left(v_{0,1}-w_{1}\right) \leq 0 \\
& \Rightarrow v_{0,0}^{2} \leq w_{0}, \text { a contradiction! }
\end{aligned}
$$

Hence, it must be that $v_{0,1}>w_{1}$.

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[^1]:    ${ }^{1}$ See https://www.facebook.com/pages/Coca-Cola-Cafe/173169246041187?sk=info
    ${ }^{2}$ See http://www.gatorade.com/history/default.aspx
    ${ }^{3}$ See http://www.time.com/time/magazine/article/0,9171,975619,00.html

[^2]:    ${ }^{4}$ See http://www.highbeam.com/doc/1P2-4568251.html

[^3]:    ${ }^{5}$ Selectivity is also the decision variable in models of labor markets with many firms and workers, such as Pissarides (1985). Firms' and workers' selectivity affect other agents through aggregate channels, but each agent is infinitesimal and has no impact on any other agent. Also, because the rents of firm-worker matches are independent across matches, being more selective has the same effect on other agents as searching less intensively: it affects the hazard rate at which opportunities arrive, not the distribution of their flow value conditional on arriving. In contrast, in our model, if one firm lowers its adoption threshold in some period (it is less selective), the distribution faced by the other firm in the same period is modified.
    ${ }^{6}$ Taylor (1995) examines the planner's optimal design problem of a search tournament: the planner chooses the number of competing firms, the entry fee and the prize. The firms then engage in simultaneous search as in Reinganum $(1982,1983$ a). The firm that finds the best innovation receives the prize.

[^4]:    ${ }^{7}$ The assumption that the prize is obtained immediately simplifies the exposition, but the analysis will be similar if the prize is obtained at the end of the commitment period.

[^5]:    ${ }^{8}$ Equivalently, for any two cumulative distributions $F^{0}$ and $F^{1}$ such that any weakly increasing and convex utility function $u$ over returns we have $\int u(v) d F^{0} \leq \int u(v) d F^{1}$, the threshold level is higher under $F^{1}$ than under $F^{0}$.

[^6]:    ${ }^{9}$ Interestingly, the inequality $w_{1}>w_{0}$ remains true in the limit where firms are infinitely patient $(\delta \rightarrow 1)$, when the firm is assumed to maximizes the expected payoff per unit of time. The firm prefers to spend more time in states 0 and 1 than in state 2 (where it is committed and cannot accept projects). Setting $w_{1}>w_{0}$ is a way to steer the system back towards state 0 , as it gets closer to state 2 .

