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Abstract

This dissertation consists of three essays in financial macroeconomics. The methodological approach common to the first two articles is the application of the search and matching theory to financial markets. The third essay builds on the literature on rare events.

In the first article, I develop a tractable two-country model in which financial contagion may arise despite a flexible exchange rate regime and substitutability between the home and foreign financial assets, contrary to the open-economy standard results under these two conditions. On the one hand, pure liquidity contractions imply negative co-movements of home and foreign outputs, in line with the literature. On the other hand, I find that non-walrasian shocks to banks' funding costs do generate the contagion.

The second essay analyzes the role of bankers' behavior in bank default. The model accounts for heterogeneity in entrepreneurs' productivity and information asymmetry at the expense of capital holders. Moral hazard arises following a productivity shock: bankers tend to choose investments that are more profitable in the short-run but whose risk is borne by the capital holders. I show that this 'risk-shifting' mechanism magnifies credit rationing in the economy and contributes to bank default since financial investors may prefer not to (re-)capitalize intermediaries as long as they cannot control for bankers' choices.

The third article examines the impact of a change in the probability of rare events on business cycles. A small and time-varying 'disaster' risk is incorporated in an otherwise standard New Keynesian model. A rise in the probability of disaster is sufficient to generate a recession without effective occurrence of the disaster. By accounting for monopolistic competition and price stickiness, the responses of consumption and wages are also reminiscent of distressed times. The article thus provides a framework of the dynamic effects of rare events which is particularly suitable for further policy analysis.

Résumé

Cette thèse comprend trois articles en macroéconomie financière. L'approche méthodologique commune aux deux premiers essais est l'application des modèles d'appariement aux marchés financiers. Le troisième article contribue à la littérature sur l'impact macroéconomique des événements rares.

Le premier article démontre qu'une contagion financière internationale est susceptible d'émerger malgré un régime de taux de change flexible et une substituabilité entre les actifs financiers nationaux et étrangers, contrairement aux résultats standards sous ces deux conditions. D'une part, des contractions pures de liquidité génèrent des co-mouvements négatifs entre les niveaux de production national et étranger, conformément à la littérature. D'autre part, des chocs non-walrasiens de coûts de capitalisation des banques sont à l'origine de la contagion financière internationale.

Le deuxième article montre que le comportement des banquiers affecte le défaut bancaire. Le modèle tient compte de l'hétérogénéité des emprunteurs et incorpore une asymétrie d'information au détriment des détenteurs de capitaux. Un aléa moral survient à la suite d'un choc de productivité : les banquiers tendent à choisir les investissements plus rentables à court terme mais dont le risque est supporté par les investisseurs. Ce mécanisme amplifie le rationnement du crédit dans l'économie et alimente le défaut bancaire dans la mesure où les investisseurs peuvent préférer ne pas (re-)capitaliser les intermédiaires tant qu'ils ne contrôlent pas leurs choix.

Le troisième article étudie l'impact macroéconomique d'une variation de la probabilité d'un événement rare. Un risque de "désastre" économique, faible mais variable au cours du temps, est introduit dans un modèle néo-keynésien par ailleurs standard. Une hausse de la probabilité suffit notamment à générer une récession sans réelle occurrence du désastre. En concurrence monopolistique et avec viscosité des prix, les réactions de la consommation et des salaires sont elles aussi cohérentes avec la récession économique. Nous proposons ainsi un cadre d'analyse des effets dynamiques des événements rares, préalable à l'investigation du rôle de la politique monétaire.

Contents

Introduction

Some non-standard macroeonomic theories, emerged in the late 1980s and in the 1990s, have gained more and more attention in the last few years as the worldwide 2007-2009 financial crisis has reinforced the need for a renewal of the economic models. Among these not so commonly used approaches are the non-walrasian analysis of financial markets, on the one hand, and the study of rare events that may affect both decision-making and aggregate outcomes, on the other hand. This dissertation builds on these two approaches and attempts to show that they provide convincing explanations to some macroeconomic phenomena, including some aspects of the recent events.

Although perfectly efficient in the main dynamic stochastic general equilibrium models, financial markets are potentially frictional and may prevent solvent economic agents with a viable project to borrow or to raise funds. In addition, these frictions may be non-walrasian if price adjustments do not allow instantaneous market-clearing following a particular shock, so that traded quantities dramatically fall. Such a financial "freeze" may stem either from mistrust due to heterogeneity among market participants and information asymmetry between them, and/or from uncertainty about aggregate economic conditions. Moreover, the size of the agents, their interconnections and the lack of anonymity also question the assumption of perfectly competitive financial markets.

The search and matching theory, also known as the Diamond-Mortensen-Pissarides framework[1] and traditionally used in the labor market literature, can provide an insightful representation of these financial market frictions. In a large sense, financial applications of the search and matching theory include the modeling of money as a medium of exchange (Kiyotaki and Wright, 1993), the credit market (Dell'Ariccia and Garibaldi, 1998), and the inter-

[1] Standing for Diamond (1982), Mortensen and Pissarides (1994), and Pissarides (2000).

bank market (Afonso and Lagos, 2012). The models account for the fact that it takes time and effort for the economic agents to find a suitable trade opportunity, because the market is decentralized and affected by the individual characteristics mentioned above. Wasmer and Weil (2004) further find that the co-existence of credit and labor market frictions creates a financial accelerator that magnifies the effects of shocks to search costs or to profits.

In line with this literature, two of the three chapters of this dissertation develop tractable search and matching models in order to reveal some interesting macroeconomic channels, reminiscent of the last financial crisis but hardly explained by the standard literature. In particular, one chapter solves an international financial contagion puzzle under flexible exchange rates while the other analyzes the role of bankers' behavior in bank default.

Another promising approach is the literature on rare events which provides key feedback effects from finance to macroeconomic business cycles. The early paper by Rietz (1988) shows in an endowment economy that large but unlikely market crashes may solve the equity premium puzzle and the riskfree rate puzzle. In turn, Gabaix (2011, 2012) and Gourio (2012) have put forward the business cycle implications of a small time-varying risk of a large economic 'disaster' *à la* Barro (2006), and reproduce the countercyclicality of the risk premia. In particular, they show that an increase in the probability of disaster, without effective occurrence of the disaster, leads investment and output to fall as capital becomes riskier, while precautionary savings lower the yield on risk-free assets. Therefore, this literature provides a rationale for the rise in spreads in distressed times. Yet few policy implications can be derived from these models. In order to do so, the last chapter of this dissertation incorporates disaster risk into a New Keynesian model. It thus provides a framework for further analysis of monetary policy facing — realized or potential — rare events.

As this dissertation addresses distinct issues in financial macroeconomics, the three chapters are presented as self-contained articles so that they can be read separately.

Chapter 1 is entitled '***International propagation of financial shocks in a search and matching environment***'. This article shows, in a two-country multi-frictional yet tractable model, that financial contagion may arise despite a flexible exchange rate regime and financial substitutability, contrary to the open-economy literature results under these two conditions.

Motivated by the 2008-2009 contagion from the United States to the Eurozone, this chapter attempts to shed some light on the limitations of the open-economy macroeconomics literature. As far as large and interdependent economies are concerned, the literature states that a monetary shock generates positive output co-movements across countries if and only if the exchange rate system is fixed or if there is complementarity between home and foreign financial assets under a floating exchange rate system. Yet the Eurozone is in neither case vis-à-vis the US. In particular financial market completeness ensures that the Eurozone distribution of financial assets is rather substitutable than complementary to the US counterpart. Solving the contagion puzzle might thus require to explore financial shocks of a different nature.

Whereas the mainstream literature hardly lets room for alternative kinds of asymmetric financial shocks, I argue here that moving away from the walrasian environment using the search and matching theory provides a tractable way to do so. First, if the economic agents have to spend time and/or effort to find a suitable trade opportunity, some non-walrasian shocks may arrive in the sense that no price adjustment mechanism ensures immediate market-clearing. In this setting, financial markets may thus be "frozen" for a significant period of time, that is, a dramatic fall in traded quantities is suddenly observed. Second, these shocks may arrive despite excess liquidity in the economy (or in the world), contrary to pure monetary contractions. Thus the decrease in trade does not stem from the rarefaction of liquidity — or capital in a wider sense — but from the rarefaction of suitable borrowers or lenders/investors. Last but not least, the search and matching theory offers an elegant framework, in which the transmission channels are easily identifiable through analytical results and very intuitive comparative statics.

In the two-country model developed in this paper, I first show that monetary contractions can be nested as a particular case and generate the negative output co-movements of the literature. Meanwhile I find that the framework lets room for another type of financial shocks, namely a rise in banks' capitalization costs in one country despite excess liquidity, that generates positive output co-movements, *i.e.* international contagion.

Chapter 2, '***A search model of bankers' greediness and bank de-***

fault' examines the role of bankers' behavior in credit rationing and bank default. In particular, I show that bank default is affected by a sectoral productivity shock if there is a combination of uncertainty and information asymmetry at the expense of capital holders in the economy. This is because bankers tend to choose investments that are more profitable in the short run but whose risk is borne by the financiers. I also find that this mechanism is responsible for a rise in credit rationing in the entire economy following the sector-specific shock even though there is no information asymmetry between bankers and entrepreneurs and even when bank default is unaffected.

Although the banking literature is flourishing, the determinants of bank default are rarely identified. Numerous macro-finance papers have introduced the banking sector in New Keynesian models but consider either that the intermediaries cannot default, or that the default rate depends on exogenous cheating from bankers. Similarly, the literature on systemic risk due to banks' interdependence generally starts from a first exogenous bank default. Moreover, the banking sector is often treated as a competitive sector while some empirical papers show that the size of banks, their interconnections, and limited anonymity create frictions on both the interbank markets and the credit markets. In this chapter, I try to give a rationale for bank default stemming from bankers' 'greediness', which significantly differs from cheating but rather results from the incentives that bankers face in a frictional economy, where information is asymmetric and productivity uncertain.

The model developed in this chapter introduces heterogeneity in entrepreneurs' productivity and information asymmetry at the expense of capital holders. Moral hazard arises following a sectoral productivity shock as bankers tend to choose investments that are more profitable in the short-run but whose risk is borne by the financiers. In the search and matching environment, bankers face a trade-off. They can invest in sectors with low productivity but high credit rationing, *i.e.* sectors in which the duration before a conclusive match is short for banks, or in high-productivity sectors where suitable unmatched entrepreneurs are relatively rare. While a no-arbitrage condition makes the two options equivalent in the long run, the first one is more profitable for the bankers in the short run.

However this type of credit relationships is also riskier, as characterized by a higher (endogenous) destruction rate. Assuming that the capital holders cannot observe entrepreneurs' characteristics or control for bankers' choices but pay for unproductive credit lines, they would have preferred that bankers invest in long-duration credit relationships. Bankers' 'risk-shifting' incentives thus have two major macroeconomic consequences. First, credit rationing, defined as the number of entrepreneurs who are — effectively but unsuccessfully — looking for a loan, is magnified in the entire economy. Second, the bank default rate rises since investors may prefer not to (re-)capitalize intermediaries as long as they do not control for bankers' choices.

Finally, Chapter 3, '***Disaster risk in a New Keynesian model***', analyzes the macroeconomic impact of a small and time-varying disaster risk in a dynamic model that is particularly appropriate for future analysis of monetary policy in response to a change in such a risk.

In the literature, the presence of rare events, sometimes called economic disasters, creates a feedback effect from asset prices to macroeconomic quantities. In particular, a rise in the probability of disaster — associated with the destruction of a fraction of capital — suffices to generate a recession without effective occurrence of the disaster. The intuition is the following. As capital becomes riskier, investment and output fall, precautionary savings lower the yield on risk-free assets, so that the spread rises when the disaster risk increases. In turn, the countercyclicality of risk premia magnifies the recession. However, since the literature on rare events is limited to endowment economies or real business cycle models so far, some modeling assumptions are quite restrictive and policy implications can hardly be derived.

In this chapter, a small and and time-varying probability of disaster is introduced in an otherwise standard New Keynesian model in order to track the changes in the responses of macroeconomic quantities that this environment may generate. The model accounts for monopolistic competition, investment adjustment costs, a non-zero steady-state inflation rate, a Taylor-type monetary policy rule, and time- or state-dependent price stickiness.

As a first result, the model is able to reproduce the recession in response to a rise in disaster risk in a full price flexibility benchmark while the New Keynesian features allow relaxing Gourio (2012)'s restriction on the variation of total factor productivity. Moreover, the fall in output is magnified under either time- or state-dependent price stickiness as compared to the flexible-price benchmark. Second, consumption and wages decrease on impact, contrary to the flexible-price case (and the literature) but probably more consistently with the empirical patterns in distressed times. The model thus improves the ability to match the expected responses of macroeconomic quantities to a change in the probability of rare events and provides a particularly suitable environment for further policy analysis.

Bibliography

[1] Gara Afonso and Ricardo Lagos. Trade dynamics in the market for federal funds. Unpublished manuscript, available at `http://sites.google.com/site/rl561a2/workinprogress`, 2012a.

[2] Robert J. Barro. Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics*, 2006.

[3] Giovanni Dell'Ariccia and Pietro Garibaldi. Bank lending and interest rate changes in a dynamic matching model. IMF Working Papers 98/93, International Monetary Fund, June 1998.

[4] Peter A Diamond. Aggregate demand management in search equilibrium. *Journal of Political Economy*, 90(5):881–94, October 1982.

[5] Xavier Gabaix. Disasterization: A simple way to fix the asset pricing properties of macroeconomic models. *American Economic Review*, 101(3):406–09, May 2011.

[6] Xavier Gabaix. Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. *The Quarterly Journal of Economics*, 127(2):645–700, 2012.

[7] François Gourio. Disaster risk and business cycles. *American Economic Review, forthcoming*, 2012.

[8] Nobuhiro Kiyotaki and Randall Wright. A search-theoretic approach to monetary economics. *American Economic Review*, 83(1):63–77, March 1993.

[9] Dale T Mortensen and Christopher A Pissarides. Job creation and job destruction in the theory of unemployment. *Review of Economic Studies*, 61(3):397–415, July 1994.

[10] Christopher A. Pissarides. *Equilibrium Unemployment Theory, 2nd Edition*, volume 1 of *MIT Press Books*. The MIT Press, 2000.

[11] Thomas A. Rietz. The equity risk premium: a solution. *Journal of Monetary Economics*, 22(1):117–131, July 1988.

[12] Etienne Wasmer and Philippe Weil. The macroeconomics of labor and credit market imperfections. *American Economic Review*, 94(4):944–963, September 2004.

Chapter 1

International Propagation of Financial Shocks in a Search and Matching Environment

Abstract

This paper develops a two-country multi-frictional model in which financial contagion arises despite a flexible exchange rate regime and substitutable financial assets, contrary to the open-economy literature results under these two conditions. The search and matching approach accounts for the time needed to restore normal functioning of financial markets following a disruption and allows dissociating two types of financial shocks: (*i*) pure liquidity contractions imply negative co-movements of home and foreign outputs, so that the model nests the standard results as a particular case; (*ii*) non-walrasian shocks to banks' funding costs in one country do generate international financial contagion.

JEL classification: C78, E44, E51, F41-42, G01, G15
Keywords: matching theory, market frictions, credit rationing, financial multiplier, international contagion, financial crises, open economy macroeconomics

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1 Introduction

The Lehman Brothers collapse in September 2008 precipitated an economic recession throughout the developed world although originated by a downturn in specific US financial markets. In particular, most of the Eurozone countries experienced severe GDP falls and unemployment rises despite their flexible exchange rate regime with the dollar and a solvent internal demand that could have made them less vulnerable to external shocks than export-led emerging market economies.

While it is widely agreed that financial shocks transmit across countries within a fixed exchange rate regime or within a monetary union because tying the hands of monetary policy, there is much less theory about similar interactions between major economic areas in a floating exchange rate context. Even more surprisingly, traditional open-economy frameworks such as Obstfeld and Rogoff (1995) generally consider that negative monetary shocks would benefit other countries, since the relative appreciation of the domestic currency resulting from an interest rate rise boosts the exports of these foreign countries. In particular, total demand for foreign goods increases because expenditure-switching effects worldwide may outweigh the negative wealth effect in the first country.

However, one may think that this reasoning could not necessarily hold in a more sophisticated setting where opposite forces drive the exchange rate dynamics implied by the shock and where financial frictions may result in a more than proportionate impact on real activity. This leads to investigate a potential *international finance multiplier* phenomenon, according to the term employed by Krugman (2008). Indeed, the recent events suggest the need for a general equilibrium model that can account both for a financial multiplier that magnifies the impact of a credit crunch on domestic real activity and for interaction mechanisms that result in a deteriorated situation in all countries, especially those in which the disturbance did not originate.

This question has been explored in models that rely on the incompleteness of financial markets in the countries to which crises are transmitted (since Allen and Gale, 2000, notably). More generally, contemporaneous open macro models with financial contagion within a floating exchange rate system assume *complementarity* between domestic and foreign financial assets. The simple intuition is that an unanticipated decline in domestic asset value damages the balance sheets of foreign investors because the latter had previously diversified their portfolio by acquiring domestic financial assets (Krugman, 2008). This is probably accurate when explaining the transmission of financial crises to emerging market economies (Dooley and Hutchison, 2009, Jotikasthira, Lundblad, and Ramadorai, 2009) which acquire US assets to alter the maturity and risk distribution of their holdings. However, no evidence supports this assumption for financial interactions between developed countries. On the contrary, it is more likely that the Eurozone distribution of financial assets is *substitutable* to the US distribution, as pointed out by Dedola and Lombardo (2012) for instance. In this case, the literature puzzle in the floating exchange rate regime remains.

This paper thus develops a two-country multi-frictional model where financial contagion across countries arises contrary to the standard open macro results when there is both a flexible exchange rate regime and substitutable financial assets. This is permitted with the occurrence of a new type of financial shocks freezing the liquidity access to commercial banks in one country. This shock creates positive output co-movements through credit rationing to firms in both countries. Assuming that credit markets are important to firms, a sudden deterioration in loan opportunities constrains their labor market participation, therefore leading to adverse employment outcomes rather than an increase in the cost of capital. Hence, the exchange rate does not appreciate as much as in the case of a monetary shock or even depreciates, in turn annihilating the traditional price competitiveness channel mentioned above. Meanwhile, the substitutability of home and foreign financial assets helps propagate the crisis by equalizing external finance conditions worldwide.

The search and matching theory is used to depict frictional credit markets in the spirit of Wasmer and Weil (2004). Its application to global financial markets further captures two important aspects here. First, this environment accounts for the fact that, after sudden financial market disruptions, restoring a normal level of confidence takes time, therefore preventing immediate market clearing by international arbitrage from financial investors. Second, it allows to nest standard monetary contractions as a particular case while giving room for non-walrasian financial shocks that arrive despite global excess savings and may reflect mistrust from financial investors about the solvency of the banking system. It turns out that the former do not propagate within the flexible exchange rate system, in accordance with the literature, whereas the latter create a recession in both countries as a reduced demand and credit rationing cause unemployment to rise while all nominal advantages generally attributed to the foreign depreciation vanish.

The paper therefore proposes a new type of asymmetrical financial shocks transmitting real effects abroad, creating an international finance multiplier, and consequently fueling a global recession. This result is absent from the current literature on contagion as far as large developed economies within a floating exchange rate system are concerned, and yet highly reminiscent of the 2008-2009 crisis. The remainder is as follows. Section 1 gives some intuition for the results. Section 2 develops the setup of the model and the equilibrium for the closed-economy. Section 3 explores the international spillovers of country-specific financial shocks, particularly the impact of a rise in bank capitalisation costs *versus* the impact of liquidity supply shocks. Section 4 discusses the contribution, and Section 5 concludes.

2 The gist of the paper

The first contribution of the paper is to develop a theoretical environment in which financial shocks may generate positive output co-movements between two interdependent economic areas despite the presence of both a flexible exchange rate regime and substitutable home and foreign assets. A second contribution lays on practical grounds as a very tractable two-country framework is provided in order to generate simple analytical results. Let us present here some of its comparative statics, as intuitive as if derived from the traditional IS-LM-BP model, while the underlying micro-foundations given by the search theory will be considered in the next Section.

Let consider the equilibrium depicted in Figure 1. On the y-axis, ξ stands for a measure of the ratio between the demand and the supply of liquidity, replacing the interest rate that would stand in a IS-LM-BP framework to let room for a friction appearing in distressed times. On the x-axis, θ measures real activity, to be defined later on. The upward-sloping II curve denotes the equilibrium condition of liquidity holders, while the downward-sloping BE curve brings together entrepreneurs' and lending banks' equilibrium conditions. The BP line is the traditional balance of payments line, and is horizontal as savings are perfectly mobile across developed countries. Point A therefore stands for the initial equilibrium, from which are now successively studied the effects of two types of financial shocks.

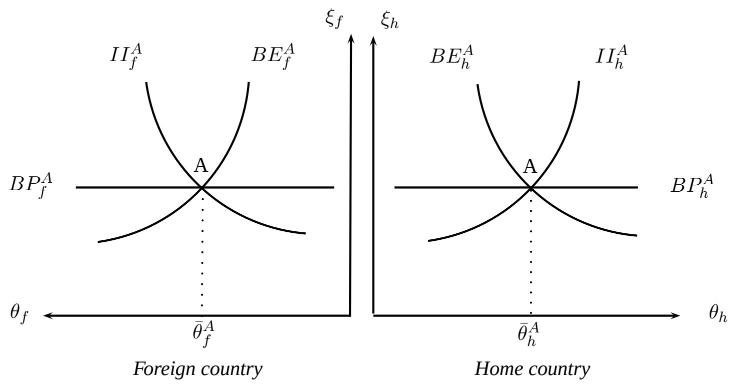

Figure 1: World initial equilibrium

Figure 2 represents the impact of a negative *liquidity supply shock*, and roughly captures the effects of a monetary contraction in the new open macroeconomics literature (see Discussion). A liquidity supply contraction in the Home country first displaces the liquidity holders' equilibrium condition II_h^A leftwards, from point A to point B at home. This point is associated with a large relative appreciation of the domestic currency which further damages the price-competitiveness and thus displaces the BE curve up to point C in the home country. Meanwhile, the symmetric foreign depreciation boosts the current account abroad. Second-round effects finally lead the economy to point D, where home and foreign financial market tightnesses are equalized again, at a higher level. Hence, this predicts that a monetary contraction at home expands activity in large interdependent countries, in strike opposition with positive output co-movements between the US and the Eurozone during the last financial crisis for instance. This result is consistent with the literature that introduced credit in the basic IS curve to create a financial multiplier magnifying the response of real activity. A shock to bank reserves as described for a closed economy by Bernanke and Blinder (1988) would generate the same result in an open-economy framework: while a simultaneous shift of the BE curve leftwards would initially moderate the effect, the qualitative spillover across countries is unchanged, that is, output co-movements would be negative.

Let now consider in Figure 3 another type of financial shocks which displaces the BE curve alone as a first-order effect — as to be proved later on.

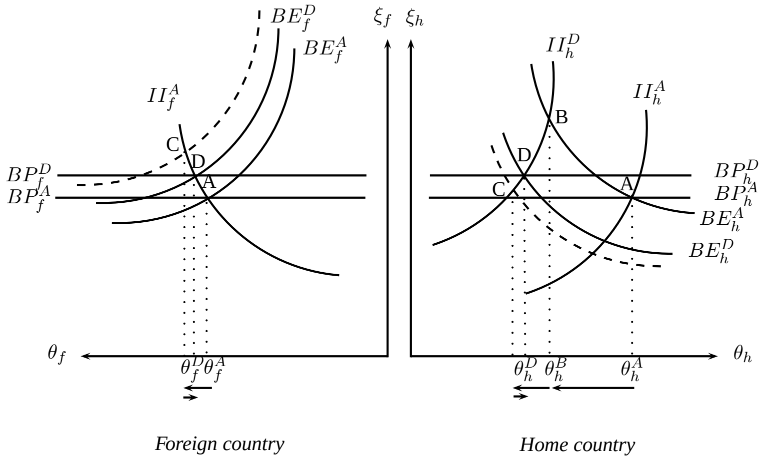

Figure 2: World effect of a (negative) domestic liquidity supply shock

Then the domestic currency depreciates and some additional export opportunities are given to productive entrepreneurs, taking back the BE curve from point B to point D at home. Meanwhile, the relative appreciation of the foreign currency combined with the domestic recession narrows foreign entrepreneurs' export opportunities. In point D, capital access is easier by the financial integration channel, yet a recession hits both countries.

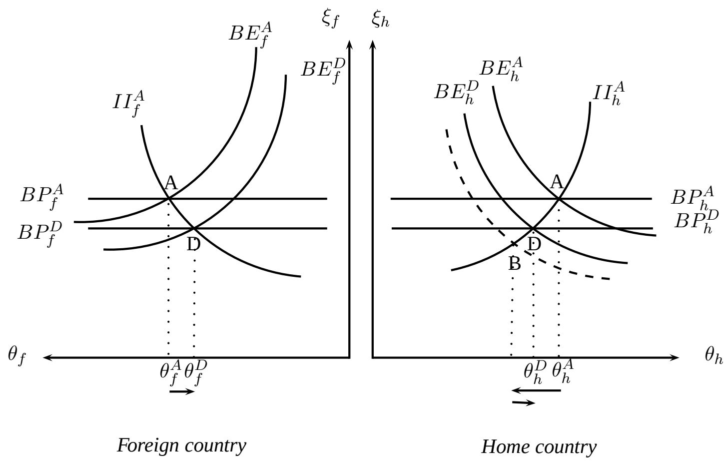

Figure 3: World effect of a domestic rise in bank capitalisation cost

The objective of the paper is thus to find the conditions under which such a shock does not depict an exogenous displacement of an 'IS-like' curve but captures a disturbance of financial nature. More precisely, both shocks should affect the relationship between liquidity holders and liquiditydemanding lending banks. The intuition is that the first shock type affects a liquidity injection parameter at home, everything else equals, while the second shock type is a shock that make financial relationships frictional despite the presence of excess liquidity in the economy, potentially due to a mistrust about the creditworthiness of the banking system.

3 Setup and equilibrium for the closed-economy

3.1 A sequential search and matching process

Each of the two economies produces one output and is composed of four types of infinitely-lived risk-neutral agents: financial investors, commercial banks, entrepreneurs, and workers. These agents interact in three (potentially) frictional markets: the financial market, the credit market and the labor market. It is assumed that a sequential multi-bargaining process takes place with the timing of events given in Figure 4.

In stage 0, commercial banks look for investors in the financial market in order to raise funds before lending to entrepreneurs. Financiers are endowed with capital but are assumed to have not the competence to generate long-term investment opportunities.[1] On the other hand, I assume that commercial banks are in need of capital at this stage, which may correspond to periods in which banks have already steep leverage ratios and hence have to find buyers to their outstanding debt before conceding new loans. One should note that these matches can however been made at infinitely high rates if investment and commercial banking activities are integrated.

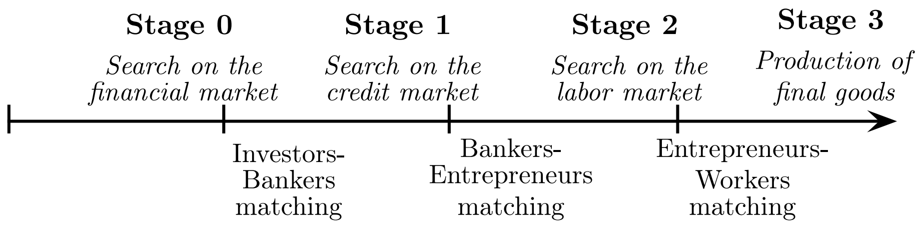

Figure 4: Timing of Events

In stage 1, bankers look for a profitable loan opportunity among en-

[1]This category can encompass very different actors such as investment banks, hedge funds, foreign sovereign funds, but also the Central Bank via quantitative easing.

trepreneurs who seek a credit to create a firm. Wasmer and Weil (2004) constructed a model in which such a credit market stage precedes a labor market stage, leading to a situation where frictions on both markets reinforce one another. Following them, I assume that entrepreneurs have no proper wealth *ex ante* and must find a credit before entering the worker recruitment stage. Acemoglu (2001) indeed argues that credit market frictions significantly constrain job creation for new firms, especially in Europe. Moreover, credit dependence of firms may be particularly relevant when a deep financial shock prevents even large firms from issuing equities as a perfectly substitutable fund-raising means.[2] In stage 2, entrepreneurs and workers look for each other in a usual search and matching approach. Finally, in stage 3, the newly created firm produces and pays back the banker who in turn pays for the former investor's services.

Considering capital transactions (stage 0) as the beginning of the sequential process allows representing limited access to liquidity for financial intermediaries.[3] A recent empirical contribution by Hale and Santos (2010) shows, first, that even US banks that do not rely on the bond market to fund their activities (but exclusively on deposits) have become exposed to the conditions in the bond market, and second, that banks pass debt market shocks to all their borrowers, whether these borrowers have themselves access to the bond market or not. This suggests that a rarefaction of liquidity holders willing to finance long-term projects may have harmful consequences for real activity through credit rationing, even in a world where all agents do not depend on external finance to fund their business and where banks can discriminate the creditworthiness of entrepreneurs addressing them for a loan. The present paper studies real effects induced by a shock freezing these financial relationships, with a significant rise of external capitalization cost to banks stemming from the inability of potential investors to assess their solvency. If the process would have started at the so-called stage 1, simulating a shock to an exogenous liquidity access of commercial banks would be of the same kind than a traditional monetary restriction by the Central

[2]Campello, Graham and Harvey (2010) found that 86% of constrained US firms declared having restrained, canceled or postponed planned investment in attractive projects during the crisis of 2008, and so did almost half of unconstrained firms.

[3]As funds only come from financial operations in this model, we can equally call them *liquidities* from the standpoint of the banker; however modeling the investor-banker relationship make them closer to long-term assets on a frictional financial market here.

Bank and lead to the counterintuitive results mentioned earlier.

In the spirit of Mortensen and Pissarides (1994), the labor market is characterized by a finite number N_U of unemployed workers looking for a job and a finite number N_V of open job vacancies. A constant returns-to-scale matching function $m_L(N_U, N_V)$ determines the flow of new firms, *i.e.* new contracts between one entrepreneur and one worker. The ratio of vacancies to unemployed workers defines the *labor market tightness*, $\theta \equiv N_V/N_U$, from which are inferred the instantaneous probabilities $q_L(\theta) = m_L(N_U, N_V)/N_V$ for an entrepreneur to find a suitable worker and $\theta q_L(\theta) = m_L(N_U, N_V)/N_U$ for a worker to find a job.

A recent literature depicts the credit market similarly, arguing that information asymmetry between borrowers and lenders makes the creation of new credit relationships time- and effort-consuming. As a consequence, some bankers are screening for credit applications while a pool of unmatched entrepreneurs is waiting for a loan in each period. At a macro level, this corresponds to a situation of credit rationing *à la* Stiglitz and Weiss (1981). Dell'Ariccia and Garibaldi (2005) and Craig and Haubrich (2006) have provided empirical evidence of a departure of gross from net credit flows persisting over time, that is, coexistent credit creation and credit destruction flows. Therefore, I consider that the ratio of N_E entrepreneurs looking for a bank to N_C bankers seeking a desirable loan opportunity measures the *credit market tightness*, denoted ϕ. A constant returns-to-scale technology, $m_C(N_E, N_C)$, increasing in both arguments, then determines the instantaneous probabilities $q_C(\phi) = m_C(N_E, N_C)/N_E$ for an entrepreneur to obtain a loan and $\phi q_C(\phi) = m_C(N_E, N_C)/N_C$ for a banker to find a project.

Finally, I construct financial market relationships similarly here. This does not preclude the case where financial markets are perfectly efficient but simultaneously allows for the representation of non-walrasian shocks, *i.e.* shocks freezing financial activities by creating frictions that cannot be immediately cleared by capital return adjustments. Moreover, this modeling accounts for the time that elapses between the shock arrival and the restoration of a normal functioning of financial markets, given the distribution of participants' creditworthiness. Therefore, with N_I financiers seeking investment opportunities and N_B commercial banks looking for capital, the ratio $\xi \equiv N_B/N_I$ defines a comparable measure of *financial market tightness* in each period. Total flows are then derived from a similar matching function $m_F(N_I, N_B)$, which specifies the Poisson time-varying arrival rates $q_F(\xi) = m_F(N_I, N_B)/N_B$ at which a banker raises funds on the financial market and $\xi q_F(\xi) = m_F(N_I, N_B)/N_I$ at which an investor meets a suitable banker. In each market, the matching rate decreases in the tightness on the demand side ($\partial q_L(\theta)/\partial\theta, \partial q_C(\phi)/\partial\phi$, and $\partial q_F(\xi)/\partial\xi < 0$), while the reverse holds on the supply side ($\partial\theta q_L(\theta)/\partial\theta, \partial\phi q_C(\phi)/\partial\phi$, and $\partial\xi q_F(\xi)/\partial\xi > 0$).

3.2 Individual behaviors

Hereafter are the equilibrium conditions for entrepreneurs, bankers, and financial investors in such a sequential process, given their proper constraints and search activities; proofs are given in Appendix. The setup is such that it is possible to abstract from the equilibrium condition of workers in the labor market without loss of generality, since I assume that unemployed workers encountering a vacancy accept the job as long as the wage is sufficiently high to allow for consumption, given the goods prices, the discount rate and the separation rate, *i.e.* when $(w/P)/(r+s) > 0$. The intuition lies in the fact that during financial crises the prime concern is involuntary — associated with credit constraints — rather than frictional unemployment.[4]

Entrepreneurs

The entrepreneurs enter in stage 1 since they have productive ideas but not the necessary wealth to start recruitment on their own and thus rely on the credit market to open a job vacancy. Searching for a loan involves a non-pecuniary flow cost (effort cost), denoted c_E. Once matched with a banker with probability $q_C(\phi)$, an entrepreneur starts looking for a suitable worker in stage 2. This stage implies a recruitment flow cost γ_L however not directly borne by entrepreneurs but by bankers once they have conceded the credit line to entrepreneurs in the previous stage. In stage 3, the one entrepreneur-one worker constituted firm produces one unit of output, sold at price p which is taken as given by the individual firm. After paying for the worker's wage w, a fraction ρ_C of net profit is paid back to the banker. I assume that the wage rate is exogenous without loss of generality here, whereas the

[4] Michaillat (2012) recently found that unemployment is mostly explained by frictions on the labor market when the rate is near to 5% in the US but that this frictional part falls to less than 2% when the rate goes to 9%, so that the cyclical component is the quasi-exclusive source of unemployment in bad times.

determination of the repayment rate is discussed further below.

Entrepreneurs optimally decide whether to start looking for a loan in order to launch a business if present-discounted expected cash flows in stage 3 exceed present-discounted expected costs while searching for the loan. Free entry further implies strict equality in equilibrium (zero-profit condition). Given the riskfree rate r and the conditional probabilities of transition from one stage to the other, c_E is discounted by $\frac{1}{r+q_C(\phi)}$ in stage 1 while $(p-w-\rho_C)$ is discounted by $\frac{q_C(\phi)}{r+q_C(\phi)}\frac{q_L(\theta)}{r+q_L(\theta)}\frac{1}{r+s}$ in stage 3 (see Appendix for computational details). It thus gives an *equilibrium condition* for entrepreneurs as

$$\frac{c_E}{q_C(\phi)} = \frac{q_L(\theta)}{r+q_L(\theta)}\frac{p-w-\rho_C}{r+s} \tag{1}$$

with the periodic search cost times the average duration of the credit search stage $1/q_C(\phi)$ on the left hand side, expected net profits given the riskfree rate r, the firm separation rate s, and the average duration of the recruitment stage (as a function of the labor market tightness θ) on the right hand side.

Bankers

Commercial banks enter the process one stage earlier since they have to raise funds before lending to entrepreneurs. Let denote c_B the periodic non-pecuniary search cost borne in stage 0 while looking for an investor with excess savings. It may stand for the effort made by the bank to gather proofs of its solvency to this investor for instance. If the match is agreed, the investor provides the banker with the required capital while searching for a suitable entrepreneur and the amount that will be lent to this entrepreneur. During the production stage 3, bankers receive ρ_C from entrepreneurs, from which is extracted an instantaneous payout to the investor, at (endogenous) rate ρ_F. Hence a similar equilibrium condition from bankers' free entry is

$$\frac{c_B}{q_F(\xi)} = \frac{\phi q_C(\phi)}{r+\phi q_C(\phi)}\frac{q_L(\theta)}{r+q_L(\theta)}\frac{\rho_C-\rho_F}{r+s} \tag{2}$$

On the left hand side, expected costs of raising funds for bankers depend on the financial market tightness ξ which gives the expected duration of stage 0. On the right hand side, bankers' output share $(\rho_C - \rho_F)$ is discounted by the respective duration of the credit and labor search stages.

Financial investors

Similarly, investors pay a non pecuniary search cost c_I per period in stage 0, while searching for a banker they consider able to turn their idle savings into a profitable long term investment opportunity. Note that when the financial market is such that the instantaneous matching rates of investors and bankers are infinitely high, this parameter simply drives the liquidity injection in the economy. In stages 1 and 2, the investors bear the flow costs γ_C and γ_L while banks are screening credit applications and while entrepreneurs are recruiting respectively. In the production stage 3, they earn ρ_F each period. Finally, they return to stage 0 if a separation comes up on the labor market. Investors' equilibrium condition is thus

$$\frac{c_I}{\xi q_F(\xi)} = \frac{-\gamma_C}{r + \phi q_C(\phi)} + \frac{\phi q_C(\phi)}{r + \phi q_C(\phi)} \left\{ \frac{-\gamma_L}{r + q_L(\theta)} + \frac{q_L(\theta)}{r + q_L(\theta)} \frac{\rho_F}{r + s} \right\} \quad (3)$$

Forward-looking investors' willingness to provide capital depends on the costs induced by search activities (γ_C and γ_L times the respective expected search durations) and the present-discounted output share ρ_F in stage 3.

3.3 Surplus sharing

The two successive rates ρ_C and ρ_F that share the surplus between entrepreneurs, bankers, and investors, are determined by Nash bargaining rules *à la* Pissarides (2000). This accounts for the fact that potential entrepreneurs need to personally interact with a bank agency in order to bargain the rate of return on capital that will be paid once the production starts in the absence of collateral wealth.

The loan repayment rate ρ_C from the firm to the banker thus maximizes the value of the match between the banker and the entrepreneur

$$\rho_C = \arg\max (B_2 - B_1)^{\delta_C} (E_2 - E_1)^{(1-\delta_C)}$$

where B_1 and B_2 (respectively E_1 and E_2) are the asset values of bankers (respectively entrepreneurs) in stages 1 and 2, according to the bargaining power δ_C of bankers relatively to entrepreneurs in the credit market.[5]

[5] Value functions are equivalent to asset values here since agents are risk-neutral.

Similarly, the flow ρ_F from bankers to investors is given by

$$\rho_F = \arg\max(I_1 - I_0)^{\delta_F}(B_1 - B_0)^{(1-\delta_F)}$$

where I_0 and I_1 are the asset values of investors in stages 0 and 1, and where δ_F is the bargaining power of investors in the financial market.

3.4 Equilibrium (closed-economy)

These two Nash expressions recursively give the equilibrium value of the tightness on each of the three frictional markets. First, the rule for ρ_F, together with zero-profit conditions for financial investors and bankers ($I_0 = 0$ and $B_0 = 0$), determines the equilibrium financial market tightness $\bar{\xi}$ as

$$\bar{\xi} = \frac{1-\delta_F}{\delta_F}\frac{c_I}{c_B} \tag{4}$$

(see Appendix for details). In equilibrium, the financial tightness thus depends on the relative flow costs: the higher is c_I, *i.e.* the costlier it is for an investor to find a commercial bank *ceteris paribus*, the less likely the former is to enter the financial market, and therefore the tighter is the market as the relative number of bankers to investors willing to trade N_B/N_I increases. The same reasoning holds for the respective bargaining powers δ_F of investors and $(1 - \delta_F)$ of bankers, with relatively more bankers willing to participate to the financial market — a higher tightness — when $(1 - \delta_F)$ is high. This simple result for $\bar{\xi}$ will be proved of particular importance in the international spillovers of shocks later on.

Recursively, the Nash rule for ρ_C and the free-entry condition for entrepreneurs ($E_1 = 0$) give the equilibrium credit market tightness $\bar{\phi}$ as

$$\bar{\phi} = \frac{1-\delta_C}{\delta_C} r \frac{c_B}{c_E}\frac{1}{q_F(\bar{\xi})} \tag{5}$$

This expression similarly says that the relative number of entrepreneurs to bankers is higher when entrepreneurs' bargaining power in the loan market is higher. On the contrary, when bankers' liquidity access is easy (high $q_F(\xi)$), numerous loan opportunities are provided to entrepreneurs, thus slackening the credit market tightness. Moreover, while $\bar{\phi}$ does not directly increase in bankers' application screening costs γ_C which are supported by the financial investor, it increases in bankers' previous costs c_B to find this former financier. Finally, for $r = 0$, all (capitalized) bankers are willing to finance an entrepreneur whatever the matching and separation probabilities because the repayment rate they earn over a loan is higher than their reservation income (the riskfree rate), consequently driving the market tightness $\bar{\phi}$ to zero. On the contrary, when $r > 0$, a banker decides to enter the credit market if the time spent in stages 1 and 2 is valuable enough to outweigh the discounting effect of the riskfree rate given that search activity is costly.[6]

Finally, the steady-state unemployment rate $\bar{u}$ equalizes flows into unemployment $s(1-u)$ and flows out of unemployment $\theta q_L(\theta)u$, that is

$$\bar{u} = \frac{s}{\theta\, q_L(\theta) + s} \tag{6}$$

The recursive system of equations (1)–(6) in six unknowns $\{\theta,\ \phi,\ \xi,\ \rho_C,\ \rho_F,\ u\}$ characterizes the closed economy. In particular, replacing $\bar{\xi}$ (4) and $\bar{\phi}$ (5) in (1)–(3) simultaneously gives the equilibrium repayment rates $\bar{\rho}_C$ and $\bar{\rho}_F$, as well as the equilibrium labor market tightness $\bar{\theta}$ through

$$\Big(\frac{c_B}{q_F(\bar{\xi})} + \frac{c_I}{\bar{\xi} q_F(\bar{\xi})} + \frac{\gamma_C}{r + \bar{\phi} q_C(\bar{\phi})}\Big)\frac{r + \bar{\phi} q_C(\bar{\phi})}{\bar{\phi} q_C(\bar{\phi})} + \frac{c_E}{q_C(\bar{\phi})} = \frac{q_L(\theta)}{r + q_L(\theta)}\frac{p - w}{r + s} - \frac{\gamma_L}{r + q_L(\theta)} \tag{7}$$

from which the steady-state unemployment rate $\bar{u}$ is determined by (6).

This equilibrium is depicted in the space formed by the labor market tightness θ and the financial market tightness ξ in Figure 5.[7] In order to represent the three representative agents in this two-dimensional space, equations (1) and (2) are put together by determining ρ_C to obtain a joint equilibrium condition for bankers and entrepreneurs (constrained agents) as

$$\Big(\frac{c_B}{q_F(\xi)}\frac{r + \bar{\phi} q_C(\bar{\phi})}{\bar{\phi} q_C(\bar{\phi})} + \frac{c_E}{q_C(\bar{\phi})}\Big)\frac{r + q_L(\theta)}{q_L(\theta)} = \frac{p - w - \rho_F}{r + s} \tag{BE}$$

which corresponds to the downward-sloping curve BE in Figure 5. A high ξ corresponds to a high demand for funds from commercial banks relatively to the available financial market supply. Therefore, at a given equilibrium credit market tightness $\bar{\phi}$, the tighter the labor market, the slacker the capital

[6]Similarly in the labor literature, an unemployed worker is willing to search for a job if he/she knows that the wage will be higher than his/her income flow while unemployed.

[7]Alternatively, the (θ,ϕ) and (ϕ,ξ) spaces provide less intuitive comparative statics.

access must be to remain on the same banker-entrepreneur's joint condition. A relatively short duration of stage 2 while looking for a suitable worker conversely offsets longer durations of the fund-raising stages. The upward-sloping II curve in Figure 5 stands for the financial investors' condition (3), which states that a tight financial market is required when the labor tightness θ is high. A large ξ increases investors' instantaneous probability of matching with a suitable banker, thus reducing the expected duration of search in the financial market, in order to outweigh the fact that stage 2 is time-consuming (θ is high), all other things equal.[8] The intersection between BE and II represents the initial aggregate equilibrium A of the closed economy, which satisfies $\{\bar{\theta}, \bar{\phi}, \bar{\xi}, \bar{\rho}_C, \bar{\rho}_F, \bar{u}\}$.

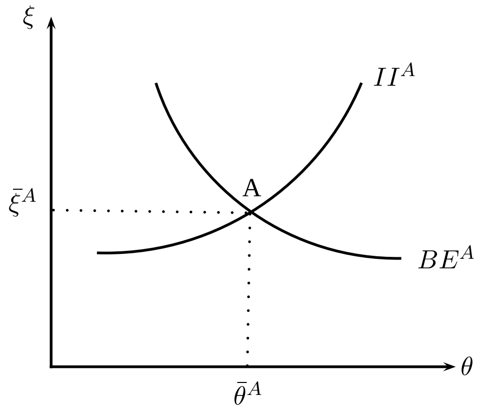

Figure 5: Initial equilibrium

After incorporating some additional mechanisms arising with the openness of the economy, next Section will study the impact of two types of shocks that may come up in the financial market of such a stylized economy:

(i) a shock to c_I — the parameter driving liquidity injection in the economy everything else equal —, interpreted as a *liquidity supply shock*. Its effects are comparable to a monetary shock though in the presence of financial accelerators that let room for credit rationing, and

(ii) a shock to c_B — the flow opportunity cost of commercial banks in the fund-raising stage —, that makes financial relationships more frictional in spite of excess savings in the economy.[9]

[8]In particular, a higher financial market tightness always means a higher matching probability for investors in equilibrium assuming that the level of information asymmetry about the participants or participants' creditworthiness is unaltered in the long run.

[9]One could think of it as a *confidence shock* due to an exogenous change in the degree

4 International spillovers of financial shocks

4.1 International relationships and world equilibrium

Each economy is specialized in the production of one good which is fully internationally mobile — with both goods being potentially perfectly substitutable without altering the model predictions. Each firm produces one unit of the good corresponding to its location every period during stage 3. The entrepreneur now chooses the respective parts of his output that will be sold at home and exported in addition to his search entry decision (1), according to the exchange rate and the competitively determined prices $p_{i_{h,t}}$ and $p_{i_{f,t}}$ of his good i ($i = h, f$) in country j ($j = h, f$) at time t, where h and f stand for Home and Foreign henceforth.

Neither the entrepreneurs nor the workers are mobile here. The country-specificity of entrepreneurs can be justified by the inertia in the production relocating decisions following unexpected financial shocks. In fact, entrepreneurs could migrate to the foreign country conditional on paying for sunk costs — due to a change in their production specialization for example — without changing the model predictions but I do not make this outside option explicit here for simplicity. Workers, whether employed or not, are supposed immobile since observed labor mobility between large economic areas within the floating exchange rate world — think of US and EU — is very limited as compared to labor mobility between countries within each of these areas. This allows to abstract from a comparable outside option for workers, and to normalize to one each national working population in order to compare variations in unemployment rates.

Commercial bankers who lend to a unique small and local entrepreneur are also primarily local since they *de facto* belong to the country where their borrower is. Formally, no direct outside option abroad is made explicit here but a generalization preserving initial free entry and free exit conditions worldwide is possible. Theoretical and empirical literatures have widely documented that geographical distance between lenders and (potential) borrowers indeed affect loan decisions. Degryse, Cerqueiro and Ongena (2007) and Agarwal and Hauswald (2010) recently summarized the two mechanisms through which distance matters. On the one hand, transportation costs hin-

of heterogeneity among bankers or a change in the degree of information asymmetry at the expense of liquidity holders.

der matching between remote credit market participants: a potential borrower has to spend time and effort to personally interact with loan officers or to look for a suitable loan (because of product differentiation) while banks endure costs in assessing loan applicants or in monitoring loans that both increase with physical distance (Sussman and Zeira, 1995). On the other hand banks' capacity to collect critical information about expected returns and probabilities of default of potential borrowers is enhanced by proximity, thus encouraging banks to concentrate on a limited geographical area to benefit from the monopoly power created by this informational advantage (Hauswald and Marquez, 2006). This results in spatial price discrimination and geographical credit rationing, empirically supported at a micro level (Degryse and Ongena, 2005; Agarwal and Hauswald, 2010) and within a medium size country (Casolaro and Mistrulli, 2008). Although the transposition at the world level would require a specific analysis, two arguments support it here. First, if transaction costs and informational advantages are decisive channels in very limited areas, they are probably deeper between major areas because additional differences, in regulation for instance, are likely to prevent from collecting private information or detecting credit delinquency. Second, both banks and firms are small ones and new ones (think of a particular bank agency), two major characteristics for which the aforementioned channels are particularly strong in this literature.

Finally, financial capital is perfectly mobile across countries. Investors are not assigned a particular location *ex ante* but search for a banking investment in the country where their intertemporal value, denoted I, is the highest, given the costs and transition rates. Therefore, an instantaneous no-arbitrage condition implies that $\bar{I}_{j,t} = \bar{I}_t$ for all j.[10] This implies that investors choose to locate their assets in the country where their matching rate is the highest, *i.e.* where the financial market is the tightest (since $\partial \xi q_F(\xi)/\partial \xi > 0$) everything else equals. Equilibrium is thus characterized by a unique financial tightness at the world level, that is, an integrated financial market and external finance conditions equalized worldwide.

Given these international relationships, the balance of payments and the subsequent expression for the exchange rate are as follows. The current

[10] Further, note that $\bar{I} = 0$ at equilibrium by the free entry condition.

account of the home country expressed in domestic currency is standard as

$$CA_t \equiv C_{h_{f,t}} S_t \, p_{h_{f,t}} - C_{f_{h,t}} p_{f_{h,t}}$$

where $C_{i_{j,t}}$ denotes the level of consumption of good i in country j at time t, and where S is the nominal floating exchange rate defined as the price of the domestic currency in terms of the foreign currency. The law of one price is assumed to always hold, such that the price of the each good at home depends on its price abroad times the exchange rate: $p_{i_{h,t}} = S_t \, p_{i_{f,t}}$.

As financial investors already matched with a banker cannot immediately withdraw their capital after a shock, the definition of the financial account comes down to the inter-country difference in new investor-banker relationships, *i.e.* the difference of financial match flows at home and abroad

$$FA_t \equiv m_F(N_{I_t}, N_{B_{h,t}}) - m_F(N_{I_t}, N_{B_{f,t}})$$

Re-expressed in terms of matching rates (by definition, $\xi_i q_F(\xi_i) \equiv \frac{m_F(N_I, N_{B_i})}{N_I}$), the inter-country financial market tightness differential replaces the traditional interest rate differential in driving international capital flows

$$FA_t = N_{I_t} \xi_{h,t} q_F(\xi_{h,t}) - N_{I_t} \xi_{f,t} q_F(\xi_{f,t})$$

Note that, even if financial flows are driven by the financial tightness differential, unmatched investors will not necessarily immediately pour their savings into the foreign market following a shock at home as general equilibrium effects will also imply a rarefaction of suitable bankers in the foreign country, and therefore similar movements of the financial tightness abroad. Scarcity of real business opportunities and internal credit frictions are magnified everywhere by the inertia in creating new financial relationships.

The balance of payments identity $CA + FA \equiv 0$ finally gives the expression for the exchange rate as follows, with time subscripts dropped

$$S = \frac{p_{f_h} C_{f_h} - N_I \xi_h q_F(\xi_h) + N_I \xi_f q_F(\xi_f)}{p_{h_f} C_{h_f}} \tag{8}$$

Unsurprisingly, the domestic currency appreciates (S decreases) with exports and relative financial opportunities to capital holders at home, while it depreciates with imports and relative financial advantages abroad.

The general equilibrium in the two-country case is now determined by adding to the previous symmetric set of unknowns for both countries $\{\theta_h, \theta_f, \phi_h, \phi_f, \xi_h, \xi_f, \rho_{C_h}, \rho_{C_f}, \rho_{F_h}, \rho_{F_f}, u_h, u_f\}$ the consumption levels $\{C_{h_h}, C_{f_h}, C_{h_f}, C_{f_f}\}$, prices $\{p_{h_h}, p_{f_h}, p_{h_f}, p_{f_f}\}$, and the exchange rate $\{S\}$. Two first order conditions from consumers' optimization problem, the two laws of one price, two aggregate constraints on goods, and each country's resource constraint (standard equations relegated in Appendix) are added to the set of equations, as well as the balance of payments (8).

This equilibrium is point A in the former Figure 1, where the balance of payments BP horizontal line depicts the financial market integration through which capital flows take advantage of all opportunities.[11]

4.2 Effect of liquidity supply shocks

Let first consider a shock to the liquidity injection parameter c_I in Figure 6. A domestic increase in c_I displaces the liquidity holders' equilibrium condition II_h^A leftwards, from point A to point B at home. Bankers are hit in turn because the rarefaction of liquidity both raises the interest rate and creates contagion effects that make financial matching even more time-consuming. This implies moving along the BE_h^A curve, except for the fraction of bankers that are not profitable anymore and move BE leftwards as they exit.

The relative appreciation of the domestic currency at point C further drives a price-competitiveness recession in the home country, displacing again the BE curve leftwards. Meanwhile, the depreciation abroad boosts the economic activity in the second country. At point D, the reduced demand in the first country subsequently generates negative second-round effects abroad but not large enough to be detrimental to the foreigners, since the economy *in fine* stabilizes at the general equilibrium where home and foreign financial market tightnesses are equalized, that is, at point E.

A multiplier here magnifies the impact of the rarefaction of liquidity hold-

[11] The perfect international mobility of financial investors assumed here for analytical and graphical simplicity ensures that financial market tightnesses, ξ_h and ξ_f, are instantaneously equalized between countries. With a slightly upward-sloping BP curve, the qualitative results will remain even if it may quantitatively reduce the financial transmission. In another setting where financial investors would be strongly immobile, portfolio composition effects would substitute to ensure capital mobility, producing similar contagion effects though via different channels (Krugman, 2008, Dedola and Lombardo, 2012).

ers via longer fund-raising stage durations for credit intermediaries. However, note that the international transmission channels do not qualitatively differ from the literature even in the presence of a financial multiplier because the solvency of commercial banks was not the source of the shock. The spillover effect of pure liquidity supply shocks is therefore similar to the one of traditional monetary shocks, that is, negative output co-movements.

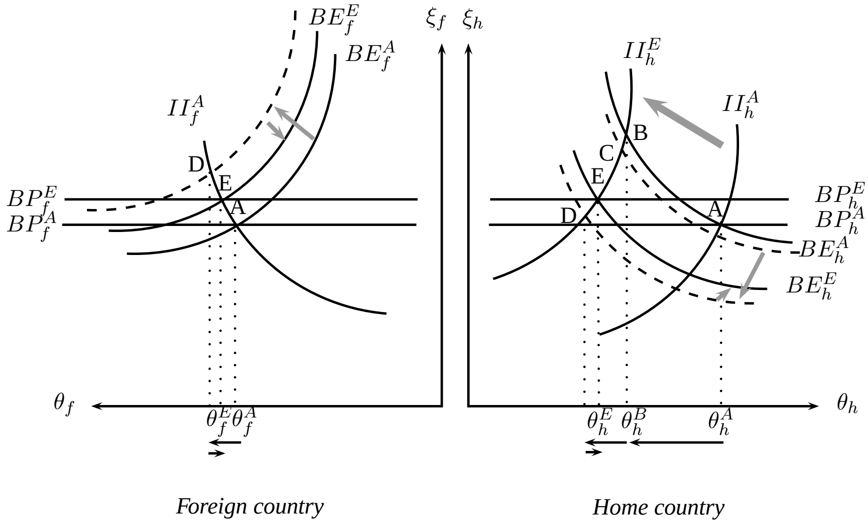

Figure 6: World effect of a domestic liquidity supply shock

The theoretical framework allows to corroborate the picture by simple analytical results. Let define the equilibrium elasticity of each matching function as $\eta_L \equiv -q_L'(\bar{\theta})\bar{\theta}/q_L(\bar{\theta})$ (labor market), $\eta_C \equiv -q_C'(\bar{\phi})\bar{\phi}/q_C(\bar{\phi})$ (credit market), and $\eta_F \equiv -q_F'(\bar{\xi})\bar{\xi}/q_F(\bar{\xi})$ (financial market), $\eta_L, \eta_C, \eta_F \in (0, 1)$. The expressions for $\bar{\xi}$, $\bar{\phi}$, and $\bar{\theta}$ are loglinearized around the two-country steady-state and the analytical responses to asymmetric liquidity supply shocks are derived. Results are summarized in Table 1, where a hatted variable denotes its loglinear deviation from steady-state ($\hat{x} = \frac{x-\bar{x}}{\bar{x}}$).

The first two rows present intuitive results. A negative domestic liquidity supply shock (c_{I_h} rises) tightens the *financial* market (row 1) via a negative wealth effect. As financiers are internationally mobile, the same effect is obtained worldwide. The *credit* market also tightens in both countries (row 2) because less capitalized banks are able to finance a given mass of entrepreneurs, hence ϕ rises. However, the sign of the *labor* market response to the shock differs across countries in row 3 (assuming that the elasticity in the southwestern cell is negative, see Appendix), confirming the literature puzzle. Further loglinearization of (6) makes clear that unemployment decreases in the labor market tightness in one country, as

$$\hat{u} \approx -(1-\eta_L)(1-\bar{u})\hat{\theta}$$

while output increases in the labor market tightness since output is proportional in employment in each country. Therefore, the unemployment rate increases in the country where the negative liquidity supply shock arrives but decreases abroad. Note that the magnitude of the effect does not depend on the consumer preference parameters and thus even a strong national consumption bias does not question the present transmission channels.

Table 1: Home and foreign elasticities to local liquidity supply shocks

	Home country	**Foreign country**
$\partial\hat{\xi}_j/\partial\hat{c}_{I_h}$	1	1
$\partial\hat{\phi}_j/\partial\hat{c}_{I_h}$	η_F	η_F
$\partial\hat{\theta}_j/\partial\hat{c}_{I_h}$	$\frac{q_L(\bar{\theta})}{\gamma_L\eta_L}\Big\{-\frac{\bar{c}_{B_h}\eta_F}{q_F(\xi_h)}-\frac{\bar{c}_{I_h}\eta_F}{\xi_h q_F(\xi_h)}-\frac{\bar{c}_{E_h}\eta_C\eta_F}{q_C(\phi_h)} +\frac{\gamma_C(1-\eta_C)\eta_F}{\phi_h q_C(\phi_h)}-(1-\bar{N}_I)(1-\eta_F)\Big\}$	$\frac{q_L(\bar{\theta})}{\gamma_L\eta_L}(1-\bar{N}_I)(1-\eta_F)$

These results nest the standard open economy framework as a particular case by predicting that an expansion prevails in the foreign area following a monetary contraction at home within a flexible exchange rate system, even if this sounds puzzling in terms of financial contagion. This is simply the reverse case of an expansive 'beggar-thy-neighbor' monetary policy or competitive devaluations by which short-term booms are expected at the expense of trading partners. As far as monetary shocks are concerned, the new open-economy macro literature confirms the early Mundell-Fleming negative output co-movements, that is, no financial contagion emerges as long as either fixed exchange rates or financial market incompleteness (financial asset complementarity) is imposed. Again, standard results are confirmed here despite the presence of financial multiplier mechanisms. This potentially explains why the literature on international contagion within a floating exchange rate system has not integrated more sophisticated financial channels, or on the contrary, why the credit constraints literature has focused on closed economy or monetary union settings. The present setup confirms that incorporating financial multipliers into existing models is not sufficient to account for positive output co-movements following liquidity shocks.

4.3 Effect of an exogenous rise in bank fund-raising costs

In order to let room for international transmission of financial shocks, the present paper argues for considering financial shocks of a different nature, departing from the walrasian perspective in the sense that they prevent interest rate movements to immediately clear the market. In particular, this is the case if it is suddenly more difficult for banks to raise funds while there is no change in the amount of idle liquidities. Hence, let now consider in Figure 7 a shock to the search cost c_B of banks in the fund-raising stage. One can imagine that bank capitalization is suddenly costlier because higher (real or perceived) heterogeneity in the banking sector requires either that banks make a sustained effort to gather the proofs of their creditworthiness to investors and/or that they bear a higher opportunity cost in the climate of mistrust due to prohibitive information asymmetry to investors.[12]

The shock to c_B primarily shifts the BE_h curve to the left because zero-profit conditions directly imply that the number of banks willing to enter decreases with a higher liquidity search cost c_B, and so does the number of credit-constrained entrepreneurs. The number of investors therefore also contracts because it becomes more difficult to find a suitable banker, slightly displacing the II curve to the left (congestion effects). The domestic currency now depreciates at point C, providing with potential export opportunities the rare but yet newly created firms — entrepreneurs successfully matched with the remaining banks. Meanwhile, the relative appreciation abroad associated with the domestic recession damages the foreign current account. The situation is aggravated abroad by the fact that global investors choose not to pour their savings into the foreign economy, even if the match with commercial bankers would comparatively be easier because the rarefaction of business opportunities is magnified by internal credit frictions. Both countries are therefore kept away from new leverage facilities in spite of a lower financial market tightness at the new equilibrium (point D).

Three lessons can be drawn. First, the labor market tightness at home

[12]The underlying informational structure of the banking sector that would allow to determine the reasons of the rise in c_B is not made explicit here for simplicity.

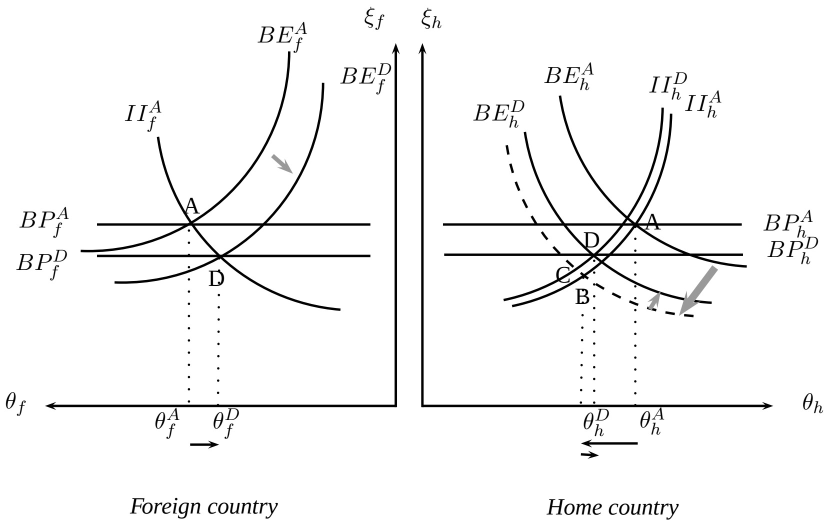

Figure 7: World effect of a domestic rise in bank fund-raising costs

is lower from its initial level (θ_h^A) and therefore unemployment higher. Investors' decision to exit the financial market when banking heterogeneity rises in turn forces more banks to exit the market because the rarefaction of funds adds to the fact that their search for funds was already costlier. This effect is itself passed on entrepreneurs and the three frictions reinforce one another creating internal financial accelerator mechanisms. Second, the unemployment rate also goes up in the foreign country even though the financial distress was local and despite the flexible exchange rate regime. There is thus a worldwide downturn that temper real adverse effects in the first area by transmitting them partly to the other zone. Last but not least, these overall effects are concomitant to a slackening of the financial tightness at the world level, confirming that the nature of the financial shock is different from a standard monetary contraction in DSGE models.

In Table 2, the first row indicates that the *financial* tightness is lower, which means that the reason why bank have less opportunities to raise funds is not liquidity scarcity but larger frictions on the financial market. The second row reflects *credit* rationing worldwide as in the case of liquidity supply shocks (the magnitude only differs by the elasticity of the financial market to both types of shocks). Finally, home and foreign *labor* market tightnesses, and therefore home and foreign outputs, appear here with positive co-movements as one would expect from a financial contagion.

Table 2: Home and foreign elasticities to a rise in bank fund-raising costs

	Home country	**Foreign country**
$\partial\hat{\xi}_j/\partial\hat{c}_{B_h}$	-1	-1
$\partial\hat{\phi}_j/\partial\hat{c}_{B_h}$	$1-\eta_F$	$1-\eta_F$
$\partial\hat{\theta}_j/\partial\hat{c}_{B_h}$	$\frac{q_L(\bar{\theta})(1-\eta_F)}{\gamma_L\eta_L}\Big\{-\frac{\bar{c}_{B_h}}{q_F(\bar{\xi}_h)}-\frac{\bar{c}_{I_h}}{\bar{\xi}_h q_F(\bar{\xi}_h)}-\frac{\bar{c}_{E_h}\eta_C}{q_C(\bar{\phi}_h)} + \frac{\gamma_C(1-\eta_C)}{\bar{\phi}_h q_C(\bar{\phi}_h)}+(1-\bar{N}_I)\Big\}$	$-\frac{q_L(\bar{\theta})(1-\eta_F)}{\gamma_L\eta_L}(1-\bar{N}_I)$

4.4 Quantitative evaluation

This subsection estimates the magnitude of international spillovers resulting from both types of financial shocks. As most of the financial parameters considered in this new approach lack of empirical counterparts, they are chosen so that steady-state values are realistic, while discussion about micro measures of missing data could constitute further research.[13] The calibration of labor markets is standard and kept as simple as possible here.[14]

The matching functions are supposed to be Cobb-Douglas

$$m_F(N_B, N_I) = \mu_F N_B^{\eta_F} N_I^{1-\eta_F}$$

$$m_C(N_E, N_C) = \mu_C N_E^{\eta_C} N_C^{1-\eta_C}$$

$$m_L(N_U, N_V) = \mu_L N_U^{\eta_L} N_V^{1-\eta_L}$$

where μ_F, μ_C, and μ_L, stand for matching efficiency measures in the financial market, credit market, and labor market, respectively. On the financial and credit markets, let assume that this efficiency parameter is normalized to unity, that the tightness elasticity of the matching functions is 0.5, and that the bargaining powers of investors and bankers which characterize the Nash bargaining rules, δ_C and δ_F, are also equal to 0.5. On the labor market, the tightness elasticity η_L is set up to 0.66, as I will also assume that two third of the surplus is earned by workers at equilibrium, in order to be consistent

[13]Afonso and Lagos (2012) use Fedwire data to calibrate a search version of the US interbank market; however, private data about financial amounts transfered from international investors to country-specific commercial banks both by capitalization and debt is not available to our knowledge.

[14]For specific discussions about the quantitative performance of search and matching models of labor markets for macroeconomic analysis, see Yashiv (2009) or Cardullo (2010).

with the Hosios rule. The matching efficiency on the labor market is allowed to vary between 1.1 and 1.5, a range around Shimer (2005)'s estimation at 1.355. The quarterly separation rate is 0.1 and the riskless rate is 0.05.

For the moment, let see what the equilibrium labor market outcomes would be with moderate financial frictions and moderate credit frictions. Let set $c_I = c_B = 0.1$ so that the equilibrium financial market tightness in (4) is equal to 1. With an entrepreneurs' non pecuniary cost c_E at 0.005, the equilibrium credit market tightness is then also equal to 1 by (5). Finally, the flow cost γ_C of bankers screening credit applications is equally set to 0.1. With a flow cost γ_L of job vacancies at 1.5, it results by (7) and (6) that the predicted unemployment rate is 4.68% when $\mu_L = 1.5$ or 7.26% when $\mu_L = 1.1$, according to the labor market structural efficiency.

Now let consider an initial situation in which unemployment rates are of similar magnitude but in a context where banks find liquidities at very high rates, whereas entrepreneurs are indeed moderately credit constrained. In other words, the steady-state is re-parameterized in order to make a distinction between the credit market, where the information about entrepreneurs' creditworthiness is not immediately available to bankers, on the one hand and the financial market, where banker-investor relationships are essentially frictionless in normal times, on the other hand. Assuming that investors' and bankers' bargaining powers on the financial market are now $\delta_F = 0.995$ and $(1-\delta_F) = 0.005$ respectively, with unchanged values for the search costs ($c_I = c_B = \gamma_C = 0.1$), it results from (4) that bankers now raise funds immediately as the Poisson rate at which they match with a financier ($q_F(\xi)$) is now 14 times larger. This can be interpreted as the existence of large excess savings in the pre-crisis equilibrium, modeled here by much more financial investors initially entering the process. The credit tightness remains at 1, implying that entrepreneurs' flow cost must now equal 0.00035 from (5). Therefore, with $\gamma_L = 0.5$, the initial unemployment rates are now evaluated at 4.94% and 7.66% (when $\mu_L = 1.5$ and $\mu_L = 1.1$ respectively), that is, quite close to the previous numbers. This verifies that the model is quantitatively able to reproduce frictionless financial markets in normal times and may provide a more realistic initial equilibrium.

In order to stay consistent with the symmetry of the model, the average labor market efficiency ($\mu_L = 1.355$) is used to compare the quantitative impact of the different financial shocks from a unique initial unemployment rate at 5.72%. The elasticity of the labor market tightnesses to *liquidity supply shocks* are thus evaluated at -0.82 at home while $+0.19$ abroad. In line with the qualitative analysis, it confirms the literature negative co-movements between domestic and foreign responses, unlike the recent events. In the case of shocks to the cost of bank capitalization, the elasticities respectively become -0.44 and -0.19, implying that this type of shocks is transmitted across countries.[15] In terms of unemployment rate, the response to negative confidence shocks is $+0.0083$ in the country where the shock arrives while $+0.0036$ in the foreign country. These numbers are not directly interpretable since there is no data equivalence for the parameters considered here but give a sense of the magnitude of the contagion as they indicate that the relative unemployment deviation is 2.3 times bigger in the home country.

5 Discussions

5.1 How much does this apply to the 2008-2009 episode?

Even though this model is of course not directly transposable to the 2008-2009 crisis, it brings qualitative insights that differ from the existing literature. In particular, some stylized facts suggest that the new non-walrasian type of financial shocks considered here is more relevant than traditional liquidity supply shocks. First, the contraction in international trade has been sharper in the Eurozone than in the US, in contradiction to the standard open economy results in a flexible exchange rate case.[16] Second, the increase in unemployment rate has been transmitted but dampened abroad, with a rise by 111% in the US between February 2008 and October 2009 while by 36% in the Eurozone in the same period, that is, relative real effects being about three times higher in the US.[17] Finally, the model predicts that the Euro would appreciate vis-a-vis the dollar in case of a shock to the bank capitalization cost. Although the spot eurodollar rate has depreciated in that period, a rigorous decomposition shows that the macro variables considered here are almost not accountable for the observed movements. Clarida (2012)

[15] Both financial shocks have the same negative real effect at the world level (-0.63).

[16] Annual exports of goods fell by 18% in value from 1,277 billions in 2008 to 1,046 billions of US dollars in 2009 in the US, while by 23% from 2,312 billions of US dollars to 1,791 billions in the Eurozone in the same period. Source: OECD.

[17] From 4.8% in February 2008 to 10.1% in October 2009 in the US, and from 7.2% to 9.8% in the same period in the Eurozone. Source: Eurostat.

gives a measure of the 'risk neutral fair value' of the exchange rate and finds that it is stable around 1.37 from October 2008, while the carry trade risk premium explains most of the observed variations in the period.

The aim of this paper is neither to provide a realistic quantitative analysis — which would of course require many additional sophistications such as risk averse agents and specific policy implementation in addition to a precise calibration of all market characteristics of each area — nor to give some clues to the more recent events related to fragilities inherent to the Eurozone. However, it provides a tractable analytical framework that allows to understand the conditions under which financial contagion may emerge when the exchange rate regime is flexible and when financial assets are substitutable across countries, reminiscent of the recession of October 2008-February 2009.

5.2 The specific role of each market friction

In addition to reinforce one another to create a financial accelerator[18], each of the frictions considered here play a distinct role. First, in the absence of *financial* friction ($q_F(\xi)$ goes to infinity), the only way to simulate a monetary contraction would be to introduce money in the model but without any real effect here since prices are fully flexible. A two-country model with sticky prices introducing some type of liquidity constraints could potentially reach the same conclusions but at the expense of simplicity and tractability. Here the arrival of (and the distinction between) the shocks to c_I and c_B requires a positive friction (delay) even if very small in steady-state.

Second, in the absence of *credit* friction, the equilibrium labor market tightness would be higher because entrepreneurs would not be financially constrained.[19] The fact that the credit friction is local captures the idea that, because of moral hazard effects, new entrepreneurs cannot raise funds from global financial investors directly but have to address commercial banks that are able to reduce the informational gap locally. As an alternative to *ex post* monitoring costs *à la* Townsend, the search cost accounts for *ex ante* screening costs here in order to keep tractability while expressing the optimal output sharing as the result of a Nash bargaining process.

Finally, the *labor* market friction does not play a central role here but

[18] See Wasmer and Weil (2004)

[19] See Wasmer and Weil (2004) for a formal description of how the existence of a credit friction makes the labor market tightness deviate from the Pissarides equilibrium.

has two practical advantages. On one hand, it allows to express disturbances to real activity in terms of employment outcomes instead of assuming that output is exactly proportional to credit rationing (which may be a stronger assumption since the latter is a variable of "financial" type). On the other hand, it makes analytical results as simple as possible in the case where only employed workers consume (the equivalent of their exogenous wage). The results do not qualitatively change if all agent types consume — their respective (endogenous) income — but a wealth effect of financial shocks would then come from individual income variations in addition to the wealth effect that results from the fact that some agents are unmatched and in addition to the substitution expenditure-switching effects. Since there is no consumption/savings decisions here, the tractable case is preferred.

The multi-frictional framework therefore allows for the arrival of a very particular type of shock that generates the positive co-movement result. This shock is different from a standard monetary contraction and could not be reproduced in standard DSGE models. The only way to create positive co-movements in a DSGE would be to simulate an ad hoc exogenous decrease in future cash flows of producers. Here, both types of shocks are still exogenous but of "financial" nature since they hinder the relationship between liquidity holders and intermediaries, while entrepreneurs' entry decision is hit because of the rarefaction of financiers as a second-round effect.

5.3 Literature

This paper contributes to three separate bodies of literature. A first issue is related to the macroeconomic role of a financial multiplier by which credit constrained firms overreact to a change in borrowing conditions from commercial banks. This paper has accounted for such a mechanism — although modeled in a different manner than in Bernanke and Blinder (1989), Bernanke, Gertler and Gilchrist (1999), or Kiyotaki and Moore (1997) — and has showed that this is yet not sufficient to make international financial contagion emerge. Credit frictions adopted here follow the tractable closed-economy formalizations by Den Haan, Ramey and Watson (2003) and Wasmer and Weil (2004), but the case of liquidity supply shocks has indicated that they do not solve the contagion puzzle within a floating exchange rate system when simply extended to an open-economy framework.

Second, this paper finds that the combination of non-walrasian financial shocks and financial markets integration considerably alters the standard mechanisms of the two-country models that do not introduce specific financial relationships. Major recent two-country frameworks reached the same conclusions than the early Mundell-Fleming-Dornbusch models as far as monetary shocks were concerned because they ignored financial frictions, and particularly the fact that information asymmetry and agent heterogeneity create potential occurrence of non-walrasian shocks. Boivin, Kiley and Mishkin (2010) have recently reminded that "the core channels of policy transmission (...) have remained steady from early policy-oriented models to modern DSGE models" while "in contrast, non-neoclassical channels, such as credit-based channels, have remained outside the core models", and added that the exchange rate channel was the sole neoclassical channel resulting from the openness of the economy.

For instance, the well-known model by Obstfeld and Rogoff (1995) that notably provided the Keynesian analysis with microeconomic foundations in a two-country model lead to counterintuitive results in light of the recent events when concluding that monetary expansions in one country imply negative co-movements between home and foreign outputs because of exchange rate fluctuations. In their own words, following a unilateral increase in home money supply, "the world real interest rate falls and world demand rises, but because the domestic currency depreciates, some world demand is shifted toward home products at foreign producers' expense."[20] I follow the new open-macroeconomics literature by calling this *substitution* mechanism the "expenditure-switching channel" although the *wealth* effect of interest rate variations of course differs in my model since the cost of capital constrains production rather than altering the level of consumption first as it would be the case in the new open-economy macroeconomics literature.

Later improvements of the Obstfeld-Rogoff monopolistic competition framework did not change the co-movement predictions. Betts and Devereux (2000a, 2000b) included local price stickiness to depart from the law of one price hypothesis, and confirmed the negative output co-movement in-

[20] Obstfeld and Rogoff (1995) precise this does not necessarily mean 'beggar-thy-neighbor' effects in terms of welfare because foreigners "enjoy more leisure, improved terms of trade, and consumption higher than income" when output falls. But it is likely that, for a large and prolonged disruption, individuals perceive more disutility from reduced consumption and potential unemployment than utility from leisure, so that welfare implications are not explored here to focus on the output co-movements puzzle.

duced by monetary shocks in the presence of pricing-to-market. Even more surprisingly, in one-area estimated models used until very recently by the Federal Reserve and the European Central Bank, monetary contractions lead to significant and persistent nominal and real appreciations of the domestic currency (for e.g. Eichenbaum and Evans (1995) for the US, Smets and Wouters (2003) for the Eurozone). This implies that financial disruptions at home, as represented through monetary contractions in those frameworks, would benefit a second foreign large country, contrary to the recent events.

The few existing two-country papers with more sophisticated monetary mechanisms have kept a fixed exchange rate regime and thus eluded the issue of the US-Eurozone financial contagion. For instance, a financial multiplier is at work in Gilchrist, Hairault and Kempf (2002) but in the context of a monetary union; more recently Devereux and Yetman (2010) studied the international transmission of shocks when investors are highly levered in one country but did not question the exchange rate regime.

Finally, aside from the macro papers, a literature on financial contagion has attempted to account for the complexity of modern financial interrelations but relying on the incompleteness of financial markets in the countries to which crises are transmitted. This representation is convenient to study the effect of shocks from developed to emerging market economies, from the famous paper by Allen and Gale (2000) — underlying the claims that banking systems have on one another due to regional incompleteness of financial markets as observable in Asia or in the US in the late nineteenth century — to sudden stops in capital flows (Calvo, Izquierdo and Mejía, 2004) and the current evidence about recoupling movements with US financial circumstances for large and prolonged US financial distress (Dooley and Hutchison, 2009). Recent papers following this approach to explain the last financial contagion have highlighted the weakening of effects due to the international trade-based mechanism but still cannot account for the transmission channels across developed economies. In the partial equilibrium model by Krugman (2008) notably, highly leveraged institutions hold domestic and foreign assets, and international cross-holding is thus the main propagation channel. But as far as developed countries are concerned, it is more likely that domestic and foreign assets are *substitutes* rather than *complements*, and that the equalization of external finance premia across countries is instead the source of international propagation.

Dedola and Lombardo (2012) thus developed a two-country general equilibrium model, where "financial and real interdependence can be very strong even with minimal balance sheet exposure to foreign risky assets, if asset markets are integrated across the board". Yet, they also need a minimum level of asset cross-holdings even in the presence of internal financial accelerators to propagate the financial disruption. The current paper has thus taken a different approach, by assuming that leveraged banks issue equities on perfectly integrated financial markets, in order to prove that there is room for international contagion without relying on cross-holdings effects. Home and foreign financial assets are considered as perfect substitutes here, which is likely to be the case for Euro and US financial assets. In particular, this means that interest rate distributions are comparable even though there may exist a home bias in equity or bond portfolio as suggested by Coeurdacier, Kollman, and Martin (2010).

In its methodological aspects, my paper has used the search and matching modeling to represent frictions in different markets. Kiyotaki and Wright (1993) and Dell'Ariccia and Garibaldi (1998) adopted the Mortensen and Pissarides (1994) formalization to deal with rationing in monetary and credit markets. The further theoretical developments for macroeconomic purposes (Wasmer and Weil, 2004) have provided the foundations for the closed-economy version of the economy presented here, while some empirical papers (Dell'Ariccia and Garibaldi, 2005, and Craig and Haubrich, 2006) have supported this representation of credit market frictions.

Three major advantages of this approach have been revealed here. On practical grounds, it has allowed for a particularly tractable model while introducing a financial market and considering a two-country model where both home and foreign variables are endogenized. Second, it has pointed out that liquidity market disruptions are sudden while restoring confidence between investors and banks as well as between banks and credit borrowers is time consuming, due to heterogeneity and informational asymmetries, thus creating a period of time in which the reinforcement between financial, credit and labor market frictions is economically painful. Third, this modeling has permitted to depart from traditional monetary shocks to represent shocks of a different nature, for which interest rate adjustments cannot immediately clear the market. The study of non walrasian financial shocks has provided new interesting insights in terms of financial tightness and exchange rate dynamics conducive to crisis propagations.

6 Conclusion

This paper has constructed a tractable multi-frictional model whereby an asymmetric financial shock is transmitted between major economic areas with complete financial markets and within a floating exchange rate regime.

On the one hand, it nests a standard result of the literature — from the early Mundell-Fleming models to the DSGE recently used by Central Banks — by predicting a negative correlation between home and foreign outputs following asymmetric liquidity supply shocks, even in the presence of internal financial accelerators. Therefore, it attempts to explain why the open macroeconomic literature — whether neoclassic or including market imperfections — has not integrated so far the literature on financial frictions and sophisticated monetary transmission mechanisms, and *vice versa*.

On the other hand, it gives room to another type of financial shocks that arrives in spite of excess liquidity and does generate international propagation in otherwise similar contexts. This is permitted via the application of the search and matching approach, that allows representing perfectly efficient financial markets in normal times but frozen financial relationships in distressed times.

Several improvements to this framework could constitute further research. First, the model could be inserted in a fully dynamic setup whereby the resources of financial investors are no more an exogenous endowment but are driven by saving decisions of the different agents. This is likely to make entrepreneurs less dependent from financial intermediaries for recruitment when the credit constraint is binding, but also to amplify the impact and the persistence of unexpected financial shocks by reducing the willingness to save and invest in risky assets. Second, it would be interesting to allow for more than one-to-one relationships and study size effects on different agent types, whether firms or financiers. Crossed financial relationships worldwide could magnify again the contagion.

Finally, monetary policy implications will be of main interest. The situation studied here is roughly the one that prevails before special interventions, and Central Banks are not given a proper role besides being liquidity providers through quantitative easing operations. Introducing an interbank market along with the frictional financial market would both diversify liquidity access to banks and confer a more realistic role to monetary authorities. In a two-country framework, positive externalities could then emerge and replace the standard 'beggar-thy-neighbor' monetary policy instruments.

Bibliography

[1] Daron Acemoglu. Credit market imperfections and persistent unemployment. *European Economic Review*, 45(4–6):665–679, May 2001.

[2] Gara Afonso and Ricardo Lagos. Trade dynamics in the market for federal funds. Unpublished manuscript, available at `http://sites.google.com/site/rl561a2/workinprogress`, 2012a.

[3] Sumit Agarwal. Distance and private information in lending. *Review of Financial Studies*, 23(7):2757–2788, July 2010.

[4] Franklin Allen and Douglas Gale. Financial contagion. *Journal of Political Economy*, 108(1):1–33, February 2000.

[5] Ben S. Bernanke and Alan S. Blinder. Credit, money and aggregate demand. *American Economic Review*, 78(2):435–39, May 1988.

[6] Ben S. Bernanke, Mark Gertler, and Simon Gilchrist. The financial accelerator in a quantitative business cycle framework. *Handbook of Macroeconomics*, 1(3):1341–1393, 1999.

[7] Caroline Betts and Michael B. Devereux. Exchange rate dynamics in a model of pricing-to-market. *Journal of International Economics*, 50(1):215–244, February 2000.

[8] Caroline Betts and Michael B. Devereux. International monetary policy coordination and competitive depreciation: A reevaluation. *Journal of Money, Credit and Banking*, 32(4):722–745, November 2000.

[9] Jean Boivin, Michael T. Kiley, and Frederic S. Mishkin. How has the monetary transmission mechanism evolved over time? In Benjamin M. Friedman and Michael Woodford, editors, *Handbook of Monetary Economics*, volume 3 of *Handbook of Monetary Economics*, chapter 8, pages 369–422. Elsevier, 2010.

[10] Guillermo A. Calvo, Alejandro Izquierdo, and Luis-Fernando Mejía. On the empirics of sudden stops: The relevance of balance-sheet effects. NBER Working Paper 10520, May 2004.

[11] Murillo Campello, John R. Graham, and Campbell R. Harvey. The real effects of financial constraints: Evidence from a financial crisis. *Journal of Financial Economics*, 97(3):470–487, September 2010.

[12] Gabriele Cardullo. Matching models under scrutiny: An appraisal of the shimer puzzle. *Journal of Economic Surveys*, 24(4):622–656, September 2010.

[13] Luca Casolaro and Paolo Emilio Mistrulli. Distance, lending technologies and interest rates. 21st Australasian Finance and Banking Conference 2008 Paper, available at `http://ssrn.com/abstract=1243402`.

[14] Richard Clarida. Get real: Interpreting nominal exchange rate fluctuations. *International Journal of Central Banking*, 8(1):175–196, January 2012.

[15] Nicolas Coeurdacier, Robert Kollman, and Philippe Martin. International portfolios, capital accumulation and foreign assets dynamics. *Journal of International Economics*, 80(1):100–112, January 2010.

[16] Ben Craig and Joseph G. Haubrich. Gross loan flows. Federal Reserve Bank of Cleveland Working paper 06-04, March 2006.

[17] Luca Dedola and Giovanni Lombardo. Financial frictions, financial integration and the international propagation of shocks. *Economic Policy*, 27(70):319–359, 04 2012.

[18] Hans Degryse and Steven Ongena. Distance, lending relationships, and competition. *The Journal of Finance*, 60(1):231–266, February 2005.

[19] Hans A. Degryse, Geraldo M. Cerqueiro, and Steven Ongena. Distance, bank organizational structure and credit. Technical report, 2007.

[20] Giovanni Dell'Ariccia and Pietro Garibaldi. Bank lending and interest rate changes in a dynamic matching model. International Monetary Fund Working Paper 98-93, June 1998.

[21] Giovanni Dell'Ariccia and Pietro Garibaldi. Gross credit flows. *Review of Economic Studies*, 72(3):665–685, July 2005.

[22] Wouter J. den Haan, Garey Ramey, and Joel Watson. Liquidity flows and fragility of business enterprises. *Journal of Monetary Economics*, 50(6):1215–1241, 2003.

[23] Michael B. Devereux and James Yetman. Leverage constraints and the international transmission of shocks. *Journal of Money, Credit and Banking*, 42(1):71–105, September 2010.

[24] Michael Dooley and Michael Hutchison. Transmission of the u.s. subprime crisis to emerging markets: Evidence on the decoupling-recoupling hypothesis. *Journal of International Money and Finance*, 28(8):1331–1349, December 2009.

[25] Martin Eichenbaum and Charles L. Evans. Some empirical evidence on the effects of shocks to monetary policy on exchange rates. *Quarterly Journal of Economics*, 110(4):975–1009, November 1995.

[26] Simon Gilchrist, Jean-Olivier Hairault, and Hubert Kempf. Monetary policy and the financial accelerator in a monetary union. ECB Working Paper No. 175; FRB International Finance Discussion Paper No. 750, 2002.

[27] Galina Hale and João A. C. Santos. Do banks propagate debt market shocks? Federal Reserve Bank of San Francisco Working Paper 2010-08, February 2010.

[28] Robert Hauswald and Robert Marquez. Competition and strategic information acquisition in credit markets. *Review of Financial Studies*, 19(3):967–1000, 2006.

[29] Chotibhak Jotikasthira, Christian T. Lundblad, and Tarun Ramadorai. Asset fire sales and purchases and the international transmission of financial shocks. CEPR Discussion Papers 7595, December 2009.

[30] Nobuhiro Kiyotaki and John Moore. Credit cycles. *Journal of Political Economy*, 105(2):211–48, April 1997.

[31] Nobuhiro Kiyotaki and Randall Wright. A search-theoretic approach to monetary economics. *American Economic Review*, 83(1):63–77, March 1993.

[32] Paul Krugman. The international finance multiplier. Mimeo, Princeton University, 2008.

[33] Pascal Michaillat. Do matching frictions explain unemployment? not in bad times. *American Economic Review*, 102(4):1721–50, June 2012.

[34] Dale T. Mortensen and Christopher A. Pissarides. Job creation and job destruction in the theory of unemployment. *Review of Economic Studies*, 61(3):397–415, July 1994.

[35] Maurice Obstfeld and Kenneth S. Rogoff. Exchange rate dynamics redux. *Journal of Political Economy*, 103(3):624–660, June 1995.

[36] Maurice Obstfeld and Kenneth S. Rogoff. *Foundations of International Macroeconomics*. MIT Press, 1996.

[37] Christopher A. Pissarides. *Equilibrium Unemployment Theory*. MIT Press, 2000.

[38] Robert Shimer. The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49, March 2005.

[39] Frank Smets and Raf Wouters. An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5):1123–1175, September 2003.

[40] Joseph E. Stiglitz and Andrew Weiss. Credit rationing in markets with imperfect information. *American Economic Review*, 71(3):393–410, June 1981.

[41] Oren Sussman and Joseph Zeira. Banking and development. CEPR Discussion Paper No. 1127, 1995.

[42] Etienne Wasmer and Philippe Weil. The macroeconomics of labor and credit market imperfections. *American Economic Review*, 94(4):944–963, September 2004.

[43] Eran Yashiv. Labor search and matching in macroeconomics. *European Economic Review*, 51(8):1859–1895, November 2007.

A Mathematical appendix

A.1 Optimal consumptions and the price index

Utility is derived from consumption

$$U = \mathbb{E}_0 \int_{t=0}^{\infty} \beta^t C_{i,t}$$

where $C_{i,t}$ denotes the individual consumption level in country i ($i = h, f$) at time t, $\mathbb{E}_0$ is the mathematical expectation conditional on information available at time 0, and $0 < \beta = (1+r)^{-1} < 1$ is the common discount factor. Utility is linear in consumption in order to analyze specific financial transmission channels independently of risk aversion effects. The consumption level $C_{i,t}$ is a Dixit-Stiglitz composite index of home and foreign goods

$$C_{i,t} = [\alpha^{\frac{1}{\lambda}} C_{h_{i,t}}^{\frac{\lambda-1}{\lambda}} + (1-\alpha)^{\frac{1}{\lambda}} C_{f_{i,t}}^{\frac{\lambda-1}{\lambda}}]^{\frac{\lambda}{\lambda-1}}$$

where $C_{j_{i,t}}$ stands for the consumption level of good j ($j = h, f$) in country i ($i = h, f$) at time t, and λ is the intra-temporal elasticity of substitution across the two goods. Note that the results hold for any particular value for this degree of substitution between home and foreign aggregate outputs.

Workers' budget constraint in the home country is simply $w = p_{h_h} C_{h_h} + p_{f_h} C_{f_h}$, where w is the wage. For tractability, it is assumed that only workers consume here but all agents could consume with quite similar results. The intra-temporal first-order conditions in the home country are therefore

$$(C_{h_h}) : \alpha^{\frac{1}{\lambda}} C_{h_h}^{-\frac{1}{\lambda}} [\alpha^{\frac{1}{\lambda}} C_{h_h}^{\frac{\lambda-1}{\lambda}} + (1-\alpha)^{\frac{1}{\lambda}} C_{f_h}^{\frac{\lambda-1}{\lambda}}]^{\frac{1}{\lambda-1}} = \Lambda p_{h_h}$$

$$(C_{f_h}) : (1-\alpha)^{\frac{1}{\lambda}} C_{f_h}^{-\frac{1}{\lambda}} [\alpha^{\frac{1}{\lambda}} C_{h_h}^{\frac{\lambda-1}{\lambda}} + (1-\alpha)^{\frac{1}{\lambda}} C_{f_h}^{\frac{\lambda-1}{\lambda}}]^{\frac{1}{\lambda-1}} = \Lambda p_{f_h}$$

$$(\Lambda) : w = p_{h_h} C_{h_h} + p_{f_h} C_{f_h}$$

where Λ is the Lagrangian multiplier. Solving gives

$$\bar{C}_{h_h} = \frac{\alpha w (p_{h_h})^{-\lambda}}{\alpha (p_{h_h})^{1-\lambda} + (1-\alpha)(p_{f_h})^{1-\lambda}} \quad \text{and} \quad \bar{C}_{f_h} = \frac{(1-\alpha) w (p_{f_h})^{-\lambda}}{\alpha (p_{h_h})^{1-\lambda} + (1-\alpha)(p_{f_h})^{1-\lambda}}$$

The Consumption-Based Price Index is defined as the least expenditure that buys a unit of the consumption index on which period utility depends (Obstfeld and Rogoff, 1996). It is computed first by substituting the optimal consumption levels in the initial utility function, and then by replacing the instantaneous income w by the index, denoted P, while C is equalized to 1 as P is the minimum expenditure per single unit of consumption

$$\Big[\alpha^{\frac{1}{\lambda}}\Big(\frac{\alpha P_h(p_{h_h})^{-\lambda}}{\alpha p_{h_h}^{1-\lambda}+(1-\alpha)(p_{f_h})^{1-\lambda}}\Big)^{\frac{\lambda-1}{\lambda}}+(1-\alpha)^{\frac{1}{\lambda}}\Big(\frac{(1-\alpha)P_h(p_{f_h})^{-\lambda}}{\alpha p_{h_h}^{1-\lambda}+(1-\alpha)(p_{f_h})^{1-\lambda}}\Big)^{\frac{\lambda-1}{\lambda}}\Big]^{\frac{\lambda}{\lambda-1}}=1$$

Rearranging gives the solution for P in the home country

$$P_h=[\alpha p_{h_h}^{1-\lambda}+(1-\alpha)(p_{f_h})^{1-\lambda}]^{\frac{1}{1-\lambda}}$$

The foreign price index expressed in domestic currency, $S_tP_{f,t}$, is constructed similarly but does not need to equal $P_{h,t}$ as preferences parameters (α and λ) are allowed to differ from one country to another. Further note that the price index is taken as given by a particular consumer since markets for final goods are competitive, but is endogenous at the aggregate level.

A.2 Individual behaviors and autarkic equilibrium

The worker-consumer problem is given here for general equilibrium understanding but the international propagation mechanisms are primarily driven by interactions between investors, bankers and entrepreneurs in the simplified sequential representation. In each period, workers are either unemployed and earn no revenue (in stage 2) or working for a given wage w that allows for consumption (in stage 3).[21] When an unemployed worker encounters an entrepreneur whose job offer matches his or her characteristics, he or she can either reject the offer and wait for a new job opportunity or accept the offer and earn w until an adverse shock arrives. Worker-entrepreneur relationships end at the exogenous separation rate s. Hence, the optimal stochastic value function $W_{i,t}$ of an unemployed worker of country i at time t satisfies the following recursive problem

$$W_{i,t}(\theta_{i,t},S_t)=\max_{accept,reject}\left\{\max_{C_{h_{i,t}},C_{f_{i,t}}}\left\{U_{(*)}+\beta(1-s)W_{3_{i,t+dt}}+\beta sW_{2_{i,t+dt}}\right\};\right.$$
$$\left.\beta\left[1-\theta_{i,t}q_L(\theta_{i,t})\right]W_{2_{i,t+dt}}+\beta\,\theta_{i,t}q_L(\theta_{i,t})\,W_{3_{i,t+dt}}\right\}$$

[21]Unemployment benefits, minimal consumption levels while being unemployed, job search costs for workers or valuation of leisure activities could have been added to the framework but none is critical for the current purpose.

$$(*) \quad \text{s.t.} \quad w_i \;=\; p_{h_{h,t}} C_{h_{i,t}} + S_t \, p_{f_{f,t}} C_{f_{i,t}}$$

where W_2 and W_3 are the value functions of workers in the respective stages 2 and 3 of the process described above, and where $p_{j_{i,t}}$ is the price of good j in country i and expressed in country i currency at time t.

The consumption index obtained above (Appendix A) allows re-expressing the individual budget constraint as $w_{i,t} = P_{i,t}\bar{C}_{i,t}$, where $\bar{C}_{i,t}$ is the optimal consumption basket in country i at date t. Therefore, dropping time and country subscripts, the simplified Bellman equations for a worker in the successive stages of the sequential process are

$$rW_2 = \theta q_L(\theta)(W_3 - W_2)$$

$$rW_3 = \frac{w}{P} + s(W_2 - W_3)$$

A similar problem for entrepreneurs gives the following Bellman equations

$$rE_1 = -c_E + q_C(\phi)(E_2 - E_1)$$

$$rE_2 = -\gamma_L + \gamma_L + q_L(\theta)(E_3 - E_2)$$

$$rE_3 = p - w - \rho_C + s(E_4 - E_3)$$

with E_1, E_2, E_3 the respective intertemporal values of entrepreneurs in stages 1, 2 and 3, c_E the search cost in stage 1, and γ_L the search cost in the recruitment stage (offset by the amount borrowed from the bank).

Similarly, for the commercial banks,

$$rB_0 = -c_B + q_F(\xi)(B_1 - B_0)$$

$$rB_1 = -\gamma_C + \gamma_C + \phi\, q_C(\phi)(B_2 - B_1)$$

$$rB_2 = -\gamma_L + \gamma_L + q_L(\theta)(B_3 - B_2)$$

$$rB_3 = \rho_C - \rho_F + s(B_4 - B_3)$$

where c_B and γ_C stand for bankers' search costs in stage 0 and stage 1 respectively, and where γ_L is offset by the capital provided by the investor.

Similarly, for the financial investors, with c_I their search cost in stage 0,

$$rI_0 = -c_I + \xi\, q_F(\xi)(I_1 - I_0)$$

$$rI_1 = -\gamma_C + \phi\, q_C(\phi)(I_2 - I_1)$$

$$rI_2 = -\gamma_L + q_L(\theta)(I_3 - I_2)$$

$$rI_3 = \rho_F + s(I_4 - I_3)$$

Free entry implies that, in equilibrium, $E_1 = 0$, $B_0 = 0$, and $I_0 = 0$. The first Bellman equation for each agent therefore gives their respective *backward* value one stage after entering the process as follows

$$\text{For entrepreneurs} \quad E_2 = \frac{c_E}{q_C(\phi)};$$

$$\text{For bankers} \quad B_1 = \frac{c_B}{q_F(\xi)};$$

$$\text{For investors} \quad I_1 = \frac{c_I}{\xi q_F(\xi)};$$

Free exit ($E_4 = 0$, $B_4 = 0$, and $I_4 = 0$) similarly gives the value in stage 3 from the last Bellman equation in each group. *Forward* values for stages 1 and 2 are then obtained recursively as

$$\text{For entrepreneurs} \quad E_3 = \frac{p - w - \rho_C}{r + s}, \quad E_2 = \frac{q_L(\theta)}{r + q_L(\theta)} E_3;$$

$$\text{For bankers} \quad B_3 = \frac{\rho_C - \rho_F}{r + s}, \quad B_2 = \frac{q_L(\theta)}{r + q_L(\theta)} B_3, \quad B_1 = \frac{\phi q_C(\phi)}{r + \phi q_C(\phi)} B_2;$$

$$\text{For investors} \quad I_3 = \frac{\rho_F}{r + s}, \quad I_2 = \frac{-\gamma_L + q_L(\theta) I_3}{r + q_L(\theta)}, \quad I_1 = \frac{-\gamma_C + \phi q_C(\phi) I_2}{r + \phi q_C;(\phi)}$$

Equalizing the backward and forward values for each agent finally gives the respective *equilibrium conditions* (1) to (3).

Alternatively, these equilibrium conditions could have been obtained from the fact that free entry implies a zero-profit condition which equalizes expected present-discounted costs and gains in equilibrium for each agent type. For instance, the following expression must hold for entrepreneurs

$$\mathbb{E}_0(T_1)\Big\{\int_0^{T_1}(-c_E)e^{-rt}dt + \mathbb{E}_{T_1}(T_2)\Big[\int_{T_1}^{T_2} 0e^{-rt}dt + \mathbb{E}_{T_2}(T_3)\int_{T_2}^{T_3}(p - w - \rho_C)e^{-rt}dt\Big]\Big\} = 0$$

where T_1, T_2, and T_3 follow Poisson processes, and is thus rewritten as

$$\Leftrightarrow c_E \int_0^{\infty}\int_t^{\infty} q_C(\phi)e^{-q_C(\phi)T_1} dT_1 e^{-rt}dt = (p-w-\rho_C)\int_0^{\infty}\int_{T_1}^{\infty}\int_{T_2}^{\infty}\int_t^{\infty} se^{-s(T_3-T_2)}dT_3 \times e^{-rt}\,dt\; q_L(\theta)e^{-q_L(\theta)(T_2-T_1)}\,dT_2\; q_C(\phi)e^{-q_C(\phi)(T_1-0)}\,dT_1$$

Solving yields $\frac{c_E}{r+q_C(\phi)} = \frac{p-w-\rho_C}{r+s}\frac{q_L(\theta)}{r+q_L(\theta)}\frac{q_C(\phi)}{r+q_C(\phi)}$ which simplifies as (1).

Finally, the Nash bargaining rule for the repayment ρ_F, $(1-\delta_F)(I_1-I_0) = \delta_F(B_1-B_0)$, together with the *backward* values for B_1 and I_1 then gives the equilibrium financial market tightness as

$$\bar{\xi} = \frac{1-\delta_F}{\delta_F}\frac{c_I}{c_B}$$

Recursively, the second Nash bargaining rule for the repayment ρ_C, $(1-\delta_C)(B_2-B_1) = \delta_C(E_2-E_1)$, together with the values of the agents at the time they meet and the previous value for $\bar{\xi}$, gives the equilibrium credit market tightness as

$$\bar{\phi} = \frac{1-\delta_C}{\delta_C} r \frac{c_B}{c_E}\frac{1}{q_F(\bar{\xi})}$$

Solving (1) to (5) gives the equilibrium labor market tightness $\bar{\theta}$ in (7)

The equilibrium market tightnesses (4), (5), and (7) are loglinearized as

$$\hat{\xi} = \hat{c}_I - \hat{c}_B$$

$$\hat{\phi} = (1-\eta_F)\hat{c}_B + \eta_F\hat{c}_I - \hat{c}_E$$

$$\hat{\theta} \approx \frac{q_L(\bar{\theta})}{\eta_L\bar{\gamma}_L}\Big\{\frac{p\hat{p}}{s} - [(1-\eta_F)\hat{c}_B + \eta_F\hat{c}_I]\bar{\kappa} - \frac{\bar{\gamma}_C\hat{\gamma}_C}{\bar{\phi}q_C(\bar{\phi})} - \hat{c}_E(1-\eta_C)\Big[\frac{\bar{\gamma}_C}{\bar{\phi}q_c(\bar{\phi})} + \frac{\bar{c}_E}{q_c(\bar{\phi})}\Big]\Big\} - \frac{\hat{\gamma}_L}{\eta_L}$$

$$\text{where}\quad \bar{\kappa} = \frac{\bar{c}_B}{q_F(\bar{\xi})} + \frac{\bar{c}_I}{\bar{\xi}q_F(\bar{\xi})} + \frac{\eta_C\bar{c}_E}{q_C(\bar{\phi})} - \frac{(1-\eta_C)\bar{\gamma}_C}{\bar{\phi}q_C(\bar{\phi})}\quad \text{and with}\quad \bar{r} = 0$$

where a hatted variable denotes the loglinear deviation from its steady-state value ($\hat{x} = \frac{x-\bar{x}}{\bar{x}}$), and where $\eta_L, \eta_C, \eta_F \in (0,1)$ are the respective matching function elasticities at equilibrium ($\eta_L \equiv -q_L'(\bar{\theta})\bar{\theta}/q_L(\bar{\theta})$, $\eta_C \equiv -q_C'(\bar{\phi})\bar{\phi}/q_C(\bar{\phi})$, and $\eta_F \equiv -q_F'(\bar{\xi})\bar{\xi}/q_F(\bar{\xi})$). Loglinearizing (6) further gives the unemployment rate response as $\hat{u} \approx -(1-\eta_L)(1-\bar{u})\hat{\theta}$. Note that $\bar{\kappa}$ is assumed positive with plausible values of the parameters henceforth so that negative financial shocks realistically raise the unemployment rate in the closed economy.

A.3 International setup and financial spillovers

Aggregate constraints

Each firm of country i produces one unit of the good in which the economy is specialized ($i = h, f$) and maximizes profits by determining the optimal division of this output unit between domestic sales $C_{i_{h,t}}$ and exports $C_{i_{f,t}}$, taking prices $p_{i_{h,t}}$ and $p_{i_{f,t}}$ and the exchange rate S_t as given.

In the two-country case, the equilibrium condition (1) for entrepreneurs in country i (expressed in domestic currency) is thus rewritten as

$$\frac{c_{E_i}}{q_C(\phi_i)} = \frac{q_L(\theta_i)}{r + q_L(\theta_i)} \frac{p_{i_h} C_{i_h} + S p_{i_f} C_{i_f} - w_i - \rho_{C_i}}{r + s_i}$$

With a labor force normalized to one and one unit produced per firm, the instantaneous output of country i is merely its contemporaneous employment rate $(1 - u_{i,t})$. It gives four aggregate constraints on goods as

$$(1 - u_{i,t}) = C_{i_{h,t}} + C_{i_{f,t}}, \quad i = h, f$$

In each period the country-specific income is either devoted to the pecuniary costs induced by search activities or consumed in the home and foreign goods. Assuming for simplicity that output and search costs are constant through time, the resource constraints expressed in domestic currency are

$$p_{i_{h,t}} C_{i_{h,t}} + S_t p_{i_{f,t}} C_{i_{f,t}} - \gamma_C N_{C_{i,t}} - \gamma_L N_{E_{i,t}} = p_{h_{h,t}} C_{h_{i,t}} + S_t p_{f_{f,t}} C_{f_{i,t}}$$

where the equilibrium values of N_C and N_E are respectively obtained when flows of bankers and entrepreneurs into and out of the market are equalized

$$(1 - N_{C_i}) s_i = \phi_i q_C(\phi_i) N_{C_i} \quad \text{and} \quad (1 - N_{E_i}) s_i = q_L(\theta_i) N_{E_i}$$

Similarly, the ratio of unmatched global financial investors at equilibrium is

$$\bar{N}_I = \frac{\bar{s}_h + \bar{s}_f}{\bar{s}_h + \bar{s}_f + \bar{\xi}_h q_F(\bar{\xi}_h) + \bar{\xi}_f q_F(\bar{\xi}_f)}$$

and roughly captures the amount of global excess liquidity at time t.

Impact of financial shocks (two-country case)

Just as in the closed economy case, solving loglinear (open-economy) versions of equations (1)–(3), replacing $\hat{\xi}_i = \hat{c}_{I_i} - \hat{c}_{B_i}$ and $\hat{\phi}_i = (1-\eta_F)\hat{c}_{B_i} + \eta_F \hat{c}_{I_i}$, and further simplifying $\hat{\gamma}_{C_i} = \hat{\gamma}_{L_i} = \hat{c}_{E_i} = \hat{w}_i = \hat{s} = \hat{r} = 0$, give the following expression for the domestic labor market tightness

$$\hat{\theta}_h \approx \frac{q_L(\bar{\theta}_h)}{\eta_L \bar{\gamma}_{L_h}} \Big\{ \bar{p}_{h_h}\bar{C}_{h_h}(\hat{p}_{h_h} + \hat{C}_{h_h}) + \bar{S}\bar{p}_{h_f}\bar{C}_{h_f}(\hat{S} + \hat{p}_{h_f} + \hat{C}_{h_f}) - [(1-\eta_F)\hat{c}_{B_h} + \eta_F\hat{c}_{I_h}]\bar{\kappa}_h \Big\}$$

$$\text{where} \quad \bar{\kappa}_h = \bar{s}_h \Big[\frac{\bar{c}_{B_h}}{q_F(\bar{\xi}_h)} + \frac{\bar{c}_{I_h}}{\bar{\xi}_h q_F(\bar{\xi}_h)} + \frac{\eta_C \bar{c}_{E_h}}{q_C(\bar{\phi}_h)} - \frac{(1-\eta_C)\bar{\gamma}_{C_h}}{\bar{\phi}_h q_C(\bar{\phi}_h)} \Big]$$

Then, loglinearizing the expression for the exchange rate (8) and given that $\bar{\xi}_h q_F(\bar{\xi}_h)\bar{N}_I = (1-\bar{N}_I)\bar{s}_h$, we have

$$\bar{S}\bar{p}_{h_f}\bar{C}_{h_f}(\hat{S} + \hat{p}_{h_f} + \hat{C}_{h_f}) = \bar{p}_{f_h}\bar{C}_{f_h}(\hat{p}_{f_h} + \hat{C}_{f_h}) - \bar{s}_h(1-\bar{N}_I)(1-\eta_F)(\hat{\xi}_h - \hat{\xi}_f)$$

Substituting into the previous equation thus gives

$$\hat{\theta}_h \approx \frac{q_L(\bar{\theta}_h)}{\eta_L \bar{\gamma}_{L_h}} \Big\{ \bar{p}_{h_h}\bar{C}_{h_h}(\hat{p}_{h_h} + \hat{C}_{h_h}) - \bar{s}_h(1-\bar{N}_I)(1-\eta_F)(\hat{c}_{I_h} - \hat{c}_{B_h} - \hat{c}_{I_f} + \hat{c}_{B_f}) + \bar{p}_{f_h}\bar{C}_{f_h}(\hat{p}_{f_h} + \hat{C}_{f_h}) - [(1-\eta_F)\hat{c}_{B_h} + \eta_F\hat{c}_{I_h}]\bar{\kappa}_h \Big\}$$

Finally, loglinearizing consumers' budget constraint as

$$\bar{p}_{h_h}\bar{C}_{h_h}(\hat{p}_{h_h} + \hat{C}_{h_h}) + \bar{p}_{f_h}\bar{C}_{f_h}(\hat{p}_{f_h} + \hat{C}_{f_h}) = \bar{w}\hat{w}$$

and given that $\hat{w} = 0$[22], the labor market tightness simplifies to

$$\hat{\theta}_h \approx -\frac{q_L(\bar{\theta}_h)}{\eta_L \bar{\gamma}_{L_h}} \Big\{ \Big[(1-\eta_F)\hat{c}_{B_h} + \eta_F\hat{c}_{I_h}\Big]\bar{\kappa}_h + \bar{s}_h(1-\bar{N}_I)(1-\eta_F)(\hat{c}_{I_h} - \hat{c}_{B_h} - \hat{c}_{I_f} + \hat{c}_{B_f}) \Big\}$$

where the first member in curly brackets is the direct financial transmission channel while the second is the expenditure-switching channel resulting from real exchange rate variations. The home and foreign labor tightness responses to asymmetric liquidity supply shocks are therefore respectively

[22] This simplification stems from the fact that workers' wage is exogenous in the model. Besides composition changes due to exchange rate variations following negative financial shocks (c_{I_i} and c_{B_i}), consumption will be thus reduced via greater unemployment (direct effect). Further feedback effects on real prices will be caused by a change in firms' profits in the long run rather than via wage reductions here. This does not seem a very restrictive assumption when studying the immediate impact of financial shocks in the present sequential model, because wage adjustments are probably lagged, so that the present direct effects would probably still outweigh the indirect effects in a dynamic version of the model. Moreover, workers here supply inelastic and country-specific labor similarly to low-wage workers whose minimum wage is indeed exogenous.

given by $\partial\hat{\theta}_h/\partial\hat{c}_{I_h}$ and $\partial\hat{\theta}_h/\partial\hat{c}_{I_f}$ (subsection 3.2), while responses to asymmetric shocks to bank capitalization cost are $\partial\hat{\theta}_h/\partial\hat{c}_{B_h}$ and $\partial\hat{\theta}_h/\partial\hat{c}_{B_f}$ (3.3).

Calibration parameters

matching efficiency	$\mu_F = \mu_C = 1; \mu_L = 1.1, 1.355, 1.5$
tightness elasticity	$\eta_F = \eta_C = 0.5; \eta_L = 0.66$
bargaining power	$\delta_F = 0.995; \delta_F = 0.5; \delta_L = 0.66$
search costs on financial markets	$c_I = c_B = 0.1$
search costs on credit markets	$\gamma_C = 0.1$
search costs on labor markets	$\gamma_L = 0.5$
separation rate	$s = 0.1$
riskfree rate	$r = 0.05$

Chapter 2

A Search Model of Bankers' Greediness and Bank Default

Abstract

This paper develops a multi-frictional yet tractable model in which bank default is endogenously determined, and related to bank size and bankers' behavior. By accounting for heterogeneity in entrepreneurs' productivity and information asymmetry at the expense of capital holders, moral hazard arises following a sectoral shock: bankers tend to choose investments that are more profitable in the short-run but whose risk is borne by the financiers. This 'risk-shifting' mechanism magnifies credit rationing in the economy, particularly for safe borrowers, and contributes to bank default since financial investors may prefer not to (re-)capitalize intermediaries as long as they cannot control for bankers' choices. A search model is especially appropriate to depict a non-walrasian environment in which some shocks may slow down fund-raising from sound borrowers.

JEL classification: C78, D82, G01, G21
Keywords: bank default, market frictions, credit rationing, matching theory, information asymmetry, moral hazard

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1 Introduction

Recent years have seen renewed interest for the study of financial markets and the banking sector. The 2007-2009 crisis has emphasized the need for a better understanding of the role of financial intermediaries, from banks' behavior to general equilibrium effects of credit crunch and regulation issues. The scope of investigation is still important as far as the interactions between three types of well-known but theoretically isolated aspects are concerned, namely, the structure of the banking sector *per se* (size, leverage, incentives), real macroeconomic outcomes of banking activities (credit rationing), and financial interconnections (systemic risk, financial regulation). Some of these aspects have been extensively studied, however bank default is still hardly explained in the literature.

This paper analyzes how a downturn in a specific real sector — the housing market for instance — may lead to mistrust about banks' investment choices, and consequently fuels bank default and credit rationing. First, heterogeneity in entrepreneurs' productivity is introduced, both at the sectoral and at the individual levels. Second, information asymmetry at the expense of capital holders about banks' investment opportunities — or equivalently about bankers' choices between alternative investments — is accounted for. A sectoral productivity shock creates a 'risk-shifting' incentive through which bankers over-invest in the entrepreneurial market hit by the downturn at the expense of the financiers. As the latter cannot control for bankers' choices but bear the investment costs, they would have preferred that the intermediaries invest in long-duration credit relationships whereas the bankers tend to favor short-run profitability.

This 'risk-shifting' mechanism has two major macroeconomic consequences. As sound financial intermediaries find it more difficult to raise funds, credit rationing is in turn magnified, even for viable entrepreneurs, since there are fewer banks able to finance investment projects. The introduction of financial and credit market frictions allows to stress out the potential inefficiency due to the fact that bankers' behavior thus contributes to excessive credit rationing in distressed times. Second, the bank default rate goes up as fewer financiers are willing to (re-)capitalize the banking sector following the shock. More precisely, I find that the combination of uncertainty and information asymmetry makes the bank default rate react to a sectoral productivity shock.

This paper applies the Diamond-Mortensen-Pissarides search and matching theory to financial markets, in the spirit of Wasmer and Weil (2004). First, this allows to capture some non-walrasian characteristics of financial shocks by creating periods of time in which even sound borrowers cannot raise funds following a shock. As long as under-capitalized banks offer less vacant credit lines to entrepreneurs, credit rationing is magnified. Second, assuming that screening credit applications from entrepreneurs is costly in terms of time and/or effort for banks, the search and matching framework allows to depict a potential misalignment between financial investors' and banks' interests following a shock. On the one hand, bankers tend to choose sectors in which productivity may be lower and investments riskier but in which the number of entrepreneurs looking for a loan is high so that finding an individually suitable entrepreneur is easy. On the other hand, capital holders would prefer long-duration credit relationships at the bank, even if they are risk-neutral, because they are not compensated for riskier investments or bank default in case of a sectoral productivity downturn while bear the cost of vacant credit lines. Finally, the Nash bargaining contract allows to get rid of costly state verification effects by expressing the rate of return on capital as a function of the observable surplus of the match, so that bankers' 'greediness' stems from the incentives they face instead of cheating.

The traditional literature have highlighted three main concerns related to the banking sector, but two of them are out of the scope of this paper. The first one is the liquidity problem due to the difference of maturity between banks' assets and liabilities, causing bank runs from depositors (Diamond and Dybvig, 1983), not studied here. The second is information asymmetry, creating both adverse selection and moral hazard distortions. Information shortages may concern either the banks (lack of information on potential borrowers), resulting in credit rationing (Stiglitz and Weiss, 1981), or the capital holders (lack of information about intermediaries' lending opportunities), leading to moral hazard from bankers themselves and sometimes called 'risk-shifting'.[1] While I account for firms' credit rationing, this paper only introduces information asymmetry at the expense of capital holders while banks do identify suitable entrepreneurs. A last issue in the banking literature is optimal contract design, given costly state verification *à la* Townsend

[1] See Allen and Gale (2000) for instance.

(1979). Here bankers' moral hazard inefficiency is due to the sequentiality of bankers' negotiations instead of costly monitoring.

Some recent macro-finance papers have built on this core literature to analyze the macroeconomic implications of financial disruptions. In particular, introducing financial disturbances that endanger the health of the intermediation sector in New Keynesian frameworks allows to discuss the need for alternative monetary policy rules or unconventional interventions. Cúrdia and Woodford (2010) emphasize two sources or "purely financial disturbances" that are able to reproduce the 2007 increase in credit spreads: on the one hand, some real resources are consumed in the process of originating loans, and, on the other hand, a quantity of loans is defaulted upon each period and this increases in the quantity of loans that is provided by the intermediaries. In this literature however, banks cannot default or default for exogenous reasons. In Gertler, Kiyotaki and Queralto (2011) for instance, financial intermediaries may default if they divert assets for personal gains. This requires a huge direct delinquency rate from bankers in steady-state and does not capture default stemming from riskier investment choices instead. Finally, papers on the systemic risk resulting from banks' interdependence generally take the first bank default as exogenous (Gai and Kapadia, 2010, Krause and Giansante, 2011). In addition to account for the two Cúrdia-Woodford sources of financial disturbances here, I provide an analytical framework which is particularly convenient to explore bank solvency issues, bank default, and credit rationing. Moreover, while both banks' liquidity problem and interconnections (and thus systemic risk) are out of the scope of this paper,[2] the search frictions allow to depart from a perfectly competitive and centralized market, in line with the evidence that banks' size, banks' network and limited anonymity matter for trade outcomes in the banking sector.[3]

In the rest of this paper, Section 2 presents the model, Section 3 analyzes the effects of a sectoral productivity shock and proves that bank default is affected only when uncertainty and information asymmetry are combined, Section 4 concludes.

[2]Margaretic and Pasten (2012) investigate bank default through sequential bank runs resulting from a signal about liquidity concerns in the first bank.

[3]See for instance Gabrieli (2011) for the role of banks' network effects or Afonso and Lagos (2012b) for the role of bilateral trade in the interbank market.

2 Model

2.1 Notations and sequence of events

Let us consider three types of agents, namely financial investors, bankers, and entrepreneurs, who are all infinitely-lived and risk-neutral. Let assume that the financial investors (alternatively, 'capital holders' or 'saving banks') are endowed in capital while neither the banks ('intermediaries' or 'lending banks') nor the entrepreneurs have proper wealth at the beginning of the sequence of events considered here. The intermediaries thus have to raise funds from the financial investors before opening credit lines to the entrepreneurs. It is assumed that an entrepreneur needs a unique indivisible credit line from a bank in order to produce. However, a bank contracts with a continuum of entrepreneurs. There is a number k of productive sectors across which entrepreneurs are not mobile.

The credit market is potentially frictional as a pool of entrepreneurs looking for a loan to launch a business and a pool of banks screening credit applications from entrepreneurs coexist at each period of time. The search theory provides a tractable representation of such credit market frictions — allowing to capture all friction degrees, including frictionless markets, and supported by the data.[4] Thus, let assume that a constant returns-to-scale matching function $m_C(N_{E^u_{k,t}}, N_{C^v_{k,t}})$ determines the flow of new credit relationships from the number $N_{E^u_{k,t}}$ of entrepreneurs looking for a loan in sector k and the number $N_{C^v_{k,t}}$ of vacant credit lines that bankers open to sector-k entrepreneurs at time t. Therefore, a sector-specific *credit market tightness* is defined as

$$\phi_{k,t} \equiv N_{E^u_{k,t}} / N_{C^v_{k,t}}$$

The instantaneous probability for an entrepreneur to get a loan is thus $q_C(\phi_{k,t}) = m_C(N_{E^u_{k,t}}, N_{C^v_{k,t}})/N_{E^u_{k,t}}$ and the instantaneous probability for a bank to fill a sector-k vacant credit line is $\phi_{k,t} q_C(\phi_{k,t}) = m_C(N_{E^u_{k,t}}, N_{C^v_{k,t}})/N_{C^v_{k,t}}$, with $\partial q_C(\phi_{k,t})/\partial \phi_{k,t} < 0$.

In addition, let consider that the financial market in which intermediaries raise funds is also characterized by potential search frictions.[5] New

[4]See Dell'Ariccia and Garibaldi (2005) and Craig and Haubrich (2006) for the evidence.

[5]This market can be thought of as an interbank market or as a private financial market through which large investors (re-)capitalize commercial banks. Bilateral trade is relevant

financial relationships are determined from a similar matching function $m_F(N_{L_t^u}, N_{C_t^u})$, increasing in the mass $N_{L_t^u}$ of financiers looking for investment opportunities and in the mass $N_{C_t^u}$ of credit lines that bankers would like to finance at time t. This allows to represent a non-walrasian financial market in distressed times — in the sense that there may be no price adjustments immediately able to clear the market — without excluding efficiency (with infinite matching rates) otherwise. The *financial market tightness* is defined as

$$\xi_t \equiv N_{C_t^u}/N_{L_t^u}$$

and gives the instantaneous probabilities $q_F(\xi_t) = m_F(N_{L_t^u}, N_{C_t^u})/N_{C_t^u}$ for a banker to raise funds and $\xi_t q_F(\xi_t) = m_F(N_{L_t^u}, N_{C_t^u})/N_{L_t^u}$ for a financier to capitalize a credit line at a commercial bank, with $\partial q_F(\xi)/\partial \xi < 0$. Note that, unlike the credit market tightness, the financial market tightness is not k-specific because of some information asymmetry at the expense of financial investors, specified further below.

A bank therefore accumulates credit lines, that are in three possible states: 'unfunded' — as soon as the bank finds it valuable to expand but is capital-constrained —, 'open' (or 'vacant') to applications from (sector-specific) entrepreneurs, and 'productive' — once matched with an entrepreneur. Individual productive entities (credit relationships) may separate. In this case, the entrepreneur becomes unmatched, *i.e.* starts looking for a bank again or exit, while the credit line turns 'vacant'. Moreover, both productive and vacant credit lines may terminate if the bank defaults. In this case, all credit lines either turn 'unfunded' — if the bank looks for recapitalization — or are simply destroyed — if the bank is shut down (exit), while the financial investors and entrepreneurs also become unmatched (have to search again or exit). Both the sector-specific separation probability, denoted s_k, and the bank default probability, d, are endogenous variables, associated with an optimal decision rule to described later on. Figure 1 sums up the sequential matching and destruction probabilities.

to depict such markets since the actors are neither atomic nor anonymous (See Afonso and Lagos, 2012a, 2012b, for the interbank market for instance). The market for bank deposits, which is likely to be more competitive than bilateral, is also characterized by long-term relationships so that search frictions might be relevant (Tripier, 2012). However since the model is expressed in real terms and is not aimed at explaining bank runs, an equity-like fund-raising market seems more relevant than a market for liquidity here.

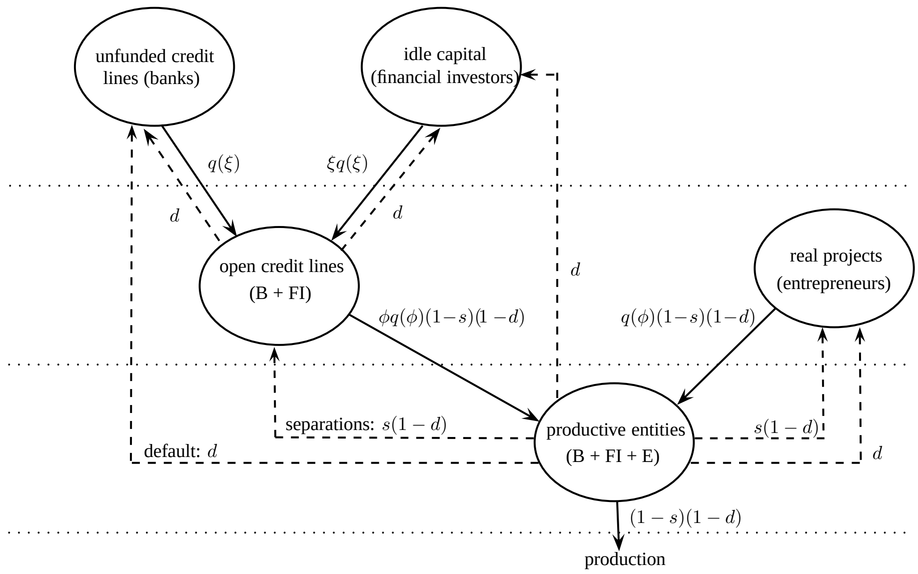

Figure 1: transition probabilities

Let further assume that productive entities generate output flows $A_{k,t}p_{j,k,t}$, where $A_{k,t}$ is sector-specific productivity, and $p_{j,k,t}$ is idiosyncratic productivity of the entrepreneur j in sector k, once he/she has obtained a credit line. $p_{j,k,t}$ is drawn every period and in advance of production from a time-invariant cumulative distribution function $F(\cdot)$ with positive support and density $f(\cdot)$. This output is used to reimburse the banker at a rate $\psi_{i,j,k,t}$ that is determined by a Nash bargaining rule that maximizes the surplus created by the match between the entrepreneur j and the bank i, in which $0 < \delta_C < 1$ is the bargaining power of the bankers.[6] The commercial bank receiving $\psi_{i,j,k,t}$ in turn pays back a return on capital services $\rho_{i,t}$, negotiated at the time of the match with the financial investor according to a similar Nash bargaining rule with bargaining powers $0 < \delta_F < 1$ and δ_F for capital owners and intermediaries respectively. At the time of the negotiation between the banker and the financier, the additional credit line is not associated with a particular entrepreneur yet, so that the financial repayment rate is not indexed by j, nor with a particular sector k which depends on the banker' later choice.

Finally, some pecuniary and non pecuniary flow costs are associated with

[6]The credit market is thus formally closer to an equity-like rather than to a debt-like contract. Thereby, Nash-bargaining is a reduced-form for an optimal contracting problem, not developed here for simplicity but still preventing from costly state verification issues.

search activities as follows. While financial investors are looking for suitable banks, they bear a flow cost c_I. This is the opportunity cost for — hence determines the willingness of — financiers capitalizing commercial banks instead of keeping capital idle, given discounting at a riskfree rate r and the transition probabilities.[7][8] Meanwhile banks bear a flow cost c_B while searching capital, which is non pecuniary since they do not have proper wealth ex ante. Symmetrically, entrepreneurs pay a non pecuniary flow cost c_E per period while looking for a loan. Finally, when a financier encounters a suitable bank, he/she finances c per period for a vacant credit line, while the effective application screening cost of this vacant credit line is $c'_{i,k,t}$. This cost depends on both the entrepreneurial sector and the size of bank i at time t, as banks' ability to screen credit applications may vary with their number of sector-k credit relationships.[9]

2.2 Particular case: constant idiosyncratic productivity and exogenous destruction rates

This economy follows Wasmer and Weil (2004) in that there are three agent types and sequential interactions into two search-and-matching frictional markets.[10] However a number of additional characteristics are also included here so as to stress out the behavior and the role of financial intermediaries:

(i) time-varying idiosyncratic productivity (p) and two optimal destruction rules for individual separations (s) and a bank default rate (d),

(ii) two Nash bargaining rules, *i.e.* the two endogenous repayments ρ and ψ,

(iii) several relationships at each bank so that bank optimal size is key,

[7]Other asset types in which financial investors could invest in could be easily introduced without changing the results. Since we focus on the role of intermediaries' capital constraints here, the financiers just decide to capitalize bank activities or to exit without loss of generality.

[8]With a model in real terms, c_I can be either pecuniary or a utility cost interchangeably.

[9]If c'_k is concave, banks tend to specialize in some entrepreneurial activities, whereas if c'_k is convex they tend to diversify their vacancies across sectors. Both cases are considered here.

[10]While Wasmer and Weil have bankers, entrepreneurs, and workers, with frictions in the credit and labor markets, I focus on the behavior of bankers as financial intermediaries, so that the frictions are on the financial and credit markets here.

(iv) entrepreneurs' heterogeneity and information asymmetry between banks and financiers so that the amount given by the financiers while financing vacant credit lines c is not equal to the effective cost of vacancies at the bank c'_k. A possible interpretation is that only banks observe the characteristics of the entrepreneurial sectors — *i.e.* their productivity and/or the tightness, hence the search duration before a conclusive match — so that financiers have to choose between capitalizing bankers given this information asymmetry or keeping their capital idle. Alternatively, the sectoral characteristics are common knowledge but the financiers cannot control for bankers' sectoral choices while opening new credit lines once the financial match is done (no specific contract).[11]

In a deterministic economy, with constant productivity and exogenous destruction rates, but maintaining (ii)-(iv), Wasmer and Weil (2004)'s methodology can be extended quite easily to solve the model such that, given free entry,

(i) the equilibrium financial and credit market tightnesses are given by

$$\bar{\xi} = \frac{1-\delta_F}{\delta_F}\frac{c_I}{c_B} \quad \text{(financial market tightness), and recursively} \tag{1}$$

$$\bar{\phi}_k = \frac{1-\delta_C}{\delta_C}\Big[\frac{c_B}{c_E}\frac{r+d}{q_F(\bar{\xi})} + \frac{c'_k - c}{c_E}\Big] \quad \text{(credit market tightness)} \tag{2}$$

(ii) the equilibrium repayment rates are the solution to the pair of equations

$$\psi = \frac{\delta_C A A_k p_k - (\rho + c - c'_k)[r + s_k(1-d) + d](1-\delta_C)}{r + s_k(1-d) + d + \delta_C \phi_k q_C(\phi_k)(1-s_k)(1-d)} \tag{3}$$

$$\rho = \delta_F - (\delta_F c'_k - c)\frac{r + d + s_k(1-d)}{\phi_k q_C(\phi_k)(1-s_k)(1-d)} \tag{4}$$

(iii) an equilibrium condition for each agent type — a credit creation condition for banks, a bank capitalization condition for financiers, and a search condition for entrepreneurs — can be derived analytically (see Appendix)[12]

[11] In this case $f(\cdot)$ can be known by financial investors as long as they are not able to discriminate individual entrepreneurs so that they have to intermediate their investments.

[12] Similarly to firms' job creation condition in the standard labor market literature.

Equation (1) expresses that the financial market tightness — *i.e.* the ratio of the number of banks willing to raise funds over the number of units of capital provided financial investors to the banking sector — increases in financial investors' costs c_I relatively to banks' costs c_B and in banks' (relatively to financial investors') bargaining power $(1-\delta_F)$. The credit market tightness in (2) similarly depends on entrepreneurs' and banks' search costs and bargaining powers. It also increases in the average time that banks need to raise funds on the financial market $(1/q_F(\bar{\xi}))$, in the bank default rate d as this corresponds to fewer banks for a given number of entrepreneurs, and in the riskfree rate r which is the opportunity cost associated with vacant credit lines.[13]

Finally, note that bankers make a trade-off when deciding the sector to which they will open new credit lines. In equilibrium a no-arbitrage condition must hold so that the asset value of opening new credit lines must be equalized across sectors. However, following a negative sectoral productivity shock, bankers may prefer to reallocate their vacant credit lines to the sector hit by the shock even it this sector has low productivity because entrepreneurs looking for a loan in this sector are numerous — hence the sectoral credit market tightness higher and bankers' search duration shorter — instead of investing in high-productivity sectors. However, this will not be sufficient to observe a misalignment between bankers' and financial holders' interests as both find it profitable that the credit line becomes productive rapidly (see Section 3). Thus introducing some source of uncertainty in the model is necessary to make bankers' moral hazard emerge.

2.3 Random idiosyncratic productivity and endogenous separations and default rates

Let us now write the full model and characterize the equilibrium when idiosyncratic productivity is random and the destruction rates are endogenous. Time is discrete. Figure 2 describes the timing of events for capitalized banks: firms' productivity is drawn every morning, then potential separations from existing credit relationships at the bank and potential bank de-

[13]If there were only one entrepreneurial sector in the economy, no bank default nor variable size, equation (2) would be the same than in my first chapter, and would further collapse to Wasmer and Weil (2004)'s credit market tightness if there were only one Nash rule.

faults are determined, according to optimality rules which will be described later on. Production then occurs according to the number of remaining filled credit lines. New credit relationships at the end of the day become productive from the next day onward.

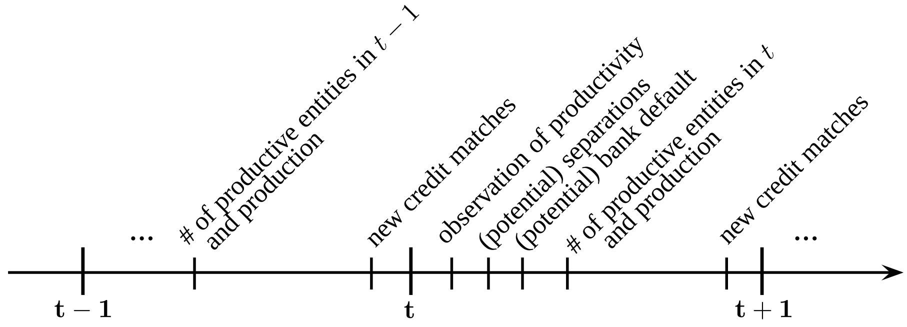

Figure 2: Timing of Events

Surplus sharing

The credit repayment rate that shares the (gross) surplus created by the match between an entrepreneur and a vacant credit line is given by a Nash bargaining rule (dropping i and j subscripts) as

$$\psi_{k,t} = \arg\max(C^p_{k,t} - C^v_{k,t})^{\delta_C}(V^p_{k,t} - V^u_{k,t})^{(1-\delta_C)} \tag{5}$$

where $V^p_{k,t}$, respectively $V^u_{k,t}$, denotes the value function of a sector-k entrepreneur who is producing, respectively looking for a loan (unmatched), at time t, $C^v_{k,t}$, respectively $C^p_{k,t}$, is the value function of a credit line which is vacant, respectively productive, and where $0 < \delta_C < 1$, respectively $1 - \delta_C$, is the bargaining power of the bank, respectively of the entrepreneur, on the credit market.[14]

Given the search costs and the transition probabilities given in Section 2.1, the Bellman equations standing for sector-k entrepreneurs — while unmatched and matched/productive respectively — are given by

[14]This Nash bargaining rule gives tractable analytical results because the agents are risk-neutral here, however the same steady-state could be obtained with risk-averse agents by replacing this equation by a "surplus-splitting bargaining rule" ensuring that the agents exactly get δ_C and $1 - \delta_C$ percent of the surplus at any point in time (Merz (1995), Andolfatto (1996), Cooley and Quadrini (1999)).

$$V_{k,t}^{u} = -c_E + \beta E_t \left\{ q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1}) \int_{p_{k,t+1}^R}^{\infty} \frac{V_{k,t+1}^{p}(p)f(p)}{1-F(p^R)}dp \right\}$$
$$+\beta E_t \left\{ [1-q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1})]V_{k,t+1}^{u} \right\}$$

$$V_{k,t}^{p}(p) = A_{k,t}p_{k,t} - \psi_{k,t} + \beta E_t\{[s_{k,t+1}(1-d_{t+1}) + d_{t+1}]V_{k,t+1}^{u}\}$$
$$+\beta E_t \left\{ (1-s_{k,t+1})(1-d_{t+1}) \int_{p_{k,t+1}^R}^{\infty} \frac{V_{k,t+1}^{p}(p)f(p)}{1-F(p^R)}dp \right\}$$

where p_{ikt}^R is the optimal reservation level for sector-k entrepreneurs' idiosyncratic productivity chosen by bank i at time t, such that matches producing $p_{ijkt} < p_{ikt}^R$ are not profitable and terminate (the optimal separation rule is made explicit further below).[15] The value functions for (sector-k) credit lines, which are respectively unfunded, vacant, and productive, are given by

$$C_{k,t}^{u} = -c_B + \beta q_F(\xi)C_{k,t+1}^{v} + \beta[1-q_F(\xi)]C_{k,t+1}^{u}$$

$$C_{k,t}^{v} = c - c_{k,t}' + \beta E_t\{[1-\phi_{k,t}q_C(\phi_{k,t})(1-s_{k,t+1})](1-d_{t+1})C_{k,t+1}^{v} + d_{t+1}C_{k,t+1}^{u}\}$$
$$+\beta E_t \left\{ \phi_{k,t}q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1}) \int_{p_{k,t+1}^R}^{\infty} \frac{C_{k,t+1}^{p}(p)f(p)}{1-F(p^R)}dp \right\}$$

$$C_{k,t}^{p}(p) = \psi_{k,t} - \rho_t + \beta E_t \left\{ (1-s_{k,t+1})(1-d_{t+1}) \int_{p_{k,t+1}^R}^{\infty} \frac{C_{k,t+1}^{p}(p)f(p)}{1-F(p^R)}dp \right\}$$
$$+\beta E_t\{s_{k,t+1}(1-d_{t+1})C_{k,t+1}^{v}\} + \beta E_t\{d_{t+1}C_{k,t+1}^{u}\}$$

Symmetrically, the repayment rate ρ_t that shares the financial surplus from investors' and bankers' match is given by a second Nash rule as

$$\rho_t = \arg\max(I_t^v - I_t^u)^{\delta_F}(C_{kt}^v - C_t^u)^{(1-\delta_F)} \tag{6}$$

[15] Similar Bellman equations when idiosyncratic productivity is stochastic, the separation rate is endogenous and time is discrete can be found in Krause and Lubik (2007) for instance.

where δ_F is the bargaining power of investors in the financial market[16], and where the value functions I_t^u and I_t^v of financial investors, respectively looking for banking opportunities and financing vacant credit lines, are

$$I_t^u = -c_I + \beta \xi q_F(\xi) I_{t+1}^v + \beta[1 - \xi q_F(\xi)] I_{t+1}^u$$

$$\begin{aligned} I_t^v = -c + \beta E_t\{ & d_{t+1} I_{t+1}^u + (1-d_{t+1})[\phi_{kt} q_C(\phi_{kt})(1-s_{kt+1}) I_{t+1}^p \\ & +(1-\phi_{kt} q_C(\phi_{kt})(1-s_{kt+1})) I_{t+1}^v]\} \end{aligned}$$

In addition, investors' value function while credit lines get productive is

$$I_t^p = \rho_t + \beta E_t\{d_{t+1} I_{t+1}^u + (1-d_{t+1})[s_{kt+1} I_{t+1}^v + (1-s_{kt+1}) I_{t+1}^p]\}$$

Again, the amount c that is given by the financiers to bankers in order to finance a vacant credit line is not sector-specific since application screening is the competence of commercial banks and the motive for intermediation here. Because of the information asymmetry, c is generally different from the effective application screening cost c'_k derived below. Moreover, because bankers' sectoral choice is taken after the financial bargaining, neither the financial repayment rate, ρ, nor the financiers' value functions are indexed by k. However, the credit surplus is observable so that bankers cannot cheat on the financial repayment once the credit line becomes productive.[17]

Bank screening technology and optimal size

A bank wants to expand as soon as creating new productive credit lines increases its expected present-discounted profits net of search costs. Since every credit line is 'vacant' before being 'productive' here, a sufficient condition to determine an optimal bank size is to consider that the marginal cost of vacant credit lines depends on the size of bank i. Let us assume that the application screening cost is increasing in the number $N_{C_{ikt}^v}$ of sector-k

[16]Since financial investors and bankers decide how to share the financial surplus at the time they meet, *i.e.* before the banker-entrepreneur negotiation, financial investors know that they will get a fraction $\tilde{\delta}_F$ of the net total surplus, and negotiate backward a share $\delta_F = \tilde{\delta}_F/[1-(1-\tilde{\delta}_F)(1-\delta_C)]$ of the financial surplus.

[17]Without costly state verification issues, bankers' 'greediness' differs from delinquency.

vacant credit lines at bank i as

$$c_k = \kappa (N_{C^v_{ikt}})^{\epsilon}$$

so that banks are more and more efficient in screening credit applications from sector-k entrepreneurs if $\epsilon < 1$, less and less otherwise.[18] Given the law of motion for $N_{C^v_{ikt}}$, further derivation (detailed in Appendix) gives the equilibrium flow cost of an additional vacant credit line opened to sector-k entrepreneurs as

$$c'_{kt} = \frac{\kappa}{q(\phi_{kt})} \frac{\epsilon N^{1-\epsilon}(N_{C^v_{kt}})^{\epsilon}}{(1 - s_{kt+1})(1 - d_{t+1}) N_{E^u_{kt}}} \tag{7}$$

where N is the total number of banks.

Optimal destruction rules

- Individual credit relationship separations

It is optimal for banks to terminate the credit relationship with a particular productive entrepreneur if the continuation value of remaining matched is smaller than the continuation value of having a vacant credit line, *i.e.* if

$$C^p_{kt}(p) < C^v_{kt}$$

Following the labor market literature, this rule is equivalent to determine an endogenous reservation threshold for idiosyncratic productivity that banks require from the entrepreneurs, such that matches producing $p_{ijkt} < p^R_{ikt}$ are not profitable and separate. After some computations (given in Appendix), substituting the time-varying expression for the Nash-bargained credit repayment rate into $C^p_{ikt}(p) - C^v_{kt} = 0$, the optimal threshold is

$$p^R_{ikt} = \frac{1}{A_{kt}} \Big[\rho_{it} + (c - c'_{ikt}) - \frac{c_E}{q_C(\phi_{kt})} \frac{1 - \delta_C \phi_{kt} q(\phi_{kt})}{1 - \delta_C} \Big] \tag{8}$$

As one would expect, this threshold decreases in the sectoral productivity level A_{kt}: bankers require from entrepreneurs a higher idiosyncratic productivity to compensate for a low sectoral productivity, everything else equal. It increases in the financial rate ρ that bankers have to pay back to investors.

[18] Similarly, Rotemberg (2006) considers that firms have recruitment costs that are concave in the number of job vacancies and defines the equilibrium as if the costs were convex.

It decreases in both search cost c'_{ikt} and c_E on the credit market as a rise in these costs increases the continuation value of remaining matched today. It increases in the sectoral credit market tightness since, when there are many searching entrepreneurs relatively to bankers, the search duration for bankers is short and this tends to increase the minimum idiosyncratic productivity required from entrepreneurs.

Therefore the sector-specific separation rate s_k is given by

$$s_{kt} = s_k(p^R) = F(p^R) = \int_0^{p^R_{kt}} f(p)dp \qquad (9)$$

- Bank default

Symmetrically, a bank defaults if the stockholders' continuation value of remaining matched with this bank is smaller than the continuation value of having idle capital — given the costs of financing vacant credit lines and the expected duration before finding a suitable entrepreneur in particular —, *i.e.* if

$$I^v_t < I^u_t, \quad \text{where } I^u_t = 0 \text{ by free entry.}$$

However it is not possible to determine a similar threshold of idiosyncratic productivity in order to infer the default rate, even though a time-varying expression for the financial repayment rate is determined. Therefore, it is necessary to consider the equilibrium default rate directly, stemming from $I^v_t = 0$ and the Nash rule for the financial repayment rate, so as to obtain

$$E_t d_{t+1} = 1 + \frac{\xi_t q_F(\xi_t)}{c_I}\left[\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}c_E\phi_{kt} - c\right] \qquad (10)$$

Bank default decreases in c_I since higher costs while financial investors are searching for a suitable bank increases the value of remaining matched today, *i.e.* not impose on the bank to default. The same rationale holds for the cost c borne by the financiers as long as the credit line is vacant. On the contrary, a higher matching rate $\xi q_F(\xi)$ increases financial investors' outside options so that the value of remaining matched with the same bank decreases, leading more banks to default. The higher the bargaining power $\tilde{\delta}_F$ of the investors on the financial market vis-à-vis the bankers $(1-\tilde{\delta}_F)$, the higher the bank default rate since bargaining with a new bank becomes more profitable.

Finally, when the separation rate increases, the bank default rates increases along with the credit market tightness.

Equilibrium credit rationing

Finally, let us define some aggregate quantities that can easily be derived at equilibrium. In particular, credit rationing in a productive sector is defined as the number of entrepreneurs within the sector who are currently — effectively but unsuccessfully — looking for a loan. By normalizing the total number of entrepreneurs to unity and equalizing flows into and out of the pool of unmatched entrepreneurs, we get an equilibrium level of credit rationing in sector k as

$$N_{E_k^u} = \frac{s_k(1-d)+d}{s_k(1-d)+d+q_C(\phi_k)(1-s_k)(1-d)} \tag{11}$$

More separations in sector k or bank default increase credit rationing in sector k whereas a higher matching probability $q_C(\phi_k)$ for sector-k entrepreneurs decreases credit rationing in sector k.[19]

Moreover, the number of productive entities in each sector is $N_{E_k^p} = 1 - N_{E_k^u}$ and is also equal to the number $N_{C_k^p}$ of productive credit lines, given by

$$N_{C_k^v} = \frac{s_k(1-d)+d}{\phi_k q_C(\phi_k)} N_{C_k^p} \tag{12}$$

Recursively, the number $N_{C_k^u}$ of unfunded credit lines (bank projects) is

$$N_{C^u} = \frac{[d+\phi_k q_C(\phi_k)(1-s_k)(1-d)]N_{C_k^v} - s_k(1-d)N_{C_k^p}}{q_F(\xi)} \tag{13}$$

Regarding financial investors, the number of capital units involved in production is equal to the number of productive entities summed up across sectors ($N_{I^p} = \sum_k N_{C_k^p}$), while the number of capital units financing vacant credit lines is equal to the sum of vacant credit lines ($N_{I^v} = \sum_k N_{C_k^v}$). As we know from the transition matrix that the law of motion for $N_{I_{t+1}^v}$ is[20]

$$N_{I_{t+1}^v} = (1-d_{t+1})[1-\phi_{kt}q_C(\phi_{kt})(1-s_{kt})N_{I_t^v} + s_{kt}(1-d_{t+1})N_{I_t^p} + \xi_t q_F(\xi_t)N_{I_t^u}$$

[19] If $d = 0$, this expression reminds the Beveridge curve in the labor market literature.

[20] The law of motion could easily be generalized to allow for other types of financial assets, including bonds with an endogenous riskfree rate r for instance, without changing the results.

the equilibrium number of financiers looking for a banking investment is

$$N_{I^u} = \frac{[d + \phi_k q_C(\phi_k)(1 - s_k)(1 - d)]N_{I^v} - s_k(1 - d)N_{I^p}}{\xi q_F(\xi)} \tag{14}$$

This is the mass of capital that is available for the banking sector but not immediately allocated to a particular bank because of the financial friction stemming from the information asymmetry between the financiers and individual banks.

Equilibrium

Given free entry, $i.e$ with $V_k^u = 0$, $C_k^u = 0$, and $I^u = 0$, the Nash bargaining rules (5) and (6) imply that (see Appendix for details)

(i) the equilibrium financial and credit market tightnesses are similar to (1) and (2) in the deterministic case despite time-with varying idiosyncratic productivity and endogenous destruction rates here,

(ii) the agents' equilibrium conditions (for searching for a loan, creating an additional credit opportunity, and capitalizing banks) are also similar to the deterministic case

(iii) in addition to (3) and (4) that still hold in equilibrium, time-varying expressions for the two repayment rates can be derived as

$$\psi_{kt} = \delta_C A_{kt} p_{kt} + (1 - \delta_C)(c - c'_{kt} + \rho_t) + \delta_C c_E \phi_{kt} \tag{15}$$

$$\rho_t = \frac{\tilde{\delta}_F}{1 - \tilde{\delta}_F} \frac{1}{1 - \delta_C}(A_{kt} p_{kt} - \psi_{kt} + c_E \phi_{kt}) - c \tag{16}$$

Further, since the repayment rate is linear in the idiosyncratic productivity level and since credit relationships producing $p_{jk} < p_{ik}^R$ separate, the average repayment rate observed at time t on the credit market is given by

$$E_t[\psi_{ijkt}(p)|p_{jkt} \geq p_{ikt}^R] = \delta_C A_{kt} E_t(p_{jkt}|p_{jkt} \geq p_{ikt}^R) + \delta_C c_E \phi_{kt} + (1 - \delta_C)[\rho_t + (c - c'_{ikt})]$$

Equilibrium is thus characterized by equations (1),(2), (7)–(16) and the set of unknowns $\{\xi, \phi_k, \psi_k, \rho, c'_k, p_k^R, s_k, d, N_{E_k^u}, N_{C_k^v}, N_{C^u}, N_{L^u}\}$. The next Section analyzes the effects of a permanent sectoral productivity shock.

3 Effects of a sectoral productivity shock

3.1 Case 1: No information asymmetry

As a first case, let suppose that there are several entrepreneurial sectors but no information asymmetry in the economy. The financiers know (or can control for) the particular sectors in which the bankers open new credit lines as well as the size of the bank so that they pay the exact amount c'_k that is necessary for each vacancy. For further simplicity, let assume that $c'_k = c$ for all credit lines as bank size becomes irrelevant in this particular case.

Let consider a negative sectoral productivity shock. As A_1 falls, the threshold for idiosyncratic productivity that bankers require from entrepreneurs rises (equation (8)). Since the distribution $f(\cdot)$ is time-invariant, more entrepreneurs in sector 1 fall below the new threshold and more separations occur by (9). Therefore credit rationing increases in sector 1 by (11). However, because the cash flow ψ_1 from the entrepreneurs to the bankers falls with the surplus by (15) — while separations are not constrained — bankers increase the number of vacant credit lines in sector-1 to increase their profits. Thus the credit market tightness in sector 1, ϕ_1, is unaffected by the shock. The financial repayment ρ also decreases (16) so that, given that there is no information asymmetry, the financiers capitalize more vacant credit lines to offset the profit loss on individual relationships. This is the case because they know the cost associated with each vacancy and because there is no costly state verification (the surplus is known every period so that the banks cannot cheat on the repayment to the financiers even though ρ is not sector-specific). The financial market tightness ξ is thus unchanged as well. (See Appendix A.8.)

If the economy is deterministic (constant idiosyncratic productivity), the effects on the two repayment rates ϕ_1 and ρ are identical, however, all the other variables — including the sector-1 credit rationing and credit market tightness — are unaffected by the shock.[21] Thus the response of credit rationing to a sectoral productivity shock is due to the presence of uncertainty, captured by random draws from $f(\cdot)$ here. Since there is no risk premium in the model — that would bankers to increase ψ_1 above its current value

[21] On the contrary, in the (deterministic) Wasmer and Weil (2004)'s economy, a fall in output affects the labor market tightness — and a financial accelerator stemming from the existence of two search frictions magnifies the effect — because workers' wage is exogenous.

following the shock —, bankers rise their reservation threshold, so that credit relationships are more fragile (their duration is subjected to changes in individual productivity) in the stochastic economy. However, as long as there is no information asymmetry at the expense of financial investors, the ability of banks to raise funds is the same than before the shock and bank default is unaffected, even though the turn-over of credit relationships is more important in the sector hit by the shock.

3.2 Case 2: information asymmetry and moral hazard

The simplest way to capture information asymmetry in this model is to assume that $c'_k \neq c$ such that the financiers pay c for each vacant credit line whatever the sector. If c'_k differs from c but is exogenous, the effects of a sectoral productivity shock is the same than if $c'_k = c$. Therefore it is assumed that this cost depends on the number of vacant credit lines at the bank, $c_k(N_{C^v_{ikt}}) = \kappa(N_{C^v_{ikt}})^{\epsilon}$. To this respect, bank size matters but one could imagine other determinants of c'_k so that information asymmetry would hold with constant bank size.

When idiosyncratic productivity is constant (deterministic economy), the effects of a sectoral productivity shock are identical whether information is asymmetric or not and whether there the bank default rate is exogenous or not. More precisely the two repayment rate increase following a negative sectoral productivity shock, while all the other variables — including credit rationing — are not affected (See Appendix A.9.1). Information asymmetry is thus irrelevant because the variation in the credit repayment rate captures the fall in sectoral productivity and, since there is no costly state verification, the sectoral choice made by bankers does not affect the financiers' earnings.

On the contrary, the combination of an endogenous size of banks and an optimal threshold for idiosyncratic productivity (stochastic economy) makes information asymmetry matter, because it allows bankers to benefit from the high profitability of short-duration credit relationships at the expense of the financiers. The sign of the responses differs if bankers' application screening costs are concave ($\epsilon < 1$) or convex ($\epsilon > 1$). In particular, the credit market tightness, ϕ_k, goes up if the marginal cost of opening new vacant credit lines is convex since the rise in c'_k makes bankers better off if they stop searching for new entrepreneurs. On the contrary, if the cost is concave (and low enough), the tightness falls because banks have more incentive to expand their size as long as the entrepreneurs meet the higher idiosyncratic productivity threshold (See Appendix A.9.2). However the financial market tightness ξ is unchanged so that banks' expansion in the sector hit by the shock reduces the number of vacant credit lines for other sectors in the economy (everything else equal).

Overall, bankers benefit from the fact that (*i*) a higher threshold for idiosyncratic productivity increases the credit repayment rate relatively to the deterministic economy, despite lower sectoral productivity, (*ii*) the period during which credit lines are vacant is less costly, either because the match is quicker when relatively more entrepreneurs are looking for a loan (c'_k convex) or because application screening is more and more effective (c'_k concave). Nonetheless, the drawback of a negative sectoral productivity shock is that separations are more frequent (since $f(\cdot)$ is time-invariant but the threshold is higher, less entrepreneurs are viable next periods). As the financiers have to pay a fix amount per vacancy each time credit relationships separate, they would prefer that the banks invest in sectors with long-duration credit relationships instead. Following the shock, bankers thus become "greedy" in the sense that they make investment choices that are neither aligned with the financiers' interests — with a higher risk since the entrepreneurs who are solvent today are more likely to fall below the reservation threshold tomorrow — nor profitable for the economy — as they provide relatively more loans to the low-productivity sector at the expense of the 'good' sectors.[22]

The combination of information asymmetry and uncertainty in the economy creates a 'risk-shifting incentive' for bankers, with major macroeconomic consequences as the initial sectoral productivity shock is transmitted through credit rationing to the other sectors in the economy. Besides, bank default only reacts to sectoral downturns under these two conditions, while it remains unaffected as long as there is either information asymmetry or uncertainty in the economy (Appendix A.9.2). When a bank defaults, the costs of vacant credit lines that never get productive or have become productive for a short period of time are borne by the financiers while the bankers only stop making profits. Since there is no bailout of the defaulting banks here, more of the entrepreneurs who are individually creditworthy or belong to the 'good' sectors of the economy get credit constrained since all credit relationships are destroyed in the banks which are shut down. Finally, in

[22] A social planner's problem would be necessary to assess the welfare loss more precisely.

case the application screening costs are concave, the size of the banks which do not default but invest in short-run profitability at the expense of safer borrowers increases following the shock. Therefore bank default not only goes up because of the risk-shifting mechanism but is also likely to increase in turn the inefficiency resulting from bankers' moral hazard.[23]

4 Conclusion

This paper develops a multi-frictional yet tractable model in which bankers' behavior crucially affects the macroeconomic outcomes, including credit rationing and bank default. By introducing alternative investment choices for bankers and information at the expense of financial investors, a risk-shifting mechanism arise. Following a sectoral productivity shock, bankers tend to choose investments that are riskier, because they are more profitable in the short run, even if they are not aligned with the financiers' interests nor suitable for the economy as credit constraints become more binding for high-productivity borrowers.

Moreover, it allows to determine an endogenous bank default rate and shows that the sectoral productivity shock affect the bank default rate if and only if information asymmetry and uncertainty are combined in the economy. Because of entrepreneurs' heterogeneity and information asymmetry, financial investors cannot observe real investment opportunities. Therefore, if a negative sectoral productivity shock arrives but financial investors cannot control for banks' investment choices, they may prefer not to (re-)capitalize the banking sector even though the intermediaries are still able to appraise the idiosyncratic productivity of potential borrowers within or outside of the sector hit by the shock.

The search and matching environment is especially appropriate to depict financial markets that are almost frictionless in normal times but can remain frictional for an extended period of time in case of a disruption. Hence, distressed times are characterized by a significant slow down of fund-raising from sound borrowers, and credit rationing is magnified for all sectors in the

[23]This may also increase the systemic risk or the cost of a public bailout but these issues are out of the scope of this paper. Bank default inefficiently magnifies credit rationing by destructing the relationships with viable entrepreneurs at the defaulting banks. An extension of the model in which banks' balance sheets would be interrelated could however provide a rationale to the systemic risk with the same underlying mechanism.

economy.

Some extensions of the model could add to the results presented here. For instance, considering the problem of a social planner that is able to discriminate suitable credit relationships (similarly to banks) but suffers from capital losses in case of default (similarly to capital holders) would allow to assess the size of the inefficiency and to determine the desirability of policy interventions that would provide liquidity access to the banking sector. Furthermore, including depositors in the model is not likely to change the main predictions. In the absence of deposit insurance, threats to the solvency of a particular bank would give a signal for a run, so that the bank would effectively defaults under the conditions presented here. In the presence of a deposit insurance scheme or a public bank bailout conditional on the departure of the shareholders, a drop in the stock value of the bank would replace the bank default in practice.

Bibliography

[1] Gara Afonso and Ricardo Lagos. Trade dynamics in the market for federal funds. Unpublished manuscript, available at `http://sites.google.com/site/rl561a2/workinprogress`, 2012a.

[2] Gara Afonso and Ricardo Lagos. An empirical study of trade dynamics in the fed funds market. Unpublished manuscript, available at `http://sites.google.com/site/rl561a2/workinprogress`, 2012b.

[3] Franklin Allen and Douglas Gale. Bubbles and crises. *Economic Journal*, 110(460):236–55, January 2000.

[4] David Andolfatto. Business cycles and labor-market search. *American Economic Review*, 86(1):112–32, March 1996.

[5] Thomas F. Cooley and Vincenzo Quadrini. A neoclassical model of the phillips curve relation. *Journal of Monetary Economics*, 44(2):165–193, October 1999.

[6] Ben Craig and Joseph G. Haubrich. Gross loan flows. Federal Reserve Bank of Cleveland Working paper 06-04, March 2006.

[7] Vasco Cúrdia and Michael Woodford. The central-bank balance sheet as an instrument of monetary policy. *Journal of Monetary Economics*, 58(1):54–79, January 2011.

[8] Giovanni Dell'Ariccia and Pietro Garibaldi. Bank lending and interest rate changes in a dynamic matching model. IMF Working Papers 98/93, International Monetary Fund, June 1998.

[9] Giovanni Dell'Ariccia and Pietro Garibaldi. Gross credit flows. *Review of Economic Studies*, 72(3):665–685, July 2005.

[10] Wouter J. den Haan, Garey Ramey, and Joel Watson. Liquidity flows and fragility of business enterprises. *Journal of Monetary Economics*, 50(6):1215–1241, September 2003.

[11] Douglas W Diamond and Philip H Dybvig. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):401–19, June 1983.

[12] Silvia Gabrieli. Too-interconnected versus too-big-to-fail: Banks' network centrality and overnight interest rates. Unpublished manuscript, available at `http://dx.doi.org/10.2139/ssrn.1801390`.

[13] Prasanna Gai and Sujit Kapadia. Contagion in financial networks. Bank of England working papers 383, Bank of England, March 2010.

[14] Mark Gertler, Nobuhiro Kiyotaki, and Albert Queralto. Financial crises, bank risk exposure and government financial policy. Unpublished manuscript, available at `http://www.econ.nyu.edu/user/gertlerm/gertlerkiyotakiqueraltomay24wp.pdf`.

[15] Marlène Isoré. International propagation of financial shocks in a search and matching environment. FIW Working Paper series 068, FIW, April 2011.

[16] Andreas Krause and Simone Giansante. Interbank lending and the spread of bank failures: A network model of systemic risk. *Journal of Economic Behavior and Organization*, 2012.

[17] Michael U. Krause and Thomas A. Lubik. The (ir)relevance of real wage rigidity in the new keynesian model with search frictions. *Journal of Monetary Economics*, 54(3):706–727, April 2007.

[18] Paula Margaretic and Ernesto Pasten. Sequential bank runs. Unpublished manuscript, available at `http://www.touteconomie.org/conference/index.php/afse/aim/paper/viewFile/503/220`.

[19] Monika Merz. Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36(2):269–300, November 1995.

[20] Dale T Mortensen and Christopher A Pissarides. Job creation and job destruction in the theory of unemployment. *Review of Economic Studies*, 61(3):397–415, July 1994.

[21] Christopher A. Pissarides. *Equilibrium Unemployment Theory*. MIT Press, 2000.

[22] Julio J. Rotemberg. Cyclical wages in a search-and-bargaining model with large firms. In *NBER International Seminar on Macroeconomics 2006*, NBER Chapters, pages 65–114. National Bureau of Economic Research, Inc, 2008.

[23] Joseph E Stiglitz and Andrew Weiss. Credit rationing in markets with imperfect information. *American Economic Review*, 71(3):393–410, June 1981.

[24] Robert M. Townsend. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293, October 1979.

[25] Fabien Tripier. Efficiency gains from narrowing banks: a search-theoretic approach. Unpublished manuscript, available at `https://sites.google.com/site/fabientripier/`, 2012.

[26] Etienne Wasmer and Philippe Weil. The macroeconomics of labor and credit market imperfections. *American Economic Review*, 94(4):944–963, September 2004.

A Mathematical appendix

A.1 Equilibrium market tightnesses and repayment rates

Following Wasmer and Weil (2004), the equilibrium financial market tightness is easily derived from the Nash bargaining rule on the financial repayment rate,

$$(1-\delta_F)(I_t^v - I_t^u) = \delta_F(C_{kt}^v - C_t^u),$$

together with the first (backward looking) Bellman equation for banks,

$$C_{k,t}^u = -c_B + \beta q_F(\xi) C_{k,t+1}^v + \beta[1-q_F(\xi)] C_{k,t+1}^u,$$

considered at equilibrium, and for financial investors,

$$I_t^u = -c_I + \beta \xi q_F(\xi) I_{t+1}^v + \beta[1-\xi q_F(\xi)] I_{t+1}^u,$$

and given that free entry implies $I^u = 0$ and $C^u = 0$, as equation (1).

Similarly, the Nash bargaining rule for the credit repayment rate

$$(1-\delta_C)(C_{k,t}^p - C_{k,t}^v) = \delta_C(V_{k,t}^p - V_{k,t}^u),$$

with the first (backward looking) Bellman equation for entrepreneurs which is, in the deterministic case,

$$\begin{aligned} V_{k,t}^u = {} & -c_E + \beta q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1}) V_{k,t+1}^p \\ & + \beta[1 - q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1})] V_{k,t+1}^u, \end{aligned}$$

the first and the second Bellman equation for credit line as

$$\begin{aligned} C_{k,t}^v = {} & c - c'_{k,t} + \beta \phi_{k,t} q_C(\phi_{k,t})(1-s_{k,t+1})(1-d_{t+1}) C_{k,t+1}^p + \beta d_{t+1} C_{k,t+1}^u \\ & + \beta[1-\phi_{k,t} q_C(\phi_{k,t})(1-s_{k,t+1})](1-d_{t+1}) C_{k,t+1}^v \end{aligned}$$

and free entry, $V^u = 0$, give the expression for the credit market tightness (2).

The two equilibrium repayment rates are the solution to the pair of equations (3) and (4) which are obtained by substituting into the two Nash rules the forward looking Bellman equations, respectively given by

$$\begin{aligned} V_{k,t}^p = {} & A_k p_k - \psi_{k,t} + \beta[s_{k,t+1}(1-d_{t+1}) + d_{t+1}] V_{k,t+1}^u \\ & + \beta(1-s_{k,t+1})(1-d_{t+1}) V_{k,t+1}^p \end{aligned}$$

$$C^p_{k,t} = \psi_{k,t} - \rho_t + \beta(1 - s_{k,t+1})(1 - d_{t+1})C^p_{k,t+1} + \beta d_{t+1}C^u_{k,t+1}$$
$$+\beta s_{k,t+1}(1 - d_{t+1})C^v_{k,t+1}$$

$$I^v_t = -c + \beta d_{t+1}I^u_{t+1} + \beta(1 - d_{t+1})[\phi_{kt}q_C(\phi_{kt})(1 - s_{kt+1})I^p_{t+1}$$
$$+\beta(1 - \phi_{kt}q_C(\phi_{kt})(1 - s_{kt+1})]I^v_{t+1}$$

$$I^p_t = \rho_t + \beta d_{t+1}I^u_{t+1} + \beta(1 - d_{t+1})[s_{kt+1}I^v_{t+1} + \beta(1 - s_{kt+1})I^p_{t+1}]\}$$

The Bellman equations in the case where idiosyncratic productivity is stochastic and the separation rate endogenous can be reduced to the Bellman equations in the deterministic case so that (1)–(4) hold from the same computation.

A.2 Individual equilibrium conditions

Banks

An equilibrium credit creation condition for banks can be obtained by equalizing the forward and backward values of C^v and C^p from the three Bellman equations for credit lines and free entry ($C^u = 0$) (see Wasmer and Weil, 2004), as

$$\frac{c_B}{q_F(\xi)} = \frac{(\psi_k(p) - \rho)(1 - s_k)(1 - d)\phi_k q_C(\phi_k) + (c - c'_k)[r + d + s_k(1 - d)]}{(r + d)[r + d + s_k(1 - d) + \phi_k q_C(\phi_k)(1 - s_k)(1 - d)]}$$

where the left-hand side is the flow cost of fundraising c_B times the average duration $1/q_F(\xi)$, and the right-hand side is the expected present-discounted profits earned from productive credit lines depending on the search costs and duration of application screening (vacant credit lines), the separation and default rate, and the riskfree rate $r = 1/\beta - 1$.

An alternative method to obtain this equilibrium condition is to maximize over $N_{C^u_{i,t}}$, $N_{C^v_{i,k,t+1}}$, $N_{C^p_{i,k,t+1}}$, and $p^R_{i,k,t}$, bank i's profits given by

$$E_0 \sum_0^\infty \beta^t \left\{ \Psi_{i,t} - \rho_{i,t}N_{C^p_{i,t}} + cN_{C^v_{i,t}} - c(N_{C^v_{i,t}}) - c_B N_{C^u_{i,t}} \right\}$$

subject to

$$N_{C^p_{i,k,t+1}} = (1 - s_{i,k,t+1})(1 - d_{i,t+1})[N_{C^p_{i,k,t}} + N_{C^v_{i,k,t}} \phi_{k,t} q_C(\phi_{k,t})],$$

$$N_{C^v_{i,k,t+1}} = (1 - d_{i,t+1})[1 - \phi_{k,t} q_C(\phi_{k,t})(1 - s_{i,k,t+1})] N_{C^v_{i,k,t}} + q_F(\xi_t) N_{C^u_{i,t}} + (1 - d_{i,t+1}) s_{i,k,t+1} N_{C^p_{i,k,t}}$$

$$\text{and} \quad \Psi_{i,k,t} = N_{C^p_{i,k,t}} \int_{p^R}^{\infty} \frac{\psi_{k,t}(p) f_k(p)}{1 - F(p^R_{i,k,t})} dp,$$

where the first two constraints are the laws of motion for filled and vacant credit lines respectively, to be summed up across sectors with $N_{C^p} = \sum_k N_{C^p_k}$, $N_{C^v} = \sum_k N_{C^v_k}$, and where the third equation is the sum of the repayment rates at the bank (total instantaneous earnings) obtained from Nash bargaining with individual entrepreneurs given their idiosyncratic productivity draws (with $\Psi = \sum_k \Psi_k$).[24] The first-order conditions for this problem are

$$(N_{C^u_{i,t}} :) \quad \lambda_t = \frac{c_B}{q_F(\xi_t)}$$

$$(N_{C^v_{i,k,t+1}} :) \quad \lambda_t = \beta E_t \left\{ c - c'_{ikt} + \lambda_{t+1}(1 - d_{it+1})[1 - \phi_{kt} q_C(\phi_{kt})(1 - s_{ikt+1})] + \mu_{t+1} \phi_{kt} q_C(\phi_{kt})(1 - s_{ikt+1})(1 - d_{it+1}) \right\}$$

$$(N_{C^p_{i,k,t+1}} :) \quad \mu_t = \beta E_t \left\{ \frac{\partial \Psi_{ikt}}{\partial N_{C^p_{ikt}}} - \rho_{it} + \mu_{t+1}(1 - s_{ikt+1})(1 - d_{it+1}) + \lambda_{t+1} s_{ikt+1}(1 - d_{it+1}) \right\}$$

$$(p^R_{ijkt} :) \quad \frac{\partial \Psi_{ikt}}{\partial p^R_{ijkt}} = (\mu_t - \lambda_t) \frac{\partial s_{ikt}}{\partial p^R_{ijkt}} (1 - d_{it})[N_{C^p_{ikt-1}} + \phi_{kt-1} q(\phi_{kt-1})) N_{C^v_{ikt-1}}]$$

where λ and μ are the Lagrange multipliers associated with the constraints. Solving for the first three equations at equilibrium would give the same credit

[24] If all credit lines had the same productivity, total earnings at time t at bank i would simply be $\psi_{ikt} N_{C^p_{ikt}}$.

creation condition for bankers.

Financial investors

With free entry, $L^u = 0$, the first Bellman equation gives the forward value $I^v = \frac{c_I}{\beta \xi q_F(\xi)}$. The second and third Bellman equations can be solved together at equilibrium to obtain the backward value of I^p and I^v. Equalizing the forward and backward values of I^v finally gives financial investors' condition as

$$\frac{c_I}{\xi q_F(\xi)} = \frac{\rho\, \phi_k q_C(\phi_k)(1-s_k)(1-d) - c[r+d+s_k(1-d]}{(r+d)[r+d+s_k(1-d)+\phi_k q_C(\phi_k)(1-s_k)(1-d)]}$$

On the left hand side is the cost of entering the financial market that depends on the periodic cost times the duration before a conclusive match. On the right hand side are the expected gains that depend on the periodic return on capital ρ received from the banks minus the cost paid while the bank is screening entrepreneurs' applications, given the discount rate and the transition rates.

Entrepreneurs

The free entry condition, $V^u = 0$, and the first Bellman equation, gives the forward value for V^p. Free exit and the last Bellman equation give the backward value for V^p. Hence the equilibrium condition is as follows

$$\frac{c_E}{q(\phi_k)} = \frac{[AA_k p_k - \psi_k(p)](1-s_k)(1-d)}{r+d+s_k(1-d)}$$

On the left hand side are expected costs for sector-k entrepreneurs while seeking a loan (the flow cost c_E time the duration of the search $1/q(\phi_k)$). On the right hand side are the expected profit flows (value of production minus credit repayments), discounted by the riskfree rate r, the destruction rates s_k and d.

A.3 Bank screening technology and optimal size

Following the derivation in Rotemberg (2006) for the labor market, let derive the non-linear cost $c_{ikt} = c_k(N_{C^v_{ikt}})$ that is paid while bank i is screening credit applications from (sector-k) entrepreneurs and that helps to determine the size of bank i. In particular, assuming that $c_k(N_{C^v_{ikt}}) = \kappa N^{\epsilon}_{C^v_{ikt}}$ and given that $\phi_{kt} q(\phi_{kt}) \equiv q(\phi_{kt}) \frac{N_{E^u_{kt}}}{N_{C^v_{kt}}}$, we can reexpress the law of motion for lending relationships at bank i as

$$N_{C^v_{i,k,t}} = \left[\frac{N_{C^p_{i,k,t+1}}}{(1-s_{i,k,t+1})(1-d_{i,t+1})} - N_{C^p_{i,k,t}} \right] \frac{N_{C^v_{kt}}}{q(\phi_{kt}) N_{E^u_{kt}}}$$

From $c_k(N_{C_{ikt}}) = \kappa \left\{ \left[\frac{N_{C^p_{i,k,t+1}}}{(1-s_{ikt+1})(1-d_{it+1})} - N_{C^p_{i,k,t}} \right] \frac{N_{C^v_{kt}}}{q(\phi_{kt}) N_{E^u_{kt}}} \right\}^{\epsilon}$, the marginal cost of creating lending relationships for bank i becomes

$$c'_{ikt} = \frac{\kappa \epsilon}{(1-s_{ikt+1})(1-d_{it+1})} \left[\frac{N_{C^p_{i,k,t+1}}}{(1-s_{ikt+1})(1-d_{it+1})} - N_{C^p_{i,k,t}} \right]^{\epsilon-1} \left[\frac{N_{C^v_{kt}}}{q(\phi_{kt}) N_{E^u_{kt}}} \right]^{\epsilon}$$

With symmetric banks, $\frac{N_{C^v_{ikt}}}{N_{C^v_{kt}}} q(\phi_{kt}) N_{E^u_{kt}} = \frac{1}{N} q(\phi_{kt}) N_{E^u_{kt}}$, where N is the number of banks, such that the equilibrium marginal cost is given by equation (7).

A.4 Time-varying credit repayment rate

The time-varying expression for the (sector-k) credit repayment rate ψ_{kt} will allow to compute the optimal reservation threshold that determines the the separation rate thereafter. Let derive it from the first Nash bargaining rule, $V^p_{kt}(p) - V^u_{kt} = \frac{\delta_C}{1-\delta_C}(C^p_{kt}(p) - C^v_{kt})$, as follows.

The Bellman equation standing for the surplus of the credit match is

$$CS_{kt}(p_k) = V^p_{kt}(p_k) - V^u_{kt} + C^p_{kt} - C^v_{kt}$$

With $V^u_{kt} = 0$ by free entry, replacing by the (time-varying) Bellman equations for the credit lines and entrepreneurs gives, after some simplification,

$$\begin{aligned} CS_{kt} = A_{kt} p_{kt} + c'_{kt} - c - \rho_t + \beta E_t \{ (1-s_{kt+1})(1-d_{t+1}) [(V^p_{kt+1} + C^p_{kt+1} \\ - C^v_{kt+1}) - \phi_{kt} q_C(\phi_{kt}) (C^p_{kt+1} - C^v_{kt+1})] \} \end{aligned}$$

Since $(V^p_{kt+1} + C^p_{kt+1} - C^v_{kt+1}) = (CS)_{kt+1}$ and $(C^p_{kt+1} - C^v_{kt+1}) = \delta_C (CS)_{kt+1}$,

$$CS_{kt} = A_{kt} p_{kt} + c'_{kt} - c - \rho_t + \beta E_t \{ (1-s_{kt+1})(1-d_{t+1}) [1 - \phi_{kt} q_C(\phi_{kt}) \delta_C] (CS)_{kt+1}$$

Then, from the first Bellman equation for entrepreneurs, we know that

$$V^p_{kt+1} = \frac{c_E}{\beta q_C(\phi_{kt}) E_t[(1 - s_{kt+1})(1 - d_{t+1})]}$$

and since $V^p_{kt+1} = (1 - \delta_C)(CS)_{kt+1}$, we have

$$CS_{kt} = (A_{kt}p_{kt} + c'_{kt} - c - \rho_t) + \frac{1 - \phi_{kt} q_C(\phi_{kt})\delta_C}{1 - \delta_C} \frac{c_E}{q_C(\phi_{kt})} \quad \text{(A)}$$

From the last Bellman equation for entrepreneurs (10), we also have

$$V^p_{kt} = A_{kt}p_{kt} - \psi_{kt} + \beta E_t[(1 - s_{kt+1})(1 - d_{t+1})V^p_{kt+1}]$$

so that,

$$CS_{kt} = \frac{A_{kt}p_{kt} - \psi_{kt}}{1 - \delta_C} + \frac{c_E}{(1 - \delta_C)q_C(\phi_{kt})} \quad \text{(B)}$$

Equalizing (A) and (B) finally gives

$$\psi_{ikt} = \delta_C A_{kt} p_{jkt} + (1 - \delta_C)(c - c'_{ikt} + \rho_{it}) + \delta_C c_E \phi_{kt} \quad \text{(C)}$$

At equilibrium

$$\bar{\psi}_k = \delta_C \bar{A}\bar{A}_k \bar{p}_k + (1 - \delta_C)(c - \bar{c}'_k + \bar{\rho}) + \delta_C c_E \bar{\phi}_k$$

The credit repayment rate depends on the relative bargaining powers of entrepreneurs and bankers in the credit market (δ_C), the productivity of the match, the costs involved by the credit search period (c'_k and c_E), the sectoral credit market tightness (ϕ_k), and the rate of return on capital (ρ).[25]

A.5 Separation rule and optimal reservation threshold for idiosyncratic productivity

The time-varying expression for the credit repayment is further used to compute the threshold as follows. By definition, the credit relationship terminates if its asset value for the bank is negative, $C^p_{kt}(p) - C^v_{kt}(p) < 0$. The

[25] In the labor market search literature, a similar equation gives the wages as a function of the bargaining powers, the productivity, the search costs, and the labor market tightness. However, ρ has no counterpart and is due to the multi-search framework considered here.

reservation level for idiosyncratic productivity is such that

$$C^p_{kt}(p^R) - C^v_{kt} = 0$$

From (B) and given that $C^p_{kt} - C^v_{kt} = \delta_C (CS)_{kt}$, we have

$$A_{kt} p_{kt} - \psi_{kt}(p^R) + \frac{c_E}{q_C(\phi_{kt})} = 0$$

Replacing $\bar{\psi}_k$, we get the time-varying threshold required by banks for entrepreneurs' idiosyncratic productivity as

$$p^R_{ikt} = \frac{1}{A_{kt}}\Big[\rho_{it} + (c - c'_{ikt}) - \frac{c_E}{q_C(\phi_{kt})}\frac{1 - \delta_C \phi_{kt} q(\phi_{kt})}{1 - \delta_C}\Big]$$

which further determines the credit separation rate at bank i.

A.6 Time-varying financial repayment rate

The sequence of events is such that the shares that each agent type effectively gets from the net surplus (NS) at the end is as follows: $(1 - \delta_C)(1 - \tilde{\delta}_F)$ for entrepreneurs, $\delta_C(1 - \tilde{\delta}_F)$ for bankers, and $\tilde{\delta}_F$ for financial investors. As bankers know that future bargaining with entrepreneurs on the credit market will determine their effective share of the net surplus, they take this effect into account at the time they bargain with financial investors, so that where

$$\delta_F = \frac{\tilde{\delta}_F}{1 - (1 - \tilde{\delta}_F)(1 - \delta_C)}$$

The net surplus is given by

$$NS_t = C^p_{kt} - C^v_{kt} + V^p_{kt} - V^u_{kt} + I^p_t - I^v_t = CS_{kt} + I^p_t - I^v_t$$

Replacing by the Bellman equations for financial investors we have

$$NS_t = CS_{kt} + \rho_t + c + \frac{c_E}{q_C(\phi_{kt})}\frac{1 - \phi_{kt} q_C(\phi_{kt})}{1 - \delta_C}\frac{\tilde{\delta}_F}{1 - \tilde{\delta}_F} \quad \text{(D)}$$

From the fact that $I^p_t - I^v_t = \tilde{\delta}_F NS_t$, we also have

$$NS_t = \frac{\rho_t + c}{\delta_F} + \frac{c_E}{q_C(\phi_{kt})}\frac{1 - \phi_{kt} q_C(\phi_{kt})}{1 - \delta_C}\frac{1}{1 - \tilde{\delta}_F} \quad \text{(E)}$$

Therefore, equalizing (D) and (E), with CS_{kt} given by (B), we get

$$\rho_t = \frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}[A_{kt}p_{kt} - \psi_{kt} + c_E\phi_{kt}] - c$$

A.7 Bank default

It is more profitable for the financial investors to impose default on banks if the continuation value of remaining matched is less than the continuation value of being unmatched, *i.e.* if $I_t^v - I_t^u < 0$. Since $I_t^u = 0$ by free entry, $I_t^v = 0$ gives

$$-c + \beta E_t\{(1-d_{t+1})\phi_{kt}q_C(\phi_{kt})(1-s_{kt+1})(I_{t+1}^p - I_{t+1}^v) + (1-d_{t+1})I_{t+1}^v\} = 0$$

Since $I_{t+1}^p - I_{t+1}^v = \tilde{\delta}_F NS_{t+1}$, $NS_{t+1} = \frac{CS_{t+1}}{1-\tilde{\delta}_F}$, and $I_{t+1}^v = \frac{c_I}{\beta\xi_t q_F(\xi_t)}$, we get

$$E_t d_{t+1} = 1 + \frac{\xi_t q_F(\xi_t)}{c_I}\left[\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}c_E\phi_{kt} - c\right]$$

A.8 Equilibrium conditions: no information asymmetry

Here is the loglinearized system of equilibrium conditions in the particular case where bankers' application screening cost is linear in the number of vacant credit lines and exactly equal to the amount provided by financiers: $c = c'_k$, ie. the case where there is no information asymmetry. It is also assumed for simplicity that $\bar{r} = 0$ and $\bar{A} = 1$. The system consists in 10 equations with the following set of variables: $\{\xi, \phi_k, \psi_k, \rho, p_k^R, s_k, d, N_{E_k^u}, N_{C_k^v}, N_{C_k^p}\}$.

$$\hat{\xi} = \hat{c}_I - \hat{c}_B$$

$$\hat{\phi}_k = \hat{c}_B + \hat{d} + \eta_F\hat{\xi}$$

$$\bar{A}_k\bar{p}_k^R(\hat{A}_k + \hat{p}_k^R) = \bar{\rho}\hat{\rho} - \frac{c_E}{q_C(\bar{\phi}_k)}\frac{\eta_C - \delta_C\bar{\phi}_k q_C(\bar{\phi}_k)}{1-\delta_C}\hat{\phi}_k$$

$$\hat{s}_k = \sigma_k\hat{p}_k^R$$

$$\bar{d}\hat{d} = \frac{1-\delta_F}{\delta_F}\frac{q_F(\bar{\xi})}{\bar{c}_B}\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{c_E\bar{\phi}_k}{1-\delta_C}(-\eta_F\hat{\xi} - \hat{c}_B + \hat{\phi}_k)$$

$$\hat{N}_{E_k^u} = \hat{\phi}_k + \hat{N}_{C_k^v}$$

$$\frac{\bar{N}_{C_k^v}\bar{\phi}_k q_C(\bar{\phi}_k)}{\bar{N}_{C_k^p}}[\hat{N}_{C_k^v} - \hat{N}_{C_k^p} + (1-\eta_C)\hat{\phi}_k] = \bar{s}_k(1-\bar{d})\hat{s}_k + \bar{d}(1-\bar{s}_k)\hat{d}$$

$$\bar{\psi}_k\hat{\psi}_k = \delta_C\bar{A}_k\bar{p}_k\hat{A}_k + (1-\delta_C)\bar{\rho}\hat{\rho} + \delta_C c_E\bar{\phi}_k\hat{\phi}_k$$

$$\bar{\rho}\hat{\rho} = \frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}(\bar{A}_k\bar{p}_k\hat{A}_k - \bar{\psi}_k\hat{\psi}_k + c_E\bar{\phi}_k\hat{\phi}_k)$$

$$\bar{N}_{C_k^p}\hat{N}_{C_k^p} = -\bar{N}_{E_k^u}\hat{N}_{E_k^u}$$

where an overbar indicates the equilibrium value of a variable, a hat indicates the log-deviation from equilibrium of a variable, and where η_F, η_C, and σ_k are respectively the elasticities of $q_F(\bar{\xi})$, $q_C(\bar{\phi}_k)$, and $s_k(p_k^R)$ with respect to their argument. Below are the effects of a sectoral productivity shock when $c_k' = c$.

$$\frac{\partial\hat{N}_{E_k^u}}{\partial\hat{A}_k} = \frac{\partial\hat{N}_{C_k^v}}{\partial\hat{A}_k} = -\frac{\sigma_k\bar{s}_k(1-\bar{d})\bar{N}_{C_k^p}(1-\tilde{\delta}_F\bar{p}_k/\bar{p}_k^R)}{\bar{s}_k(1-\bar{d})+\bar{d}} \quad < 0 \quad \text{if} \quad \frac{\bar{p}_k^R}{\bar{p}_k} > \tilde{\delta}_F$$

$$\frac{\partial\hat{\psi}_k}{\partial\hat{A}_k} = \frac{\bar{A}_k\bar{p}_k}{\bar{\psi}_k}[\delta_C(1-\tilde{\delta}_F) + \tilde{\delta}_F] > 0$$

$$\frac{\partial\hat{\rho}}{\partial\hat{A}_k} = \frac{\tilde{\delta}_F\bar{A}_k\bar{p}_k}{\bar{\rho}} > 0$$

$$\frac{\partial\hat{s}_k}{\partial\hat{A}_k} = -\sigma_k\left(1-\tilde{\delta}_F\frac{\bar{p}_k}{\bar{p}_k^R}\right) \quad < 0 \quad \text{if} \quad \frac{\bar{p}_k^R}{\bar{p}_k} > \tilde{\delta}_F$$

$$\frac{\partial\hat{p}_k^R}{\partial\hat{A}_k} = -1 + \tilde{\delta}_F\frac{\bar{p}_k}{\bar{p}_k^R} \quad < 0 \quad \text{if} \quad \frac{\bar{p}_k^R}{\bar{p}_k} > \tilde{\delta}_F$$

$$\frac{\partial\hat{\xi}}{\partial\hat{A}_k} = \frac{\partial\hat{\phi}_k}{\partial\hat{A}_k} = \frac{\partial\hat{d}}{\partial\hat{A}_k} = 0$$

If the economy is deterministic (constant idiosyncratic productivity and exogenous separations), and whether bank default is exogenous or not, $\partial\hat{\rho}/\partial\hat{A}_k$ and $\partial\hat{\psi}_k/\partial\hat{A}_k$ are the same than in the stochastic case, however $\partial\hat{N}_{E_k^u}/\partial\hat{A}_k = 0$.

A.9 Equilibrium conditions: information asymmetry

In the presence of information asymmetry, it must be that $c'_k \neq c$ and bank size matters to determine c'_k. For simplicity that, $\bar{r} = 0$, $\bar{A} = 1$, and $\bar{N} = 1$. The system consists in 11 equations in $\{\xi, \phi_k, \psi_k, \rho, p_k^R, s_k, d, c'_k, N_{E_k^u}, N_{C_k^v}, N_{C_k^p}\}$.

$$\hat{\xi} = \hat{c}_I - \hat{c}_B$$

$$\hat{\phi}_k = \hat{c}_B + \hat{d} + \eta_F\hat{\xi} + \frac{\bar{c}'_k}{c_B}\frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{xi})}{\bar{d}}\hat{c}'_k$$

$$\hat{c}'_k = \eta_C\hat{\phi}_k + \epsilon\hat{N}_{C_k^v} - \hat{N}_{E_k^u} + \bar{s}_k(1-\bar{d})\hat{s}_k + \bar{d}(1-\bar{s}_k)\hat{d}$$

$$\bar{A}_k\bar{p}_k^R(\hat{A}_k + \hat{p}_k^R) = \bar{\rho}\hat{\rho} - \frac{c_E}{q_C(\bar{\phi}_k)}\frac{\eta_C - \delta_C\bar{\phi}_k q_C(\bar{\phi}_k)}{1-\delta_C}\hat{\phi}_k$$

$$\hat{s}_k = \sigma_k\hat{p}_k^R$$

$$\bar{d}\hat{d} = \frac{\bar{\xi}q_F(\bar{\xi})}{\bar{c}_I}\frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{c_E\bar{\phi}_k}{1-\delta_C}[(1-\eta_F)\hat{\xi} - \hat{c}_I + \hat{\phi}_k)$$

$$\hat{N}_{E_k^u} = \hat{\phi}_k + \hat{N}_{C_k^v}$$

$$\frac{\bar{N}_{C_k^v}\bar{\phi}_k q_C(\bar{\phi}_k)}{\bar{N}_{C_k^p}}[\hat{N}_{C_k^v} - \hat{N}_{C_k^p} + (1-\eta_C)\hat{\phi}_k] = \bar{s}_k(1-\bar{d})\hat{s}_k + \bar{d}(1-\bar{s}_k)\hat{d}$$

$$\bar{\psi}_k\hat{\psi}_k = \delta_C\bar{A}_k\bar{p}_k\hat{A}_k + (1-\delta_C)\bar{\rho}\hat{\rho} + \delta_C c_E\bar{\phi}_k\hat{\phi}_k - (1-\delta_C)\bar{c}'_k\hat{c}'_k$$

$$\bar{\rho}\hat{\rho} = \frac{\tilde{\delta}_F}{1-\tilde{\delta}_F}\frac{1}{1-\delta_C}(\bar{A}_k\bar{p}_k\hat{A}_k - \bar{\psi}_k\hat{\psi}_k + c_E\bar{\phi}_k\hat{\phi}_k)$$

$$\bar{N}_{C_k^p}\hat{N}_{C_k^p} = -\bar{N}_{E_k^u}\hat{N}_{E_k^u}$$

Effects of a shock to A_k in the deterministic economy

When idiosyncratic productivity is constant, the effects of a sectoral productivity shock are identical whether information is asymmetric or not and whether there the bank default rate is exogenous or not. More precisely we get

$$\partial\hat{N}_{E_k^u}/\partial\hat{A}_k = 0,\ \partial\hat{d}/\partial\hat{A}_k = 0,\ \partial\hat{\phi}_k/\partial\hat{A}_k = 0,\ \partial\hat{c}'_k/\partial\hat{A}_k = 0,$$

$$\frac{\partial\hat{\psi}_k}{\partial\hat{A}_k} = \frac{\bar{A}_k\bar{p}_k}{\bar{\psi}_k}[\delta_C(1-\tilde{\delta}_F) + \tilde{\delta}_F] > 0,$$

$$\frac{\partial\hat{\rho}}{\partial\hat{A}_k} = \frac{\tilde{\delta}_F\bar{A}_k\bar{p}_k}{\bar{\rho}} > 0$$

Effects of a shock to A_k in the stochastic economy (endogenous threshold and separation)

- Exogenous bank default

$$\frac{\partial\hat{N}_{E_k^u}}{\partial\hat{A}_k} = -\frac{\sigma_k\bar{s}_k(1-\bar{d})N_{C_k^p}(1-\tilde{\delta}_F p_k/p_k^R)\,B}{D}$$

$$\frac{\partial\hat{s}_k}{\partial\hat{A}_k} = -\frac{\sigma_k(1-\tilde{\delta}_F\bar{p}_k/\bar{p}_k^R)[\bar{s}_k(1-\bar{d})+\bar{d}]\,F}{D}$$

$$\frac{\partial\hat{p}_k^R}{\partial\hat{A}_k} = -\frac{(1-\tilde{\delta}_F\bar{p}_k/\bar{p}_k^R)[\bar{s}_k(1-\bar{d})+\bar{d}]\,F}{D}$$

$$\frac{\partial \hat{c}'_k}{\partial \hat{A}_k} = -\frac{\sigma_k \bar{s}_k (1-\bar{d})(1-\tilde{\delta}_F \bar{p}_k/\bar{p}_k^R)\, G}{D}$$

$$\frac{\partial \hat{\phi}_k}{\partial \hat{A}_k} = -\frac{\sigma_k \bar{s}_k (1-\bar{d})(1-\tilde{\delta}_F \bar{p}_k/\bar{p}_k^R)\frac{\bar{c}'_k}{c_B}\frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{\xi})}{\bar{d}}\, G}{D}$$

$$\text{where} \quad B = 1 + \frac{\bar{c}'_k}{c_B}\frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{\xi})}{\bar{d}}\left[\epsilon - \eta_C(1-\bar{s}_k)(1-\bar{d})\right],$$

$$\text{where} \quad F = 1 + \frac{\bar{c}'_k}{c_B}\frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{\xi})}{\bar{d}}\left[\epsilon(1-\eta_C) - \eta_C(1-\epsilon)\bar{N}_{E_k^u}\right],$$

$$\text{where} \quad G = \bar{s}_k(1-\bar{d}) + \bar{d} - (1-\epsilon)(1-\bar{N}_{E_k^u}),$$

$$\text{and} \quad D = \left[\bar{s}_k(1-\bar{d}) + \bar{d}\right] F + \left\{ \frac{\delta_C}{1-\delta_C}\frac{q_F(\bar{\xi})}{c_B \bar{d}}\frac{c_E}{q_C(\bar{\phi}_k)(1-\delta_C)}\left[-\tilde{\delta}_F \right.\right.$$
$$\left.\left. +\eta_C - \bar{\phi}_k q_C(\bar{\phi}_k)(\delta_C + \tilde{\delta}_F(1-\delta_C))\right]\right\} \frac{\bar{c}'_k \sigma_k \bar{s}_k (1-\bar{d})}{\bar{A}_k \bar{p}_k^R}\, G$$

The sign of these expressions depends on the range for $\tilde{\delta}_F$ and for ϵ. In particular having concave ($\epsilon < 1$) or convex ($\epsilon > 1$) application screening costs matters.

- Endogenous bank default

The expressions of the derivatives including all of the effects disentangled so far can hardly be interpreted. A numerical estimation could be helpful, but because the parameters to be included in the calibration are generally unknown, this would only serve as an exercise to simulate the effects discussed above. However, it is found that it is the only case where $\partial \hat{d}/\partial \hat{A}_k \neq 0$.

Chapter 3

Disaster Risk in a New Keynesian Model

This article is joint with Urszula Szczerbowicz, PhD student at LUISS Guido Carli and Sciences Po

Abstract

This paper incorporates a small and time-varying "disaster risk" à la Gourio (2012) in a New Keynesian model. A change in the probability of disaster may affect macroeconomic quantities and asset prices. In particular, a higher risk is sufficient to generate a recession without effective occurrence of the disaster. By accounting for monopolistic competition, price stickiness, and a Taylor-type rule, this paper provides a baseline framework of the dynamic interactions between the macroeconomic effects of rare events and nominal rigidity, particularly suitable for further analysis of monetary policy. We also set up our next research agenda aimed at assessing the desirability of several policy measures in case of a variation in the probability of rare events.

JEL classification: E2, E3
Keywords: disaster risk, rare events, DSGE models, business cycles

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1 Introduction

A recent but growing literature studies how the risk of rare events — sometimes called economic "disasters" — affects the dynamic interactions between macroeconomic quantities and asset prices — risk premia in particular. However, disaster risk is still rarely accounted for in general equilibrium models, especially in the models used to conduct monetary policy where variations in the expected returns are generally entirely driven by variations in the risk-free interest rate. Yet understanding the efficiency and the desirability of monetary policy facing — realized or potential — rare events is of main interest. In order to design an appropriate intervention, studying the effects of a time-varying disaster risk in this class of models is a prerequisite.

Early papers on disaster risk were restricted to endowment economies (Rietz, 1988, Barro, 2006, Gabaix, 2012) such that policy implications could have hardly been derived. Gourio (2012) has gone a step further by introducing a small and stochastically time-varying risk premium into a real business cycle model. His model has thus provided a tractable way to analyze the feedback effects between changes in aggregate risk and the macroeconomic variables, as well as to reproduce some important empirical facts in terms of asset pricing including the countercyclicality of the risk premia. In particular, an increase in the probability of disaster leads investment and output to fall as capital becomes riskier. Meanwhile precautionary savings lower the yield on risk-free assets, such that the spread rises in distressed times.

This paper builds on Gourio's approach and introduces a time-varying risk of disaster in an otherwise standard New Keynesian DSGE model, providing a baseline framework that will allow to evaluate the role of monetary policy facing changes in the probability of rare events. The occurrence of a disaster is associated with the destruction of a share of capital, but the appealing feature of the model is that business cycles are significantly affected by the disaster risk even when disasters do not effectively arrive. We especially focus on the responses of macroeconomic quantities to a sudden rise in the probability of disaster, and get some interesting preliminary results.

First, we are able to relax one essential assumption in Gourio's work which consists in imposing a reduction in total factor productivity by exactly the same amount than the capital stock to replicate the data. We show that the output fall may be large enough by introducing investment adjustment costs and monopolistic competition in intermediate goods instead. The response of output is much more important under time-dependent price stickiness, however, since firms may be more inclined to adjust their prices when the aggregate risk rises (Caplin and Leahy, 1991), we also allow for some state-dependent price adjustment.

Second, we find that consumption falls on impact in case of a rise in disaster risk while Gourio found the opposite response with a more stylized model. Similarly, we get a drop in wages which is not observed in the pure flexible-price but otherwise similar version of the model, that seems more reminiscent of distressed economic times, whether under time-dependent or state-dependent price stickiness. Finally, we compare the responses of the model to standard monetary, fiscal, and productivity shocks, with and without the presence of a disaster risk.

This version of the model does not study the feedback effects between these macroeconomic quantities and the impact of disaster risk on asset pricing yet. However, the set-up is such that we will be able to do so quite easily by already incorporating a stochastic discount factor from which the term premium will be derived and some features that proved effective in replicating the variations of equity premia, including habit formation.[1] Gourio (2012) shows that the presence of a time-varying disaster risk allows to replicate well the first- and second moments of asset returns, as well as their correlation with the macroeconomic quantities. This suggests that the degree of risk aversion or the amount of risk in the economy has a significant impact on macroeconomic dynamics while Tallarini (2000)'s "observational equivalence" only holds when the probability of disaster is constant over time. In our model, solved under certainty-equivalence so far, linear and nonlinear approximations give almost identical results since we study the responses of macroeconomic quantities to a (small) change in the probability of disaster instead of the responses to a (large) disaster shock. Asset pricing in the presence of a time-varying disaster risk would however require the combination of nonlinear methods and aggregate uncertainty.[2]

The remainder of the paper is as follows. Section 2 develops the model, Section 3 discusses how the steady state is affected by the presence of a disaster risk and presents the calibration, Section 4 describes the response

[1] See Campbell and Cochrane (1999) and Uhlig (2007).

[2] See Bloom (2009) for a model with uncertainty shocks for instance.

functions to a shock to the probability of disaster as well as to standard shocks. Section 5 gives our further research agenda, and Section 6 concludes.

2 Model

2.1 Households

Households consume goods, supply labor, and save through risk-free bonds and capital accumulation so as to maximize the expected discounted sum of utility flows given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\phi}}{1+\phi} \right) \tag{1}$$

where β is the subjective discount factor, E_0 the expectation operator, C and L consumption and labor flows respectively, h a habit formation parameter, γ the coefficient of relative risk aversion or the inverse of the intertemporal elasticity of substitution, and ϕ the inverse of the elasticity of work effort with respect to the real wage. Households own the capital stock K_t and lease a fraction u_t of it to the firms. Thus their budget constraint is

$$C_t + I_t + \frac{B_{t+1}}{p_t} \leq W_t L_t + (1 + i_{t-1}) \frac{B_t}{p_t} + R_t^k u_t K_t + \Pi_t - T_t \tag{2}$$

where I_t is investment, B_t are one-period bonds, w_t is the real wage, Π_t are profits from firms, and R_t^k is the real rental rate of capital, at time t.

Capital is considered as a risky asset here in the sense that it may be hit by a "disaster". In Barro (2006) and Gourio (2012)'s spirit, a disaster occurrence may be either a war which physically destroys a part of the capital stock, the expropriation of capital holders, a technological revolution that make it worthless, or the loss of intangible capital due to a prolonged recession. We assume that the disaster destroys a share b_k of the capital stock if realized.[3] Therefore the law of capital accumulation is given by

$$K_{t+1} = \left\{ (1 - \delta_t) K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t \right\} (1 - x_{t+1} b_k) \tag{3}$$

[3] As a disaster lowers the return on capital because investing in capital is riskier one can equally consider ex ante that this is the price or the quantity of capital which is affected by the disaster.

where $\delta_t = \delta u_t^\eta$ is the depreciation rate increasing with capital utilization (Burnside and Eichenbaum, 1996), and $S = \frac{\tau}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ is a capital adjustment cost function which verifies the usual properties ($S(0) = 0$, $S'(0) = 0$, and $S''(.) > 0$). The disaster is captured by the indicator x_{t+1} which is equal to 1 with probability θ_t and equal to 0 otherwise. Gourio (2012) argues that this probability can be considered as strict rational expectations or more generally account for time-varying beliefs which may differ from the objective probability.[4] We consider that the log of the probability of disaster follows a first-order autoregressive process as

$$\log \theta_t = (1 - \rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \sigma_\theta \varepsilon_{\theta_t} \tag{4}$$

and assume that the shocks θ_{t+1} and x_{t+1} are independent, conditional on θ_t, in line with the evidence that a disaster occurrence tomorrow is not likely if there is a disaster today (Gourio, 2008).

We relax Gourio (2012)'s assumption that total factor productivity is reduced by exactly the same amount than the capital (b_k) in case of a disaster here. This assumption has been made for two reasons. First, detrending the capital by the (stochastic) technology level gives a stationary variable and reduces the dimension of the state space, so as to obtain analytical results and simplify the numerical analysis. Second, it delivers an empirically relevant magnitude for the recession. However, the combination of adjustment costs and monopolistic competition allows us to replicate a large enough fall in output following a rise in disaster risk without having to maintain this assumption here. Moreover, while Gourio argues that some disasters were associated with a fall in TFP (South America since 1945, Russia in 1917), some papers find, on the contrary, that TFP may rise in recessions as the least productive firms are shut down (for instance Petrosky-Nadeau, 2010).

Maximizing (1) subject to (2), (3), and (4) gives standard first-order conditions for consumption, labor, and the riskfree bonds, respectively as

$$\lambda_t = (C_t - hC_{t-1})^{-\gamma} - \beta h E_t (C_{t+1} - hC_t)^{-\gamma} \tag{5}$$

[4]Building on the behavioral macroeconomics literature would help to disentangle whether this probability is objective or stemming from agents' sentiments or "animal spirits" (waves of optimism or pessimism) but this is out of the scope of our paper for now (see Section 5).

$$\chi L_t^{\phi} = w_t \lambda_t \quad (6)$$

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + i_t)(1 + \pi_{t+1})^{-1} \quad (7)$$

in which $1 + \pi_{t+1} \equiv \frac{p_{t+1}}{p_t}$ where π is the (net) inflation rate, whereas the first-order conditions for capital and capital utilization are both affected by the disaster probability and the disaster size effect, $\theta_t b_k$, as follows[5]

$$\mu_t = \beta E_t \left[\lambda_{t+1} R_{t+1}^k u_{t+1} + \mu_{t+1} \left(1 - \delta u_{t+1}^{\eta}\right) (1 - \theta_{t+1} b_k) \right] \quad (8)$$

$$\lambda_t R_t^k = \mu_t \delta \eta u_t^{\eta - 1} (1 - \theta_t b_k) \quad (9)$$

Finally the first-order condition on investment, also affected by the disaster risk, is

$$\lambda_t = \mu_t \left(1 - \theta_t b_k\right) \left[1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \tau \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \beta E_t \mu_{t+1} \left(1 - \theta_{t+1} b_k\right) \tau \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \quad (10)$$

Without investment adjustment cost ($\tau = 0$), the Euler equation would be

$$\beta E_t \frac{\lambda_{t+1}}{\lambda_t} = E_t \left\{ \left[R_{t+1}^k u_{t+1} + (1 - \delta_{t+1}) \right] (1 - \theta_t b_k) \right\}^{-1}$$

and would further reduce to the standard Euler equation if the probability of disaster was equal to zero.[6] This states that the marginal utility from consumption tomorrow λ_{t+1} will be greater than the marginal utility from consumption today λ_t if the probability θ_t drawn today that a disaster arrives tomorrow increases given that the disaster would destroy a share of capital tomorrow. Consumption may fall or rise on impact following a shock to the disaster risk depending on the value of the elasticity of intertemporal substitution (see Sections 3 and 4).

[5]These expressions hold under certainty-equivalence, such that disaster risk is not an *uncertainty* shock in this version of the paper. See Section 5 and Appendix.

[6]For the purpose of the quantitative exercise, we keep adjustment costs positive ($\tau > 0$) though, in order to get a more gradual response of investment to changes in the probability of disaster, without qualitative impact on the Euler equation.

Moreover, complete markets imply that there is a unique stochastic discount factor, denoted $Q_{t,t+1}$ such that

$$1 + i_t = (E_t Q_{t,t+1})^{-1} \tag{11}$$

If $\tau = 0$, we can easily derive, from the first-order condition on bonds and the Euler equation above, that

$$E_t Q_{t,t+1} = E_t \left\{ (1 + \pi_{t+1}) \left[R^k_{t+1} u_{t+1} + (1 - \delta_{t+1}) \right] (1 - \theta_t b_k) \right\}^{-1}$$

such that the stochastic discount factor also accounts for the disaster risk, while remains standard if $\theta_t = 0$.[7]

The existence of a risk of disaster on capital also affects the level of the Tobin's q. Defined as the ratio of the market value of one additional unit of investment to the marginal replacement cost of installed capital,[8] it is given by the ratio of the Lagrange multipliers on (3) and (4), that is,

$$q_t = \frac{\mu_t}{\lambda_t} \tag{12}$$

Without disaster risk, the first-order condition on investment would imply that, in steady-state, $\bar{\lambda} = \bar{\mu}$, and thus $\bar{q} = 1$. Therefore whenever $q_{t+s} > 1$ in any period $t + s$ more investment would then add to the value of the firm, whereas with $q_{t+s} < 1$ it would be optimal for firms to disinvest. Here the disaster risk implies that $\bar{\lambda} = \bar{\mu}(1 - \bar{\theta} b_k)$, and thus

$$\bar{q} = \frac{1}{1 - \bar{\theta} b_k} \quad > 1 \quad \text{if} \quad \bar{\theta} > 0$$

The higher the disaster risk in steady-state, the higher the Tobin's q: the threshold value for (dis-)investment incentives is higher in the presence of a disaster risk. This is because a rise in disaster risk today leads to a higher marginal replacement cost of capital tomorrow, associated with a rise in the level of investment that is required to increase firms' net market value.

[7]Our time-varying stochastic discount factor however differs from Gourio (2012)'s because we do not assume that total factor productivity is reduced by the same amount than the capital stock in case of a disaster.

[8]In microeconomic terms, the ratio of the marginal benefit in terms of utility of an extra unit of investment over the marginal benefit in terms of utility of sacrificing a unit of current consumption in order to have an extra unit of investment.

2.2 Firms

The production block is roughly similar to the New Keynesian literature,[9] except that we will allow the price adjustment to depend on the disaster risk. Production is split into a monopolistic competition market producing intermediate goods and a competitive sector producing the final consumption good as a CES composite of the intermediate goods.

Final goods producers

With intermediate goods indexed by j over a continuum of unit interval, the aggregate is given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

which corresponds to a downward sloping demand curve for each good j as

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} y_t$$

and to an aggregate price index given by

$$p_t = \left(\int_0^1 p_{j,t}^{1-\nu} dj \right)^{\frac{1}{1-\nu}}$$

Intermediate goods producers

Intermediate goods are produced with capital and labor, according to a standard Cobb-Douglas production function

$$Y_{j,t} = A_t \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha}$$

in which the capital leased to the firms is

$$\tilde{K}_t = u_t K_t \tag{13}$$

where u_t is the variable utilization rate of capital, and in which total factor productivity, denoted A_t, is driven by

$$\log A_t = (1 - \rho_A) \log \bar{A} + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A_t} \tag{14}$$

[9] See for instance Fernández-Villaverde and Rubio-Ramírez (2006)

where the shocks are small and normally distributed (ε_t is i.i.d. $N(0,1)$).

There is a two-step problem for firms producing the intermediate goods. First, each firm j minimizes capital and labor costs at each date, independently of price adjustment, subject to the restriction of producing at least as much as the intermediate good is demanded at the selling price, that is,

$$\min_{L_{j,t},\tilde{K}_{j,t}} p_t(w_t L_{j,t} + R_t^k \tilde{K}_{j,t})$$

$$\text{s.t.} \quad A_t \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \geq \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

The first-order conditions for this problem give a capital-labor ratio which holds at the aggregate level since it is the same across all firms

$$\left(\frac{\tilde{K}_{j,t}}{L_{j,t}}\right)^* = \frac{w_t}{R_t^k}\frac{\alpha}{(1-\alpha)}$$

and allows to write the optimal marginal input costs as

$$mc_t^* = w_t^{1-\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\frac{R_t^{\alpha}}{A_t}$$

from which the aggregate first-order conditions are expressed as

$$w_t = mc^*\,(1-\alpha)\,A_t\left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha} \tag{15}$$

$$R_t^k = mc^*\alpha A_t\left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha-1} \tag{16}$$

Then, given the optimal input mix, some firms maximize their profits by choosing their selling price $p_{j,t}$. We consider two alternative ways to introduce nominal stickiness. One is standard Calvo time-dependent pricing so that firms in the intermediate sector face a constant probability ζ_0 of being unable to change their price at each time t despite the disaster risk. The other one is to assume that firms' price adjustment increases in the aggregate risk, *i.e.* the gap between the current value of the probability ζ_t of being unable to change one's price and the Calvo probability ζ_0 is given by

$$\zeta_t - \zeta_0 = -\theta_t^{\iota}$$

where ι is the elasticity of the gap to the probability of disaster.[10] [11]

Writing ζ as standing either for ζ_0 in the first case or for ζ_t in the second, the profit-maximizing problem in both cases is

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\left(\frac{p_{j,t}}{p_{t+s}} \right)^{1-\nu} y_{t+s} - mc^*_{t+s} \left(\frac{p_{j,t}}{p_{t+s}} \right)^{-\nu} y_{t+s} \right)$$

The solution to this problem holds at the aggregate level ($p^*_t = p^*_{j,t}$). The gap between this optimal price p^*_t and the consumer price index p_t is

$$\frac{p^*_t}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t} \right)^{\nu} Y_{t+s} mc^*_{t+s}}{\sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\frac{p_{t+s}}{p_t} \right)^{\nu-1} Y_{t+s}}$$

This expression can finally be rewritten recursively in order to stress out the price adjustment dynamics and in terms of an inflation gap to allow for a non-zero inflation steady-state, such that

$$\frac{1+\pi^*_t}{1+\pi_t} = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1t}/p^{\nu}_t}{\Xi_{2t}/p^{\nu-1}_t} \tag{17}$$

with $\pi_t = \frac{p_t}{p_{t-1}} - 1$ the net inflation rate, π^*_t the net reset inflation rate, and

$$\frac{\Xi_{1t}}{p^{\nu}_t} = \frac{Q_{t+s}}{\beta} Y_t mc^*_t + \zeta\beta E_t \frac{\Xi_{1t+1}}{p^{\nu}_{t+1}} (1+\pi_{t+1})^{\nu}, \quad \text{and} \tag{18}$$

$$\frac{\Xi_{2t}}{p^{\nu-1}_t} = \frac{Q_{t+s}}{\beta} Y_t + \zeta\beta E_t \frac{\Xi_{2t+1}}{p^{\nu-1}_{t+1}} (1+\pi_{t+1})^{\nu-1} \tag{19}$$

All the computational details are given in Appendix.

2.3 Public authority

The public authority consumes some output G_t, charges lump sum taxes T_t to households, and issues debt D_t which pays interest i_t set up according to a standard Taylor-type rule that depends on the deviation of inflation from

[10] Note that this function requires to impose a parameter restriction so that ζ remains positive. With $\bar{\theta} = 0.01$ in particular, ι cannot be lower than 0.05.

[11] This price setting reminds the 'SS pricing' literature (Caplin and Leahy, 1991) although the firms do not react to the effective realization of aggregate shocks but to the expected risk. This is because the probability of disaster is incorporated in the forward-looking agents' optimization problem and the size of an effective disaster is constant here.

steady-state and on an output growth gap as

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\psi_\pi(\pi_t - \bar{\pi}) + \psi_Y\left(y_t - \bar{y}\right) + \bar{i}\right] + \sigma_i \varepsilon_{i_t} \tag{20}$$

in which y is the growth rate of output and where an overbar indicates the steady-state value of a variable. The public authority's budget constraint equates spending plus payment on existing debt to collected taxes plus new debt issuance[12], that is,

$$G_t + (1+i_t)\frac{D_t}{p_t} = T_t + \frac{D_{t+1}}{p_t}$$

in which G_t follows a first-order autoregressive process in the logs

$$\log G_t = (1-\rho_G)\log(\omega\bar{Y}) + \rho_G \log G_{t-1} + \sigma_G \varepsilon_{G_t} \tag{21}$$

where ω is the steady-state share of output devoted to public expenditures.

3 Equilibrium

3.1 Market clearing

Market-clearing in the bond market implies that the total amount of debt is equal to the total amount of bounds in period t

$$D_t = B_t$$

and market-clearing in output implies that

$$Y_t = C_t + I_t + G_t \tag{22}$$

Moreover, knowing the demand for individual intermediate goods firms, we are able to derive the aggregate production function as a function of the individual firms' production function and a measure of the inefficiency introduced by the dispersion in relative prices, $\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$, such that

$$Y_t = \frac{A_t \tilde{K}_t^{\alpha} L_t^{1-\alpha}}{\Omega_t} \tag{23}$$

in which the aggregate price distortion is given by the recursive equation

[12]We assume that there is no money, hence no seignorage revenue in the model.

$$\Omega_t = (1-\zeta)\left(\frac{1+\pi_t}{1+\pi_t^*}\right)^{\nu} + \zeta(1+\pi_t)^{\nu}\Omega_{t-1} \tag{24}$$

Finally, given that a fraction ζ of firms do not readjust their prices, the aggregate price index, $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$, is given by $p_t^{1-\nu} = (1-\zeta)p_t^{*1-\nu} + \zeta p_{t-1}^{1-\nu}$, further rewritten in inflation terms as

$$(1+\pi_t)^{1-\nu} = (1-\zeta)(1+\pi_t^*)^{1-\nu} + \zeta \tag{25}$$

Equilibrium is characterized by equations (3) to (25) in 23 unknowns: $\{Y, C, I, G, A, L, K, \tilde{K}, u, w, R^k, \Omega, \pi, \pi^*, \tilde{\Xi}_1, \tilde{\Xi}_2, mc^*, \lambda, \mu, i, q, Q, \theta\}$.

3.2 Calibration and steady-state analysis

Our calibration, summarized in Table 1, is mostly based on the standard New Keynesian literature (Smets and Wouters, 2003, Rudebusch and Swanson, 2008). In particular the value of the inverse of the elasticity of intertemporal substitution (EIS) ranges from 0.5 to 6 under CRRA preferences with a baseline value of 2. In addition, Barro (2006) found on historical data that the average share of capital that is destroyed in case of disaster is 43%, while Gourio (2012) estimates that the average probability of a such a disaster is 1.7% annually, backing it out from evidence on asset prices under the assumption that the fall in total factor productivity is also exactly equal to 43%. Since we use the quarterly calibration of standard New Keynesian models and are not able to replicate the estimation so far, we test for several values of $\bar{\theta}$ around a 1% benchmark, as well as for several values of b_k and of the persistence in the shock to θ, without significant changes in our results.

In our steady-state, the capital stock, output, and consumption are lower in the presence of a disaster risk as compared to the same economy without disaster for all values of risk aversion/EIS. Steady-state investment and labor may be larger in the presence of disasters if the EIS is very high ($\gamma = 0.5$), but are generally weaker, such that wages are generally lower. The firms can substitute labor to capital such that their steady-state marginal costs are unchanged even though the cost of capital is higher in case of disaster. Therefore the non-zero steady-state inflation rate is unaffected by disaster risk and equal to the public authority's target that we set at 2% annually. The main ratios, C/Y, I/Y, G/Y are in all cases slightly above or below their standard values, 60%, 20% and 20%, respectively. Finally, the steady-state risk premium in case of disaster corresponds to the wedge between the higher steady-state return on capital and the unchanged riskfree rate.

Gourio (2012) found that the model quantities shift to a lower steady-state in the economy with disaster risk (as compared to an economy without disaster) if and only if the EIS is larger than unity. Therefore, it is noteworthy to clarify at this point why we do get a lower steady-state for all values of the EIS here, on the one hand, and its further implications on the model dynamics, on the other hand. First, with Epstein-Zin preferences, *i.e.* dissociating the risk aversion coefficient from the inverse of the EIS, it would be possible to show that, when investment on capital becomes riskier, the *risk-adjusted* return on capital goes down for risk averse agents, while the effect of this change on the consumption-savings decisions depends on the value of the EIS (Weil, 1990, Angeletos, 2007). In particular, when the EIS is larger than unity ($\gamma < 1$), the substitution effect of a higher risk-adjusted return is larger than the income effect and savings fall. Therefore the steady-state capital stock and output are lower. However, when the EIS is equal to 1, both effects cancel each other out and savings are unaffected by changes in the risk-adjusted return, that is, are unaffected by changes in the return on capital even if agents are risk-averse.[13] Our specification, where risk aversion is only the inverse of the EIS, does not allow to disentangle the two effects, yet remains preferable in order to solve the equity premium puzzle by incorporating habit formation (Weil, 1989, Uhlig, 2007, Angeletos, 2007).

More importantly, the reason why we get lower steady-state macro quantities even when the EIS is unity is because we solve the model such that the disaster risk is treated as a small but certain probability of disaster instead of being a large uncertain shock. This allows to solve the model quite easily without having to maintain Gourio's assumption that the disaster is a strict combination of a depreciation shock to capital and a negative shock to the total factor productivity by the same amount. Meanwhile, this does not substantially restrict our business cycle analysis for two reasons. First, we capture the main first-moment effect of disaster risk by the fact that depreciation of capital will be higher in the future, even though we do not have the second-moment effect associated with higher uncertainty about future

[13] The EIS determines the *sign* of the effect of increased uncertainty on savings while the risk aversion only affects its *magnitude* (Weil, 1990).

depreciation.[14] Second, Gourio shows that Tallarini (2000)'s *observational equivalence* in the dynamics of the macroeconomic variables in case there is an aggregate risk or not does not hold when the probability of disaster is not constant. When the disaster risk is time-varying, Gourio finds that risk aversion matters for the macroeconomic dynamics, and this is captured here.

4 Impulse responses of the macroeconomic variables

Analyzing the effects of a time-varying risk on asset pricing would require to treat the disaster risk as an uncertainty shock and to use nonlinear methods to solve the model. However, since there is a consensus about the irrelevance of approximation beyond the first-order for the macroeconomic quantities, on the one hand, and given that we do not consider the case of a large shock, on the other hand, we maintain certainty-equivalence and first-order methods in this version of the paper, although we keep track of some second-order corrections in the Appendix.[15][16] For each (small) shock below, we compare the responses obtained in our model (solid line) to their counterpart in a flexible-price but otherwise similar model[17] (dashed line) and in a standard sticky-price New Keynesian model without disaster risk (dotted line).

4.1 A rise in the probability of disaster

Figure 1 depicts the responses of the main variables to a rise in the probability of disaster, θ. Investment and capital fall on impact as households foresee the upcoming depreciation of capital when the probability of disaster, θ, rises. These effects are much more important under Calvo price stickiness ($\zeta = 0.8$) than under flexible prices ($\zeta = 0$) as all firms do not adjust their prices downwards as much as they would optimally do to match the fall in

[14] Gourio admits that the two effects are present but cannot be disentangled in his article. In every case, both effects push the variables in the same direction, and the first-moment effect is far more important for macroeconomic quantities.

[15] Since certainty-equivalence holds, these correction terms are naturally very small.

[16] The effective occurrence of a disaster would be a large shock, whereas the rise in the probability of disaster considered here is a small one.

[17] The flexible-price model is different from Gourio's RBC with disaster risk since we have CRRA preferences with habit formation, a public authority, and variable utilization rate of capital, on the one hand, and because we do not assume a fall to TFP by the same amount as simultaneous to the rise in the probability of disaster, on the other hand.

aggregate demand. The capital stock still goes down next periods because of the depressed investment even though the probability of disaster gradually returns to its initial level (from the autoregressive process).

Labor supply decreases when prices are flexible because it is less attractive for workers to work today when the return on savings is low (intertemporal effect), despite a negative wealth effect that tends to push employment up.[18] Wages thus slightly rise. However, when prices are sticky, the firms that cannot readjust their prices downwards as much as they want face an even lower demand for their own intermediate goods, and thus in turn lower their demand of labor, leading wages to fall. Because capital and labor decrease more under sticky prices, combined with the fact that decrease in aggregate demand is more severe, the slump in output is far larger with nominal rigidity.

In the flexible case, consumption increases on impact as households substitute consumption for investment in the first period, while lower output leads consumption to fall in the next periods, for standard values of the EIS and/or risk aversion.[19] With sticky prices however, consumption falls on impact for the baseline calibration ($\gamma = 2$), or lower values of the EIS (higher risk aversion). For very low risk aversion, consumption moves up on impact similarly to the flexible-price case but a quantitative difference due to price stickiness remains, as shown in Figure 5.

As investment in capital is riskier, households' demand for safer government bonds rises, so that the short-term nominal interest rate falls ("flight to quality" effect). However, because of the inertia in the Taylor-type reaction, the interest rate — and therefore inflation — falls less under price stickiness Finally, actual inflation decreases less than reset inflation, so that the price dispersion falls, but still falls more than the nominal interest rate, so that the real rate rises.

Figures 6 to 10 present some robustness checks and alternative specifications. Figure 6 considers different values of the steady-state probability of disaster ($\bar{\theta}$). While the magnitude of the effects increases in the steady-state disaster risk, the qualitative responses are all identical. Figure 7 gives some alternative values for the persistence of the shock (ρ_θ). Figure 8 tests for

[18]The relative importance of the two effects would depend on the EIS with Epstein-Zin preferences. However this result is familiar with standard calibration of CRRA preferences.

[19]Gourio (2012) found a similar effect with a slightly different flexible-price model and a simultaneous shock to the TFP.

different values for the share of capital which is destroyed in case of disaster (b_k), including a possible negative value.[20]

More importantly, Figure 9 gives the responses under state-dependent price stickiness for different values of the parameter $\iota < 1$.[21] The responses still differ significantly from the pure flexible-price version of the model ($\zeta = 0$) and our main results hold, notably the drop in wages, including for an extreme $\iota = 0.1$.

We finally consider a fall in the probability of disaster in Figure 10. Table 2 gives the second-order correction terms associated with this shock, naturally found to be very small under the certainty-equivalence assumption.

To sum up, a rise in the probability of disaster creates a recession, a fall in inflation, a flight to quality in terms of asset demand, depressed investment and labor, as well as lower consumption for standard risk aversion. The fact that the probability of a disaster is higher suffices to generate this recession, without effective occurrence of the disaster.

4.2 Standard shocks

The responses to standard shocks in the model with disaster risk are very close to the responses in a standard New Keynesian model.

For a TFP shock (Figure 2), output and investment rise because the marginal returns on labor and capital rise. However this is slightly less important in the presence of a disaster risk which depreciates capital. Consumption rise more however from the substitution effect between investment and consumption for households. The response of labor is discussed extensively in the literature: in opposition to a RBC where labor increases because the marginal return on labor is higher, sticky prices prevent some firms from lowering their prices leading them to lower their labor demand because of the contraction in demand for their own intermediate goods (Galí, 1999). In addition, higher incomes for households make leisure more desirable so that the supply of labor does not substantially rise neither. As reset inflation is higher than actual inflation, price dispersion falls and the real interest rate goes up despite the fall in the nominal rate.

A positive shock to public expenditures (Figure 3) also replicates the very

[20] A negative value of b_k verifies that the model works symmetrically such that the rare event could be a "miracle" instead of a "disaster".

[21] When $\iota \geq 1$, the responses are almost identical to the time-dependent pricing case.

well-known reactions. In all cases, there is a temporary rise in output from the rise in aggregate demand, an eviction effect on private consumption and investment, hence a fall in capital. Thus firms rely more on labor and wages go up. High reset inflation creates more price dispersion, and the nominal rate is increased.

Finally, a monetary contraction (Figure 4) generates the standard decrease in all macro quantities, as well as in inflation and price dispersion.

5 Further research

This paper provides a baseline framework that could be used to develop a number of innovative research ideas, including the role of monetary policy to prevent self-fulfilling recessions in case of misperceptions about the disaster risk. This Section presents our research agenda, which broadly consists in three steps.

First, we would like to account for a *perceived* risk of disaster along with the *real* disaster risk. Gourio (2012) considers that the probability of disaster introduced in his model (and in ours) may result from the economic agents' perception, probably because considering that the probability taken as given by the agents is the real risk would be associated with perfect individual rationality and knowledge about disasters while one could be more agnostic by considering it as merely perceived, especially for *rare* events. We think that it would be helpful to build on the behavioral macroeconomics literature (Gabaix and Laibson, 2002, De Grauwe, 2010, Fuster, Laibson and Mendel, 2010, Angeletos and La'O, 2012, Barsky and Sims, 2012) in order to disentangle a *perceived* from a *real* disaster risk. Another mean would be the use of computational methods in order to keep the disaster variable (x_{t+1}) as an indicator in the Euler equation instead of substituting the time-varying probability θ_t of an effective future occurrence. This would allow to simulate a rise in the probability of disaster while preventing the real occurrence of a disaster by accounting for uncertainty in the model.

As a second step, we will evaluate the model predictions in terms of asset pricing, especially the countercyclicality of the risk premium. Some interactions between price rigidity and the risk of disaster may affect equity returns. The asset price volatility may in turn have important consequences on consumption volatility. In particular the perception of disaster risk may be one of the psychological mechanisms that alter the reactivity of consumption changes to asset price movements (see Lynch, 1996, or Gabaix and Laibson, 2002, for instance), in addition to habit formation (Campbell and Cochrane, 1999, Uhlig, 2007), or adjustment costs (Grossman and Laroque, 1990). On practical grounds, pricing assets requires a few more sophistications in our setup. One is to go beyond the first-order approximation in the Taylor expansion. The consensus in the literature is that these higher-order terms do not matter for the responses of macroeconomic quantities we have focused on so far but have an important role in the asset pricing in the presence of a time-varying risk. Another key element will be to add corporate bonds in the model since leverage is a standard way to make equity returns more volatile and procyclical — in line with the data — in the literature, which may be even more relevant in a model in which firms' prices are sticky.

Finally, we would like to assess the desirability of monetary policy to prevent a (self-fulfilling) recession from a sudden rise in the (perceived) probability of disaster. Several conventional and unconventional interventions could be compared with one another by incorporating a welfare function measuring their effectiveness. In particular we think of adding an extra term in the Taylor-type rule which would represent a direct response of the monetary authority in the face of a wave of pessimism. This would be a quasi-conventional intervention, making changes in the nominal interest rate more reactive but still limited by the zero lower bound. A more unconventional measure could consist in purchasing corporate bonds (which may encompass bank debt), directly affected by the disaster risk, by selling riskfree government bonds (as far as sovereign default is excluded).

6 Conclusion

This paper provides a baseline framework to analyze the business cycle responses of macroeconomic quantities in the presence of a small time-varying disaster risk in an otherwise standard New Keynesian model. While following Gourio (2012) on the description of an economic disaster, we relax the assumption that total factor productivity needs to fall by the same amount than the capital stock in case of a disaster. By incorporating investment adjustment costs and monopolistic competition, we show that the magnitude of the recession following a shock to the probability of disaster may be far increased. As compared with the early papers on rare events, we also account for the fact that consumption and wages do not rise in distressed economic times, whether nominal rigidity is time-dependent or state-dependent. More generally, this paper is a first step towards the introduction of rare events into the models used to conduct monetary policy, and will be used to compare the effectiveness of several interventions in the presence of such a risk.

Bibliography

[1] George-Marios Angeletos. Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic Dynamics*, 10(1):1–30, January 2007.

[2] George-Marios Angeletos and Jennifer La'O. Sentiments. Unpublished manuscript, available at `http://faculty.chicagobooth.edu/jennifer.lao/research/sentiments.pdf`.

[3] Robert J. Barro. Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics*, 2006.

[4] Robert B. Barsky and Eric R. Sims. Information, animal spirits, and the meaning of innovations in consumer confidence. *American Economic Review, forthcoming*, 2012.

[5] Nicholas Bloom. The impact of uncertainty shocks. *Econometrica*, 77(3):623–685, 05 2009.

[6] Craig Burnside and Martin Eichenbaum. Factor-hoarding and the propagation of business-cycle shocks. *American Economic Review*, 86(5):1154–74, December 1996.

[7] John Campbell and John Cochrane. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy*, 1999.

[8] Andrew Caplin and John Leahy. State-dependent pricing and the dynamics of money and output. *Quarterly Journal of Economics*, 106(3):683–708, August 1991.

[9] Paul De Grauwe. Top-down versus bottom-up macroeconomics. *CESifo Economic Studies*, 56(4):465–497, December 2010.

[10] Paul De Grauwe. Animal spirits and monetary policy. *Economic Theory*, 47(2):423–457, June 2011.

[11] Jesús Fernández-Villaverde and Juan F. Rubio-Ramírez. A baseline DSGE model. Unpublished manuscript, available at `http://economics.sas.upenn.edu/~jesusfv/benchmark_DSGE.pdf`, 2006.

[12] Andreas Fuster, Benjamin Hebert, and David Laibson. Natural expectations, macroeconomic dynamics, and asset pricing. NBER Working Papers 17301, National Bureau of Economic Research, Inc, August 2011.

[13] Andreas Fuster, David Laibson, and Brock Mendel. Natural expectations and macroeconomic fluctuations. *Journal of Economic Perspectives*, 24(4):67–84, 2010.

[14] Xavier Gabaix. Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. *Quarterly Journal of Economics*, 127(2):645–700, 2012.

[15] Xavier Gabaix and David Laibson. The 6D bias and the equity-premium puzzle. In *NBER Macroeconomics Annual 2001, Volume 16*, NBER Chapters, pages 257–330. National Bureau of Economic Research, Inc, 2002.

[16] Jordi Galí. Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *American Economic Review*, 89(1):249–271, March 1999.

[17] François Gourio. Disasters and recoveries. *American Economic Review*, 98(2):68–73, May 2008.

[18] François Gourio. Disaster risk and business cycles. *American Economic Review, forthcoming*, 2012.

[19] Sanford J. Grossman and Guy Laroque. Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. *Econometrica*, 58(1):25–51, January 1990.

[20] Anthony W. Lynch. Decision frequency and synchronization across agents: Implications for aggregate consumption and equity return. *Journal of Finance*, 51(4):1479–97, September 1996.

[21] Nicolas Petrosky-Nadeau. TFP during a credit crunch. GSIA working papers, Carnegie Mellon University, Tepper School of Business, 2010.

[22] Thomas A. Rietz. The equity risk premium: a solution. *Journal of Monetary Economics*, 22(1):117–131, July 1988.

[23] Glenn D. Rudebusch and Eric T. Swanson. Examining the bond premium puzzle with a DSGE model. *Journal of Monetary Economics*, 55:S111–S126, October 2008.

[24] Frank Smets and Rafael Wouters. An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1(5):1123–1175, 09 2003.

[25] Thomas D. Tallarini Jr. Risk-sensitive real business cycles. *Journal of Monetary Economics*, 45(3):507–532, June 2000.

[26] Harald Uhlig. Explaining asset prices with external habits and wage rigidities in a dsge model. *American Economic Review*, 97(2):239–243, May 2007.

[27] Philippe Weil. The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics*, 24(3):401–421, November 1989.

[28] Philippe Weil. Nonexpected utility in macroeconomics. *Quarterly Journal of Economics*, 105(1):29–42, February 1990.

A Mathematical appendix

A.1 Households

Given that next period disaster x_{t+1} is equal to 1 with probability θ_t and equal to 0 with probability $1-\theta_t$, the law of accumulation of capital can be rewritten as

$$K_{t+1} = [\theta_t(1-b_k) + (1-\theta_t)]\{(1-\delta_t)K_t + [1-S(I_t/I_{t-1})]I_t\}$$

$$= (1 - \theta_t b_k)\{(1 - \delta_t)K_t + [1 - S(I_t/I_{t-1})]I_t\}$$

Therefore the Lagrangian for the households' problem is

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{(C_t - hC_{t-1})^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\phi}}{1+\phi} \right) \right.$$
$$+\lambda_t \left(W_t L_t + (1 + i_{t-1}) \frac{B_t}{p_t} + \frac{M_t}{p_t} - \frac{B_{t+1}}{p_t} - \frac{M_{t+1}}{p_t} + R_t^k u_t K_t + \Pi_t - T_t - I_t - C_t \right)$$
$$\left. + \mu_t \left[\left((1 - \delta u_t^{\eta}) K_t + \left(1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t \right) (1 - \theta_t b_k) - K_{t+1} \right] \right\}$$

and the first-order conditions are

- Consumption: $\lambda_t = (C_t - hC_{t-1})^{-\gamma} - \beta h E_t (C_{t+1} - hC_t)^{-\gamma}$
- Labor: $\chi L_t^{\phi} = w_t \lambda_t$
- Bonds: $\lambda_t = \beta E_t \lambda_{t+1} (1 + i_t)(1 + \pi_{t+1})^{-1}$, with$1 + \pi_{t+1} \equiv \frac{p_{t+1}}{p_t}$
- Capital: $\mu_t = \beta E_t \left[\lambda_{t+1} R_{t+1}^k u_{t+1} + \mu_{t+1} \left(1 - \delta u_{t+1}^{\eta} \right) (1 - \theta_{t+1} b_k) \right]$
- Capital utilization rate: $\lambda_t R_t^k = \mu_t \delta \eta u_t^{\eta - 1} (1 - \theta_t b_k)$
- Investment: $\lambda_t = \mu_t (1 - \theta_t b_k) \left[1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \tau \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right]$
  $+\beta E_t \mu_{t+1} (1 - \theta_{t+1} b_k) \tau \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$

With no investment adjustment cost ($\tau = 0$), the FOC on investment becomes $\lambda_t = \mu_t (1 - \theta_t b_k)$, which in turn implies from the FOC on the capital utilization rate that $R_t^k = \delta_t'$. Substituting into the FOC on capital gives the Euler equation (11) in case $\tau = 0$.

A.2 Firms

- Production aggregation

The aggregate of intermediate goods is given by

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$$

so that the profit maximization problem of the representative firm in the final sector is

$$\underset{Y_{t,j}}{max}\ p_t \left(\int_0^1 Y_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}} - \int_0^1 p_{j,t} Y_{j,t} dj$$

The first-order condition with respect to $Y_{t,j}$ yields a downward sloping demand curve for each intermediate good j as

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

The nominal value of the final good is the sum of prices times quantities of intermediates

$$p_t Y_t = \int_0^1 p_{j,t} Y_{j,t} dj$$

in which Y_t is substituted to give the aggregate price index as

$$p_t = \left(\int_0^1 p_{j,t}^{1-\nu} dj \right)^{\frac{1}{1-\nu}}$$

- Cost minimization

Firms are price-takers in the input markets, facing a nominal wage $w_t p_t$ and a nominal rental rate $R_t^k p_t$ (w_t and R_t^k are in real terms). Therefore, they choose the optimal quantities of labor and capital given the input prices and subject to the restriction of producing at least as much as the intermediate good is demanded at the given price. The intratemporal problem is

$$\min_{L_{j,t}, \tilde{K}_{j,t}} \quad w_t p_t L_{j,t} + R_t^k p_t \tilde{K}_{j,t}$$

$$\text{s.t.} \quad a_t \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} \geq \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$$

The first-order conditions are

$$(L_{j,t} :) \quad w_t = \frac{\varphi_{j,t}}{p_t} (1-\alpha) A_t \left(\frac{\tilde{K}_{j,t}}{L_{j,t}} \right)^{\alpha}$$

$$(\tilde{K}_{j,t}:) \quad R_t^k = \frac{\varphi_{j,t}}{p_t} \alpha A_t \left(\frac{\tilde{K}_{j,t}}{L_{j,t}} \right)^{\alpha - 1}$$

in which the Lagrange multiplier $\varphi_{j,t}$ can be interpreted as the (nominal) marginal cost associated with an additional unit of capital or labor. Rearranging gives the optimal capital over labor ratio as

$$\left(\frac{\tilde{K}_{j,t}}{L_{j,t}} \right)^* = \frac{w_t}{R_t^k} \frac{\alpha}{(1-\alpha)}$$

in which none of the terms on the right hand side depends on j, and thus holds for all firms in equilibrium, *i.e.*, $\frac{\tilde{K}_t}{L_t} = \frac{\tilde{K}_{j,t}}{L_{j,t}}$. Replacing in the first-order conditions further gives $mc_t^* = \frac{\varphi_t}{p_t}$ as

$$mc_t^* = w_t^{1-\alpha} \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} \frac{\left(R_t^k\right)^{\alpha}}{A_t}$$

- Profit maximization

Let us now consider the pricing problem of a firm that gets to update its price in period t and wants to maximize the present discounted value of future profits. First, the (nominal) profit flow, $p_{j,t} Y_{j,t} - w_t p_t L_{j,t} - R_{j,t}^k p_t \tilde{K}_{j,t}$, can be rewritten as $\Pi_{j,t} = (p_{j,t} - \varphi_t) Y_{j,t}$, that is, in real terms, $\frac{\Pi_{j,t}}{p_t} = \frac{p_{j,t}}{p_t} Y_{j,t} - mc_t^* Y_{j,t}$. Firms will discount future profit flows by both the stochastic discount factor, $Q_t = \beta^s \lambda_{t+s}$, and by the probability ζ^s that a price chosen at time t is still in effect at time s. Replacing $Y_{j,t} = \left(\frac{p_{j,t}}{p_t} \right)^{-\nu} Y_t$, the profit maximization problem is

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} \left(\left(\frac{p_{j,t}}{p_{t+s}} \right)^{1-\nu} Y_{t+s} - mc_{t+s}^* \left(\frac{p_{j,t}}{p_{t+s}} \right)^{-\nu} Y_{t+s} \right)$$

Given that $mc_t^* = \frac{\varphi_t}{p_t}$ and factorizing, we can rewrite it as

$$\max_{p_{j,t}} E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s} \left(p_{j,t}^{1-\nu} - \varphi_t p_{j,t}^{-\nu} \right)$$

The first-order condition is

$$E_t \sum_{s=0}^{\infty} (\zeta)^s Q_{t+s} p_{t+s}^{\nu-1} Y_{t+s} \left((1-\nu) p_{j,t}^{-\nu} + \nu \varphi_t p_{j,t}^{-\nu-1} \right) = 0$$

which simplifies as

$$p_{j,t}^{*}=\frac{\nu}{\nu-1}E_{t}\frac{\sum_{s=0}^{\infty}\left(\zeta\right)^{s}Q_{t+s}p_{t+s}^{\nu}Y_{t+s}mc_{t+s}^{*}}{\sum_{s=0}^{\infty}\left(\zeta\right)^{s}Q_{t+s}p_{t+s}^{\nu-1}Y_{t+s}}$$

Note that this optimal price depends on aggregate variables only, so that $p_t^* = p_{j,t}^*$. The gap between the current price and the optimal aggregate price is thus given by

$$\frac{p_{t}^{*}}{p_{t}}=\frac{\nu}{\nu-1}E_{t}\frac{\sum_{s=0}^{\infty}\left(\zeta\right)^{s}Q_{t+s}\left(\frac{p_{t+s}}{p_{t}}\right)^{\nu}Y_{t+s}mc_{t+s}^{*}}{\sum_{s=0}^{\infty}\left(\zeta\right)^{s}Q_{t+s}\left(\frac{p_{t+s}}{p_{t}}\right)^{\nu-1}Y_{t+s}}$$

In order to stress out the recursive price adjustment, let define p_t^* as

$$p_{t}^{*}=\frac{\nu}{\nu-1}E_{t}\frac{\Xi_{1_{t}}}{\Xi_{2_{t}}}$$

in which Ξ_{1_t} and Ξ_{2_t} can be expressed recursively as

$$\Xi_{1_{t}}=Q_{t+s}p_{t}^{\nu}Y_{t}mc_{t}^{*}+\zeta E_{t}\Xi_{1_{t+1}}$$

$$\Xi_{2_{t}}=Q_{t+s}p_{t}^{\nu-1}Y_{t}+\zeta E_{t}\Xi_{2_{t+1}}$$

and rewritten as

$$\beta\frac{\Xi_{1_{t}}}{p_{t}^{\nu}}=Q_{t+s}Y_{t}mc_{t}^{*}+\zeta\beta^{2}E_{t}\frac{\Xi_{1_{t+1}}}{p_{t+1}^{\nu}}\left(\frac{p_{t+1}}{p_{t}}\right)^{\nu}$$

$$\beta\frac{\Xi_{2_{t}}}{p_{t}^{\nu-1}}=Q_{t+s}Y_{t}+\zeta\beta^{2}E_{t}\frac{\Xi_{2_{t+1}}}{p_{t+1}^{\nu-1}}\left(\frac{p_{t+1}}{p_{t}}\right)^{\nu-1}$$

Therefore, we have

$$\frac{p_{t}^{*}}{p_{t}}=\frac{\nu}{\nu-1}E_{t}\frac{\frac{\Xi_{1_{t}}}{p_{t}^{\nu}}}{\frac{\Xi_{2_{t}}}{p_{t}^{\nu-1}}}$$

A.3 Aggregation

Bonds market

Market-clearing requires that:

$$D_t = B_t$$

Aggregate demand

First replace $D_t = B_t$ into the public authority's budget constraint, and express T_t as

$$T_t = G_t + (1 + i_t)\frac{B_t}{p_t} - \frac{B_{t+1}}{p_t}$$

which can be plugged into the household budget constraint as

$$C_t + I_t + \frac{B_{t+1}}{p_t} = w_t L_t + (1 + i_t)\frac{B_t}{p_t} + R_t^k \tilde{K}_t + \Pi_t - \left(G_t + (1 + i_t)\frac{B_t}{p_t} - \frac{B_{t+1}}{p_t}\right)$$

This further simplifies to:

$$C_t + I_t + G_t = w_t L_t + R_t^k \tilde{K}_t + \Pi_t$$

where we have to verify that the RHS is equal to Y_t. Total profits Π_t must be equal to the sum of profits earned by intermediate good firms, that is

$$\Pi_t = \int_0^1 \Pi_{j,t} dj$$

Real profits earned by intermediate good firms j are given by

$$\Pi_{j,t(real)} = \frac{p_{j,t}}{p_t} Y_{j,t} - w_t L_{j,t} - R_t^k \tilde{K}_{j,t}$$

Substituting $Y_{j,t}$, we have

$$\Pi_{j,t(real)} = \left(\frac{p_{j,t}}{p_t}\right)^{1-\nu} Y_t - w_t L_{j,t} - R_t^k \tilde{K}_{j,t}$$

Therefore,

$$\Pi_{t(real)} = \int_0^1 \left(\left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t - w_t L_{j,t} - R_t^k \tilde{K}_{j,t} \right) dj = \int_0^1 \left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t dj \\ - \int_0^1 w_t L_{j,t} dj - \int_0^1 R_t^k \tilde{K}_{j,t} dj$$

$$\Pi_{t(real)} = \int_0^1 \left(\left(\frac{p_{j,t}}{p_t} \right)^{1-\nu} Y_t - w_t L_{j,t} - R_t^k \tilde{K}_{j,t} \right) dj = Y_t \frac{1}{p_t^{1-\nu}} \int_0^1 (p_{j,t})^{1-\nu} dj \\ - w_t \int_0^1 L_{j,t} dj - R_t^k \int_0^1 \tilde{K}_{j,t} dj$$

Given that

- the aggregate price level is $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$,
- aggregate labor demand must equal supply, $\int_0^1 L_{j,t} dj = L_t$, and
- aggregate supply of capital services must equal demand $\int_0^1 \tilde{K}_{j,t} dj = \tilde{K}_t$,

the aggregate profit is

$$\Pi_{t(real)} = Y_t - w_t L_t - R_t^k \tilde{K}_t$$

Plugging this expression into the household budget constraint finally gives the aggregate accounting identity as

$$Y_t = C_t + I_t + G_t$$

Inflation

Firms have a probability $1 - \zeta$ of getting to update their price each period. Since there are an infinite number of firms, there is also the exact fraction $1 - \zeta$ of total firms who adjust their prices and the fraction ζ who stay with the previous period price. Moreover, since there is a random sampling from the entire distribution of firm prices, the distribution of any subset of firm prices is similar to the entire distribution. Therefore, the aggregate price index, $p_t^{1-\nu} = \int_0^1 p_{j,t}^{1-\nu} dj$, is rewritten as

$$p_t^{1-\nu} = \int_0^{1-\zeta} p_t^{*1-\nu} dj + \int_{1-\zeta}^{1} p_{j,t-1}^{1-\nu} dj$$

which simplifies to

$$p_t^{1-\nu} = (1-\zeta) p_t^{*1-\nu} + \zeta p_{t-1}^{1-\nu}$$

Dividing both sides of the equation by $p_{t-1}^{1-\nu}$

$$\left(\frac{p_t}{p_{t-1}}\right)^{1-\nu} = (1-\zeta)\left(\frac{p_t^*}{p_{t-1}}\right)^{1-\nu} + \zeta \left(\frac{p_{t-1}}{p_{t-1}}\right)^{1-\nu}$$

and defining gross inflation as $1+\pi_t = \frac{p_t}{p_{t-1}}$ and gross reset inflation as $1+\pi_t^* = \frac{p_t^*}{p_{t-1}}$, we get

$$(1+\pi_t)^{1-\nu} = (1-\zeta)(1+\pi_t^*)^{1-\nu} + \zeta$$

Finally, from $p_t^* = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1t}}{\Xi_{2t}}$, we have

$$\frac{p_t^*}{p_t} = \frac{\nu}{\nu-1} E_t \frac{\Xi_{1t}/p_t^{\nu}}{\Xi_{2t}/p_t^{\nu-1}}$$

Rewritting the left-hand side as $\frac{p_t^*}{p_t}\frac{p_{t-1}}{p_{t-1}}$, and rearranging, we get

$$\pi_t^* = \pi_t \frac{\nu}{\nu-1} E_t \frac{\Xi_{1t}/p_t^{\nu}}{\Xi_{2t}/p_t^{\nu-1}}$$

Therefore we have

$$\frac{\Xi_{1t}}{p_t^{\nu}} = \frac{Q_{t+s}}{\beta} Y_t mc_t^* + \zeta\beta E_t \frac{\Xi_{1t+1}}{p_{t+1}^{\nu}} (1+\pi_{t+1})^{\nu}$$

$$\frac{\Xi_{2t}}{p_t^{\nu-1}} = \frac{Q_{t+s}}{\beta} Y_t + \zeta\beta E_t \frac{\Xi_{2t+1}}{p_{t+1}^{\nu-1}} (1+\pi_{t+1})^{\nu-1}$$

Aggregate supply

We know that the demand to individual firm j is given by

$$Y_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

and that firm j hires labor and capital in the same proportion than the aggregate capital to labor ratio (common factor markets). Hence, substituting in the production function for the intermediate good j we get

$$A_t \left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha} L_{j,t} = \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} Y_t$$

Then, summing up across the intermediate firms gives

$$A_t \left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha} \int_0^1 L_{j,t} dj = Y_t \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$$

Given that aggregate labor demand equals aggregate labor supply $\int_0^1 L_{j,t} dj = L_t$, we have

$$\int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj Y_t = A_t \tilde{K}_t^{\alpha} L_t^{1-\alpha}$$

Thus, the aggregate production function can be written as

$$Y_t = \frac{A_t \tilde{K}_t^{\alpha} L_t^{1-\alpha}}{\Omega_t}$$

where $\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj$ measures a distortion introduced by the dispersion in relative prices.[22] In order to express Ω_t in aggregate terms, let decompose it according to the Calvo pricing assumption again, so that

$$\Omega_t = \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\nu} dj = p_t^{\nu} \int_0^1 p_{j,t}^{-\nu}$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = p_t^{\nu} \left(\int_0^{1-\zeta} p_t^{*-\nu} dj + \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj\right)$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = p_t^{\nu} (1-\zeta) p_t^{*-\nu} + p_t^{\nu} \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = (1-\zeta) \left(\frac{p_t^*}{p_t}\right)^{-\nu} + p_t^{\nu} \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj$$

$$p_t^{\nu} \int_0^1 p_{j,t}^{-\nu} = (1-\zeta) \left(\frac{p_t^*}{p_{t-1}}\right)^{-\nu} \left(\frac{p_{t-1}}{p_t}\right)^{-\nu} + p_t^{\nu} \int_{1-\zeta}^1 p_{j,t-1}^{-\nu} dj$$

[22] This distortion is not the one associated with the monopoly power of firms but an additional one that arises from the relative price fluctuations due to prie stickiness.

$$p_t^{\nu}\int_0^1 p_{j,t}^{-\nu} = (1-\zeta)(1+\pi_t^*)^{-\nu}(1+\pi_t)^{\nu} + p_{t-1}^{-\nu}p_t^{\nu}\int_{1-\zeta}^1 \left(\frac{p_{j,t-1}}{p_{t-1}}\right)^{-\nu} dj$$

Given random sampling and the fact that there is a continuum of firms

$$\Omega_t = (1-\zeta)(1+\pi_t^*)^{-\nu}(1+\pi_t)^{\nu} + \zeta(1+\pi_t)^{\nu}\Omega_{t-1}$$

A.4 Full set of equilibrium conditions

$$K_{t+1} = \left\{(1-\delta u_t^{\eta})K_t + \left[1-\frac{\tau}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right]I_t\right\}(1-\theta_t b_k) \tag{26}$$

$$\log\theta_t = (1-\rho_\theta)\log\bar{\theta} + \rho_\theta\log\theta_{t-1} + \sigma_\theta\varepsilon_{\theta_t} \tag{27}$$

$$\tilde{K}_t = u_t K_t \tag{28}$$

$$\lambda_t = (C_t - hC_{t-1})^{-\gamma} - \beta h E_t(C_{t+1} - hC_t)^{-\gamma} \tag{29}$$

$$\chi L_t^{\phi} = w_t\lambda_t \tag{30}$$

$$\lambda_t = \beta E_t\lambda_{t+1}(1+i_{t+1})(1+\pi_{t+1})^{-1} \tag{31}$$

$$\mu_t = \beta E_t\left[\lambda_{t+1}R_{t+1}^k u_{t+1} + \mu_{t+1}\left(1-\delta u_{t+1}^{\eta}\right)(1-\theta_{t+1}b_k)\right] \tag{32}$$

$$\lambda_t R_t^k = \mu_t\delta\eta u_t^{\eta-1}(1-\theta_t b_k) \tag{33}$$

$$\lambda_t = \mu_t(1-\theta_t b_k)\left[1-\frac{\tau}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 - \tau\left(\frac{I_t}{I_{t-1}}-1\right)\frac{I_t}{I_{t-1}}\right] + \beta E_t\mu_{t+1}(1-\theta_{t+1}b_k)\tau\left(\frac{I_{t+1}}{I_t}-1\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \tag{34}$$

$$1 + i_t = (E_t Q_{t,t+1})^{-1} \tag{35}$$

$$q_t = \frac{\mu_t}{\lambda_t} \tag{36}$$

$$\log A_t = (1 - \rho_A) \log \bar{A} + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A_t} \tag{37}$$

$$w_t = mc^* \left(1 - \alpha\right) A_t \left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha} \tag{38}$$

$$R_t^k = mc^* \alpha A_t \left(\frac{\tilde{K}_t}{L_t}\right)^{\alpha - 1} \tag{39}$$

$$(1 + \pi_t^*) = (1 + \pi_t) \frac{\nu}{\nu - 1} E_t \frac{\tilde{\Xi}_{1_t}}{\tilde{\Xi}_{2_t}} \tag{40}$$

where $\tilde{\Xi}_{1_t} = \frac{\Xi_{1_t}}{p_t^{\nu}}$ and $\tilde{\Xi}_{2_t} = \frac{\Xi_{2_t}}{p_t^{\nu-1}}$.

$$\tilde{\Xi}_{1_t} = \lambda_t Y_t mc_t^* + \zeta \beta E_t \tilde{\Xi}_{1_{t+1}} \left(1 + \pi_{t+1}\right)^{\nu} \tag{41}$$

$$\tilde{\Xi}_{2_t} = \lambda_t Y_t + \zeta \beta E_t \tilde{\Xi}_{2_{t+1}} \left(1 + \pi_{t+1}\right)^{\nu - 1} \tag{42}$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[\psi_\pi (\pi_t - \bar{\pi}) + \psi_Y \left(y_t - \bar{y}\right) + \bar{i}\right] + \sigma_i \varepsilon_{i_t} \tag{43}$$

$$\log G_t = (1 - \rho_G) \log(\omega \bar{Y}) + \rho_G \log G_{t-1} + \sigma_G \varepsilon_{G_t} \tag{44}$$

$$Y_t = C_t + I_t + G_t \tag{45}$$

$$Y_t = \frac{A_t \tilde{K}^{\alpha} L_t^{1-\alpha}}{\Omega_t} \tag{46}$$

$$(1 + \pi_t)^{1-\nu} = (1 - \zeta)(1 + \pi_t^*)^{1-\nu} + \zeta \tag{47}$$

$$\Omega_t = (1-\zeta)(1+\pi_t^*)^{-\nu}(1+\pi_t)^{\nu} + \zeta(1+\pi_t)^{\nu}\Omega_{t-1} \tag{48}$$

This is a system of 23 equations in 23 unknowns: $\{Y, C, I, G, A, L, K, \tilde{K}, u, w, R^k, \Omega, \pi, \pi^*, \tilde{\Xi}_1, \tilde{\Xi}_2, mc^*, \lambda, \mu, i, q, Q, \theta\}$.

A.5 Steady-state

From the FOC on investment (34), we have

$$\bar{\lambda} = \bar{\mu}(1-\bar{\theta}b_k) \tag{49}$$

which implies by (36) that

$$\bar{q} = \frac{\bar{\mu}}{\bar{\lambda}} = \frac{1}{1-\bar{\theta}b_k} \tag{50}$$

Without disaster risk, we would have $\bar{q} = 1$ determining the threshold under which firms invest or disinvest to raise their market value. Here disaster risk implies that this threshold is greater than unity since, for a given replacement cost in terms of utility, firms find it less profitable to invest as the probability that a part of their capital turns out to be destroyed rises.

Normalizing $\bar{u} = 1$, we have $\bar{\tilde{K}} = \bar{K}$ from (28), and from (33)

$$\bar{R}^k = \delta\eta \tag{51}$$

Moreover (32) implies that

$$\bar{R}^k = \frac{1}{\beta(1-\bar{\theta}b_k)} - (1-\delta) \tag{52}$$

The last two equations imply a parameter restriction of η as

$$\eta = 1 + \frac{\frac{1}{\beta(1-\bar{\theta}b_k)} - 1}{\delta} \tag{53}$$

Therefore, with parameter values $\beta = .99$, $\delta = .025$, $\bar{\theta} = .017$, and $b_k = .43$, we have $\eta = 1.7$ (and $\eta = 1.404$ in a world without disasters).

Then from (47), and given the target inflation rate $\bar{\pi}$, we have the steady-state reset inflation rate as

$$(1+\bar{\pi}^*) = \left(\frac{(1+\bar{\pi})^{1-\nu} - \zeta}{1-\zeta}\right)^{\frac{1}{1-\nu}} \tag{54}$$

and, since from (40) we have,

$$(1+\bar{\pi}^*) = (1+\bar{\pi})\frac{\nu}{\nu-1}\frac{\tilde{\bar{\Xi}}_1}{\tilde{\bar{\Xi}}_1} \tag{55}$$

where, from (41) and (42),

$$\tilde{\bar{\Xi}}_1 = \frac{\bar{\lambda}\bar{Y}\bar{mc}^*}{1-\zeta\beta(1+\bar{\pi})^{\nu}} \tag{56}$$

$$\tilde{\bar{\Xi}}_2 = \frac{\bar{\lambda}\bar{Y}}{1-\zeta\beta(1+\bar{\pi})^{\nu-1}} \tag{57}$$

we get

$$(1+\bar{\pi}^*) = (1+\bar{\pi})\frac{\nu}{\nu-1}\bar{mc}^*\frac{1-\zeta\beta(1+\bar{\pi})^{\nu-1}}{1-\zeta\beta(1+\bar{\pi})^{\nu}} \tag{58}$$

which gives the steady-state marginal cost $\bar{mc}^*$ as

$$\bar{mc}^* = \frac{\nu-1}{\nu}\frac{1}{(1+\bar{\pi})}\frac{1-\zeta\beta(1+\bar{\pi})^{\nu}}{1-\zeta\beta(1+\bar{\pi})^{\nu-1}}\left(\frac{(1+\bar{\pi})^{1-\nu}-\zeta}{1-\zeta}\right)^{\frac{1}{1-\nu}} \tag{59}$$

Note that we must therefore restrict parameter values so that $\zeta\beta(1+\bar{\pi})^{\nu} < 1$.

With the expressions for $\bar{R}^k$ and $\bar{mc}^*$, we can express the steady-state capital-labor ratio as a function of the steady-state characteristics of disaster from (39)

$$\frac{\bar{K}}{\bar{L}} = \left(\frac{\bar{mc}^*\alpha\bar{a}}{\bar{R}^k}\right)^{\frac{1}{1-\alpha}} \tag{60}$$

Therefore the steady-state wage is given by (38)

$$\bar{w} = \bar{mc}^*(1-\alpha)\bar{a}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha} \tag{61}$$

From (48), we have

$$\bar{\Omega} = \frac{(1-\zeta)(1+\bar{\pi}^*)^{-\nu}(1+\bar{\pi})^{\nu}}{1-\zeta(1+\bar{\pi})^{\nu}} \tag{62}$$

From the law of capital accumulation (26) in steady-state, we have

$$\bar{I} = \bar{K}\left(\frac{1}{1-\bar{\theta}b_k} - (1-\delta)\right) \tag{63}$$

and given that from (44),

$$\bar{G} = \omega\bar{Y} \tag{64}$$

the accounting identity (45) becomes in steady-state

$$\bar{Y} = \frac{1}{1-\omega}\left\{\bar{C} + \bar{K}\left[\frac{1}{1-\bar{\theta}b_k} - (1-\delta)\right]\right\} \tag{65}$$

in which $\frac{1}{1-\omega}$ is the keynesian multiplier of public expenditures. Further dividing each side by $\bar{L}$ gives

$$\frac{\bar{Y}}{\bar{L}} = \frac{1}{1-\omega}\left\{\frac{\bar{C}}{\bar{L}} + \frac{\bar{K}}{\bar{L}}\left[\frac{1}{1-\bar{\theta}b_k} - (1-\delta)\right]\right\} \tag{66}$$

Replacing the left-hand side by the output-labor ratio obtained from the aggregate production function (46), we have

$$\frac{\bar{A}}{\bar{\Omega}}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha} = \frac{1}{1-\omega}\left\{\frac{\bar{C}}{\bar{L}} + \frac{\bar{K}}{\bar{L}}\left[\frac{1}{1-\bar{\theta}b_k} - (1-\delta)\right]\right\} \tag{67}$$

which can be solved for the steady-state consumption-labor ratio as

$$\frac{\bar{C}}{\bar{L}} = \frac{\bar{A}(1-\omega)}{\bar{\Omega}}\left(\frac{\bar{K}}{\bar{L}}\right)^{\alpha} - \frac{\bar{K}}{\bar{L}}\left[\frac{1}{1-\bar{\theta}b_k} - (1-\delta)\right] \tag{68}$$

Combining the FOC on consumption (29) in steady-state

$$\bar{\lambda} = [(1-h)\bar{C}]^{-\gamma}(1-\beta h) \tag{69}$$

with the FOC on labor (30) in steady-state

$$\bar{L} = \left(\frac{\bar{w}\bar{\lambda}}{\chi}\right)^{1/\phi} \tag{70}$$

we can express $\bar{L}$ as a function of the steady state consumption-labor ratio

$$\bar{L} = \left[\frac{\bar{w}(1-h)^{-\gamma} \left(\frac{\bar{C}}{\bar{L}}\right)^{-\gamma} (1-\beta h)}{\chi_L} \right]^{\frac{1}{\phi_L+\gamma}} \tag{71}$$

which gives $\bar{\lambda}$ by (69) and therefore $\bar{\mu}$. $\bar{L}$ also gives $\bar{Y}$ by (66) and $\bar{K}$ by (60). Then $\bar{G}$ is obtained by (64) and $\bar{I}$ by the accounting identity or by (63). Then we get $\bar{\tilde{\Xi}}_1$ and $\bar{\tilde{\Xi}}_2$ by (56) and (57).

Finally, from the FOC on bonds (31) we have the standard Fisher relation between the subjective discount factor, the nominal interest rate and the inflation rate, $1/\beta = (1+\bar{i})/(1+\bar{\pi})$, such that, by (35), the one-period stochastic discount factor is

$$\bar{Q} = \frac{1}{1+\bar{i}} = \frac{\beta}{1+\bar{\pi}} \tag{72}$$

Table 1: Baseline calibration parameters (quarterly values)

Utility function		
β	discount factor	0.99
γ	inverse of EIS / risk aversion coefficient	2
h	habit in consumption	0.7
ϕ	inverse of the elasticity of work effort to the real wage	1
χ	labor disutility weight	4.74
Investment		
δ	capital depreciation rate	0.025
τ	investment adjustment costs	0.5
$\bar{u}$	utilization rate of capital	1
Production		
α	capital share of production	0.33
ζ_0	Calvo probability	0.8
ν	elasticity of substitution among intermediate goods	6
Public authority		
ω	steady-state G/Y ratio	0.2
ψ_π	Taylor rule inflation weight	1.5
ψ_Y	Taylor rule output weight	0.5
$\bar{\pi}$	target inflation rate	0.005
ρ_A	TFP smoothing parameter	0.9
ρ_G	government expenditures smoothing parameter	0.85
ρ_i	interest rate smoothing parameter	0.85
Disaster risk		
$\bar{\theta}$	disaster risk	0.01
b_k	share of capital destroyed if disaster	0.43
ρ_θ	disaster risk smoothing parameter	0.85
σ	standard deviation of shocks	0.01

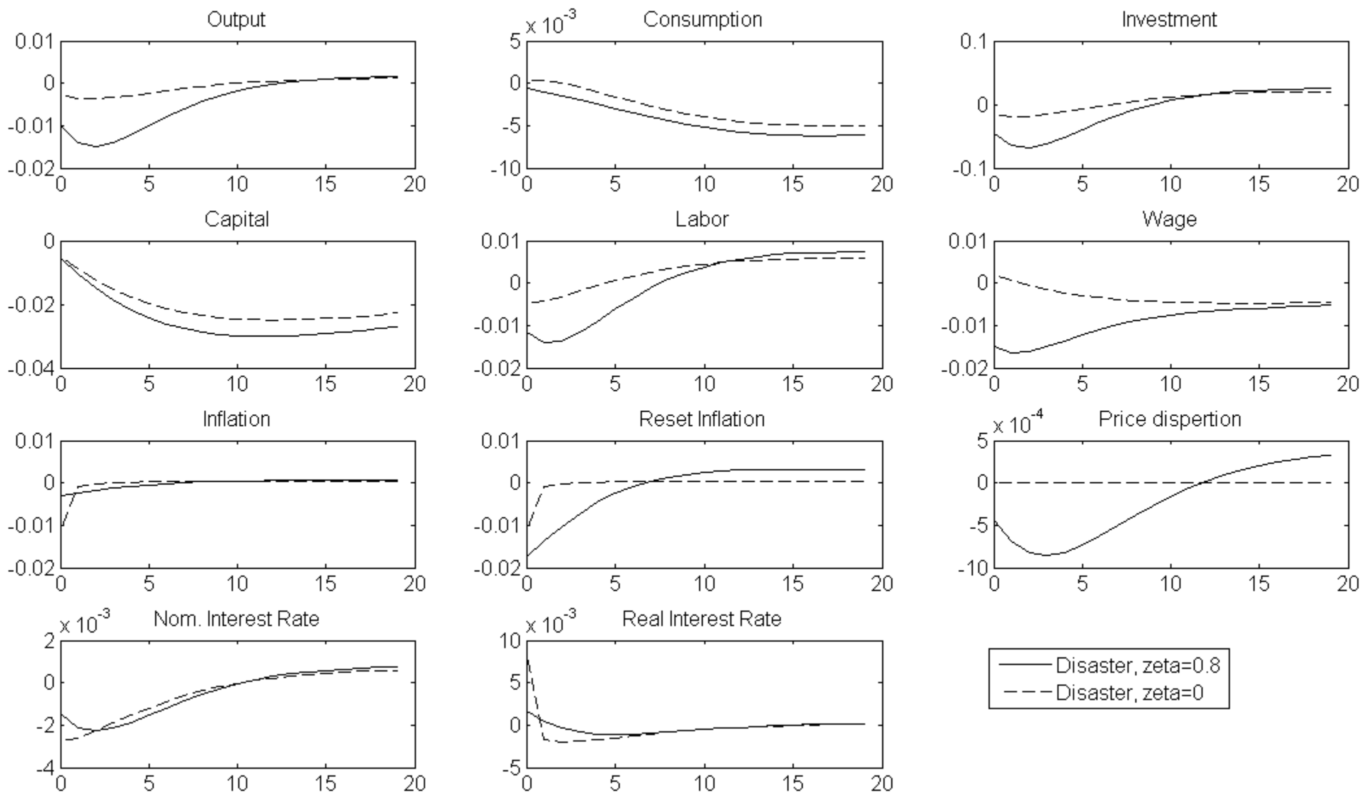

Figure 1: Standard-deviation responses to a shock to the probability of disaster (increase in θ). Solid line: model with disaster risk and sticky prices ($\zeta = 0.8$). Dashed line: model with disaster risk and flexible prices ($\zeta = 0$).

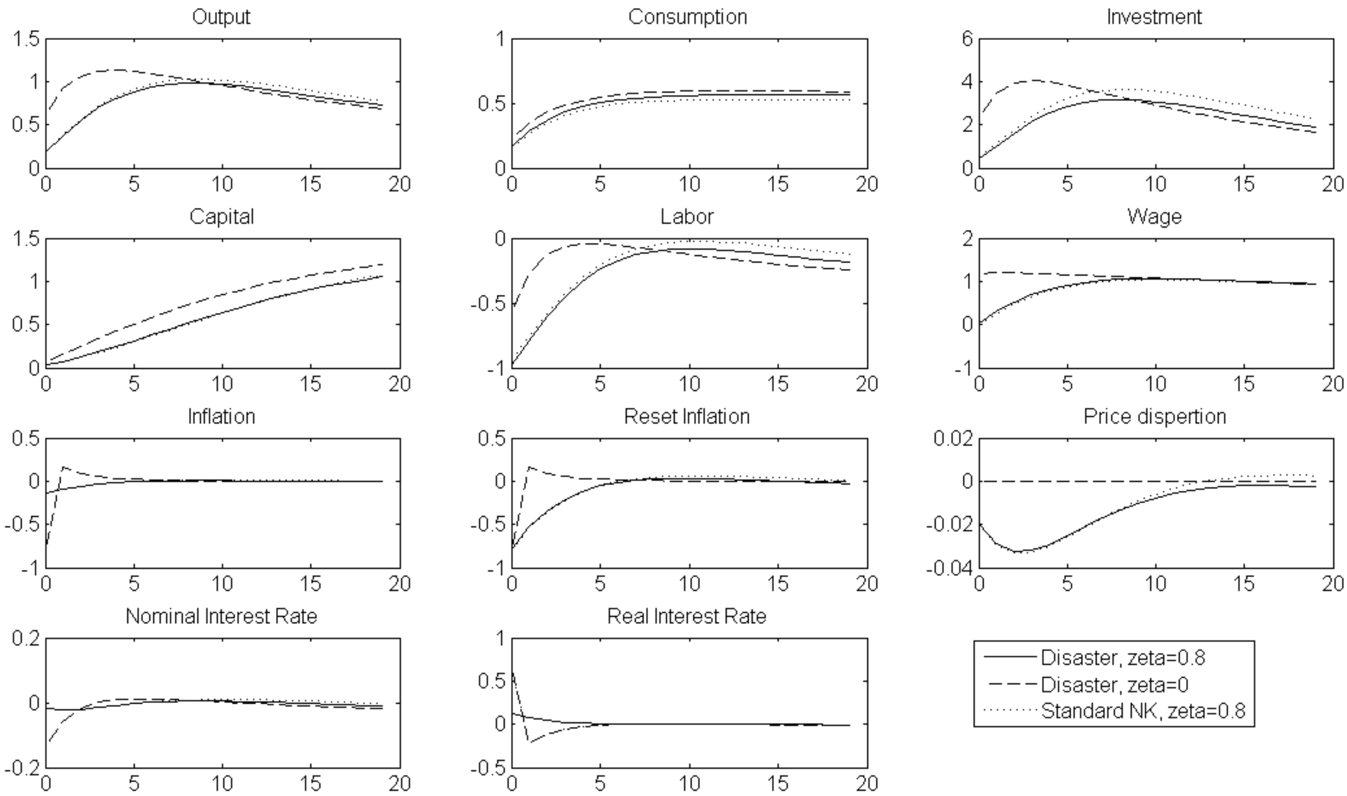

Figure 2: Standard-deviation responses to a productivity shock. Solid line: model with disaster risk and sticky prices ($\zeta = 0.8$). Dashed line: model with disaster risk and flexible prices ($\zeta = 0$). Dotted line: model without disasters, with sticky prices.

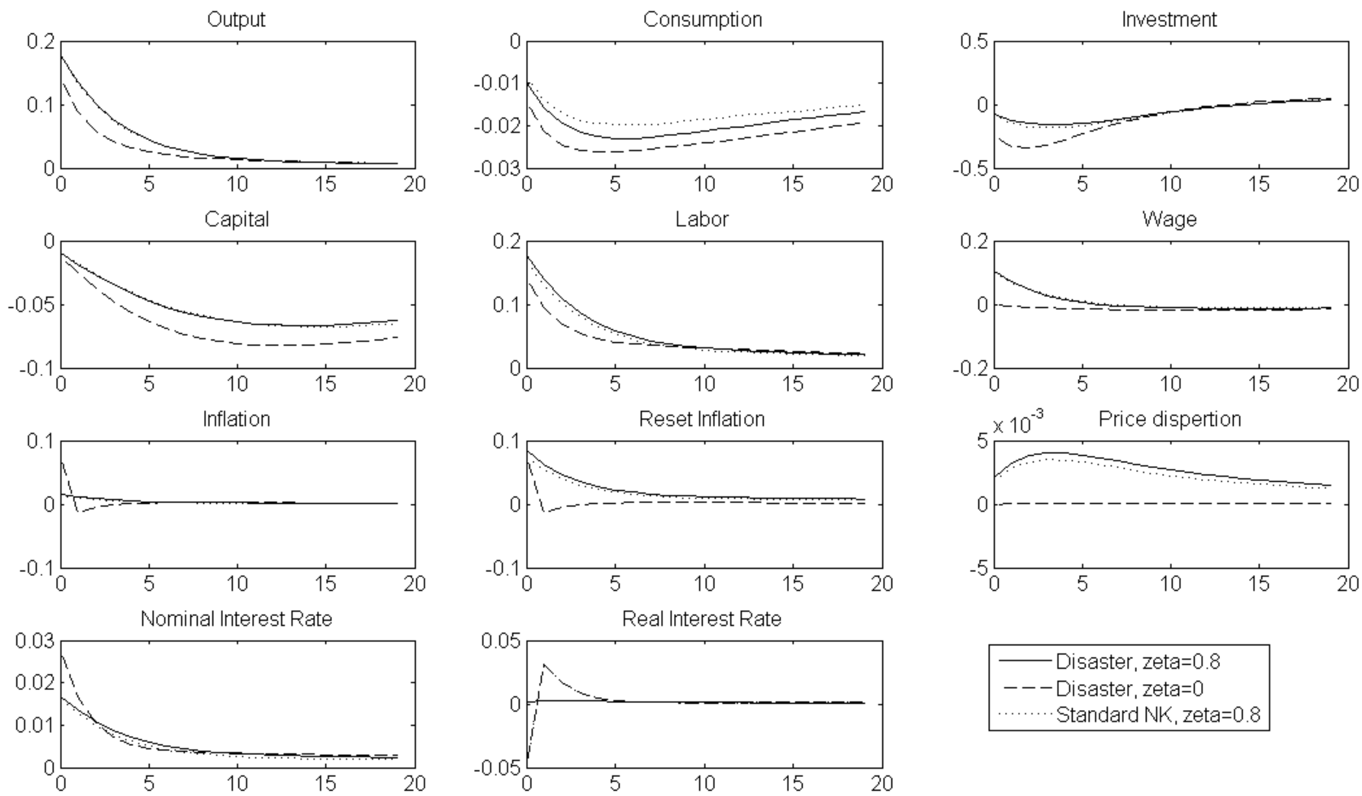

Figure 3: Standard-deviation responses to a public spending shock. Solid line: model with disaster risk and sticky prices ($\zeta = 0.8$). Dashed line: model with disaster risk and flexible prices ($\zeta = 0$). Dotted line: model without disasters, with sticky prices.

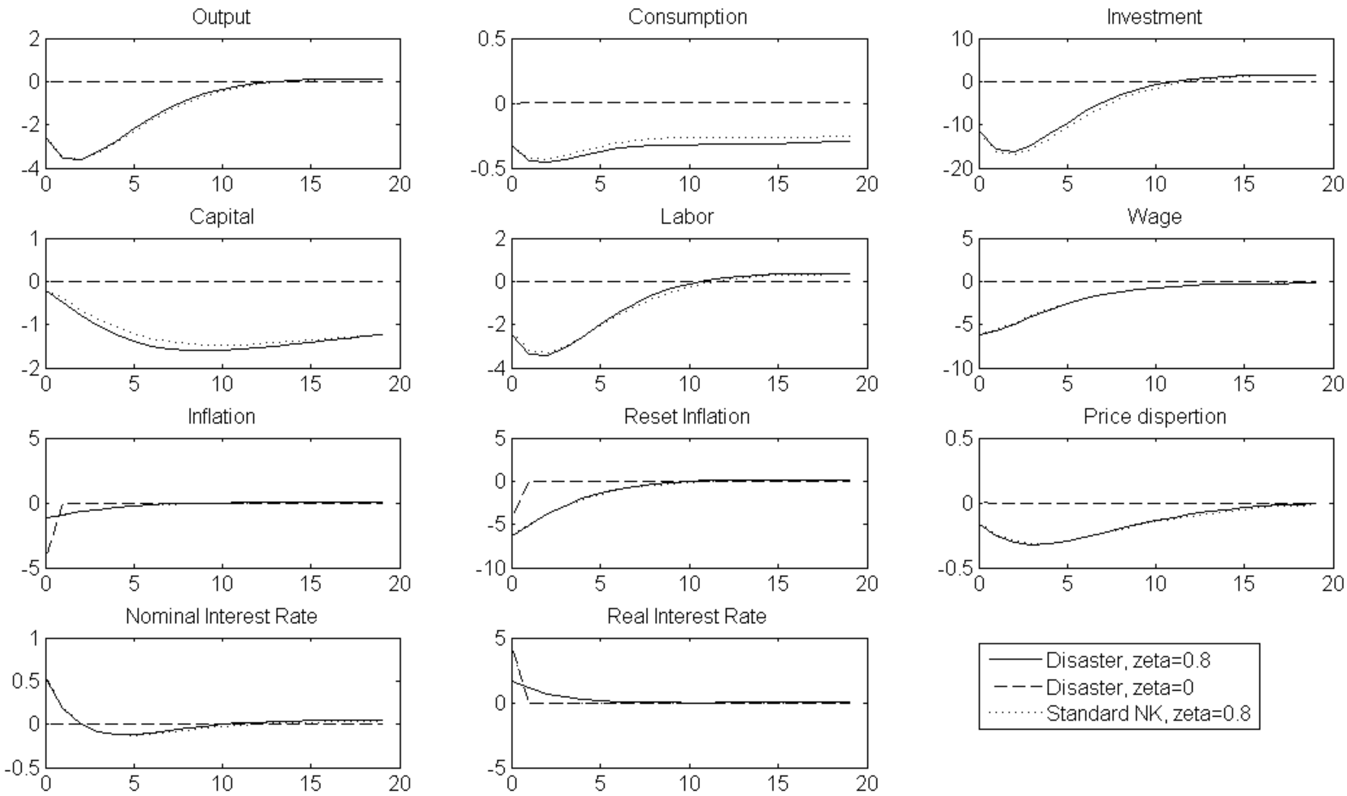

Figure 4: Standard-deviation responses to a monetary shock. Solid line: model with disaster risk and sticky prices ($\zeta = 0.8$). Dashed line: model with disaster risk and flexible prices ($\zeta = 0$). Dotted line: model without disasters, with sticky prices.

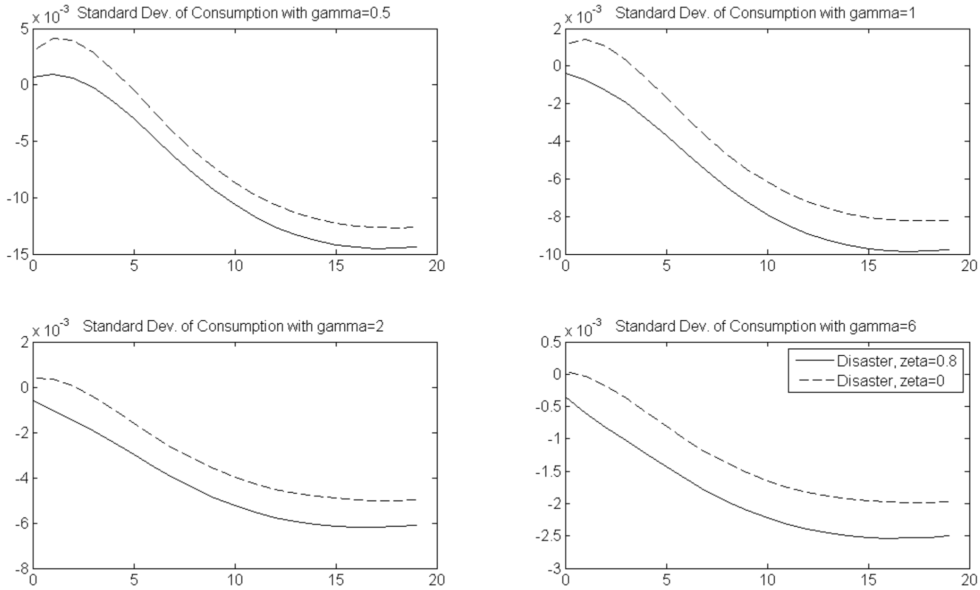

Figure 5: Standard-deviation of consumption to a shock to the probability of disaster, for different values of the risk aversion coefficient γ.

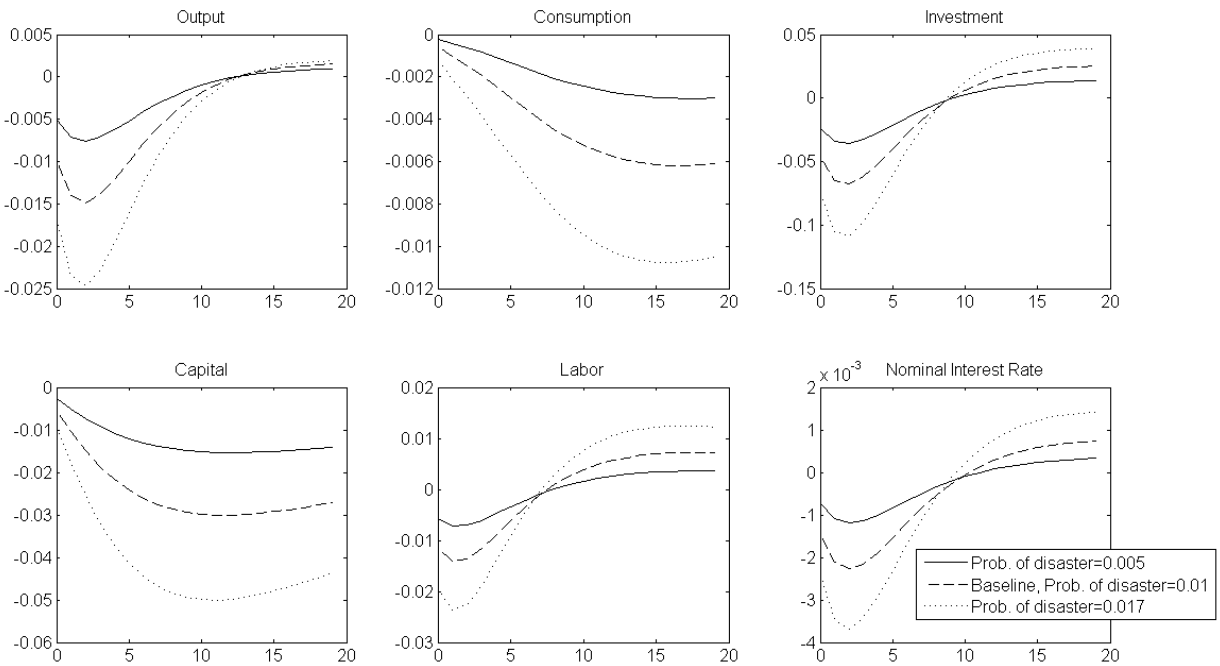

Figure 6: Standard-deviation responses to a shock to the probability of disaster, for different values of the steady-state probability of disaster, $\bar{\theta}$.

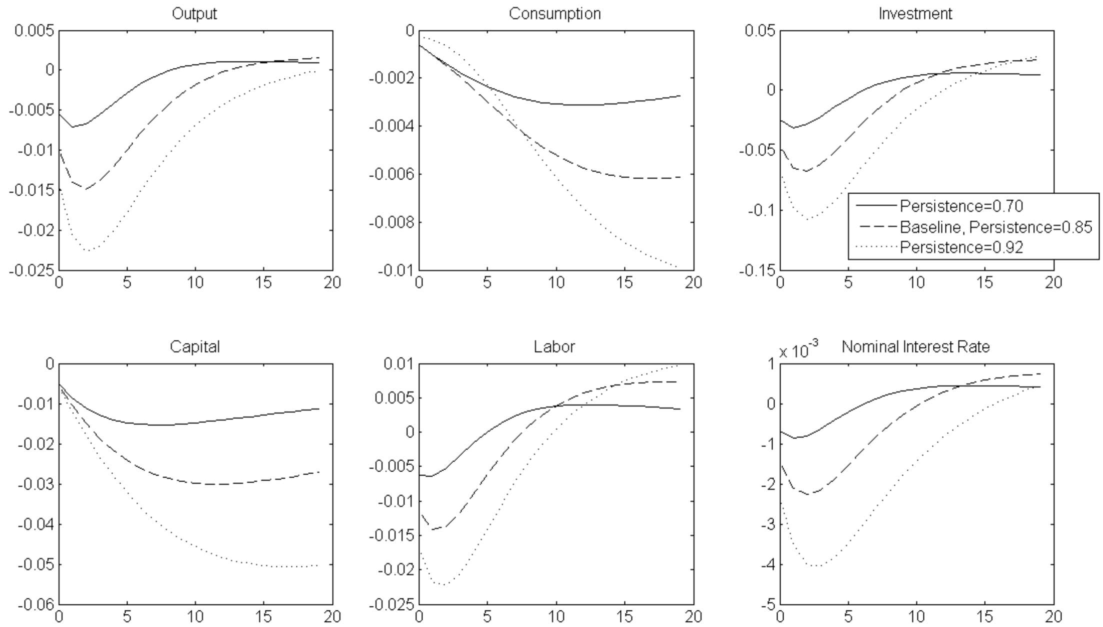

Figure 7: Standard-deviation responses to a shock to the probability of disaster, for different values of the persistence of the shock ρ_θ.

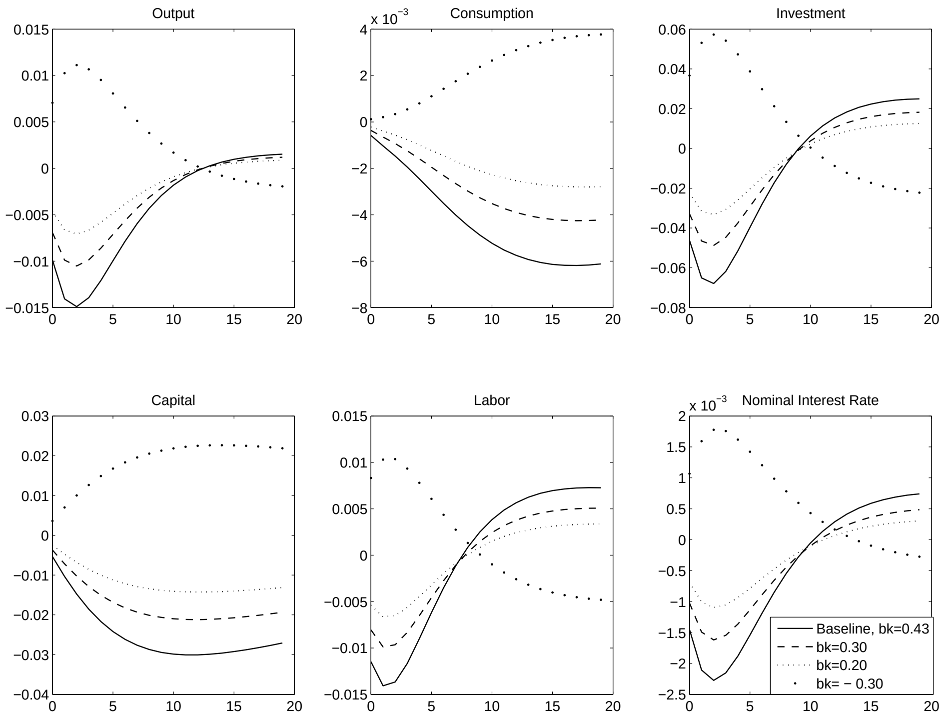

Figure 8: Standard-deviation responses to a shock to the probability of disaster, for different values of the destroyed share of capital b_k.

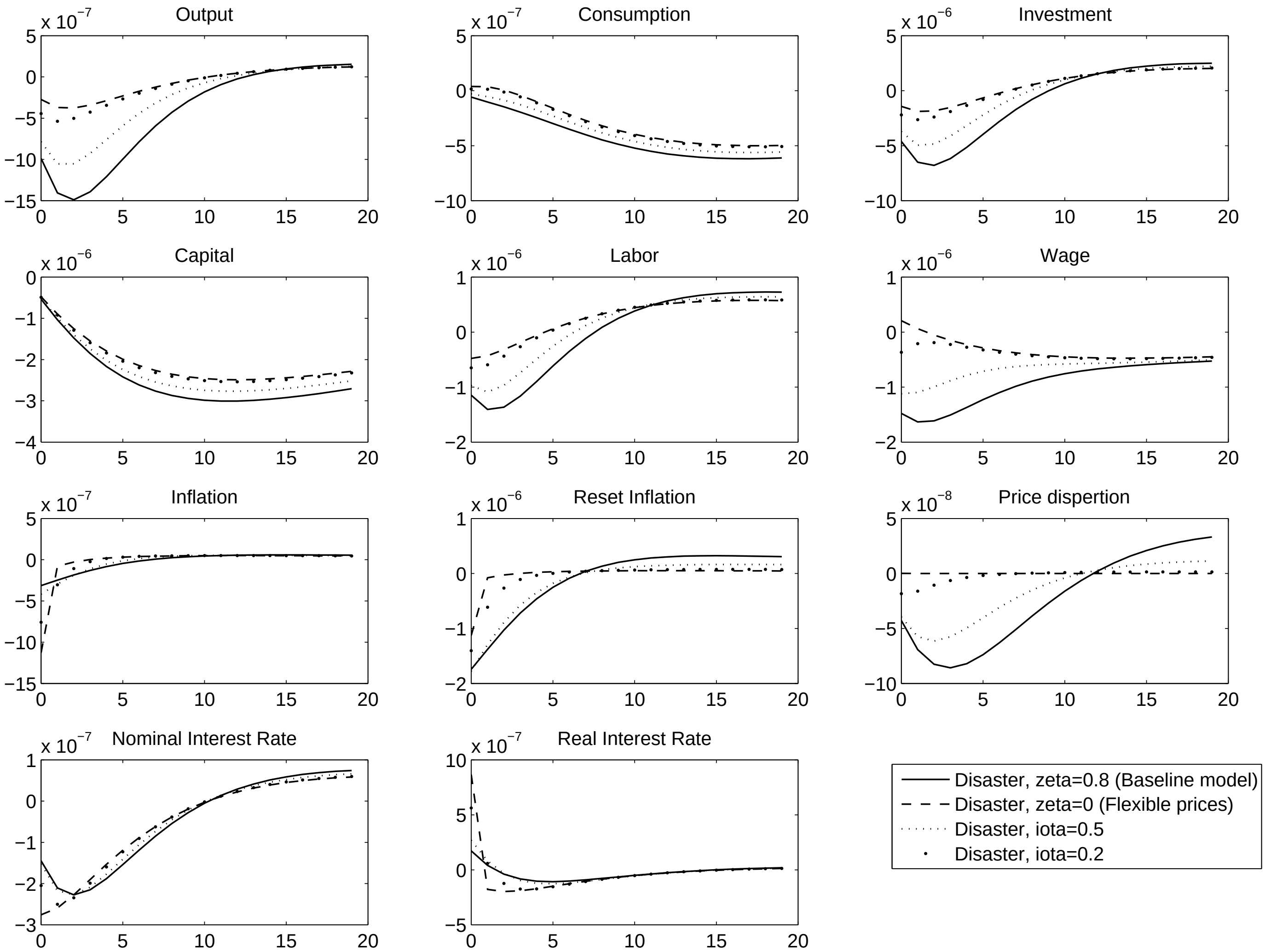

Figure 9: Standard-deviation responses to a shock to the probability of disaster, with state-dependent price stickiness. We assume that $\zeta_t = \zeta_0 - \theta_t^{\iota}$. With $\iota \geq 1$, the responses are very close to the Calvo pricing case ($\zeta = 0.8$), thus not included.

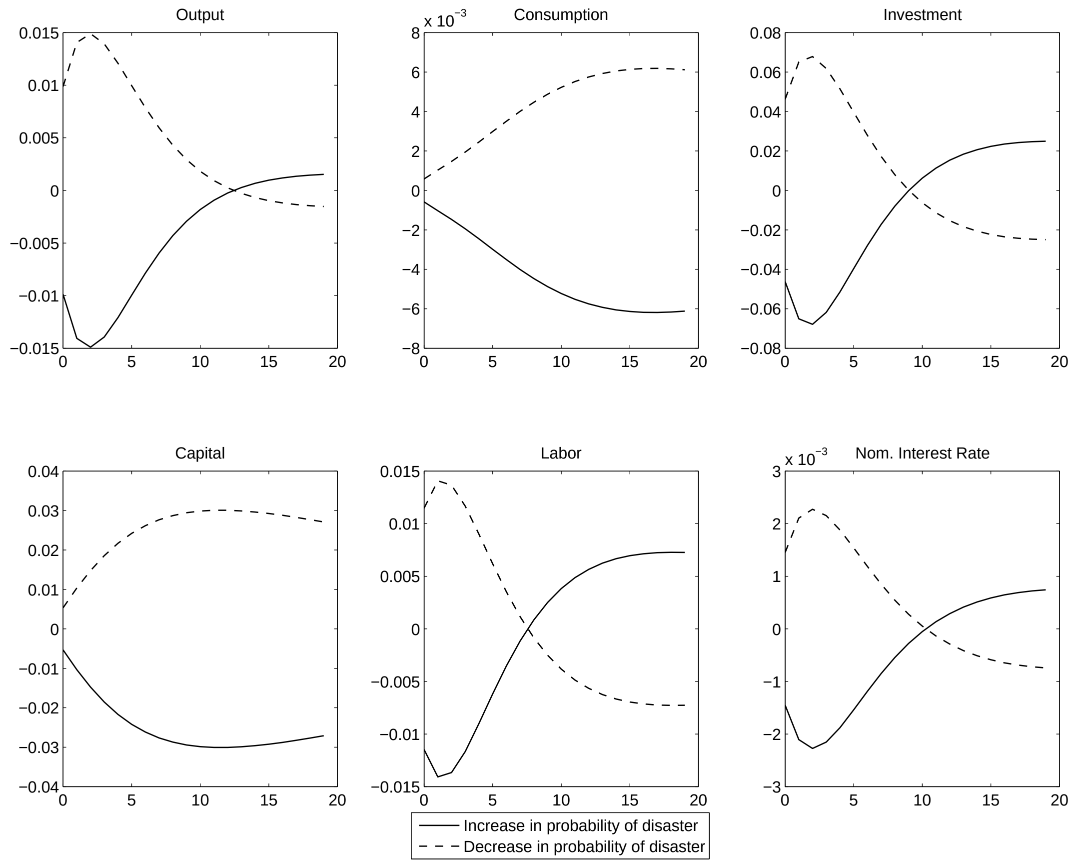

Figure 10: Standard-deviation responses to a shock to the probability of disaster. Negative and positive shocks.

Table 2: Correction terms for the second-order approximation, shock to θ

	$\hat{Y}$	$\hat{C}$	$\hat{I}$	$\hat{G}$	$\hat{K}$	$\hat{L}$	$\hat{\Omega}$
Constant	0.691892	0.169961	-0.884952	-0.917545	2.644587	-0.281788	0.001777
2nd-order correction	-0.000001	0	-0.000004	0	0	0	0
	$\hat{\pi}$	$\hat{\pi}^*$	$\hat{mc}^*$	$\hat{w}$	$\hat{R}^k$	$\hat{q}$	$\hat{Q}$
Constant	0.004987	0.026281	-0.182864	0.387128	-3.232391	0.004307	-0.015038
2nd-order correction	0	-0.000003	-0.000006	-0.000007	-0.000004	-0.000002	0